Asymptotic safety and gauge independence

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Introduction

- Asymptotic safety provides a possible continuum limit for gravity for which we expect universal scaling behaviour. Wish to compute e.g. one-loop beta function:

$$\beta_G = (d-2)G - b\,G^2$$

- Continuum limit at a UV fixed point (Weinberg 79'):
- One-loop: b > 0
- Close to two dimensions (Tsao '77, Jack and Jones '91, Kawai and Ninomiya '90): $d = 2 + \epsilon$

$$\beta = \epsilon G - \frac{38}{3} G^2$$

Exact RG approach

- Recent studies are based on the effective average action (Wetterich 94, Morris 94):

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\frac{\mathrm{k}\partial_{\mathrm{k}}\mathcal{R}_{\mathrm{k}}}{\Gamma^{(2)}_{\mathrm{k}} + \mathcal{R}_{\mathrm{k}}},$$

- Scale dependent action:

$$\mathcal{R}_k - \mathrm{IR\ regulator}$$

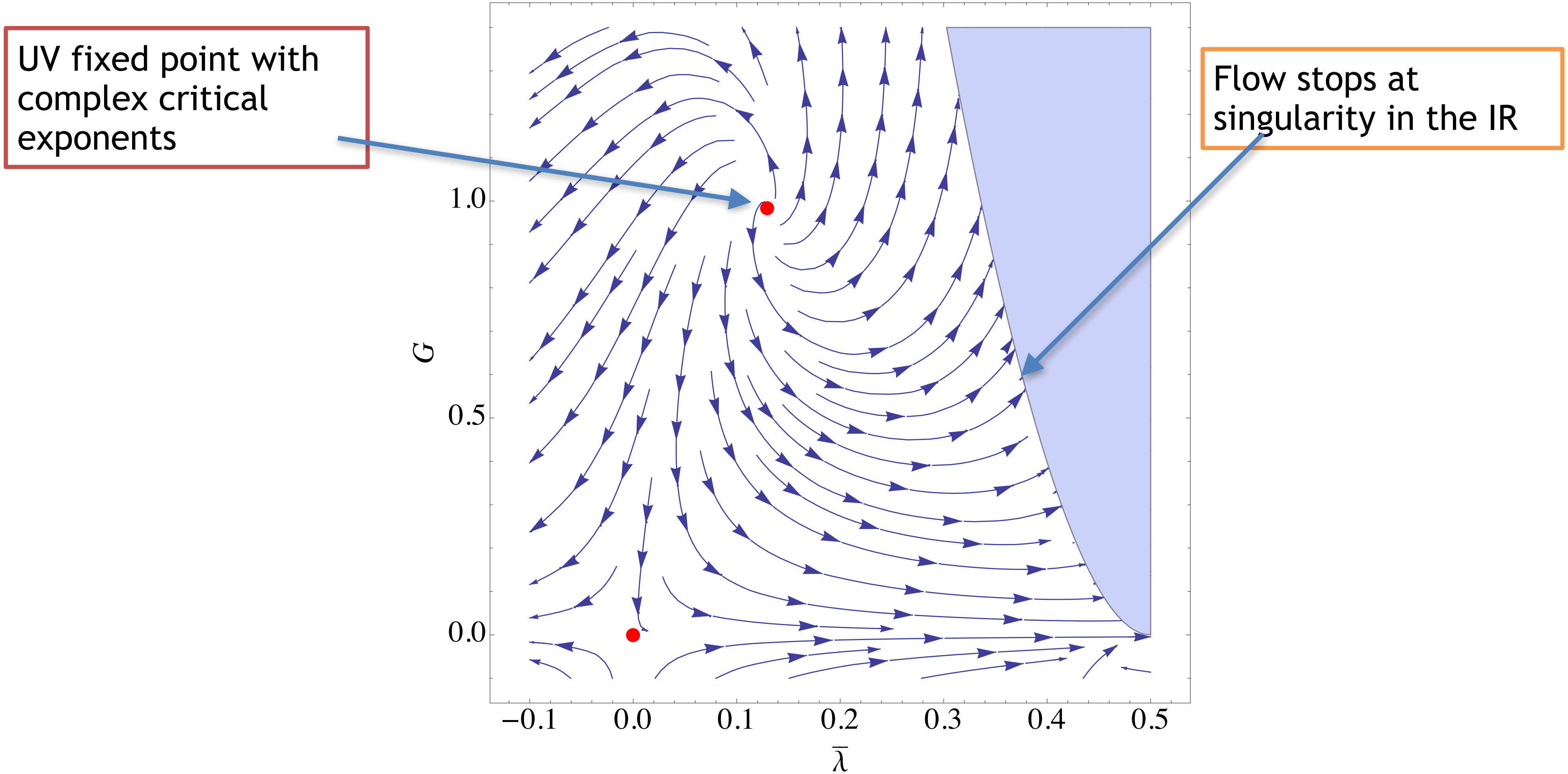
$$\Gamma_0 = \Gamma$$

$$\Gamma_\infty \approx S$$

- For gravity pioneering work by Reuter (followed by many more studies Percacci, Litim, Saueressig, Benedetti, Morris etc.)

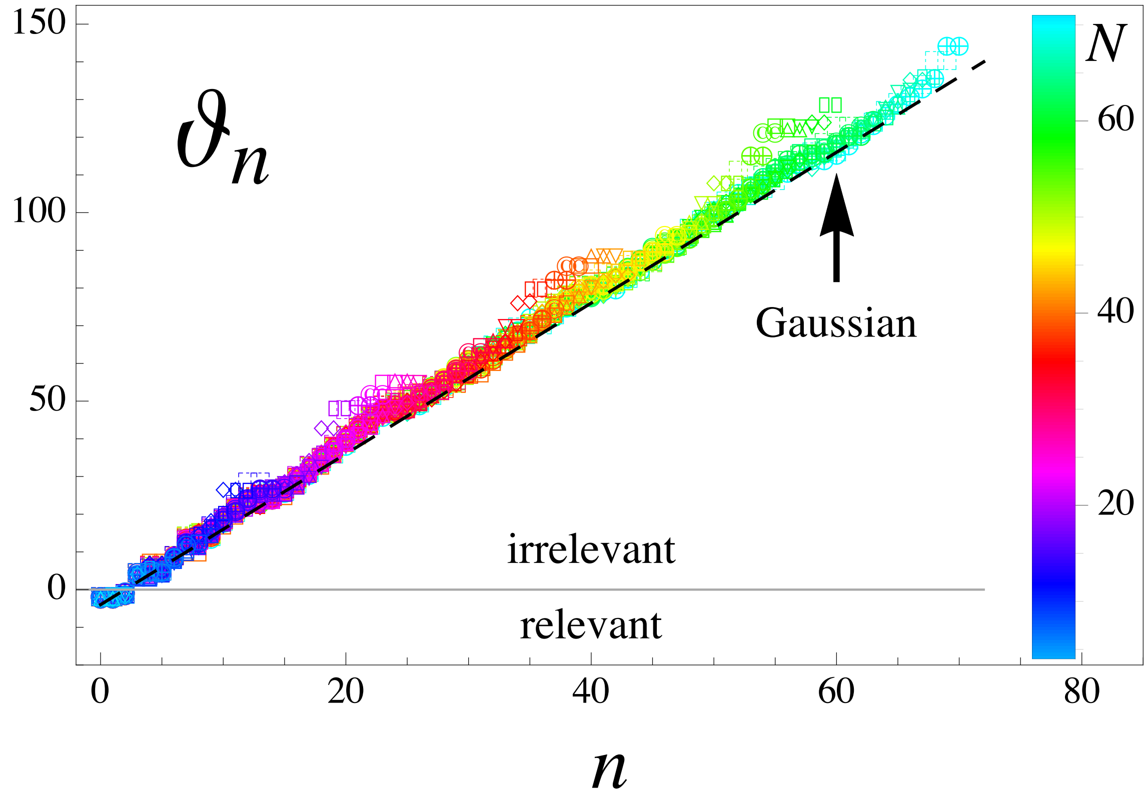
Phase diagram

Typical RG phase diagram for **dimensionless** Newton's constant and cosmological constant :



Asymptotic Safety

- Non-perturbative renormalisation.
- UV fixed point with a finite number of relevant directions.
- F(R) truncations up to R^70 (KF, Litim, Schröder):



- Three relevant directions.
- Spectrum of critical exponents is near to the gaussian one expected from perturbation theory —> Canonical mass dimension still a good guiding principle.

Issues of ERG approach

- Issues of exact renormalisation group approach:
  - gauge dependence (Falkenberg, Odintsov '98)

$$\beta = \beta(\alpha, \rho)$$

  - poles in the propagators:

$$\beta \sim \frac{1}{1 - 2\bar{\lambda}}$$

- Hypothesis: These may stem from approximations where diffeomorphism invariance is lost. This is related to the off-shell effective action being gauge dependent.
- Aim: Restore gauge invariance to improve the reliability of perturbative/non-perturbative calculations.
- Achieved by specific choice of field parameterisation.

Semi-classical theory

- Aim to find a gauge independent one-loop beta function generated by graviton fluctuations:

$$\beta_G = (d-2)G - b\,G^2$$

- One-loop effective action:

$$\Gamma - S = \frac{1}{2}\mathrm{STr}\log S^{(2)} \qquad S = \int_x \sqrt{\gamma}\,\frac{1}{16\pi G_b}(2\bar{\lambda}_b - R) + \ldots$$

- Standard approach:

$$\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

- Problem: linearised theory,

$$S_{\mathrm{EH},1} = \int d^d x\,\frac{1}{2} h \cdot S^{(2)}_{\mathrm{EH}} \cdot h$$

is not gauge invariant if background metric is off-shell (Deser, Henneaux gr-qc/0611157):

$$\nabla_\mu G_1^{\mu\nu} \neq 0$$

Semi-classical theory

- Lack of gauge invariance —> gauge dependent effective action
- Gauge independence is restored only by going on-shell (see e.g. Benedetti New J. Phys. 14 (2012) 015005)

$$R = \frac{2d}{d-2}\bar{\lambda}$$

- The linearised theory is **not unique** and depends on the parameterisation of the metric fluctuations. e.g. the exponential parameterisation (employed recently by e.g. Nink Phys. Rev. D 91, 044030 (2015)):

$$\gamma_{\mu\nu} = \bar{g}_{\mu\lambda}(e^{h})^{\lambda}_{\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + \frac{1}{2}h^{\lambda}_{\mu}h_{\lambda\nu} + \dots ,$$

- Aim to find a parameterisation which restores gauge invariance at the linear level for:

$$R \neq \frac{2d}{d-2}\bar{\lambda}$$

Restoring diffeomorphism invariance

- Essential observation: going to an arbitrary Einstein space solves all but the trace of the Einstein equations. Hence terms in the action

$$S^{(2)} = \tilde{S}^{(2)} + X\left(R - \frac{2d}{d-2}\bar{\lambda}_b\right), \qquad \text{with } R_{\mu\nu} = g_{\mu\nu}\frac{R}{d}$$

which both breaks diffeomorphism invariance and leads to poles in the propagators are proportional to the trace of the Einstein equations.

- Where X is parameterisation dependent and $\tilde{S}^{(2)}$ is independent of the cosmological constant.
- Idea: Find a parameterisation such that

$$X = 0$$

and thus restore diffeomorphism invariance of the off-shell hessians.

Restoring diffeomorphism invariance

- To remove these terms we only need to choose a parameterisation for which the cosmological constant is absent from the linearised action e.g. taking the volume element itself as a field:

$$\sqrt{\gamma(x)} = \omega(x) = \sqrt{\bar{g}(x)}\left(1 + \frac{\sigma(x)}{2}\right)$$

$$\gamma_{\mu\nu} = \left(1 + \frac{\sigma}{2}\right)^{\frac{2}{d}} \bar{g}_{\mu\lambda}(e^{\hat{h}})^{\lambda}_{\mu}$$

where $\hat{h}_{\mu\nu}$ is a traceless field.

$$S = \frac{1}{16\pi G_b}\int d^d x \sqrt{\gamma}\left(2\bar{\lambda}_b - R(\gamma_{\mu\nu})\right)$$

$$X \propto \frac{\partial}{\partial \bar{\lambda}_b}\frac{\delta^2 S}{\delta\sigma\delta\sigma} = 0$$

- Linearised theory is now gauge invariance under:

$$\hat{h}_{\mu\nu} \to \hat{h}_{\mu\nu} + \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu - \frac{1}{d} g_{\mu\nu}\nabla_\alpha \epsilon^\alpha + \ldots$$

$$\sigma \to \sigma + 2\nabla_\alpha \epsilon^\alpha + \ldots$$

Gauge independent effective action

- One loop effective action:

$$\Gamma[g_{\mu\nu}] - S[g_{\mu\nu}] = \frac{1}{2}\mathrm{Tr}[\log \Delta_2] - \mathrm{Tr}[\log \Delta_1]$$

Metric fluctuations

Ghosts

- with differential operators:

$$\Delta_2 \varphi_{\mu\nu} = \left(-\nabla^2 \varphi_{\mu\nu} - 2 R_\mu{}^\alpha{}_\nu{}^\beta \varphi_{\alpha\beta}\right)$$

$$\Delta_1 \varphi_\mu = \left(-\nabla^2 \delta_\mu^\nu - R_\mu{}^\nu\right) \varphi_\nu$$

One-loop beta function

- Two issues pointed out in the introduction have been resolved by restoring diffeomorphism invariance at the semi-classical level.
- After a suitable normalisation of the cutoff scale one can find a universal one loop beta function

$$\beta_G = (d-2)G - \frac{2}{3}(18 - N_g)\, G^2$$

$$N_g = \frac{d(d-3)}{2}$$

= Number of polarisations of the graviton

- This beta function can be obtained using covariant proper time renormalisation or the functional renormalisation group.
- Can also be found by gauge fixing the conformal fluctuations (Percacci, Vacca 2015).

Paramagnetic dominance

- As observed in the gauge dependent set-up by Reuter and Nink (jhep 1301, 062 (2013), 1208.0031) asymptotic safety results from the dominance of the terms resulting from the non-minimal coupling

$$\beta_G = (d-2)G - \frac{2}{3}(18 - N_g)\, G^2$$

Classical scaling dimension

"diamagnetic" counts physical degrees of freedom

$$N_g = \frac{d(d-3)}{2}$$

$$G = k^{d-2} G_N$$

$$\Delta_2 \varphi_{\mu\nu} = \left(-\nabla^2 \varphi_{\mu\nu} - 2R_\mu{}^\alpha{}_\nu{}^\beta \varphi_{\alpha\beta}\right)$$

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Classical scaling dimension

Non-minimal coupling aka "paramagnetic"

$$G = k^{d-2} G_N$$

$$\Delta_2 \varphi_{\mu\nu} = \left(-\nabla^2 \varphi_{\mu\nu} - 2R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\varphi_{\alpha\beta}\right)$$

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Role of convexity?

- Removed gauge dependence by a specific parameterisation.
- Have we gained anything or just shifted the problem?
- Physical results should not depend on the gauge or the parameterisation.
- However not all choices lead to a convex effective action:

$$\mathcal{G}^{-1} \cdot (\Gamma_k^{(2)} + \mathcal{R}_k) > 0$$

Role of convexity?

- Choice of parameterisation/gauge implicitly effects the RG scheme.
- Not all RG schemes lead to well defined RG flows
- Idea: Preferred gauges, parameterisations and cut-off schemes remove poles in the propagator:

$$\mathcal{G}^{-1} \cdot (\Gamma_k^{(2)} + \mathcal{R}_k) > 0$$

Field space metric (de Witt metric)

$$\mathcal{D}\gamma = d\gamma\sqrt{\det \mathcal{G}}$$

- At one loop this is equivalent to demanding that the gaussian integrands are of the right sign.

Role of convexity?

- De witt metric:

$$\mathcal{G}^{\mu\nu\alpha\beta} = \sqrt{g}\,\frac{1}{2}(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - \, g^{\mu\nu}g^{\alpha\beta})$$

'corrects' the sign of conformal modes (Mazur, Mottola '90).

- Solution to the conformal factor instability.

Beyond one-loop

- Non-perturbative Einstein Hilbert truncation

$$\Gamma_k = \int d^d x \sqrt{\gamma} \left(\lambda_k - \frac{R}{16\pi G_k} \right)$$

- Hessian's (for X=0) given by:

$$\Gamma^{(2)}_{k,s} = (16\pi G_k)^{-1} \Delta_s$$

- IR cutoff function suppresses low energy modes:

$$\mathcal{R}_{k,s}(\Delta_s) = (16\pi G_k)^{-1} k^2 C(\Delta_s / k^2)$$

- Convex for all local fluctuations

$$\mathcal{G}^{-1} \cdot (\Gamma^{(2)}_k + \mathcal{R}_k) > 0$$

Beyond one-loop

- Use the functional renormalisation group to compute non-perturbative beta function.
- Single metric truncation is gauge independent.

$$\beta_G = G(d-2+\eta_G) \qquad \eta_G = \frac{2\,(N_g-18)\,G\,\mathcal{I}_{d/2-1}}{3\,(4\pi)^{\frac{d-2}{2}}\,\Gamma\left(\frac{d}{2}-1\right)+(N_g-18)\,G\,\tilde{\mathcal{I}}_{d/2-1}}\,,$$

- Fixed point for positive Newton's constant in d<8 dimensions
- Critical exponent:

$$1/\nu \equiv -\left.\frac{\partial \beta_G}{\partial G}\right|_{G=G_*} = (d-2)+(d-2)^2\,\frac{\tilde{\mathcal{I}}_{d/2-1}}{2\,\mathcal{I}_{d/2-1}}$$

- Involves the integrals

$$\tilde{\mathcal{I}}_n = \int_0^\infty dz\,\frac{z^{n-1}C(z)}{z+C(z)}\,, \qquad \mathcal{I}_n = \tilde{\mathcal{I}}_n - \int_0^\infty dz\,z^n\,\frac{C'(z)}{z+C(z)}\,.$$

Critical scaling

- Regulator independence close to two dimensions:

$$1/\nu = \epsilon + \frac{1}{2}\epsilon^2 + \ldots \qquad\qquad d = 2 + \epsilon$$

- Both gauge and regulator independent to second order

- Comparison with two loop calculation (Aida and Kitazawa (1997), hep-th/9609077)

$$1/\nu = \epsilon + \frac{3}{5}\epsilon^2 + \ldots$$

Critical scaling

- d=4 dimensions known result from lattice calculation by Hamber (PhysRevD.61.124008 (2000), arXiv:1506.07795):

$$\langle R \rangle \equiv \left\langle \int d^d x \sqrt{\gamma}\, R \right\rangle / V \qquad\qquad \langle R \rangle \sim |G_* - G_b|^{4\nu - 1}$$

$$\nu \approx 0.335(4)$$

- Relation to beta function comes from the scaling of the free energy

$$\langle R \rangle = 16\pi G_b^2 \frac{\partial}{\partial G_b} F$$

- Scaling of the free energy can be obtained by integrating the RG flow:

$$F = \Gamma_{k=0}/V \sim |G_b - G_*|^{4\nu}$$

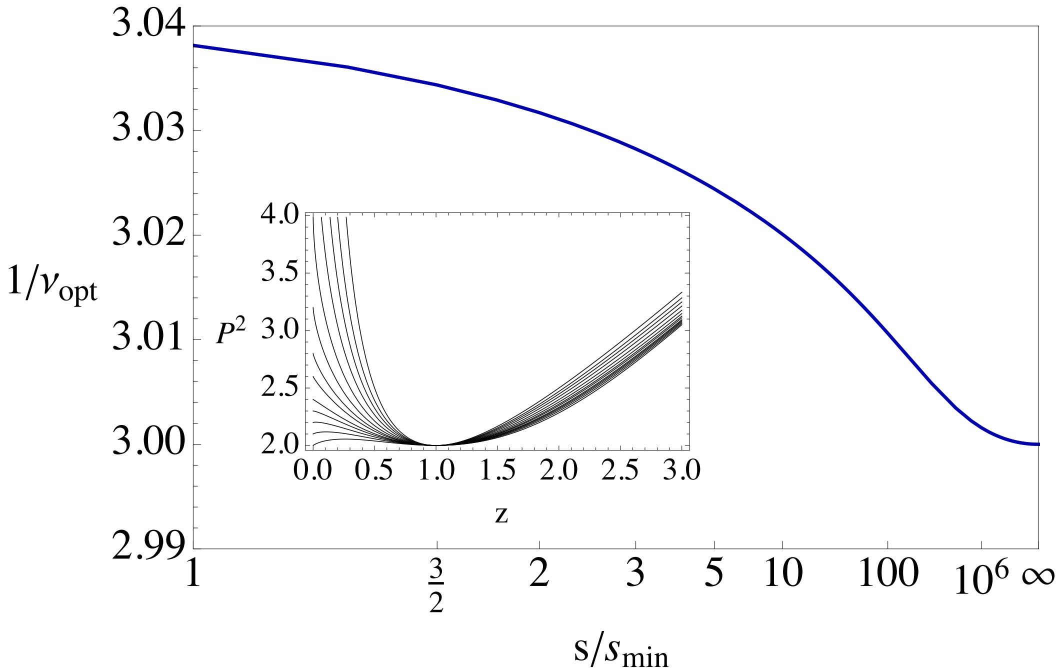
Critical scaling

- To obtain a best estimate I apply Litim's optimisation criteria:
  maximise the gap in the inverse propagator under the regulator scheme

$$P^2_{\rm gap}[C] = P^2(z_{\rm min}) \equiv z_{\rm min} + C(z_{\rm min}) \qquad P^2_{\rm gap}[C_{\rm opt}] = {\rm max}_{\rm RS}(P^2_{\rm gap}[C])$$

- Optimisation aims for better convergence of approximate solutions.
- Class of optimised regulators:

$$C_{\rm opt}(z) = s z^b \frac{1}{(1+s)^{z^b} - 1}\bigg|_{b=b_{\rm opt}}, \qquad b_{\rm opt}(s) = \frac{s}{(1+s)\log(1+s) - s}$$



Optimised ERG: $\nu \approx 1/3$

Lattice : $\nu \approx 0.335(4)$

Summary

- Restoring diffeomorphism invariance for simple approximations:
    - Gauge independence.
    - Convexity of effective action.
    - Real critical exponent in quantitative agreement with lattice studies.

- Deeper insight: Ensuring convexity of the (regulated) effective action may be necessary to obtain physical results.