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# Final Cooling For a High-Luminosity High-Energy Lepton Collider

## Super wedgies!

David Neuffer FNAL

Don Summers, T. Hart, ... U. Miss.

# Outline

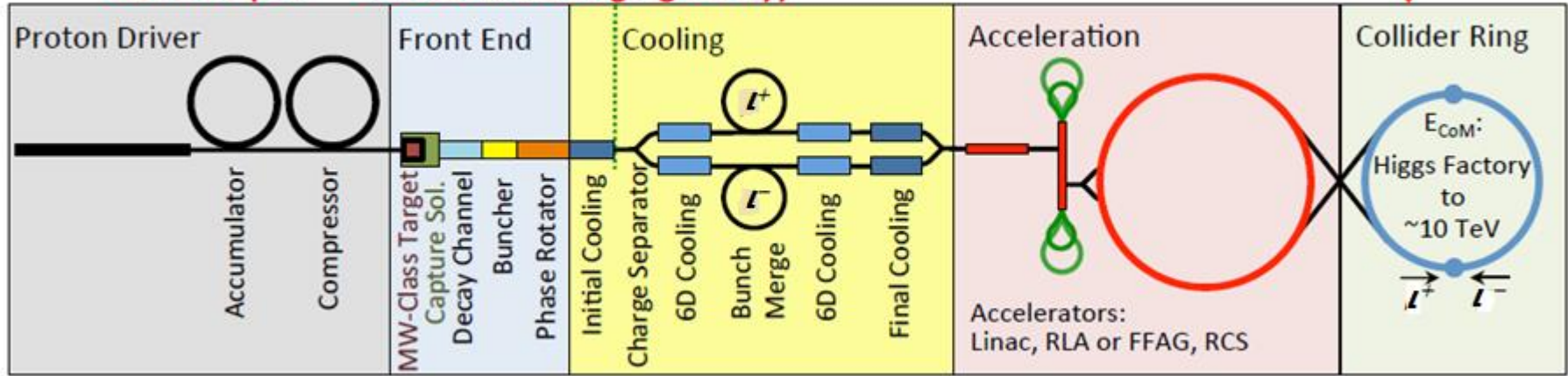
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- Motivation-**IPAC15**
- Final Cooling for a Collider & Simulation
  - R. Palmer & H. Sayed
- Final scenario variations
  - w /D. Summers & T. Hart
  - emittance exchange

# Towards multi-TeV lepton colliders

M. Palmer, FRXC3

Collider (*Lepton* Accelerator Staging Study)



Parameter	Unit	Higgs factory	3 TeV design	6 TeV design
Beam energy	TeV	0.063	1.5	3.0
Number of IPs		1	2	2
Circumference	m	300	2767	6302
$\beta^*$	cm	2.5	1	1
Tune x/y		5.16/4.56	20.13/22.22	38.23/40.14
Compaction		0.08	-2.88E-4	-1.22E-3
Emittance (Norm.)	mm-mrad	300	<b>25</b>	<b>25</b>
Momentum spread	%	0.003	0.1	0.1
Bunch length	cm	5	1	1
H. electrons/bunch	$10^{12}$	2	2	2
Repetition rate	Hz	30	15	15
Average luminosity	$10^{34} \text{ cm}^{-2}\text{s}^{-1}$	0.005	<b>4.5</b>	<b>7.1</b>

D, Neuffer

# Final cooling baseline

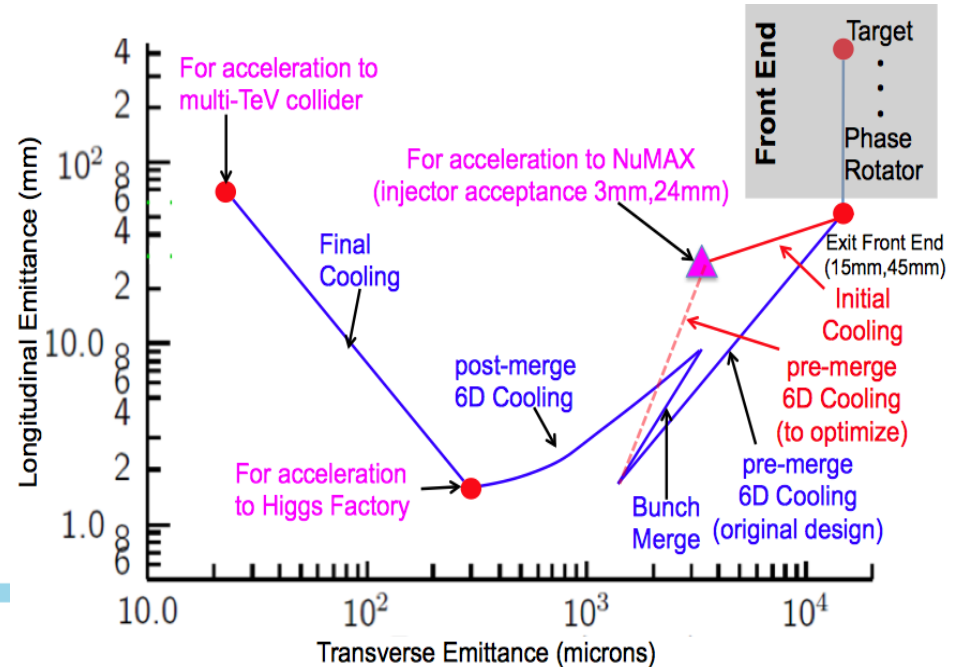
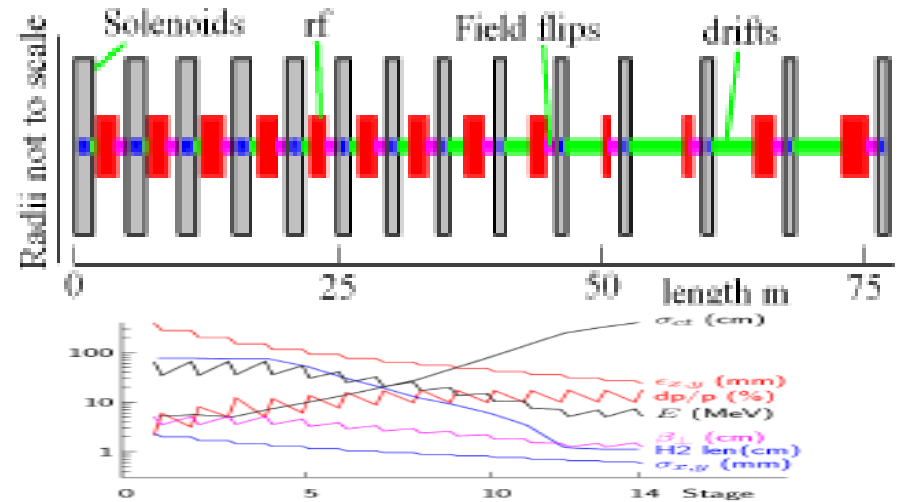
(R. Palmer et al. IPAC12)



- **Baseline Final Cooling**
  - solenoids,  $B \rightarrow 30\text{--}50\text{T}$
  - $\text{H}_2$  absorbers,
  - Low momentum
- $\epsilon_{t,N} : 3.0 \rightarrow 0.3 \times 10^{-4} \text{ m}$
- $\epsilon_L : 1.0 \rightarrow 70\text{mm}$ 
  - expensive emittance exchange

$$\epsilon_{N,eq} \cong \frac{\beta_t E_s^2}{2\beta m c^2 L_R (dE/ds)}$$

$$\beta_t \cong \frac{2P_\ell (GeV/c)}{0.3B}$$



# Detailed simulation of final cooling

(H. Sayed et al. IPAC14)

- **G4Beamline simulation of final cooling scenario**

- System is ~135m long

- $\epsilon_{t,N} : 3.0 \rightarrow 0.5 \cdot 10^{-4} \text{ m}$

- $\epsilon_L : 1.0 \rightarrow 75 \text{ mm}$

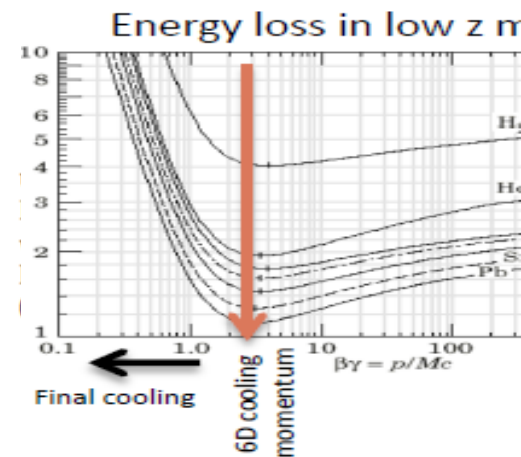
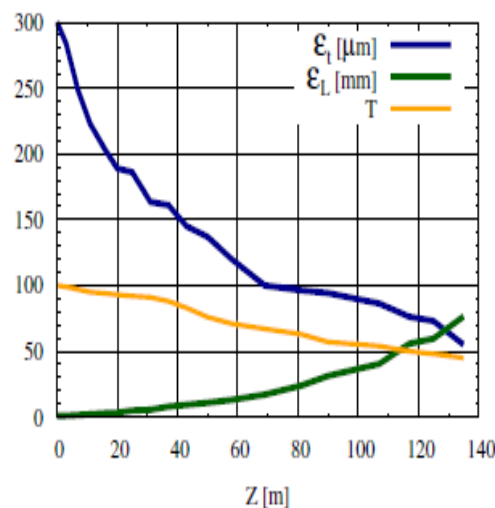
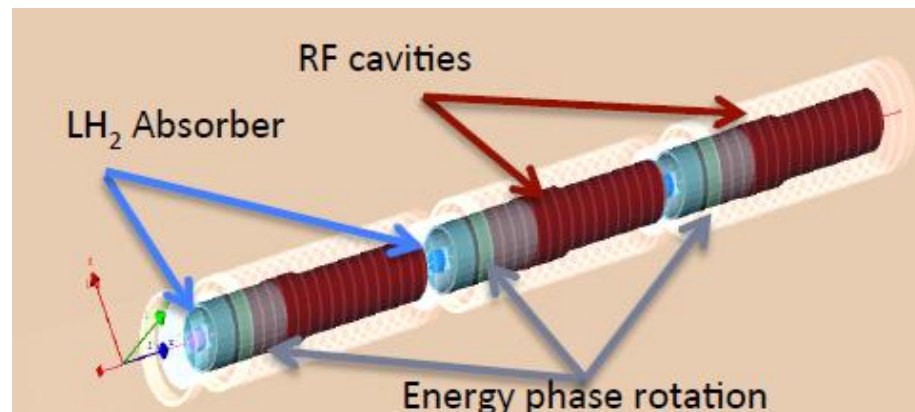
- $P_i : 135 \rightarrow 70 \text{ MeV/c}$

- $B : 25 \rightarrow 32 \text{ T}; 325 \rightarrow 20 \text{ MHz}$

- not quite specs

- Transmission ~ 50%

- **Predominantly  $\epsilon_{t,N} / \epsilon_L$  emittance exchange**

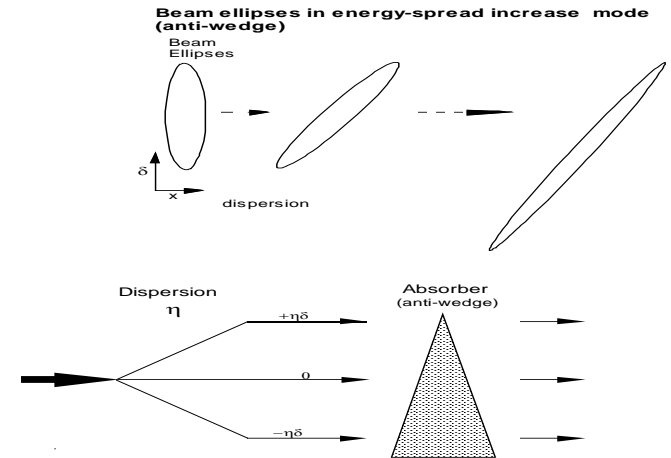
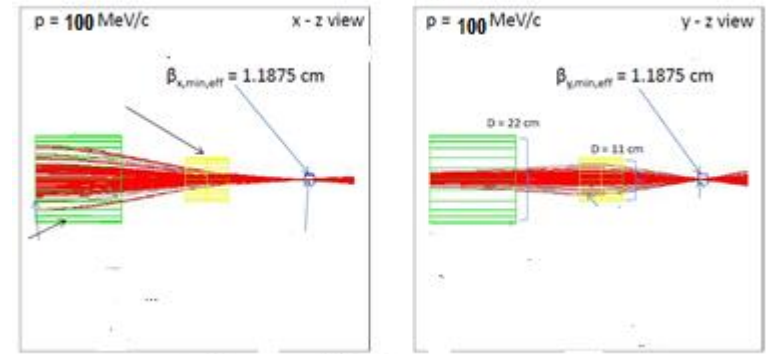


# Variant Approaches

- Keep  $P_l$ ,  $B$ ,  $E'$ ,  $f_{rf}$  within ~current technology
  - $P > 100\text{MeV}/c$ ;  $B \sim 8 \rightarrow 15\text{T}$ ;  $f_{rf} > \sim 100\text{MHz}$
  - round to flat transformation
  - beam slicing and recombination
  
- Explicitly use emittance exchange in final cooling
  - thick wedge energy loss

# Variant: “thick” wedge transform

- Use wedge to increase  $\delta p/p$ 
  - increase  $\epsilon_{\perp}$ , decrease  $\epsilon_x$
- If  $\delta p/p$  introduced by wedge  $\gg \delta p/p_{\text{beam}}$ 
  - can get large emittance exchange
    - exchanges  $x$  with  $\delta p$  (Mucool 003)
      - also in CERN 99-13, p.30
- Example:
  - 100 MeV/c;  $\delta p=0.5\text{MeV/c}$ 
    - $\epsilon_{\perp}=10^{-4}\text{m}$ ,  $\beta_0=1.2\text{cm}$
    - Be wedge 0.6cm,  $140^\circ$  wedge
  - obtain factor of  $\sim 5$  exchange
  - $\epsilon_x \rightarrow 0.2 \times 10^{-4}\text{m}$ ;  $\delta p=2.5\text{ MeV/c}$
- Much simpler than equivalent final cooling section



$$\epsilon_1 = \epsilon_0 \left[ (1 - \eta_0 \delta')^2 + \frac{\delta'^2 \sigma^2}{\delta_0^2} \right]^{-1/2}$$

# Wedge theory (MuCool-003)

- Dispersion + wedge is product of two matrices

$$\mathbf{M}_\delta = \begin{bmatrix} 1 & 0 \\ -\delta' & 1 \end{bmatrix} \quad \mathbf{M}_\eta = \begin{bmatrix} 1 & \eta_0 \\ 0 & 1 \end{bmatrix}$$

- $\delta' = dp/ds \cdot 2 \tan[\theta/2]/p$
- variables are  $[x, \delta]$ , where  $\delta = dp/p$
- transport through wedge is transport of  $[x, \delta]$  phase space ellipse, initially

$$g_0 x^2 + b_0 \delta^2 = \sigma_0 \delta_0$$

- becoming

$$g_1 x^2 + 2a_1 x\delta + b_1 \delta^2 = \sigma_0 \delta_0$$



# Results of wedge

- **new coefficients**

$$b_1 = b_0 + (\eta_0)^2 g_0$$

$$a_1 = \delta' b_0 - \eta_0 (1 - \delta' \eta_0) g_0$$

$$g_1 = \delta'^2 b_0 + (1 - \delta' \eta_0)^2 g_0$$

$$\beta_1 = m_{11}^2 \beta_0 + 2m_{11}m_{12}\alpha_0 + m_{12}^2 \gamma_0$$

$$\alpha_1 = -m_{11}m_{21}\beta_0 + (m_{11}m_{22} + m_{12}m_{21})\alpha_0 - m_{12}m_{22}\gamma_0$$

$$\gamma_1 = m_{21}^2 \beta_0 + 2m_{21}m_{22}\alpha_0 + m_{22}^2 \gamma_0$$

- **new energy width ( $\varepsilon_{L,1} = \varepsilon_{L,0} (\delta_1/\delta_0)$ )**

$$\delta_1 = \sqrt{g_1 \sigma_0 \delta_0} = \delta_0 \left[ (1 - \eta_0 \delta')^2 + \frac{\delta'^2 \sigma_0^2}{\delta_0^2} \right]^{1/2}$$

- **new transverse emittance ( $\varepsilon_{x,1} = \varepsilon_{x,0} (\delta_0/\delta_1)$ )**

$$\varepsilon_{x,1} = \sqrt{g_1 \sigma_0 \delta_0} = \varepsilon_{x,0} \left[ (1 - \eta_0 \delta')^2 + \frac{\delta'^2 \sigma_0^2}{\delta_0^2} \right]^{-1/2}$$

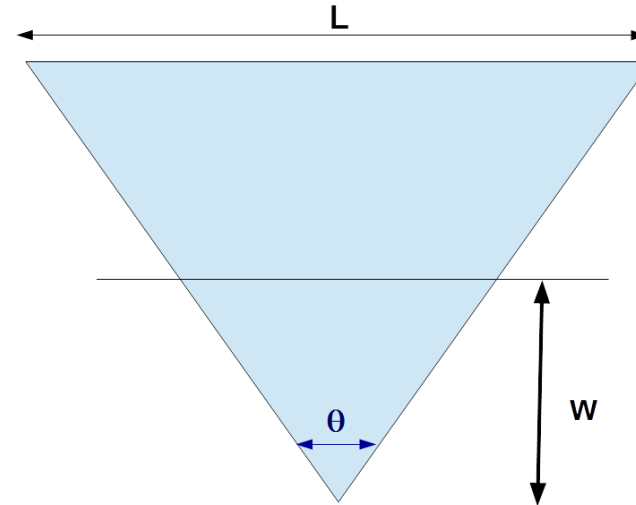
## new $\beta_x, \eta$

$$\eta_1 = -\frac{a_1}{g_1} = \frac{\eta_0 (1 - \eta_0 \delta') - \delta' \frac{\sigma_0^2}{\delta_0^2}}{(1 - \eta_0 \delta')^2 + \delta'^2 \frac{\sigma_0^2}{\delta_0^2}}$$

$$\beta_1 = \beta_0 \left[ (1 - \eta_0 \delta')^2 + \frac{\delta'^2 \sigma_0^2}{\delta_0^2} \right]^{-1/2}$$

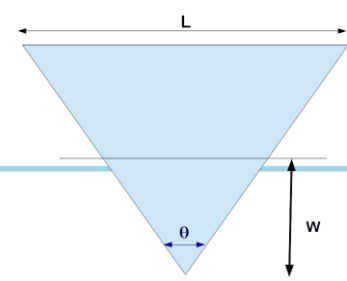
# Evaluation of Super-wedge examples

- Set reference momentum at 100 MeV/c
  - $\epsilon_x, \epsilon_y \rightarrow 10^{-4} \text{ m}$
  - $\beta_x = \beta_y = \sim 1 \text{ cm}$ 
    - $\sim 1 \text{ mm}$  beam
      - round numbers ...
- Need small  $\delta p/p$ 
  - $\delta p \sim 0.5 \text{ MeV/c}$ 
    - obtain by lengthening and flattening bunch ( $\epsilon_L = \sim 0.001 \text{ m}$ )
- Need dense low-Z wedge
  - Be ( $\rho=1.86$ )  $\rightarrow$  Diamond
    - ( $\rho=3.6 \text{ C}$ )
    - $dp/ds = 15.1 \text{ MeV/c /cm}$



- Evaluate in ICOOL
  - wedge and beam definition, match
  - emitcalc evaluates eigen emittances before/after
  - a few cm transport

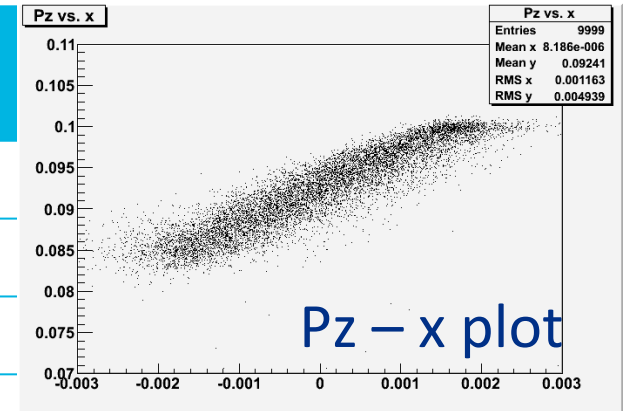
# Numerical examples



- Wedge parameters

- Diamond,  $w=1.75\text{mm}$ ,  $\theta = 100^\circ$  (4.17mm thick at center)

$z(\text{cm})$	$P_z$	$\epsilon_x(\mu)$	$\epsilon_y$	$\epsilon_L(\text{mm})$	$\sigma_E$ MeV	6-D $\epsilon$ increase
0	100	97	95.5	1.27	0.46	1.0
0.4	96.4	33.4	96.3	4.55	1.64	1.24
0.8	92.4	<b>22.7</b>	96.5	8.94	3.22	1.65



- reduces  $\epsilon_x$  by factor of 4.3,  $\epsilon_L$  increases by factor of 7.0

- first half of wedge more efficient than second half ...

- Second wedge ?

- if matched to same optics ( $P_z \rightarrow 100 \text{ MeV}/c$ ,  $\sigma_E \rightarrow 0.46 \text{ MeV}$ )

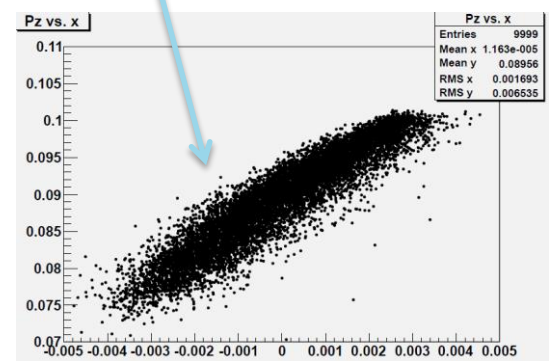
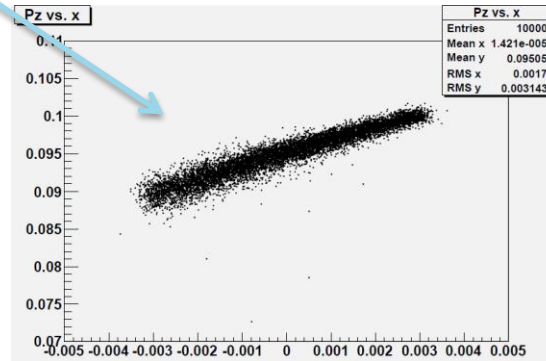
- $\epsilon_x : 23 \rightarrow 27\mu$ ;  $\epsilon_y : 97 \rightarrow 23 \mu$

# Parameter variations...

- Go to larger initial beams, larger wedge
  - $\beta_t=2.6\text{cm}$ ,  $\epsilon_t =130\mu$
  - Diamond,  $w=3.0\text{mm}$ ,  $\theta = 85^\circ$  (5.6mm thick at center)

$z(\text{cm})$	$P_z$	$\epsilon_x(\mu)$	$\epsilon_y$	$\epsilon_L(\text{mm})$	$\sigma_E(\text{MeV})$	6-D $\epsilon$ increase
0	100	129	127	0.91	0.44	1.0
0.55	95	36.6	131	4.33	2.08	1.34
1.1	89.6	<b>25.0</b>	132	8.66	4.17	1.845

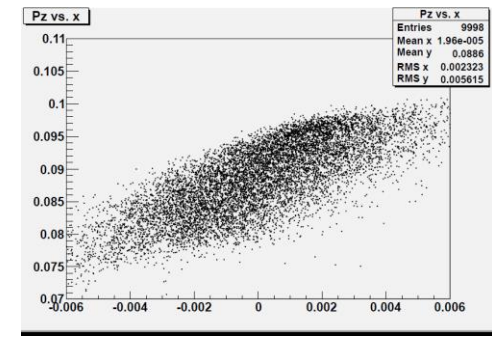
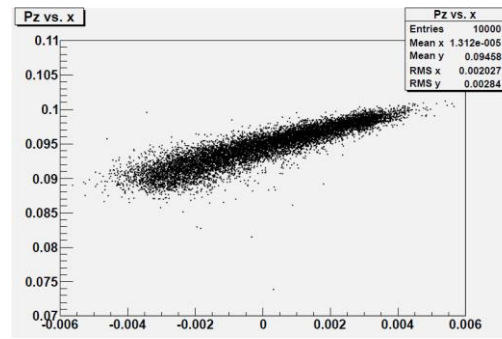
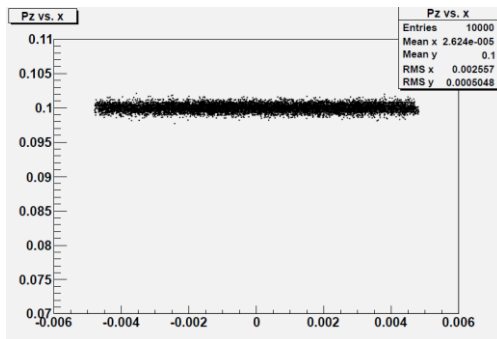
$P_z - x$  plots



# Parameter variations...

- Go to larger initial beams, larger wedge
  - $\beta_t=3.2\text{cm}$ ,  $\varepsilon_t=200\mu$
  - Graphite,  $w=4.5\text{mm}$ ,  $\theta = 100^\circ$  (9.6mm thick at center)

$z(\text{cm})$	$P_z$	$\varepsilon_x(\mu)$	$\varepsilon_y$	$\varepsilon_L(\text{mm})$	$\sigma_E$ MeV	6-D $\varepsilon$ increase
0	100	194	191	1.09	0.53	1.0
1.4	94.6	67.3	193	3.9	1.88	1.245
2.8	88.5	<b>47.3</b>	192	7.7	3.7	1.72

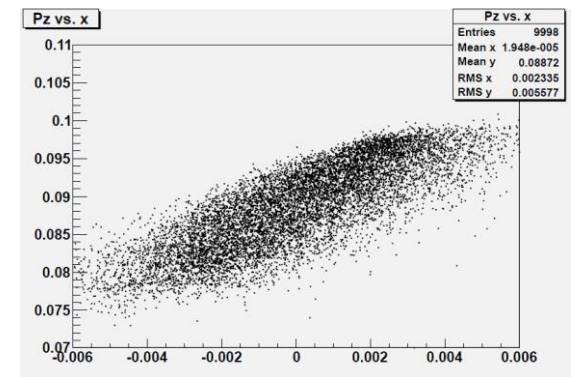
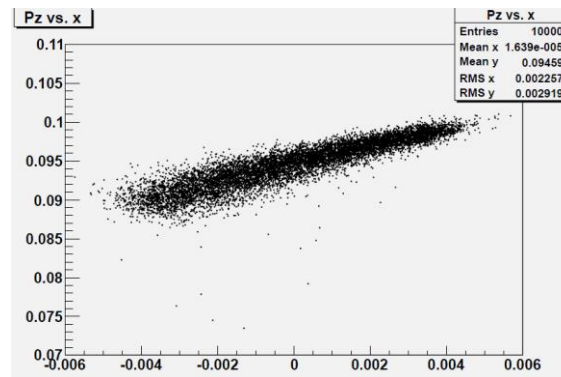
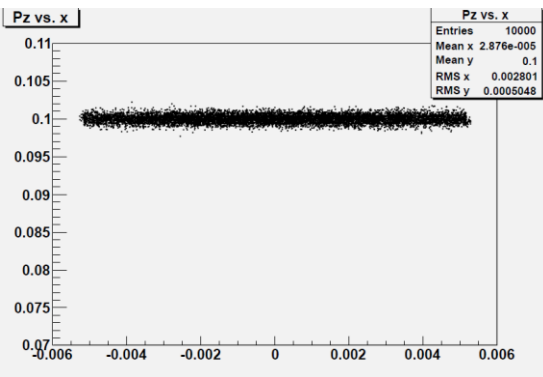


$P_z - x$  plots

# Parameter variations...

- Go to larger initial beams, larger wedge
  - $\beta_t=3.6\text{cm}$ ,  $\varepsilon_t=200\mu$
  - Be,  $w=5\text{mm}$ ,  $\theta = 110^\circ$  (14.3 mm thick at center)

$z(\text{cm})$	$P_z$	$\varepsilon_x(\mu)$	$\varepsilon_y$	$\varepsilon_L(\text{mm})$	$\sigma_E(\text{MeV})$	6-D $\varepsilon$ increase
0	100	194	191	0.99	0.53	1.0
1.4	94.6	60.2	191	4.0	1.88	1.24
2.8	88.7	<b>43.8</b>	189	7.5	3.7	1.69



# Compare with current final scenario

- last 120 m

  - 14 30 T SOLENOIDS

  - ~1 GeV rf

  - $\epsilon_{t,N}$  : 180  $\rightarrow$  55  $10^{-6}$  m

  - $\epsilon_L$  : 2.0  $\rightarrow$  75 mm

  - B: 25  $\rightarrow$  32 T; 325  $\rightarrow$  20 MHz

    - not quite specs

  - Transmission ~ 50%

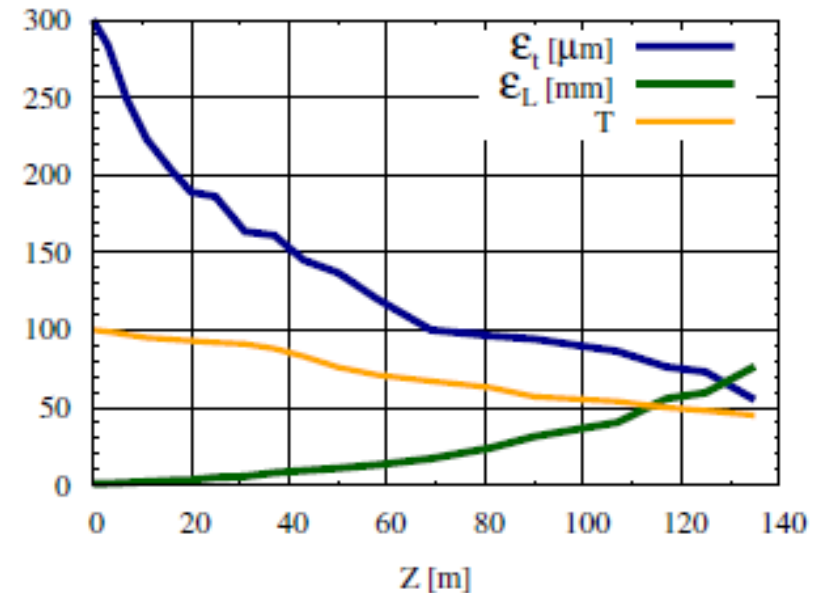
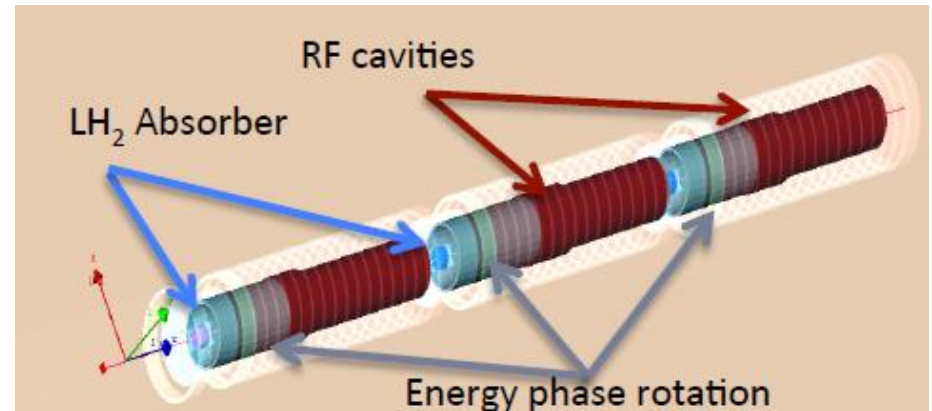
- Two Be wedges ~ 3 cm

  - $\epsilon_{t,N}$  : 200  $\rightarrow$  40  $10^{-6}$  m

  - $\epsilon_L$  : 1  $\rightarrow$  50 mm

  - Transmission ~ 95+%

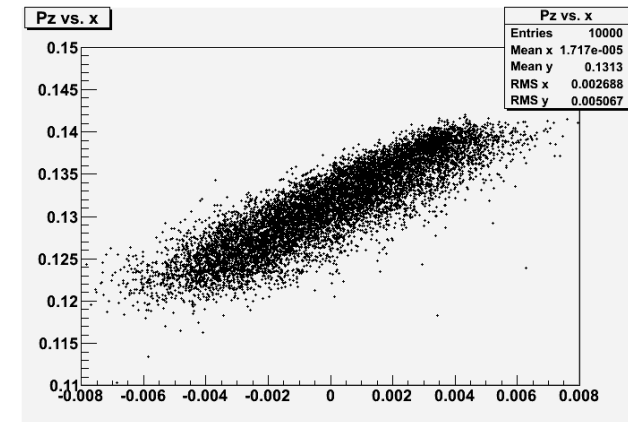
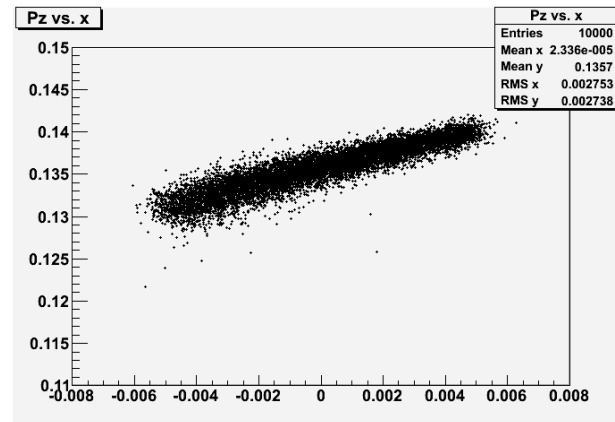
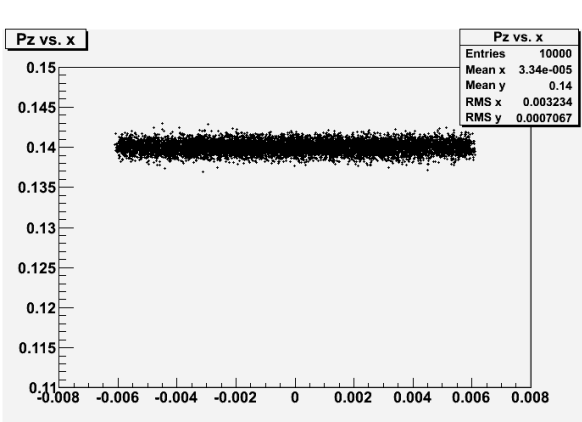
    - ~30 MeV rf



# Parameters closer to MICE

- Go to larger initial beams, larger wedge, higher P
  - $\beta_t=5\text{cm}$ ,  $\epsilon_t=272\mu$
  - $C$ ,  $w=5\text{mm}$ ,  $\theta = 105^\circ$  (14.3 mm thick at center)

$z(\text{cm})$	$P_z$ MeV/c	$\epsilon_x(\mu)$	$\epsilon_y$	$\epsilon_L$ (mm)	$\sigma_E$ MeV	6-D $\epsilon$ increase
0	<b>140</b>	272	268	1.22	0.68	1.0
1.4	135.7	102	273	4.0	2.15	1.2
2.8	131.3	<b>67.8</b>	275	7.5	3.92	1.44





# Need to complete scenario

- **Match into first wedge**
  - long  $\sim 0.15\text{m}$ ,  $\delta p = 0.5\text{MeV}/c$
  - $\beta_t \sim 3\text{cm}$   $\epsilon_t \sim 200\mu$ 
    - beam out has dispersion  
 $\eta \sim 4\text{cm}$ ,  $\delta p \sim 3\text{MeV}/c$ ,  $\beta_x \sim 0.5\text{cm}$
- **Match into second wedge**
  - Accelerate centroid to  $100(?)\text{MeV}/c$
  - long  $\sim 1\text{m}$ ,  $\delta p = 0.5\text{MeV}/c$
  - $\beta_t \sim 3\text{cm}$   $\epsilon_1 \sim 200\mu$ ,  $\epsilon_2 \sim 40\mu$
  - Dispersion match to  $\sim 0$
- **Biggest problem could be optical match out of wedge**
  - Beam has small x-emittance
  - dispersion + smaller  $\beta_x$
  - larger  $\delta p$
- **Similar to PIC match?**

# Disadvantages

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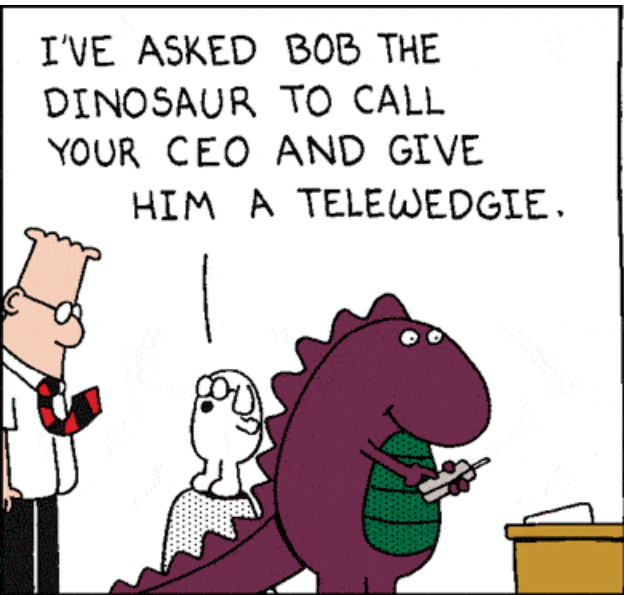
- exchange per wedge limited to  $\sim 4-6 \times$  (?)
- Does not cool
  - beam heating by  $> \sim 25\%$ ; more for larger exchange
- Optics matching is nontrivial

# Summary

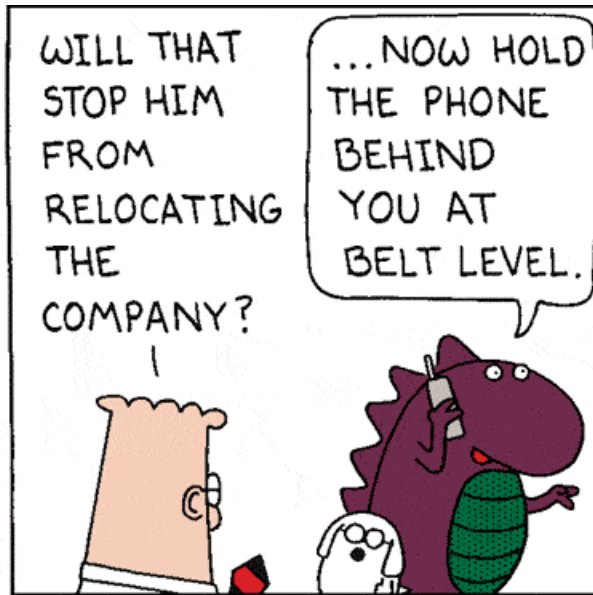
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- **Thick Wedge parameters are so attractive it has to be part of final cooling scenario**
  - Details depend on actual implementation values and matching solutions
  - how many passes possible  $2 \rightarrow 3 \rightarrow 4$  ?
  - how much longitudinal emittance increase is tolerable?
- **Not optimized ( $100\text{MeV}/c \rightarrow ???$ )**
- **Could we do an experiment at MICE ??**
  - cm-scale dense large-angle wedge, small  $\delta E$  initial beam

# Telewedgie...



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