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Preliminary study on impedance and collective effects

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Acknowledgements:

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Beam parameters

parameter	Symbol	Value
Circumference	L	100 km
Beam energy	E	45.5 GeV
Beam current	I_0	1450 mA
Bunches/beam	N_b	16700
Bunch population	N_p	1.8×10^{11}
Bunch length(*)	σ_z	2.3 mm
Energy spread(*)	σ_ε	3.7×10^{-4}
RF voltage	V_{RF}	2.5 GV
RF frequency	f_{RF}	400 MHz
Harmonic number	h	133600
Synchrotron tune	Q_s	0.46
Energy loss/turn	U_0	0.03 GeV
momentum compaction	α_c	18×10^{-5}

(*) without beamstrahlung (no collision)

Short range wakefield: Resistive wall

- To evaluate the RW impedance, we consider high conductivity such that the skin depth is much smaller than the wall thickness and

$$c\chi / b << \omega << c\chi^{-1/3} / b \qquad \chi^{1/3} b << z << b / \chi$$

$$\text{with } \chi = \frac{1}{Z_0 \sigma_c b}$$

$10.6 << \omega << 5.7 \times 10^{12}$ rad/s

$52.5 \times 10^{-6} << z << 2.8 \times 10^{7}$ m

$$Z_{//}(\omega) = [1 - i \operatorname{sgn}(\omega)] \frac{L}{2\pi b} \sqrt{\frac{Z_0 |\omega|}{2c\sigma_c}}$$

$$Z_{\perp}(\omega) = [\operatorname{sgn}(\omega) - i] \frac{L}{2\pi b^3} \sqrt{\frac{2cZ_0}{|\omega| \sigma_c}}$$

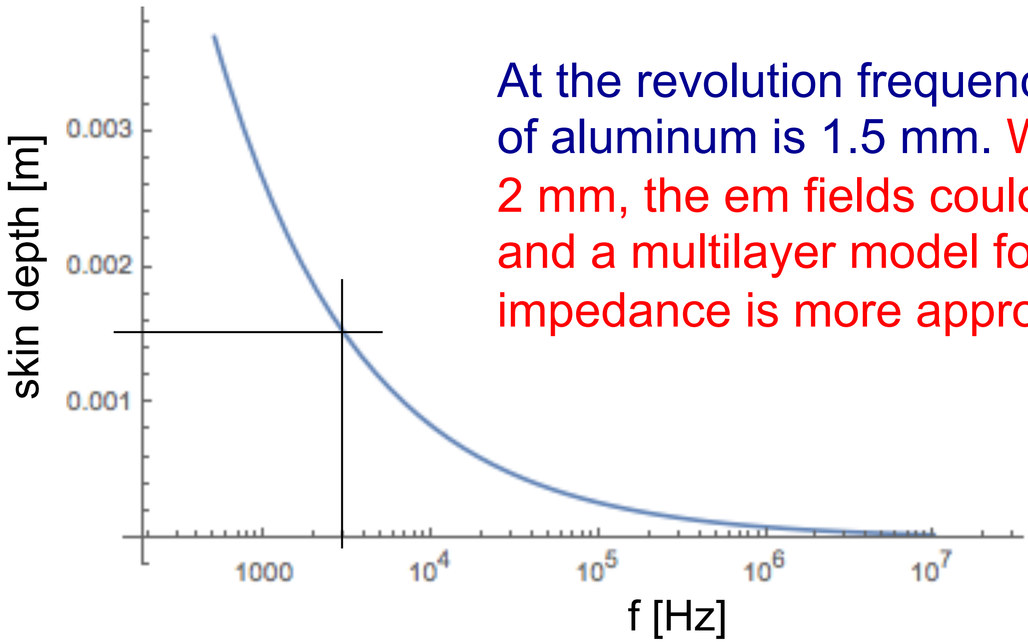
$$w_{//}(z) = -\frac{Lc}{4\pi b} \sqrt{\frac{Z_0}{\pi \sigma_c}} \frac{1}{|z|^{3/2}}$$

$$w_{\perp}(z) = \frac{Lc}{\pi b^3} \sqrt{\frac{Z_0}{\pi \sigma_c}} \frac{1}{|z|^{1/2}}$$

Short range wakefield: Resistive wall

- Vacuum pipe of aluminum

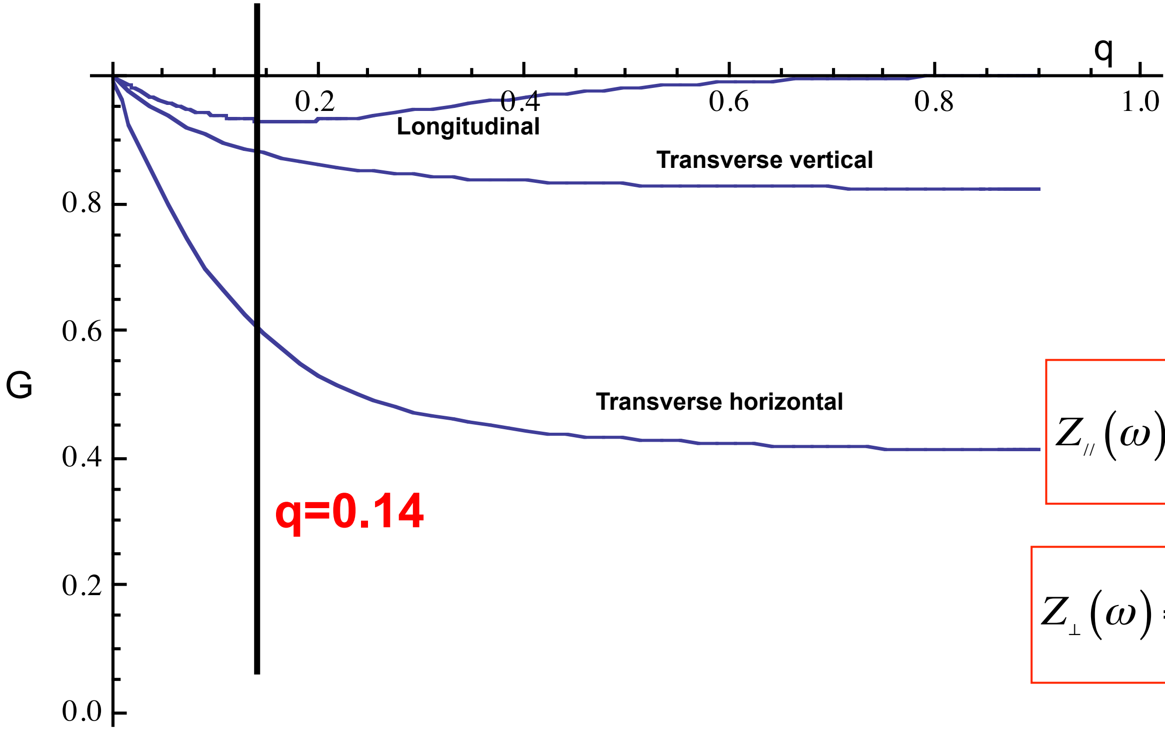
$\sigma_c = 3.7 \times 10^7$ S/m L=10^5 m

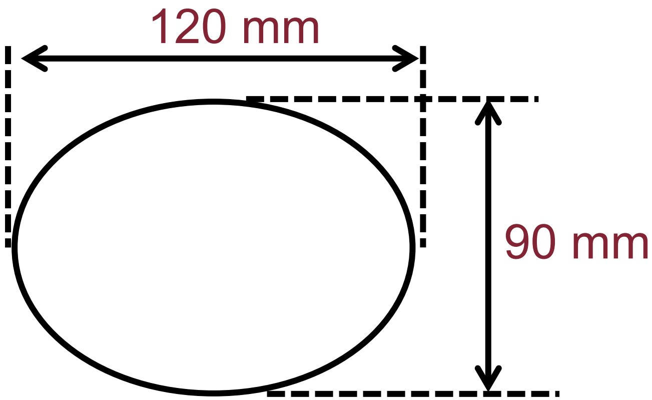


At the revolution frequency of 3 kHz, the skin depth of aluminum is 1.5 mm. With a vacuum chamber of 2 mm, the em fields could pass through the pipe and a multilayer model for evaluating the impedance is more appropriate.

Short range wakefield: Resistive wall

- Elliptic chamber





Form factors G (u_0) as a function of

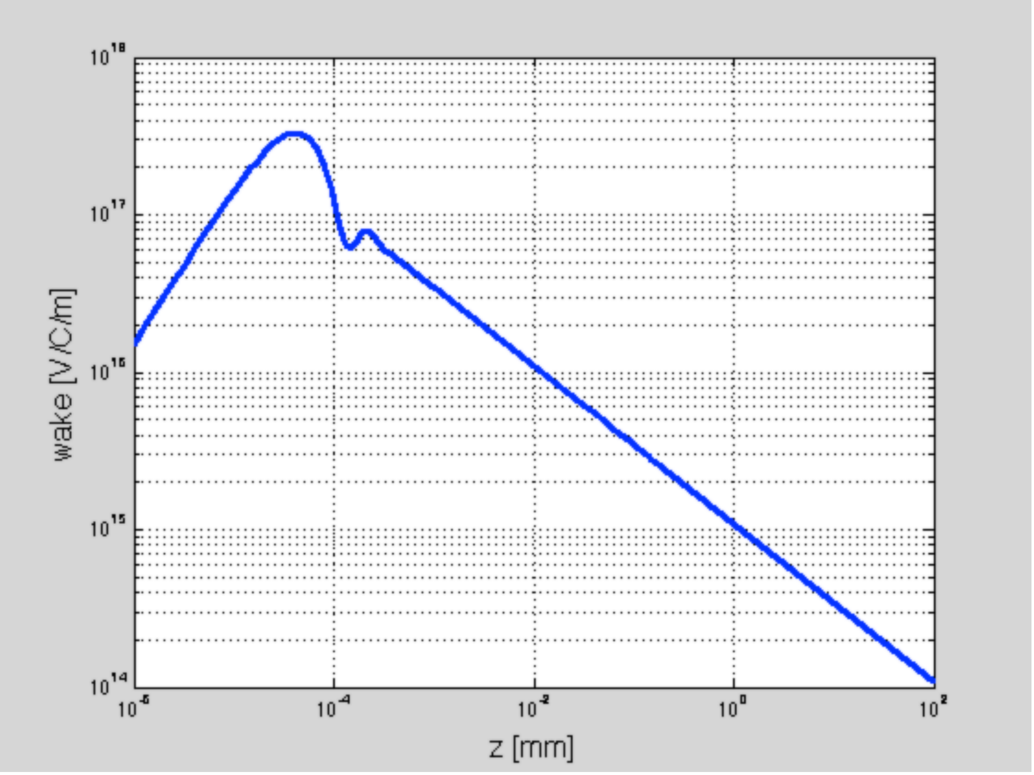
$$q = \frac{a-b}{a+b} \qquad q = e^{-2u_0}$$

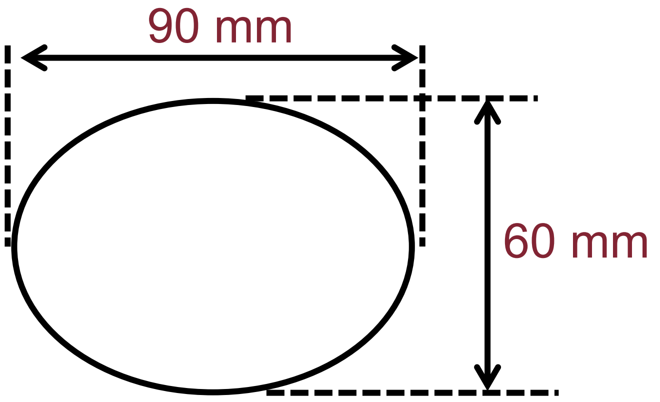
$$Z_{//}(\omega) = \left[1 - i\,\mathrm{sgn}(\omega)\right]\frac{L}{2\pi b}\sqrt{\frac{Z_0|\omega|}{2c\sigma_c}}G_{//}(u_0)$$

$$Z_{\perp}(\omega) = \left[\mathrm{sgn}(\omega) - i\right]\frac{L}{2\pi b^3}\sqrt{\frac{2Z_0 c}{|\omega|\sigma_c}}G_{\perp}(u_0)$$

Short range transverse wakefield: Resistive wall

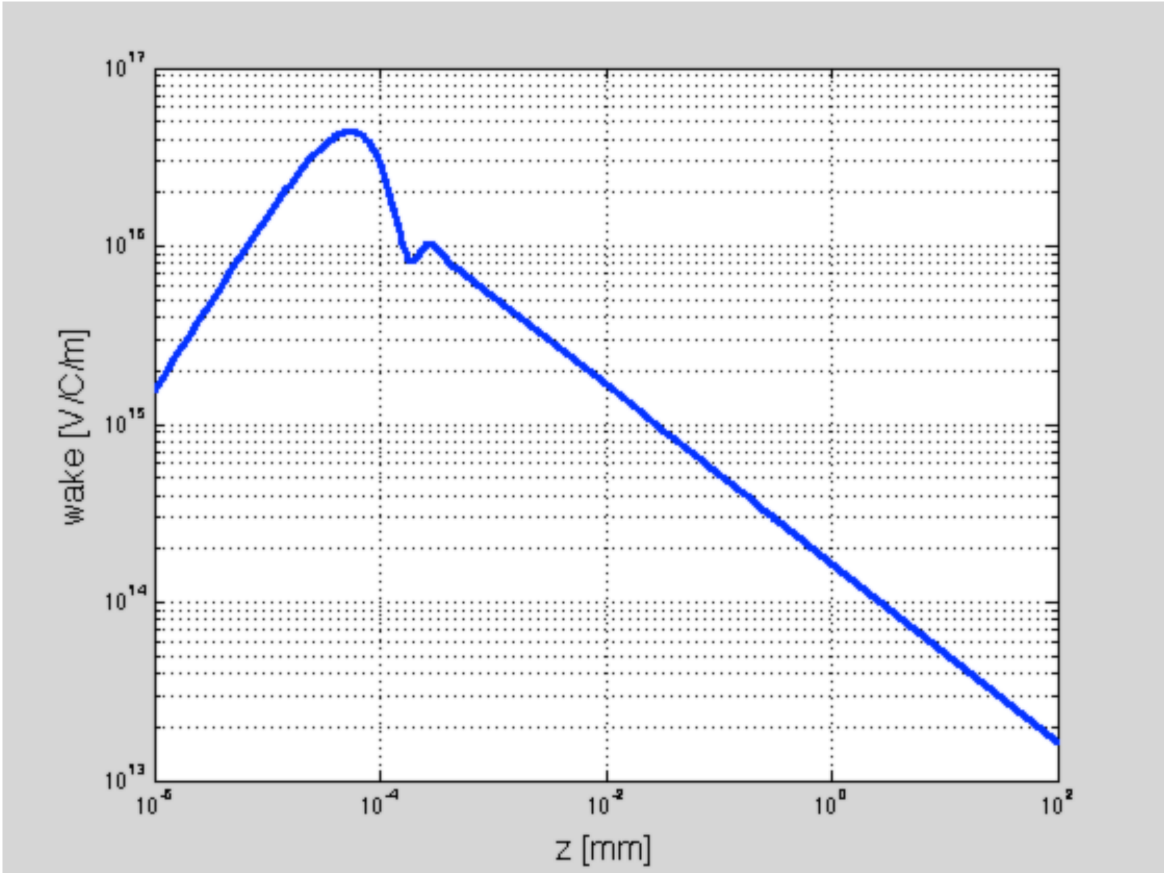
- Previous work (from N. Mounet): transverse wake due to RW





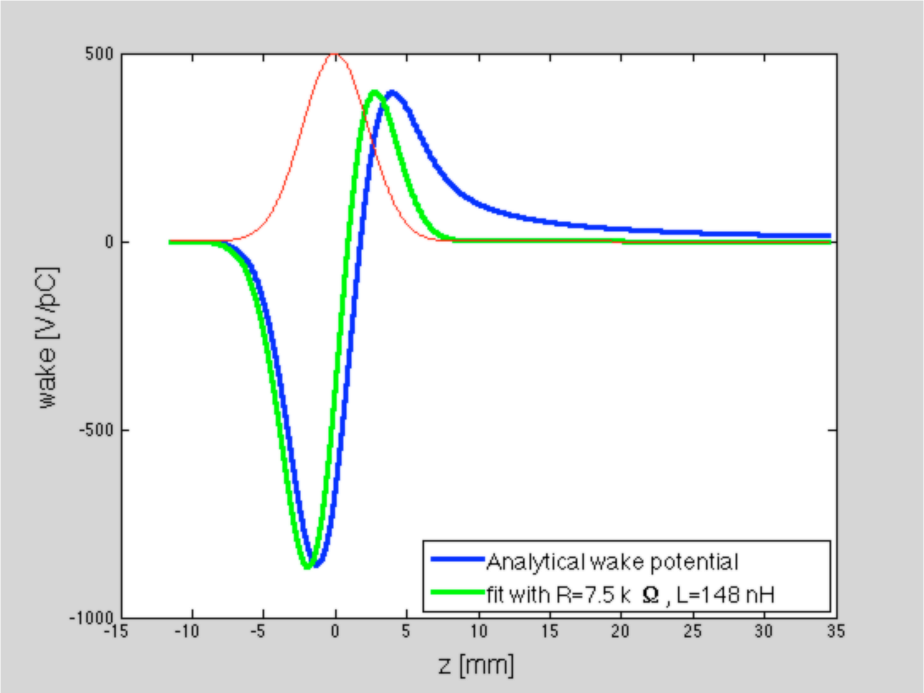
Short range transverse wakefield: Resistive wall

- With a bigger vacuum chamber the wake is reduced



Short range longitudinal wakefield: Resistive wall

- The wake potential of a Gaussian bunch with 2.3 mm bunch length can be obtained by



$$W_{//}(z) = \frac{cL}{8\sqrt{2}\pi b\sigma_z^{3/2}}\sqrt{\frac{Z_0}{\sigma_c}}F(z/\sigma_z)G_{\parallel}(u_0)$$

with

$$F(x) = |x|^{3/2} e^{-\frac{x^2}{4}}\left(I_{1/4} - I_{-3/4} \pm I_{-1/4} \mp I_{3/4}\right)$$

and I_n the modified Bessel function

fit with R-L impedance model

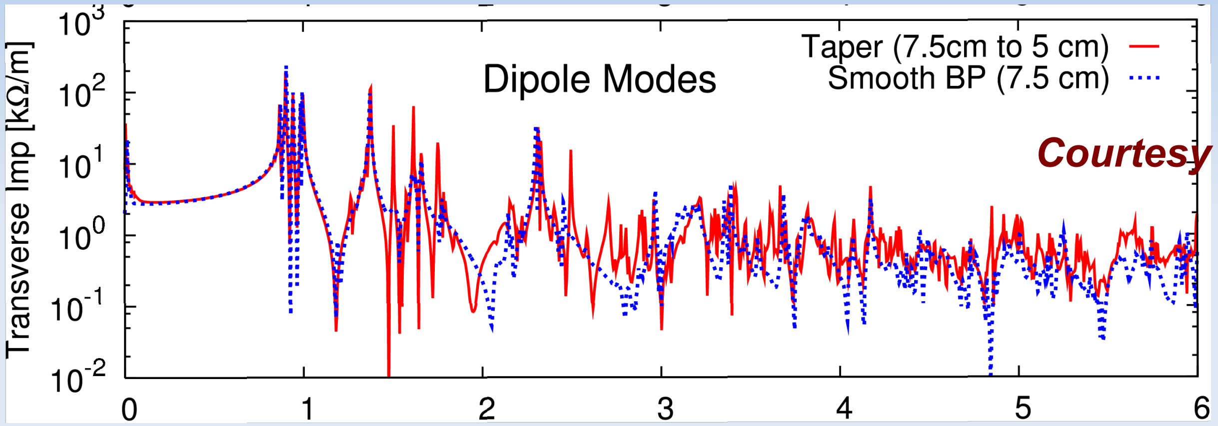
$$W(z) = -Rc\lambda(z) - Lc^2\lambda'(z)$$

R=7.5 kΩ and L=148 nH.

The loss factor is 276 V/pC

Short range wakefield: RF cavities

- Previous work: Courtey of N. Mounet
  - Broad-band resonator estimates (*Q=1*, f_r*=6* GHz):
    - RF cavities: from R. Calaga's PhD (BNL-SERL cavity, 700 MHz)



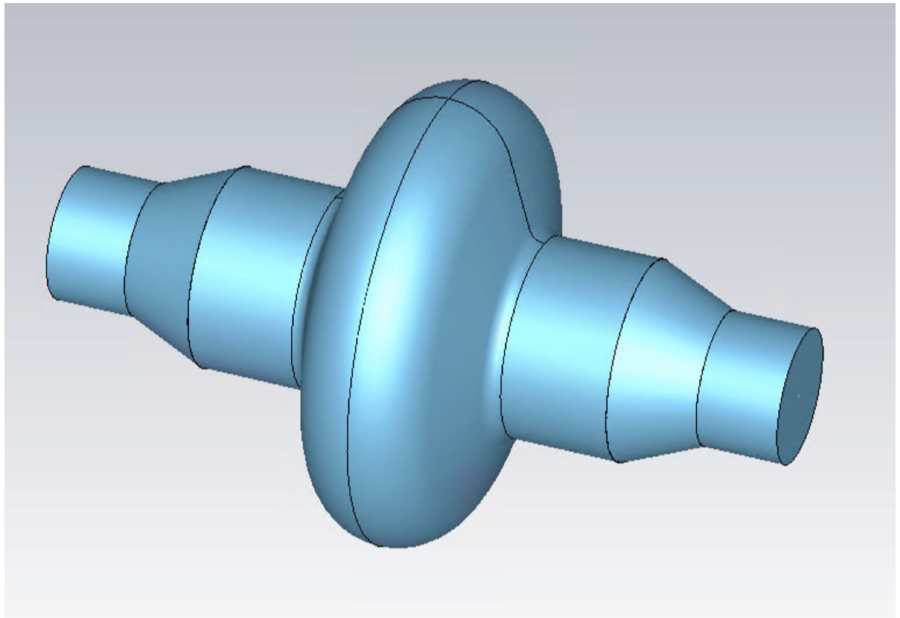
Courtesy R. Calaga

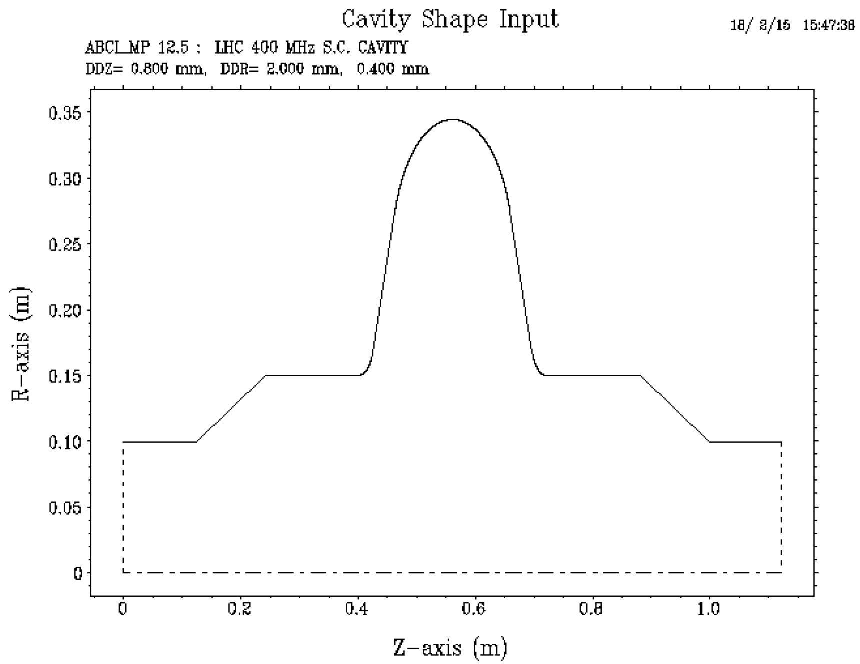
→ "fit" low-frequency part by constant inductive impedance of 3 kΩ/m, multiplied by 600 (number of cavities).

- Transitions between vacuum pipe and photon absorbers cross-sections: imp. approximated using K. Yokoya's formula for round tapers [CERN SL/90-88].

Short range wakefield: RF cavities

- 400 MHz cavities have been considered
- Both CST Microwave Studio and ABCI have been used to compare wakefields for a single cell

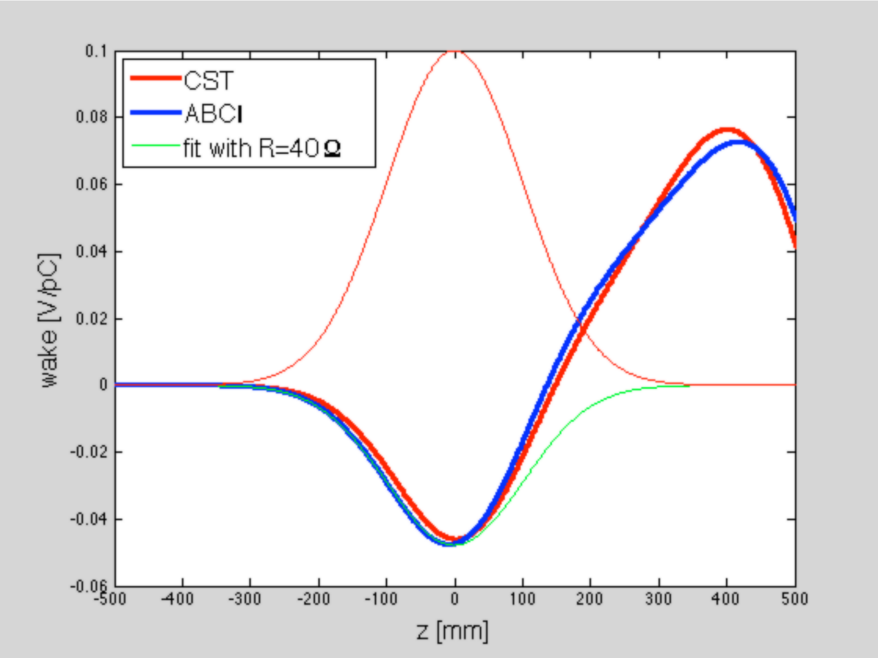




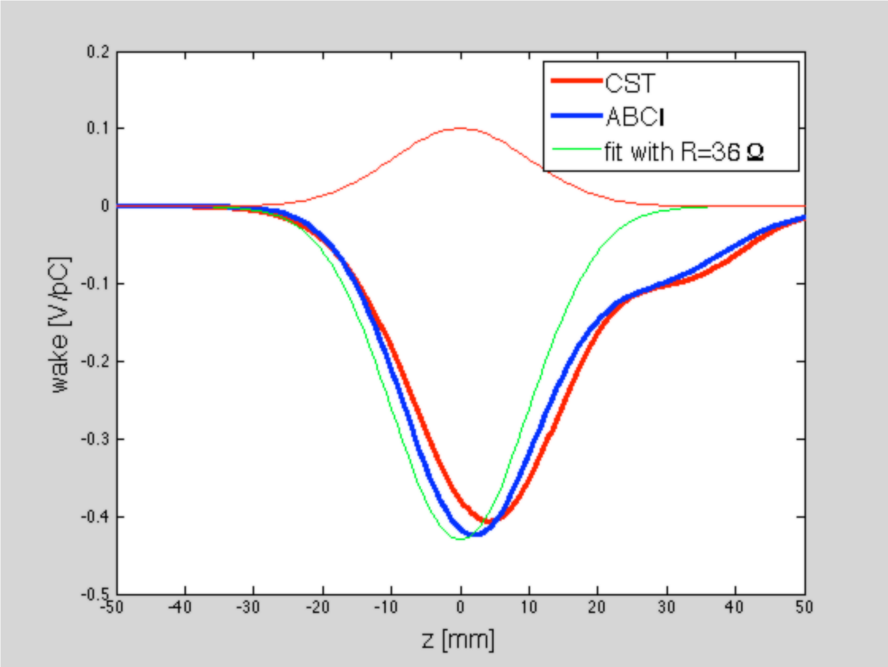
Short range longitudinal wakefield: RF cavities

- Wake potential for long bunches and fit with R-L impedance analytical model: $W(z) = -Rc\lambda(z) - Lc^2\lambda'(z)$

10 cm bunch length

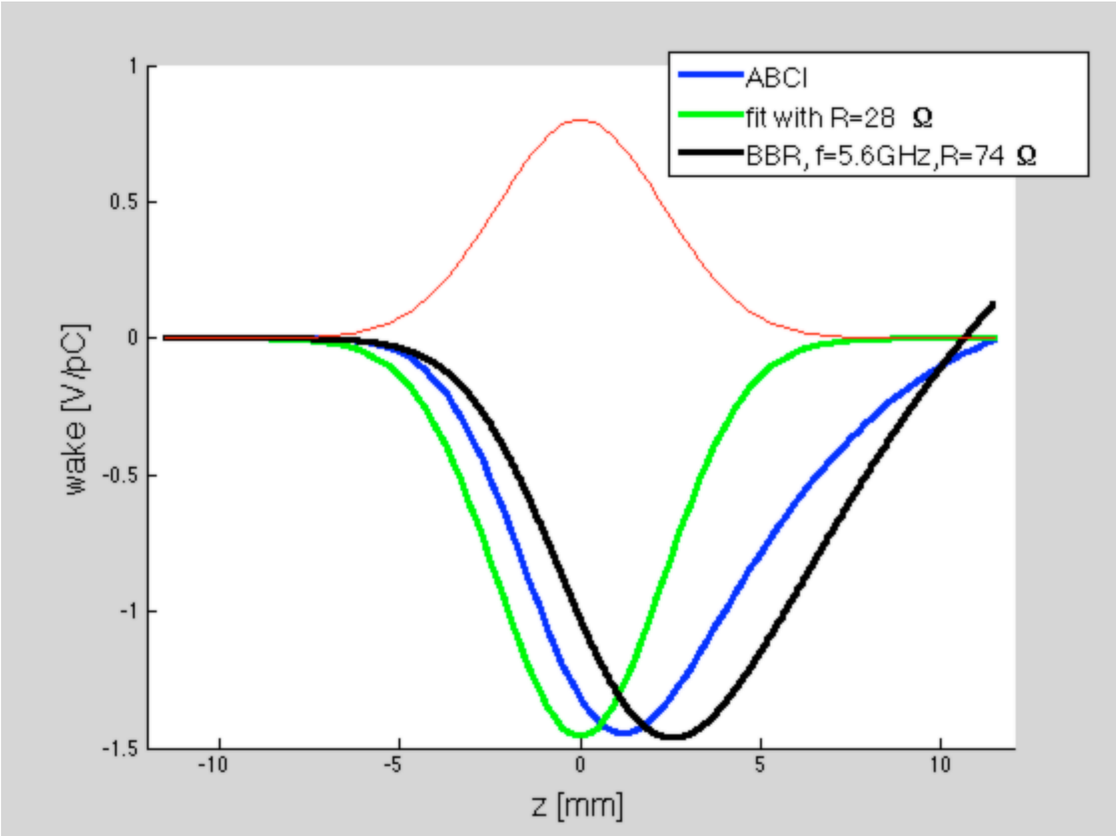


1 cm bunch length



Short range longitudinal wakefield: RF cavities

- At σ_z=2.3 mm we use ABCI results



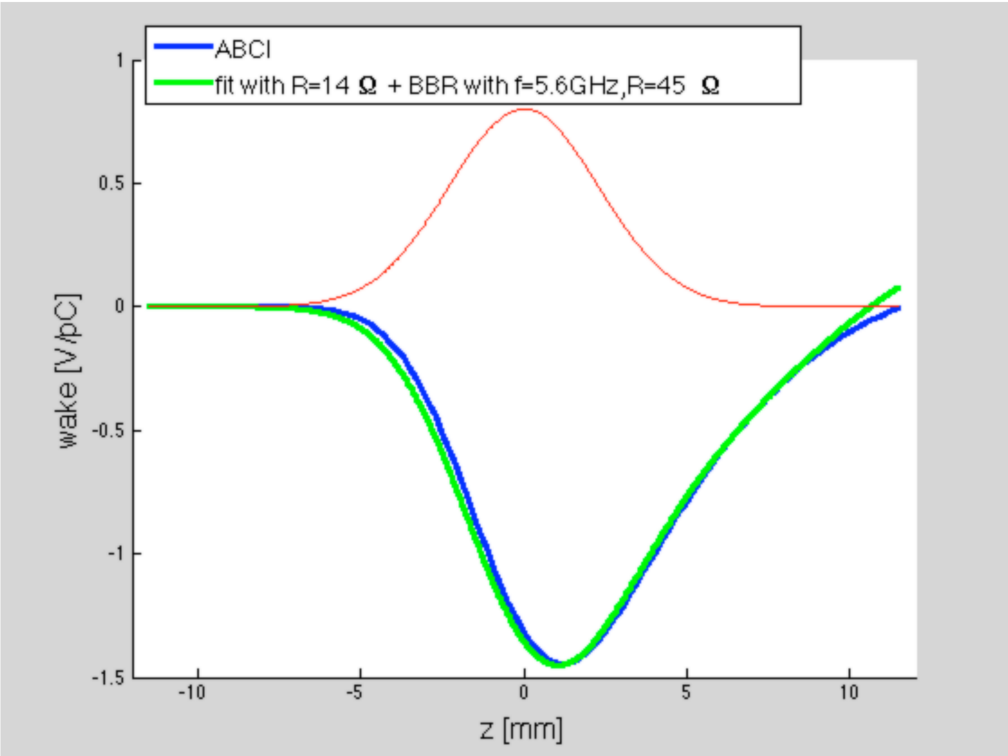
We are losing the proportionality of the wake potential with the inverse of the bunch length

The loss factor for one cell is **K_{loss}=1.043 V/pC**

Neither the R-L or a BBR impedance model can fit the wake potential

Short range longitudinal wakefield: RF cavities

- A good fit can be obtained by using a combination of BBR and R-L impedance model …



The loss factor can be evaluated by

$$k_l = \frac{Rc}{2\sqrt{\pi}\sigma_z} + \underbrace{\frac{R_s}{2Q'}\mathrm{Re}\left[\omega_1 w(\omega_1\sigma_z / c)\right]}_{\text{BBR contribution}}$$

with

$$\omega_1 = \frac{\omega_r}{Q}\left(-\frac{i}{2} + Q'\right) \qquad Q'^2 = Q^2 - 1/4$$

and w the complex error function
[Handobook of Accelerator Physics and Engineering, A. Chao, E. Tigne, p. 210]

The fit gives: **K_{loss}=1.07 V/pC**

Short range longitudinal wakefield: RF cavities

Rf Region	Frequency (MHz)	Tubes 1 MW ea	Modules per tube	Per ring Modules 2 couplers ea	Voltage (MV)	Length (m)
1	400	13	8	104	1248	260
2	400	13	8	104	1248	260
3a	400	6	8	48	576	120
3b	400	6	8	48	576	120
4a	400	6	8	48	576	120
4b	400	6	8	48	576	120
Total		**50**		**400**	**4800**	**1000**
Total for shared rf		**100**		**800**	**9600**	**2000**

Courtesy of
U. Wienands, SLAC
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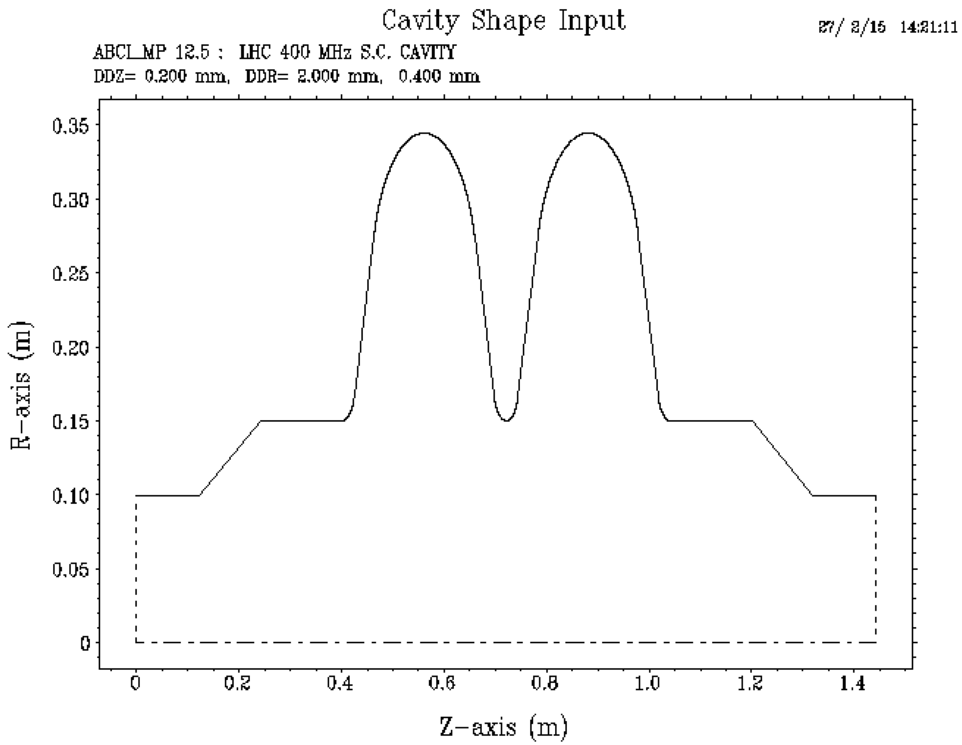
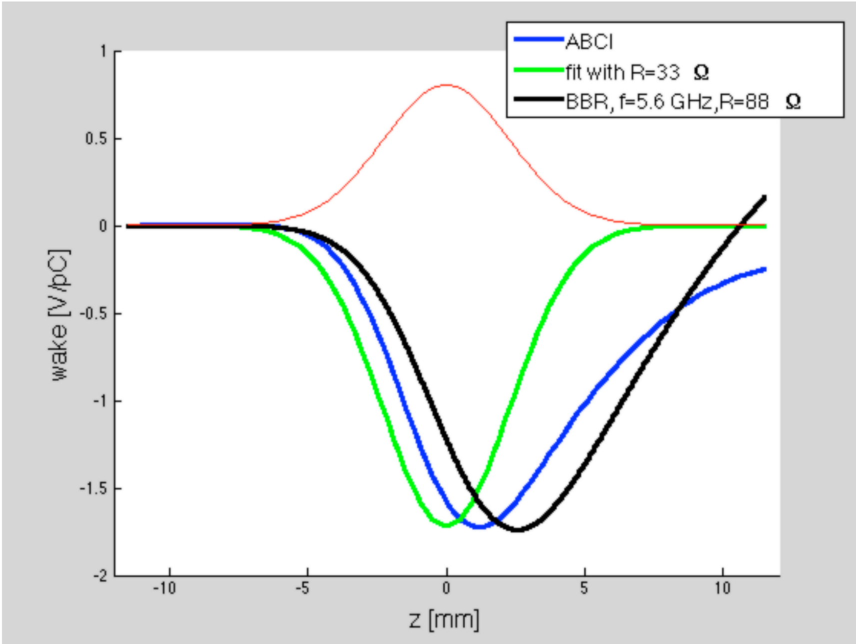
- By considering 2 two-cell cavities per module, we get a maximum of 1600 cells, obtaining a total of 44.8 kΩ
- For CEPC* ring R=28.1 kΩ (N. Wang, et al., 55th ICFA Advanced Beam Dynamics Workshop on High Luminosity Circular e+e- Colliders – Higgs Factory 2014)

* Circular Electron Positron Collider

Short range longitudinal wakefield: RF cavities

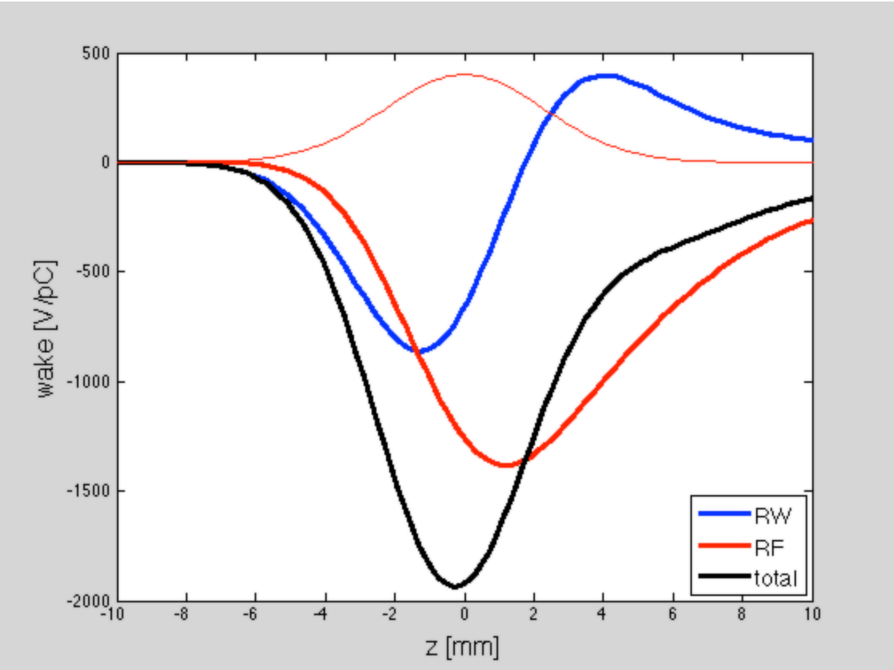
- However, if we take the short range wake of the two cell cavity, then R≅33 Ω, the loss factor is 1.25 V/pC, and the total impedance becomes 26.4 k Ω (800 cavities)





Longitudinal Impedance Budget

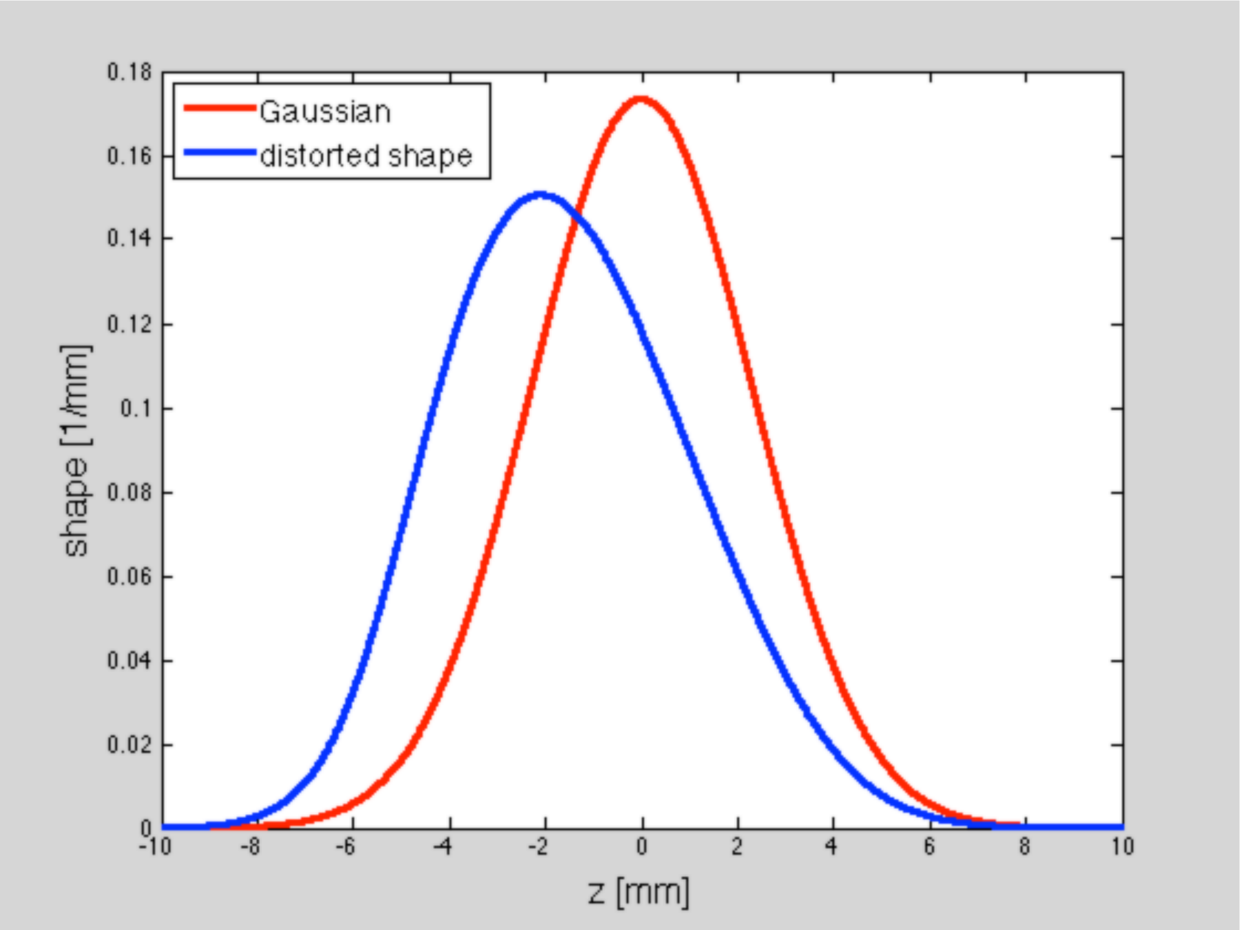
Element	R [kΩ]	L [nH]	k_{loss} [V/pC]	|Z/n| (mΩ)
Resistive wall (Al)	7.5	148	276	2.8
RF cavities	26.4	-	1000	-
total	33.9	148	1276	2.8
CEPC total	37.8	128.8	1205.1	4.4



- The total loss factor, with a bunch charge of about 29 nC gives an energy lost per turn of about 0.037 GeV.
- The RF cavities contribute mainly to the real part of the impedance and to the total loss factor.
- The impedance budget is comparable to CEPC.

Single bunch longitudinal effects

- Bunch lenghening is obtained in the potential well distortion regime by numerically solving the Haissinski equation with the R-L impedance model due to RF cavities and RW.



Initial bunch length: 2.30 mm

Final bunch length: 2.56 mm

Bunch is lengthened by about 10%

Single bunch longitudinal effects

- Microwave instability threshold is estimated according to the Boussard or Keil-Schnell criterion

$$\left|\frac{Z_{\parallel}}{n}\right| = \frac{(2\pi)^{3/2}\,\alpha_c E \sigma_\varepsilon^2 \sigma_z}{ceN_p} \cong 5\ \mathrm{m}\Omega$$

- This is a very small value … further investigation is necessary
- CEPC longitudinal impedance threshold is |Z/n|=25 mΩ
  (N. Wang, et al., 55th ICFA Advanced Beam Dynamics Workshop on High Luminosity Circular e+e- Colliders – Higgs Factory 2014)

Single bunch longitudinal effects

- Alternative crab-waist scenario using the 175 GeV optics at the Z pole (45 GeV)

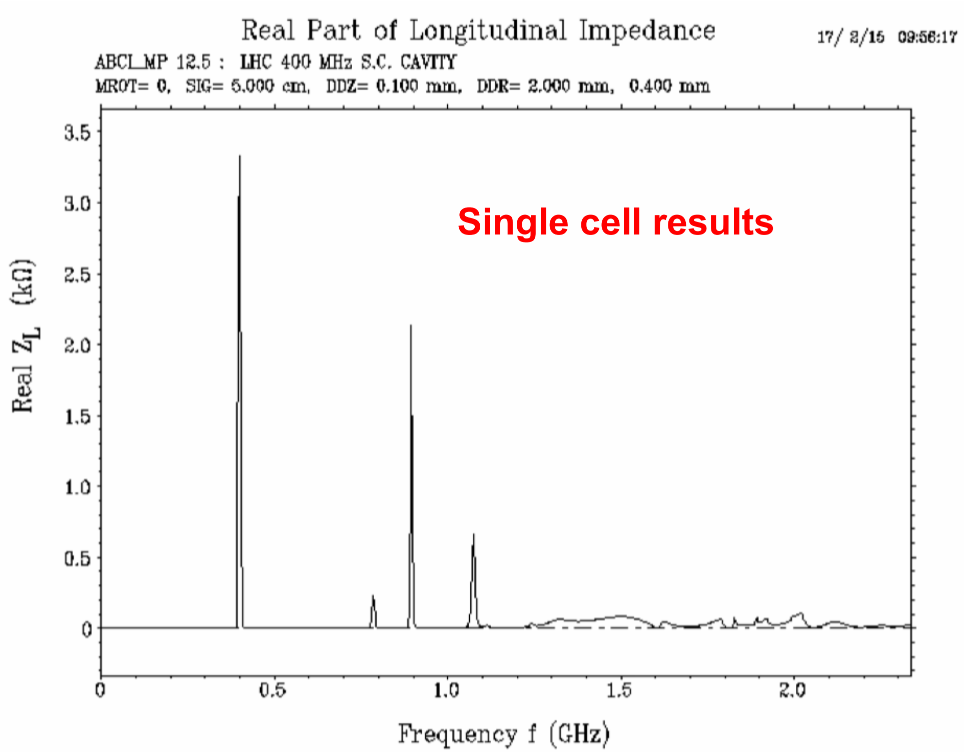
parameter	Symbol	Value
Bunch population	N_p	1.0×10^{11}
Bunch length	σ_z	2.7 mm
Energy spread	σ_ε	1.16×10^{-3}
momentum compaction	α_c	0.5×10^{-5}

$$\left|\frac{Z_{\parallel}}{n}\right| = \frac{(2\pi)^{3/2}\alpha_c E \sigma_\varepsilon^2 \sigma_z}{ceN_p} \cong 2.7\ \mathrm{m}\Omega$$

With only the RW impedance the machine is already above microwave instability threshold!

Coupled bunch instability - longitudinal

- High quality resonant modes trapped, for example, in the RF cavities can induce coupled bunch instabilities.
- The cut-off frequency of the TM01 mode for a circular pipe with radius of 10 cm (corresponding to the tubes attached to the RF cavities) is about 1.15 GHz.



- Below 1.1 GHz we can see, in addition to the fundamental mode at 400 MHz, other trapped HOMs.
- Considering the whole RF system, due to construction tolerances there is a spread in the resonant frequencies of HOMs wich reduces the maximum shunt resistance.

Coupled bunch instability - longitudinal

- In the worst case of resonant condition, by supposing the bunch as a point charge (no form factor), the grow rate of the instability is

$$\alpha = \frac{\alpha_c I_0 f R_s}{2(E/e)Q_s}$$

- Without any feedback, this grow rate can only be compensated by the natural damping rate (1320 turns) so that the maximum shunt resistance of a HOM is given by

$$R_s = \frac{2(E/e)Q_s}{\alpha_c I_0 f \tau_z} \cong \frac{365}{f[\mathrm{GHz}]}\ \mathrm{k\Omega}$$

longitudinal maximum shunt impedance