Some Basics for Particle Physics

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Outline

- Fundamental Constituents of matter
- Basic forces of the nature
- Theoretical Description
- Interactions and Feynman diagrames
- Range of forces

Elementary Particles in Nature

Force Carriers

Particles forming Matter

Spin-zero Bosons

- Higgs particles

Fermions

Quarks

Hadrons

Mesons

- Bound states of $q\ \bar{q}$
- B=0 , L=0
- Pions, $\pi^{+}: u\ \bar{d}$

Baryons

- Bound states of $q\ q\ q$
- B=1 , L=0
- Protons, $p : u\ u$ d

Leptons

- e^{-} , ν_e
- μ^{-} , ν_μ
- τ^{-} , ν_τ
- Each lepton has lepton number L=1 and Baryon number B=0.

Elementary Particles in Nature

Force Carriers (Bosons)

Particles forming Matter

Gluons (8)
Spin-1
- Strong Interactions
- Short Range
- Interact with quarks only
- Color charge

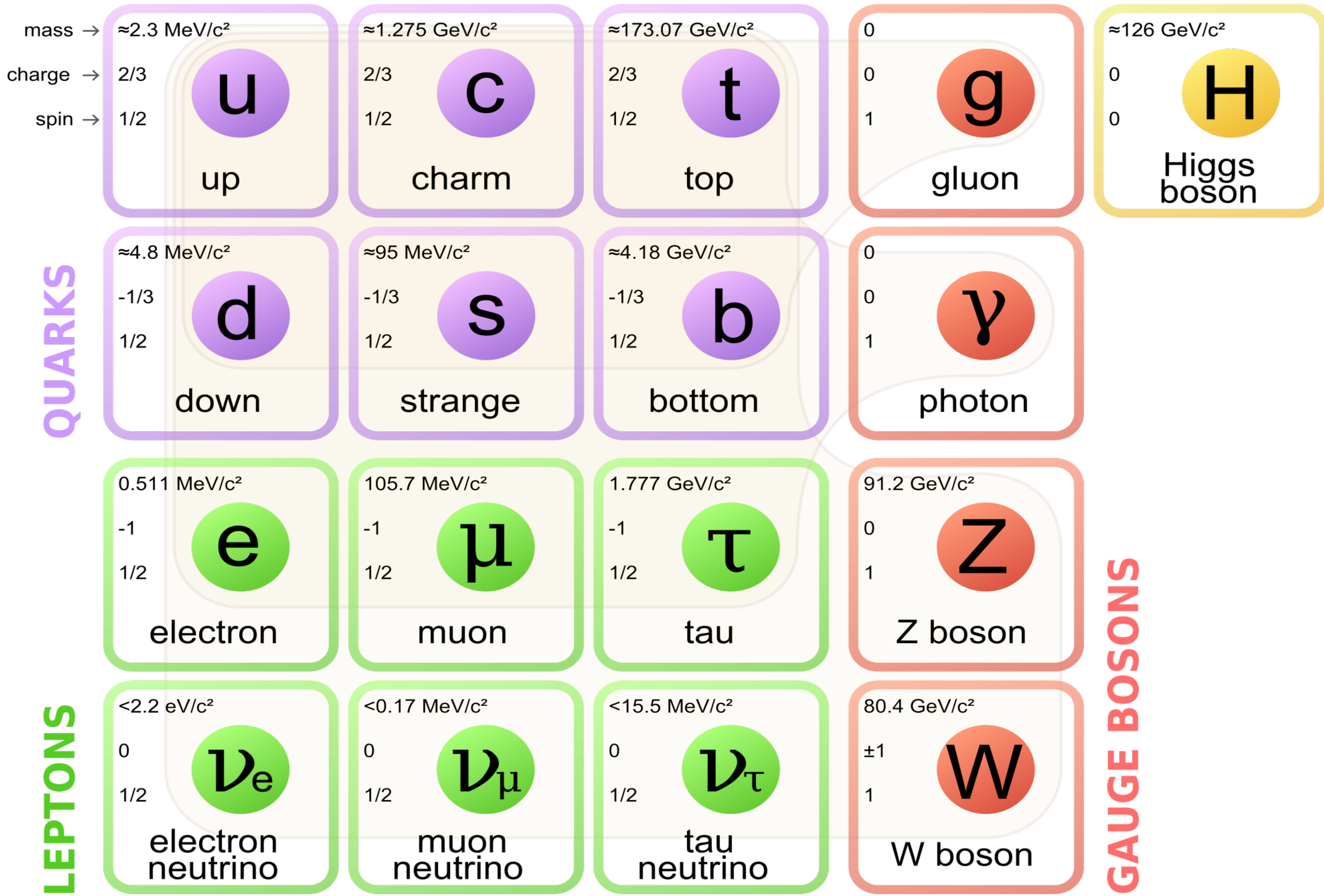
$W^{\pm}$ & Z
Spin-1
- Weak Interactions
- Short Range
- Interact with quarks, leptons and Higgs
- Iso-spin charge

Photon
Spin-1
- Electromagnetic Interactions
- Long Range
- Interact with quarks, charged leptons, $W^{\pm}$
- Electric charge

Graviton
Spin-2
- Gravitation Interactions
- Long Range
- Interact with any particle

Electroweak Theory

Grand Unified Theories



mass →
charge →
spin →
≈2.3 MeV/c²
2/3
1/2
u
up
≈1.275 GeV/c²
2/3
1/2
c
charm
≈173.07 GeV/c²
2/3
1/2
t
top
0
0
1
g
gluon
≈126 GeV/c²
0
0
H
Higgs boson
≈4.8 MeV/c²
-1/3
1/2
d
down
≈95 MeV/c²
-1/3
1/2
s
strange
≈4.18 GeV/c²
-1/3
1/2
b
bottom
0
0
1
γ
photon
0.511 MeV/c²
-1
1/2
e
electron
105.7 MeV/c²
-1
1/2
μ
muon
1.777 GeV/c²
-1
1/2
τ
tau
91.2 GeV/c²
0
1
Z
Z boson
<2.2 eV/c²
0
1/2
νe
electron neutrino
<0.17 MeV/c²
0
1/2
νμ
muon neutrino
<15.5 MeV/c²
0
1/2
ντ
tau neutrino
80.4 GeV/c²
±1
1
W
W boson
QUARKS
LEPTONS
GAUGE BOSONS

Theoretical Description

- In particle physics, we deal with high energies to create new particles and to explore the structure of composite particles like the hadrons.
- The latter requires projectiles whose wavelengths λ are at least as small as hadron radii, which are of order 10^{-15} m.
- Their momenta, $p = h/\lambda$, and hence their energies, must be several hundred MeV/c.
- Therefor, we need a quantum theory (describe microscopic systems), which is consistent with special theory of relativity.

Relativistic Quantum Theory

- In quantum mechanics the free particle is described by a plane wave

$$\Psi(\mathbf{r},t) = N\, e^{i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar}$$

- Which is a solution for the (non-relativistic) Schrödinger equation

$$i\hbar \frac{\partial \Psi(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r},t)$$

- This is corresponding to the non-relativistic energy momentum relation

$$E = p^2/2m$$

- Schrödinger equation in not consistent with special relativity, since it is first order time and second order in space derivatives.

Klein – Gordon Equation

- Relativistic energy-momentum relation

$$E^2 = p^2c^2 + m^2c^4$$

- The corresponding relativistic (Klein–Gordon) wave equation

$$-\hbar^2 \frac{\partial^2 \Psi(\mathbf{r}, t)}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \Psi(\mathbf{r}, t) + m^2 c^4 \Psi(\mathbf{r}, t)$$

with plane wave solution

$$\Psi(\mathbf{r}, t) = N \exp\left[i(\mathbf{p} \cdot \mathbf{r} - E_p t)/\hbar\right]$$

positive and negative energy solutions

$$E = E_p \equiv +(p^2c^2 + m^2c^4)^{1/2} \geq mc^2$$

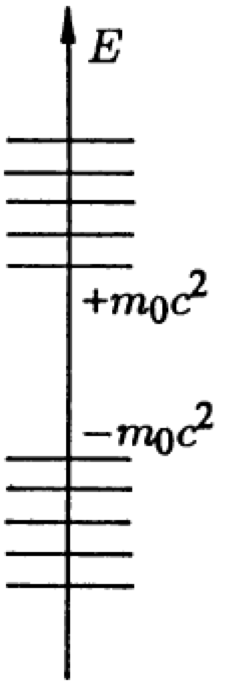
$$E = -E_p = -(p^2c^2 + m^2c^4)^{1/2} \leq -mc^2$$

Klein – Gordon Equation

- What is the interpretation of negative energy solution?

- Positive-definite probability density for position

$$\varrho = \frac{i\hbar}{2m_0c^2}\left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right)$$



Dirac Equation

- Relativistic Theory of electron

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}=H(\mathbf{r},\hat{\mathbf{p}})\Psi(\mathbf{r},t) \quad , \qquad \hat{\mathbf{p}}=-i\hbar\nabla$$

$$H=-i\hbar c\sum_{i=1}^{3}\alpha_i\frac{\partial}{\partial x_i}+\beta mc^2=c\,\boldsymbol{\alpha}\cdot\hat{\mathbf{p}}+\beta mc^2$$

Such that

$$E^2=p^2c^2+m^2c^4$$

- Therefor we have

$$\alpha_i^2=1, \quad \beta^2=1$$

$$\alpha_i\beta+\beta\alpha_i=0$$

$$\alpha_i\alpha_j+\alpha_j\alpha_i=0 \quad (i\neq j)$$

Dirac Equation

- Therefor α_i and β are hermitian 4×4 matrices and form Dirac matrices.
- Dirac equation $$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi = -i\hbar c\sum_i \alpha_i \frac{\partial\Psi}{\partial x_i} + \beta mc^2\Psi$$

$$\Psi(\mathbf{r},t) = \begin{pmatrix} \Psi_1(\mathbf{r},t) \\ \Psi_2(\mathbf{r},t) \\ \Psi_3(\mathbf{r},t) \\ \Psi_4(\mathbf{r},t) \end{pmatrix}$$

- Plane wave solution

$$\Psi(\mathbf{r},t) = u(\mathbf{p})\exp[i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar]$$

$$E = E_p \equiv +(p^2c^2 + m^2c^4)^{1/2} \geq mc^2$$

$$E = -E_p = -(p^2c^2 + m^2c^4)^{1/2} \leq -mc^2$$

Dirac Equation

- $u(\mathbf{p})$ is four component spinor satisfying

$$H_{\mathrm{p}}u(\mathbf{p}) \equiv (c\boldsymbol{\alpha}\cdot\mathbf{p} + \beta mc^2)u(\mathbf{p}) = Eu(\mathbf{p})$$

Which have four solutions describing:

1- Two with positive energy corresponding to the two possible spin states of a spin- ½ particle.

2- The other two with negative energy corresponding to the two possible spin states of a spin- ½ particle.

Dirac Equation and Hole Theory

- If states with negative energy are unoccupied, transitions from positive to negative energy states

  could occur.

- Leading to the prediction that

  atoms such as hydrogen would be

  unstable.

- Dirac postulated that the

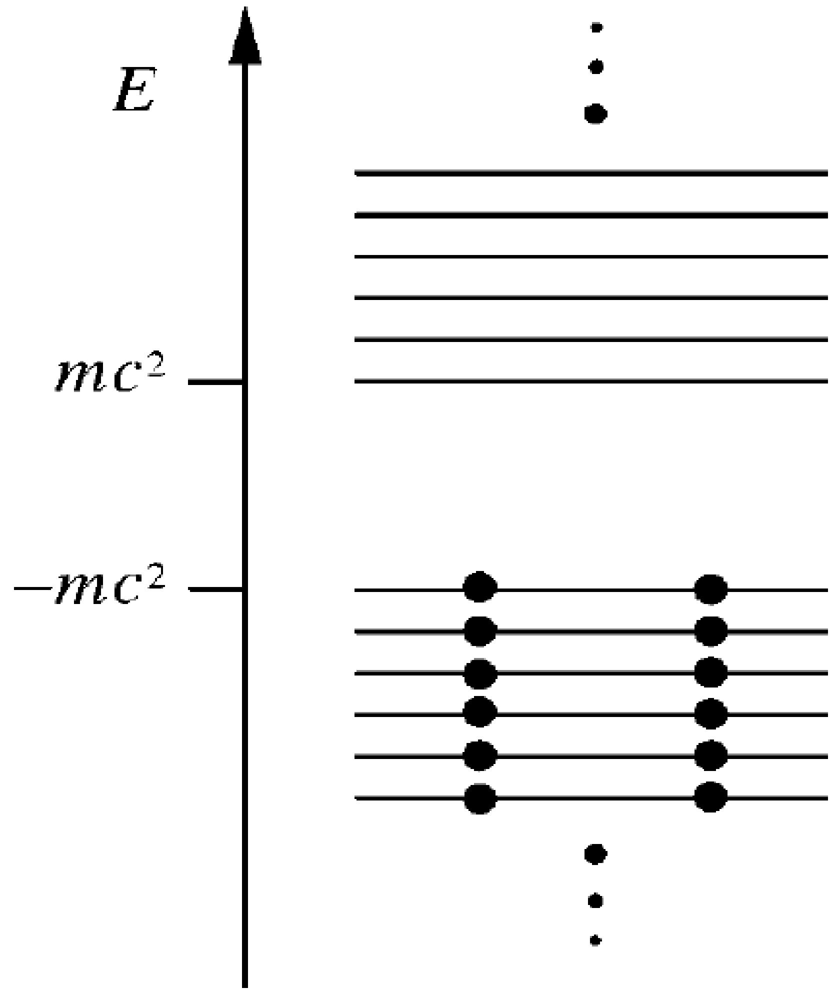
  negative energy states are almost

  always filled (Dirac Sea of negative

  energy states).

- Positive energy states are all

  Unoccupied.



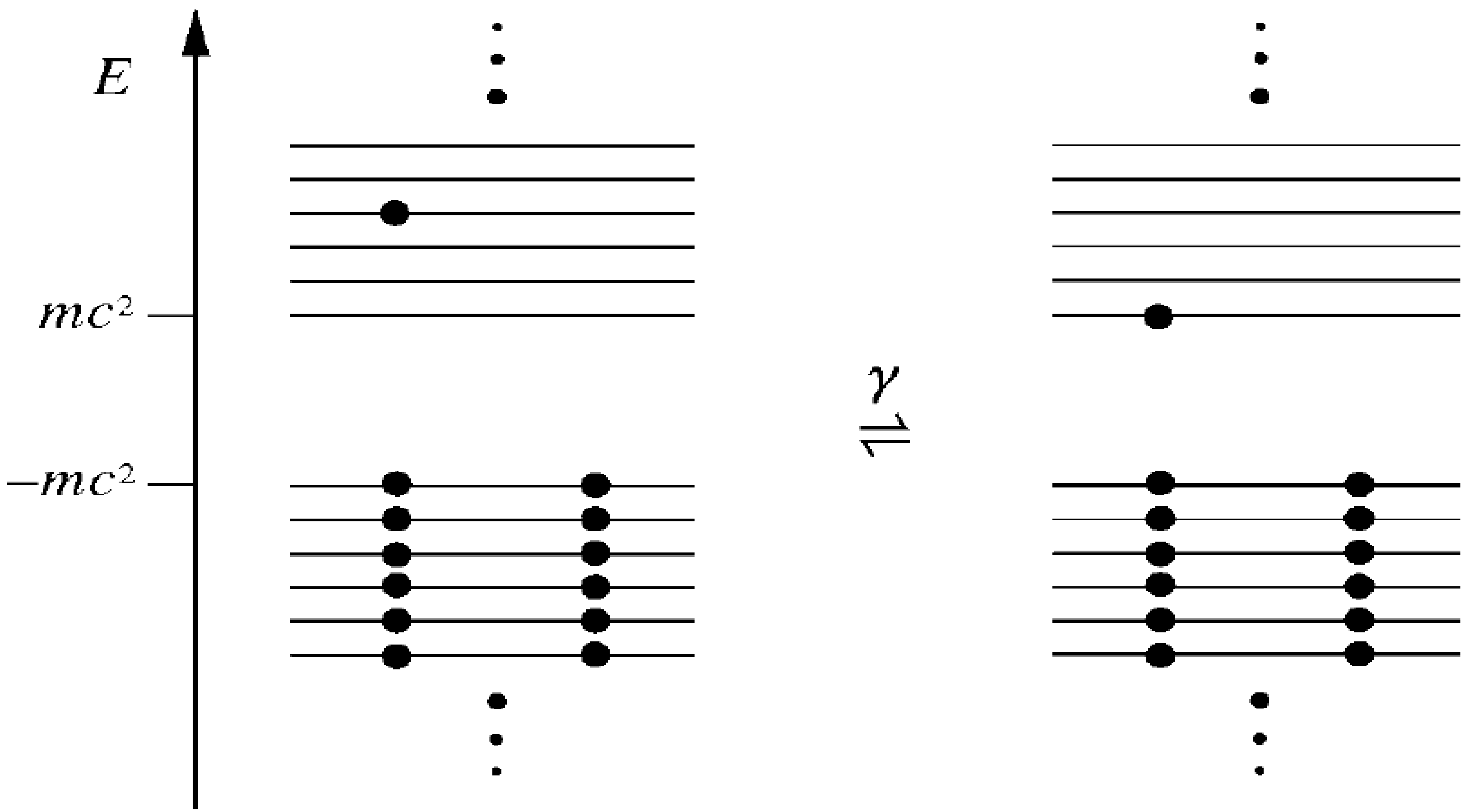
Dirac Equation and Hole Theory

- This state is indistinguishable from the usual vacuum with E_v =0, Since $p_V = \sum p = 0$; $Q_V = 0$ and the same argument apply for the spins.

- Dirac predicted the existence of a spin- ½ particle e^+ with the same mass as the electron, but opposite charge, the antiparticle of the electron.

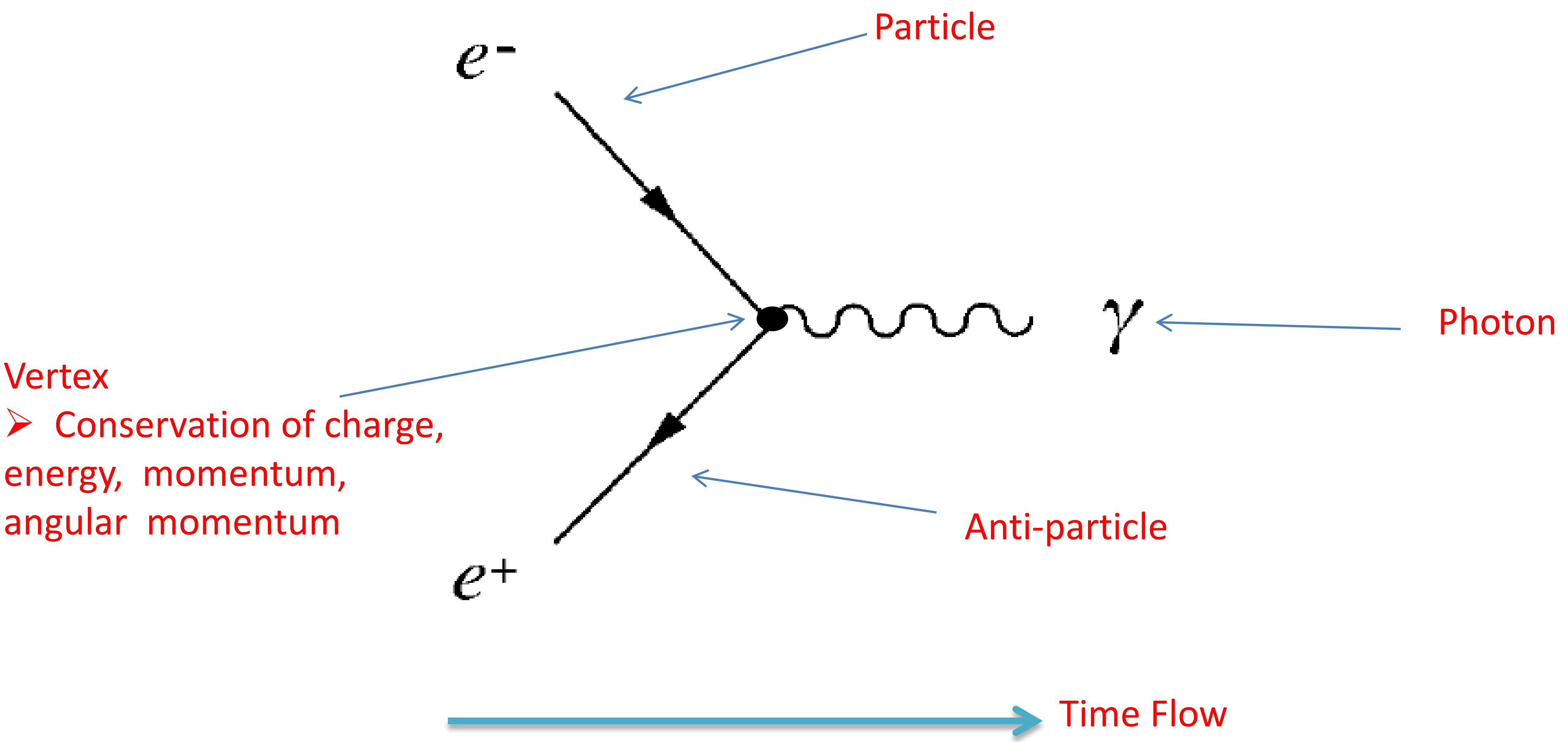
Interactions and Feynman Diagrams

- In hole theory, we can understand electromagnetic interactions of electrons and positrons by considering the emission or absorption of a single photon if transition occurs from state to another.



Interactions and Feynman Diagrams

- We use pictorial representation called Feynman diagrams

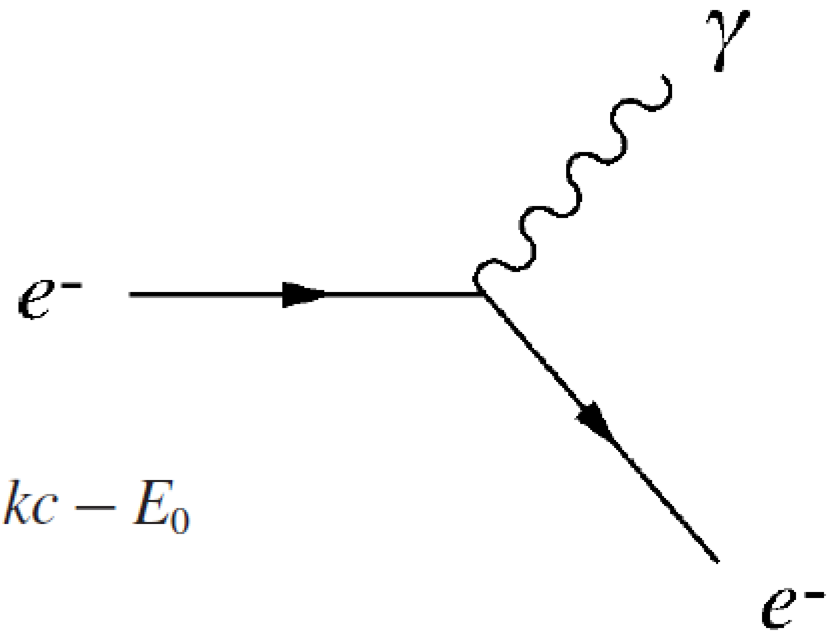


➢ Each vertex represents a basic process whose probability is of order $\alpha = \frac{1}{137} \ll 1$

Interactions and Feynman Diagrams-Real Processes

$$e^- \rightarrow e^- + \gamma$$

$$e^-(E_0, \mathbf{0}) \rightarrow e^-(E_k, -\mathbf{k}) + \gamma(ck, \mathbf{k})$$



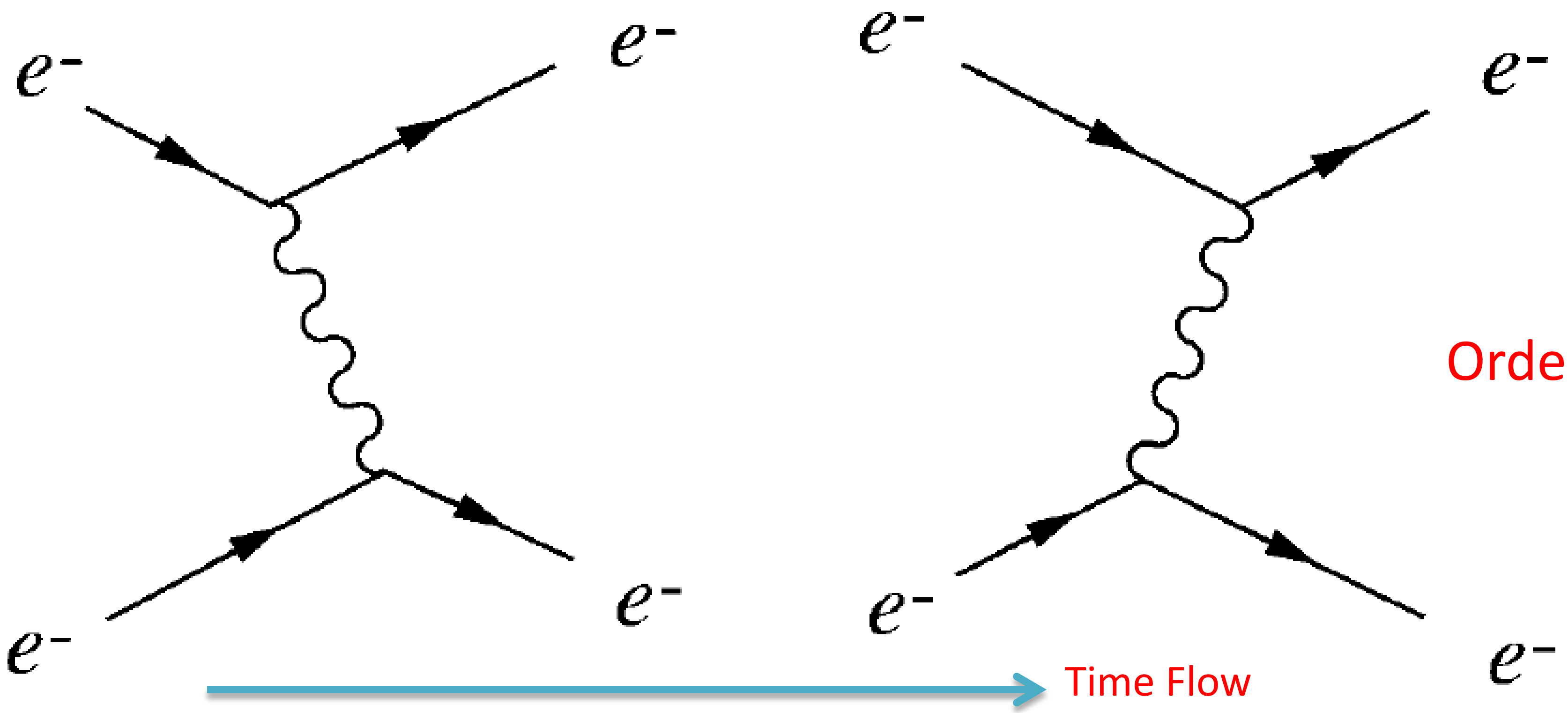
$$E_0 = mc^2,\ E_k = (k^2c^2 + m^2c^4)^{1/2} \text{ and } \Delta E \equiv E_k + kc - E_0$$

$$kc < \Delta E < 2kc$$ Energy is not conserved

- These basic processes are called virtual processes
- Real processes are built by combining two or more virtual processes such that energy conservation is only violated for a short period of time compatible with the energy–time uncertainty principle $\tau \Delta E \sim \hbar$

Interactions and Feynman Diagrams-Real Processes

- Physical elastic scattering process like $e^- + e^- \to e^- + e^-$

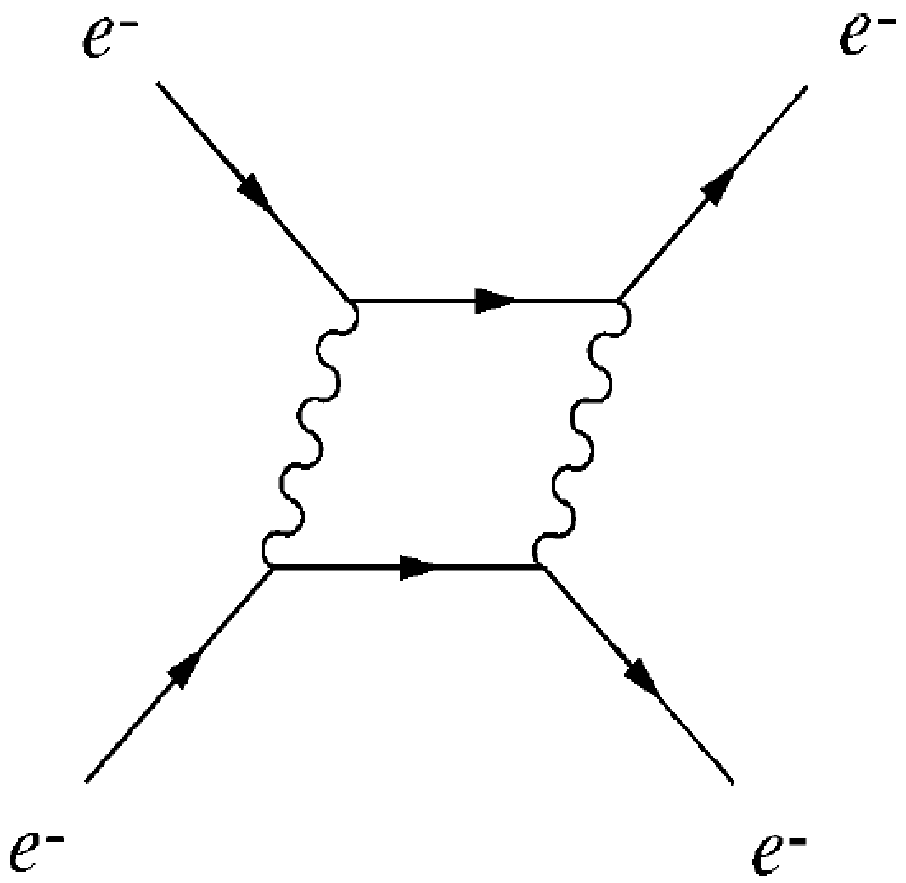


Order α^2

- The number of vertices in each diagram is called its order.
- Any diagram of order n gives a contribution of order α^n .

Interactions and Feynman Diagrams-Real Processes

- Higher order contribution to the process $e^- + e^- \to e^- + e^-$



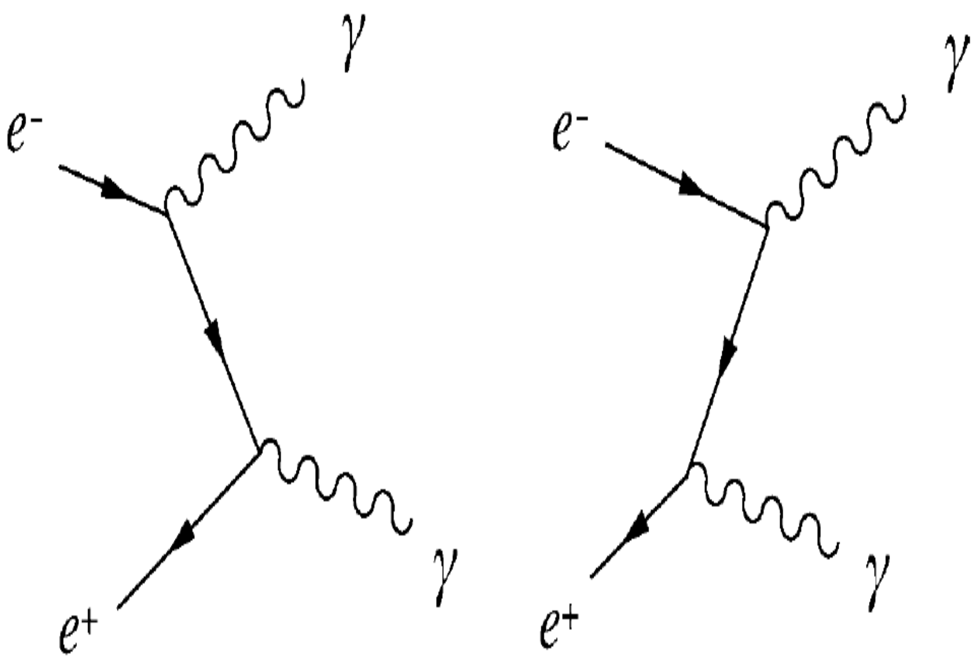
Order α^4

Time Flow

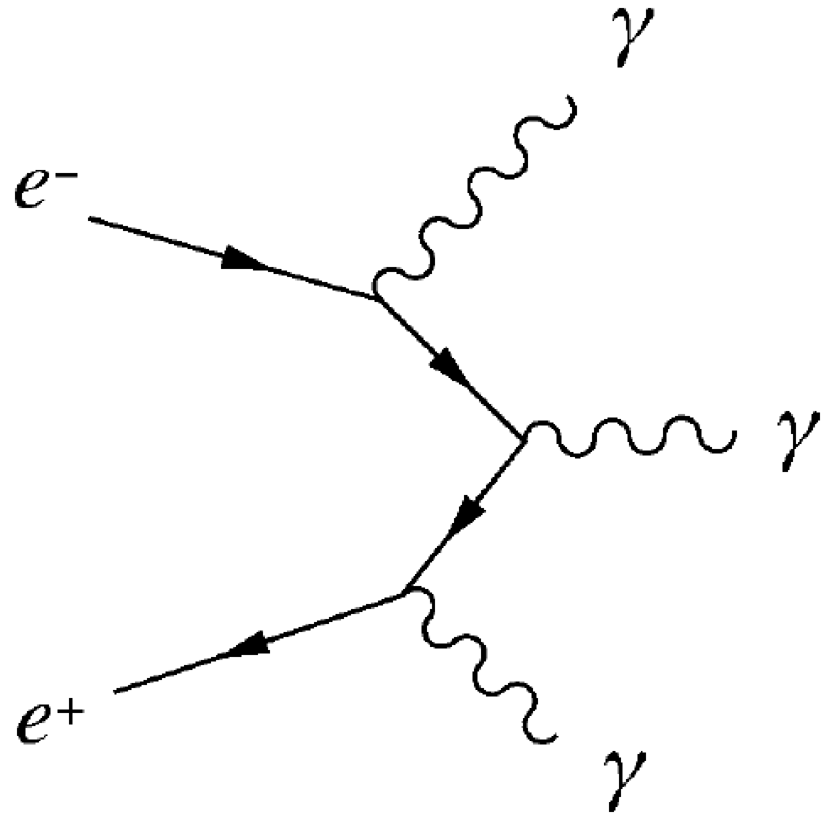
Interactions and Feynman Diagrams-Real Processes

- Electron–positron pair production and annihilation

$$e^+ + e^- \to \gamma + \gamma$$



$$e^+ + e^- \to \gamma + \gamma + \gamma$$



- In general the process $e^+ + e^- \to p\gamma$ is of order p, probability $P \sim \alpha^p$

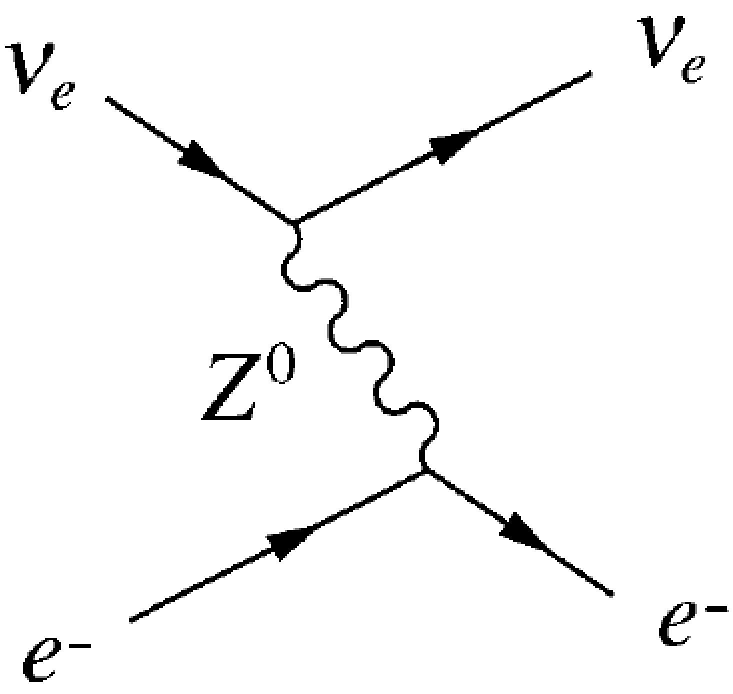
Interactions and Feynman Diagrams-Real Processes

- Define

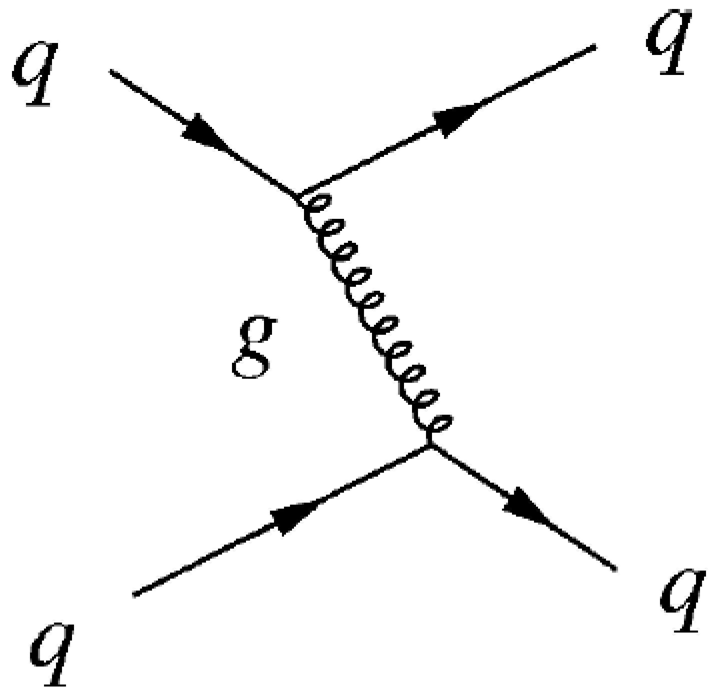
$$R \equiv \frac{\text{Rate }(e^+e^- \to 3\gamma)}{\text{Rate }(e^+e^- \to 2\gamma)} = O(\alpha)$$

- For the other interactions (weak and strong), we can have similar diagrams

$e^- + \nu_e \to e^- + \nu_e$



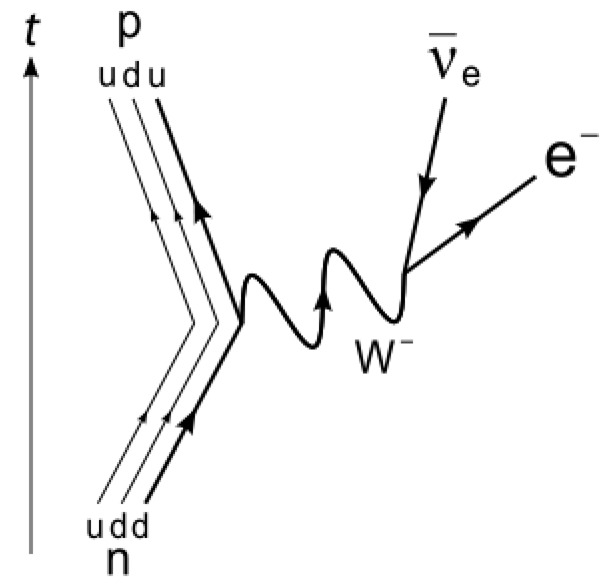
$q + q \to q + q$



Interactions and Feynman Diagrams-Real Processes

- Beta- decay

$$n \rightarrow p + e^- + \overline{\nu}_e$$



Range of forces

$$\Delta E = E_X + E_A - M_A c^2 \to 2pc, \qquad p \to \infty$$
$$\to M_X c^2, \qquad p \to 0$$

$$\Delta E \geq M_X c^2 \text{ for all } p$$

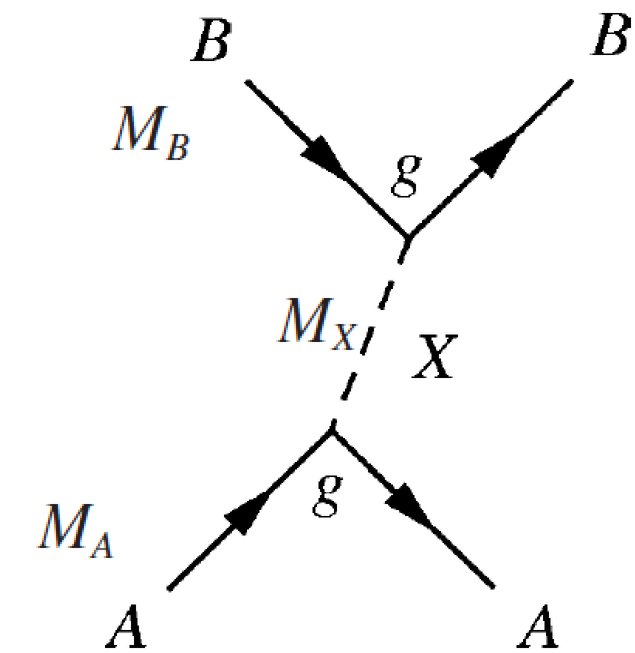
$$\tau \approx \hbar / \Delta E$$

$$r \approx R \equiv \hbar / M_X c$$

Maximum distance for propagation of *X* before being absorbed by particle *B*.

R is called the *range* of the interaction.

$$A + B \to A + B$$



$$A(M_A c^2, \mathbf{0}) \to A(E_A, \mathbf{p}) + X(E_X, -\mathbf{p})$$

$$E_A = (p^2 c^2 + M_A^2 c^4)^{1/2}$$

$$E_X = (p^2 c^2 + M_X^2 c^4)^{1/2}$$

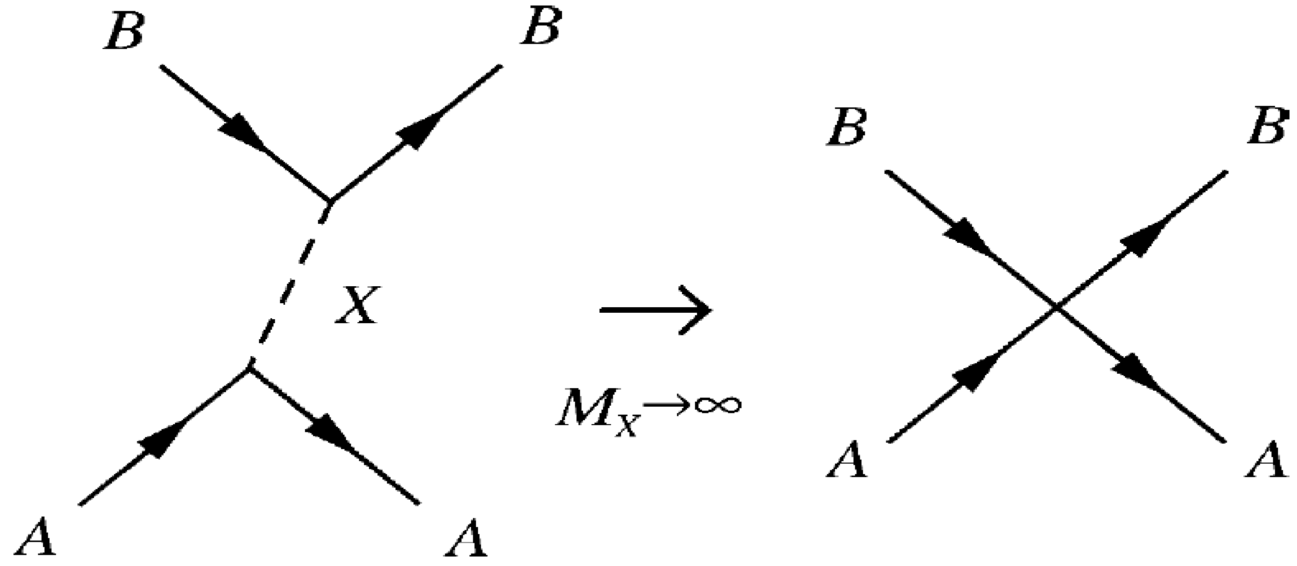
Range of forces

- Electromagnetic interactions: $M_X = 0 \rightarrow R$ is infinity (Long range).
- Weak interactions: $M_X \neq 0 \rightarrow R$ is finite (short range).

$$M_W = 80.4\,\mathrm{GeV}/c^2 \quad \text{and} \quad M_Z = 91.2\,\mathrm{GeV}/c^2 \quad (1\,\mathrm{GeV} = 10^9\,\mathrm{eV})$$

$$R_{W,Z} \equiv \frac{\hbar}{M_W c} \approx 2 \times 10^{-3}\,\mathrm{fm} \quad (1\,\mathrm{fm} = 10^{-15}\,\mathrm{m})$$

- The weak interaction can be approximated by a zero-range or point interaction in the limit $M_X \rightarrow \infty$

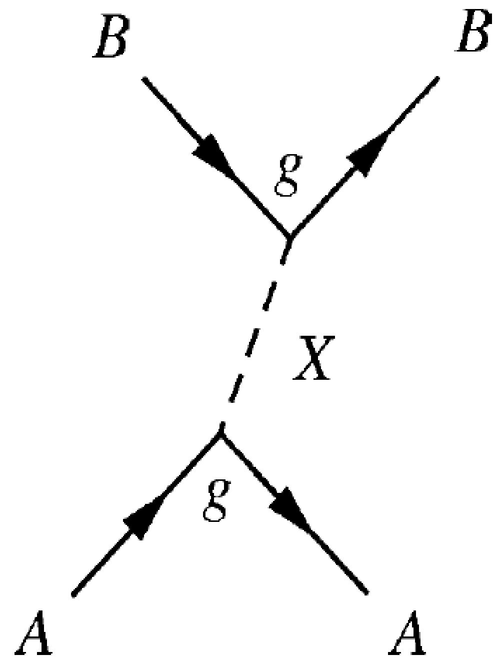


Zero-range approximation

- The probability amplitude for a particle with initial momentum $\mathbf{q}_i$ to be scattered to a final state with momentum $\mathbf{q}_f$ by potential is given by

$$\mathcal{M}(q^2) = \frac{g^2\hbar^2}{q^2 - M_X^2 c^2}$$

$$q^2 \equiv (E_f - E_i)^2 - (\mathbf{q}_f - \mathbf{q}_i)^2 c^2$$



- In the zero-range approximation, the range

$$R = \hbar / M_X c$$

is very small compared with the de Broglie wavelengths of all the particles, equivalent to

$$q^2 \ll M_X^2 c^2 \quad \Rightarrow \quad \mathcal{M}(q^2) = -G$$

Inverse energy sequared

Dimensionful coupling

$$\frac{G}{(\hbar c)^3} = \frac{1}{\hbar c}\left(\frac{g}{M_X c^2}\right)^2 = \frac{4\pi\alpha_X}{(M_X c^2)^2} \quad \Rightarrow \quad \boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} = \frac{4\pi\alpha_W}{M_W^2}}$$

References

- This lecture was essentially prepared from the text book "Particle Physics", third edition by B.R. Martin and G. Shaw.

Another references for further reading

- "Quarks and Leptons: An Introductory Course in Modern Particle Physics", by Francis Halzen and Alan D. Martin.
- "Introduction to Elementary Particles", by David Griffiths.
- "Introduction to High Energy Physics", by Donald H. Perkins.