Progress in Lattice QCD

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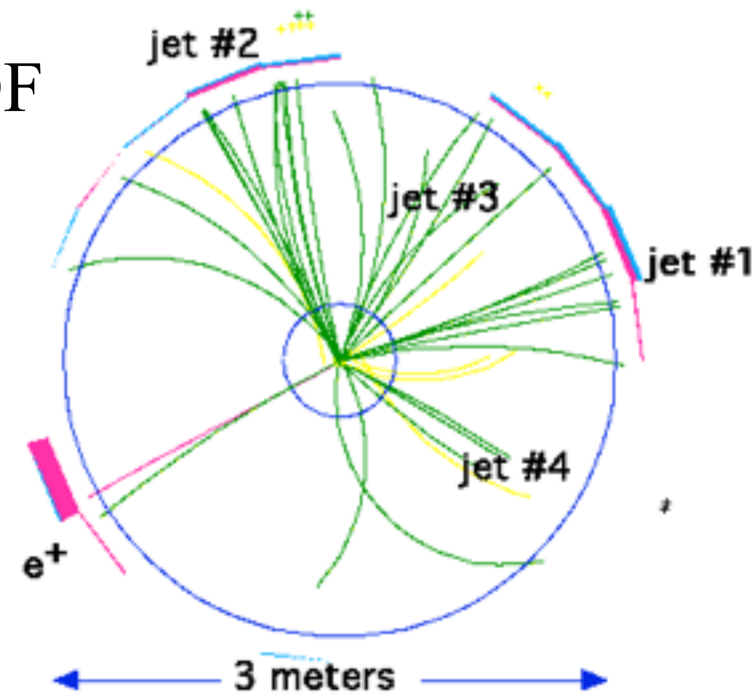
HPQCD collaboration

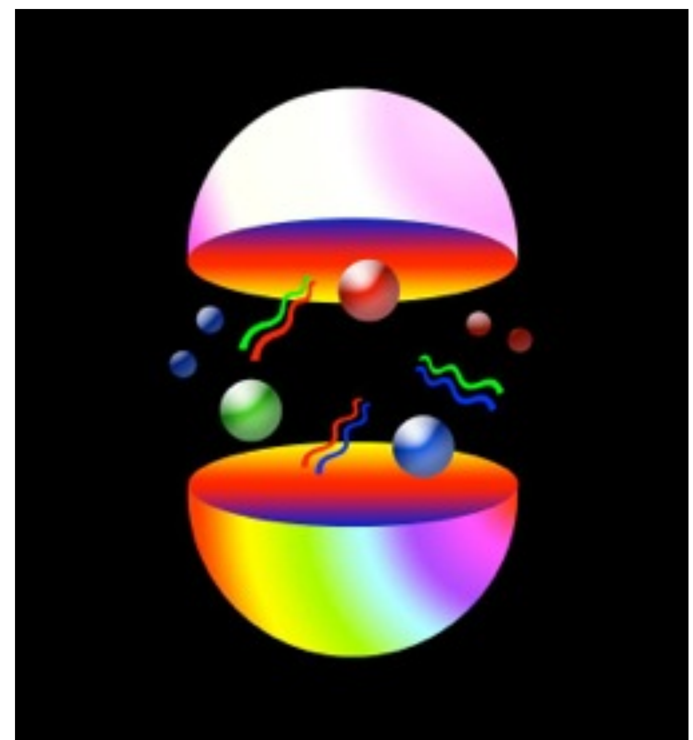
Physics in Collision
Warwick, Sept 2015

QCD is a key part of the Standard Model but quark confinement is a complication/interesting feature.

CDF



Cross-sections calculated at high energy using QCD pert. th. with ~3% errors. Also parton distribution function and hadronisation uncertainties.

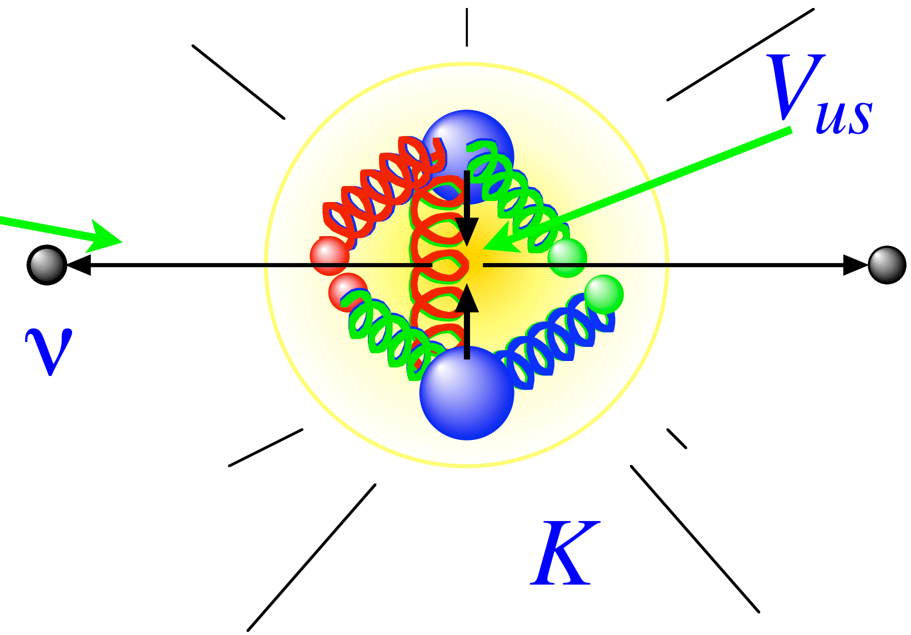


But (some) properties of hadrons much more accurately known and calculable in lattice QCD - can test SM and determine parameters very accurately (1%).

Weak decays probe meson structure and quark couplings

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to l\nu & K \to l\nu & B \to \pi l\nu \\ & K \to \pi l\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to l\nu & D_s \to l\nu & B \to D l\nu \\ D \to \pi l\nu & D \to K l\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$

CKM matrix



$$Br(M \to \mu\nu) \propto V_{ab}^2 f_M^2$$

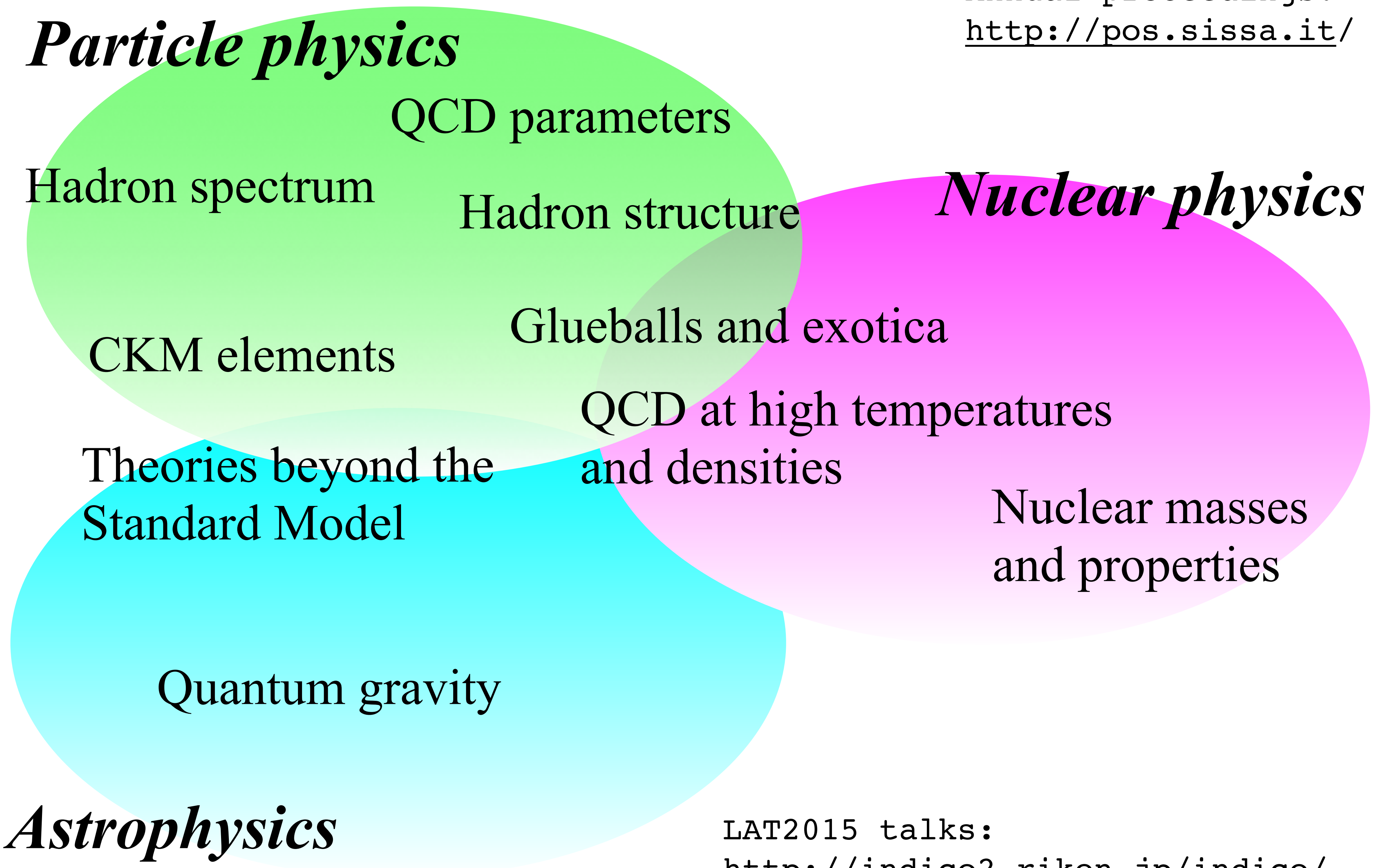
Expt = CKM x theory(QCD)

Need precision lattice QCD to get accurate CKM elements to test Standard Model (e.g. is CKM unitary?).

If V_{ab} known, compare lattice to expt to test QCD

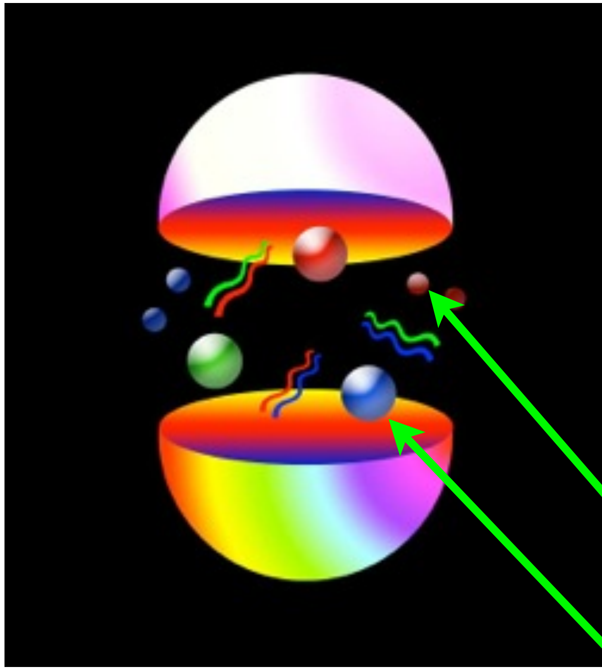
Applications of Lattice QCD/Lattice field theory

Annual proceedings:
http://pos.sissa.it/



LAT2015 talks:
http://indico2.riken.jp/indico/conferenceDisplay.py?confId=1805

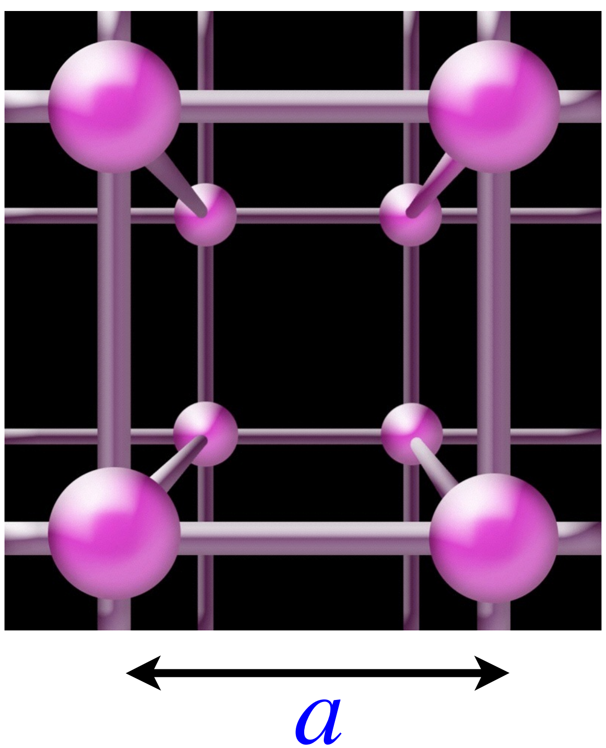
Lattice QCD = fully nonperturbative, based on Path Integral formalism



basic integral $$\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\overline{\psi}\exp(-\int \mathcal{L}_{QCD}d^4x)$$

- Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d, s (+ c) sea quarks)
- Calculate averaged "hadron correlators" from valence q props.
- Fit as a function of time to obtain masses and simple matrix elements
- Determine a and fix m_q to get results in physical units.
- extrapolate to $a=0, m_{u,d}=phys$ for real world

now able to calculate directly



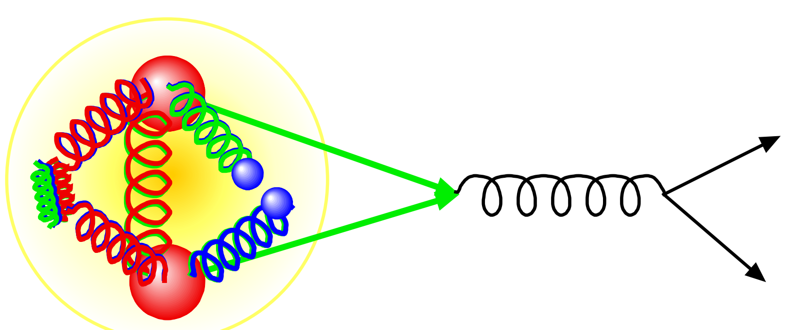
Hadron correlation functions ('2point functions') give masses and decay constants.

$$\langle 0|H^\dagger(T)H(0)|0\rangle = \sum_n A_n e^{-m_n T} \xrightarrow[T]{\text{large}} A_0 e^{-m_0 T}$$

masses of all hadrons with quantum numbers of H

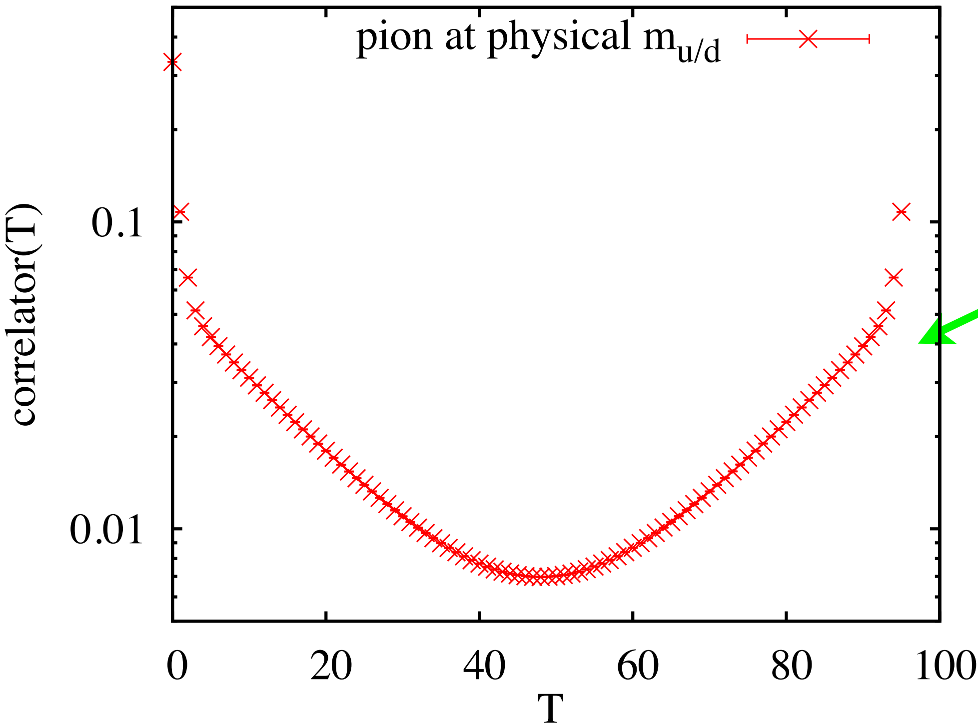
$$A_n = \frac{|\langle 0|H|n\rangle|^2}{2m_n} = \frac{f_n^2 m_n}{2}$$

decay constant parameterises amplitude to annihilate - a property of the meson calculable in QCD. Relate to experimental decay rate.



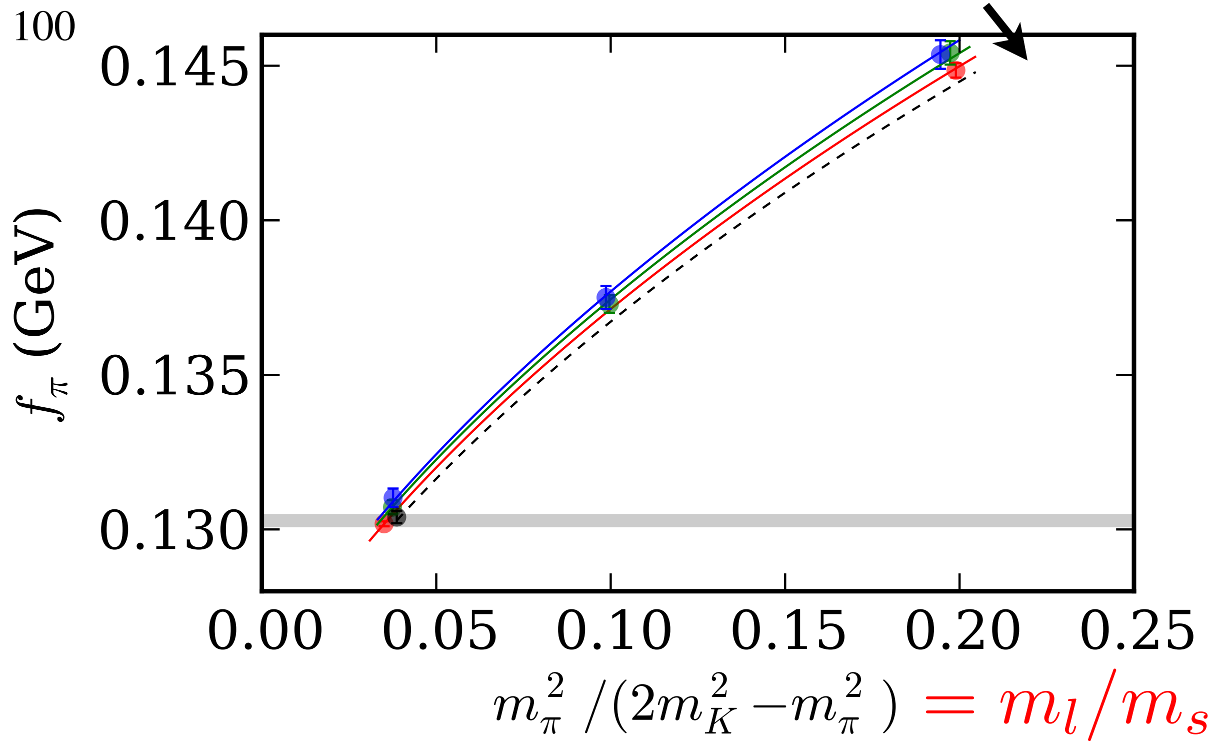
1% accurate experimental info. for f and m for many mesons! Need accurate determination from lattice QCD to match

Example (state-of-the-art) calculation



Extract meson mass and amplitude=decay constant from correlator for multiple lattice spacings and $m_{u/d}$.
Very high statistics

Extrapolation in a

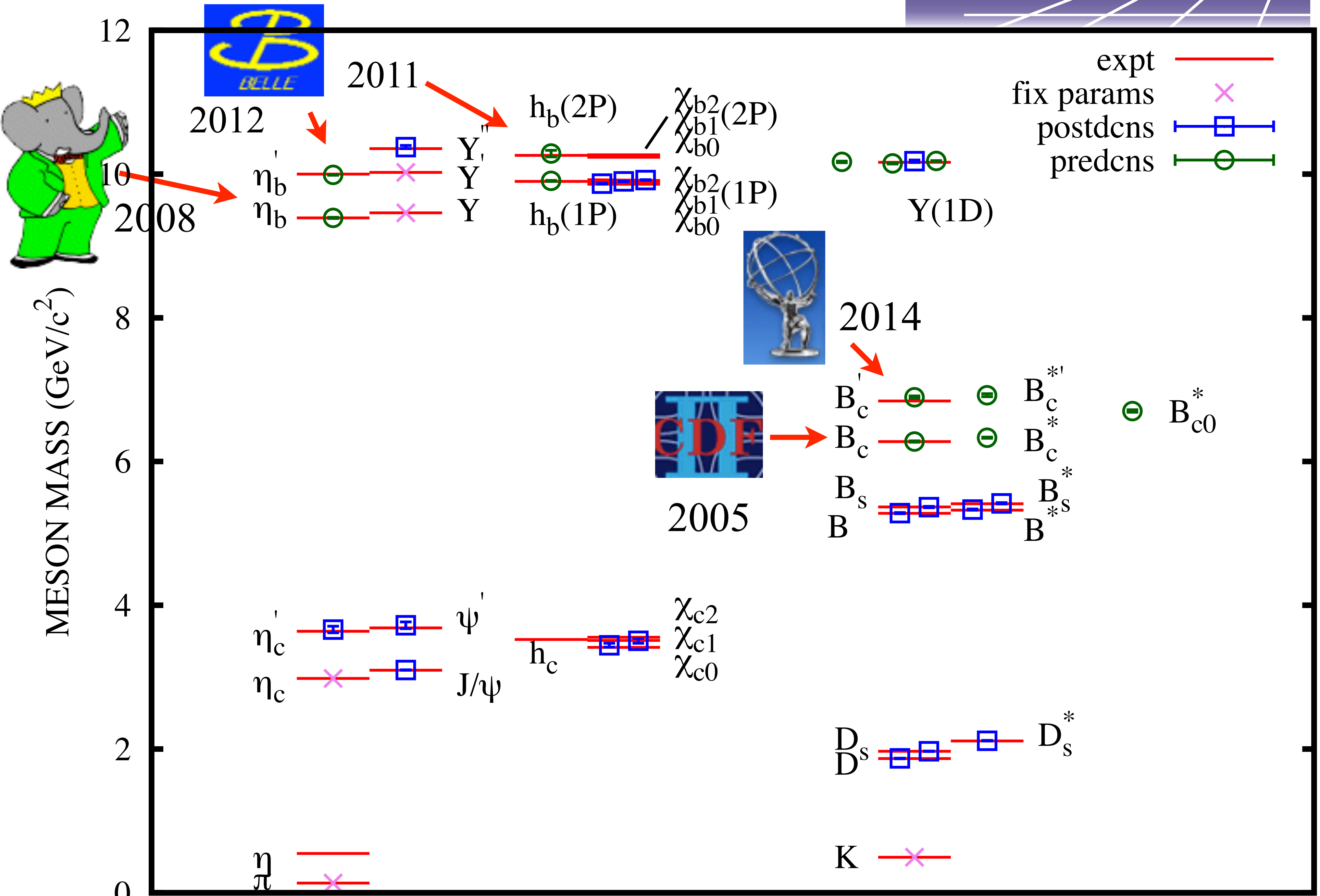


Convert decay constant to GeV units using w_0 to fix relative lattice spacing. Very small discretisation errors.

R. Dowdall et al, HPQCD, 1303.1670.

The gold-plated meson mass spectrum

HPQCD

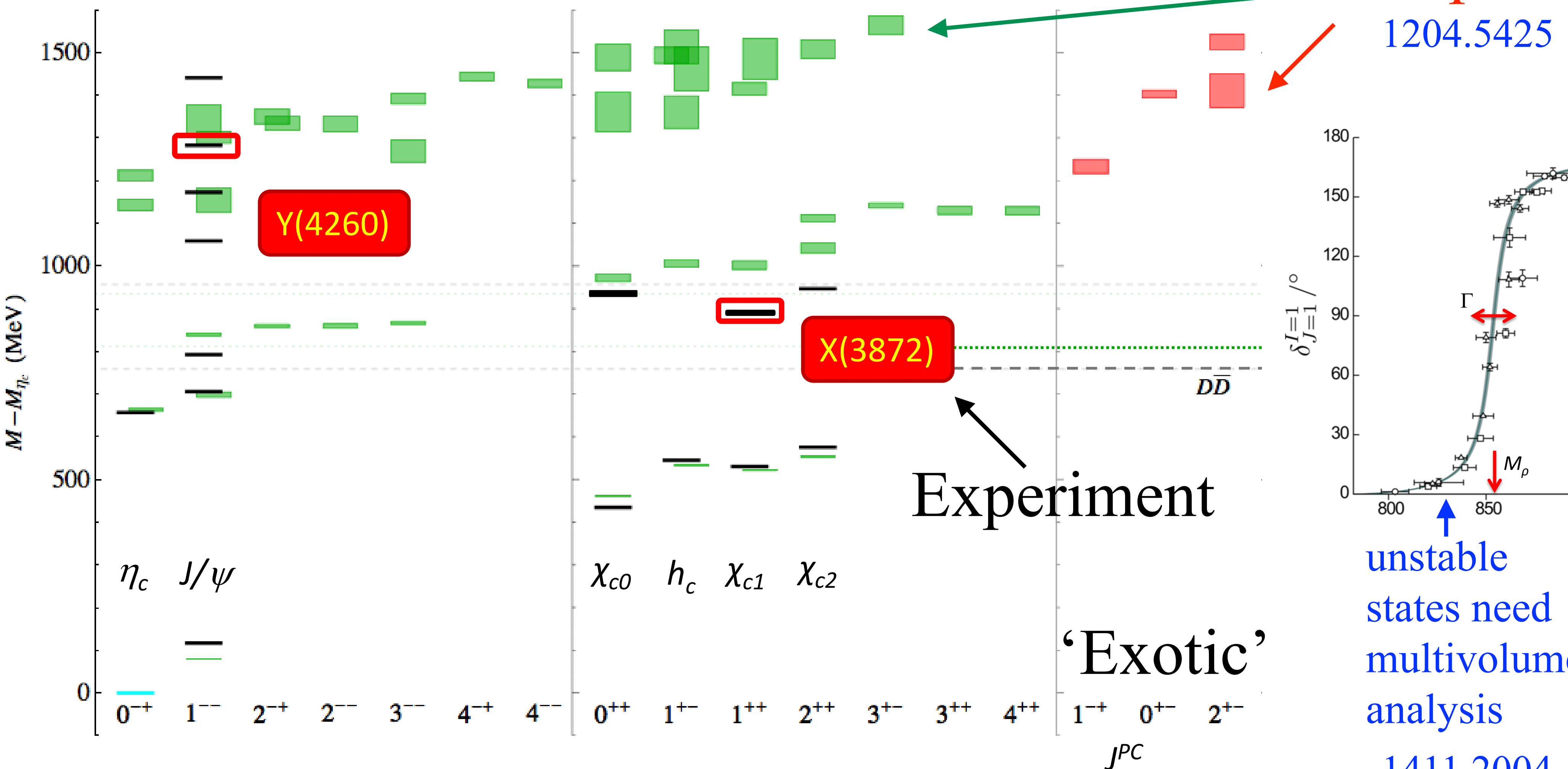


few MeV uncertainties in many cases

More detailed study of unstable and excited states important to pin down oddities now being seen (e.g. in charmonium spectrum)

Hadspec

1204.5425

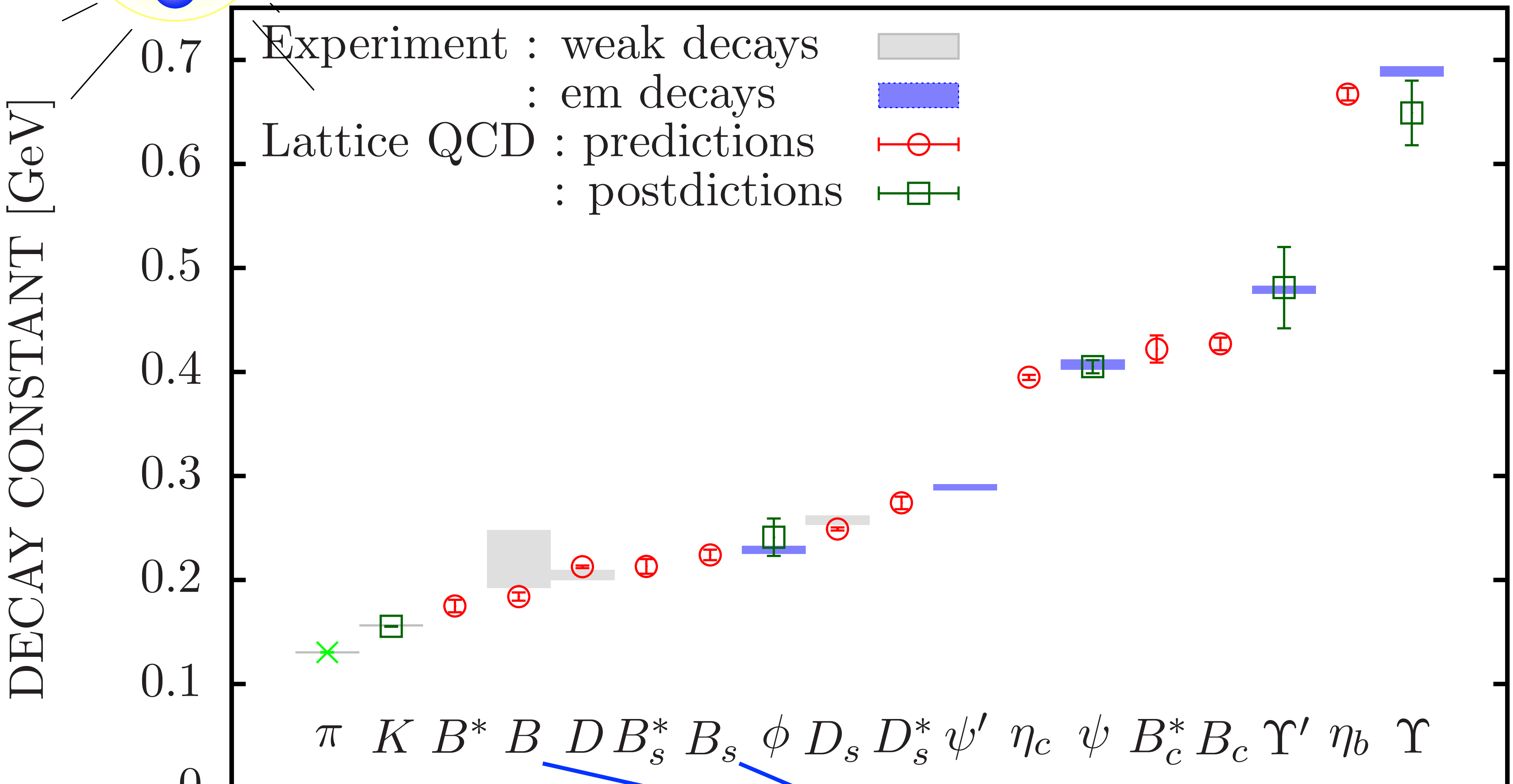


unstable states need multivolume analysis

1411.2004

Key future aim: establish whether tetra/pentaquark states, hybrids, glueballs exist - needs very high stats and large basis of operators.

The spectrum of decay constants for gold-plated mesons

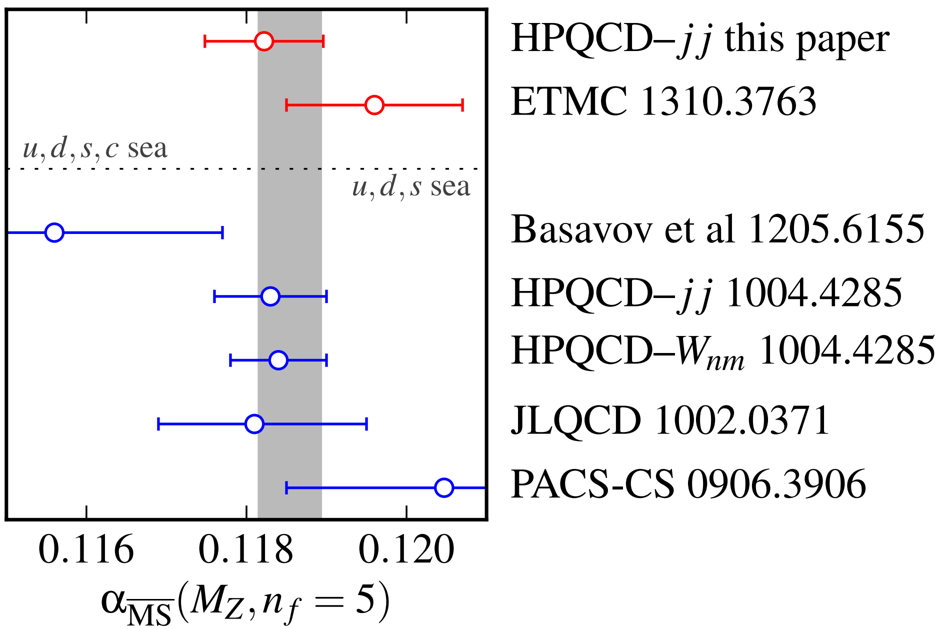


key input for SM calcn of $B_{(s)} \to \mu^+\mu^-$

HPQCD, arXiv:1302.2644, 1408.5768, 1503.05762

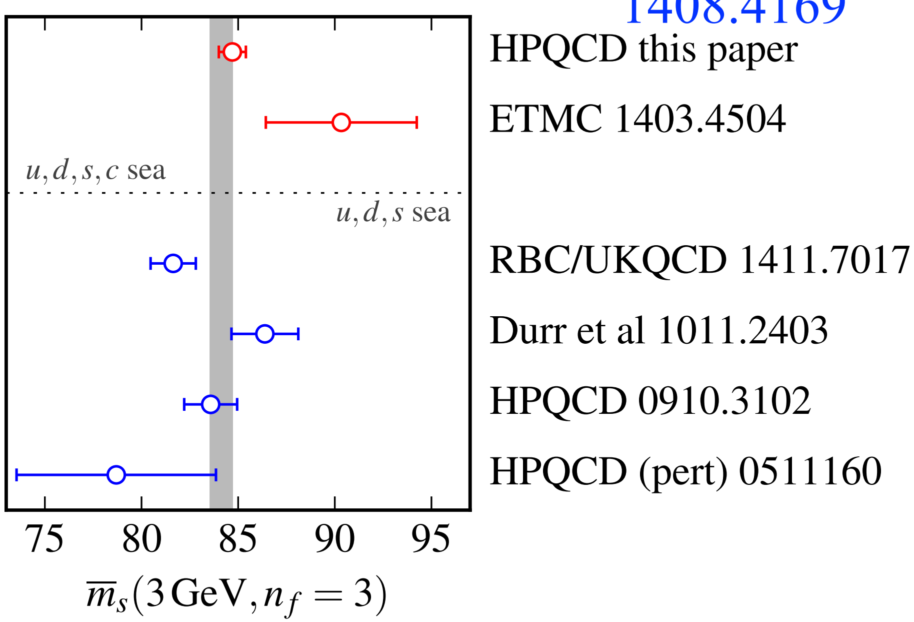
Quark masses and strong coupling constant

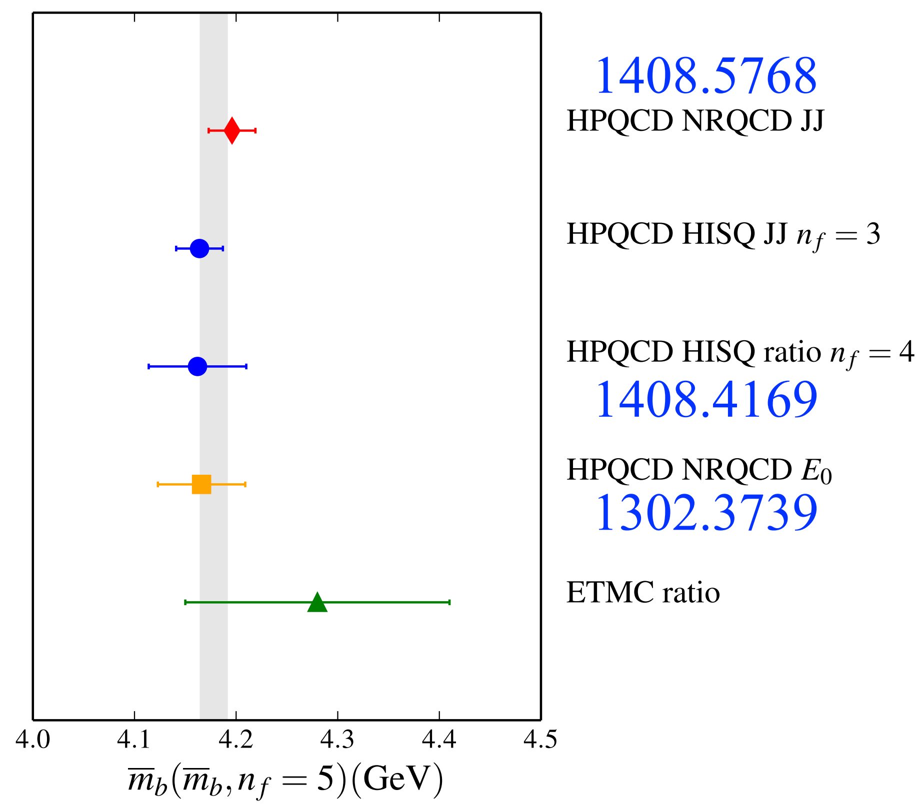
1408.4169



Lattice QCD results have transformed accuracy possible.

1408.4169





1408.5768

1408.4169

1302.3739

Future: Accurate tests of Higgs $\rightarrow b\bar{b}$ require halving uncertainty on m_b and α_s .....

Quark mass ratios

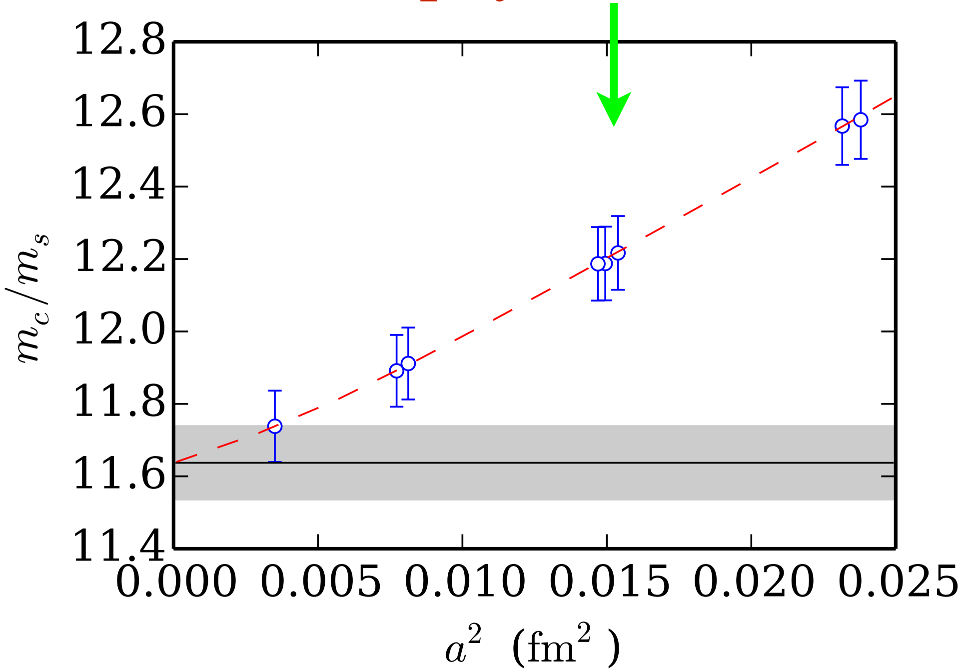
HPQCD: 1408.4169

Obtained directly from lattice QCD if same quark formalism is used for both quarks.
Ratio is at same μ and for same n_f.

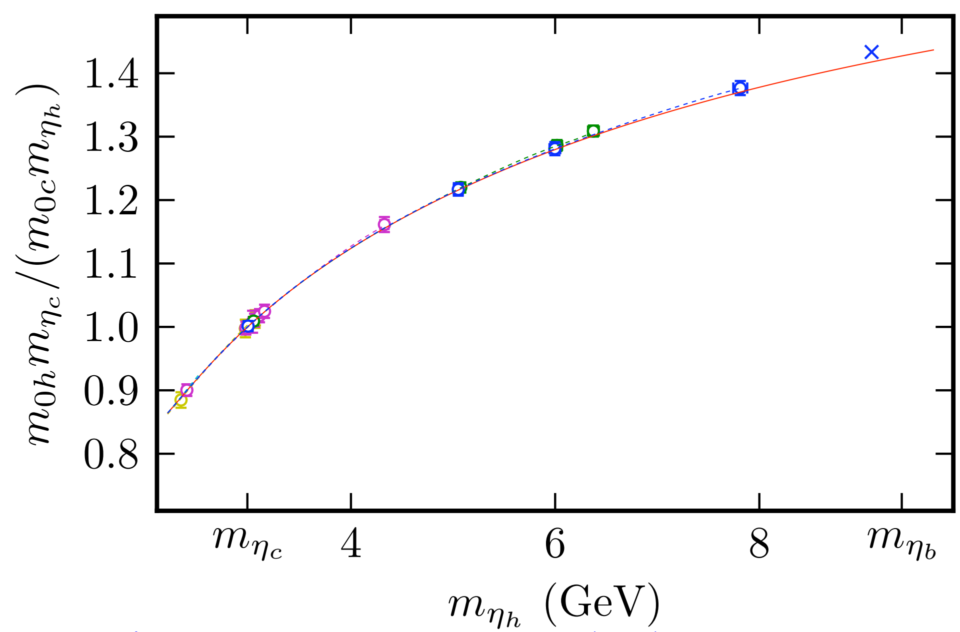
$$\left(\frac{m_{q1,latt}}{m_{q2,latt}}\right)_{a=0} = \frac{m_{q1,\overline{MS}}(\mu)}{m_{q2,\overline{MS}}(\mu)}$$

Not possible any other way ...

m_c/m_s *new* 2+1+1 with physical u.d:



m_b/m_c



$$m_b/m_c = 4.51(4)$$

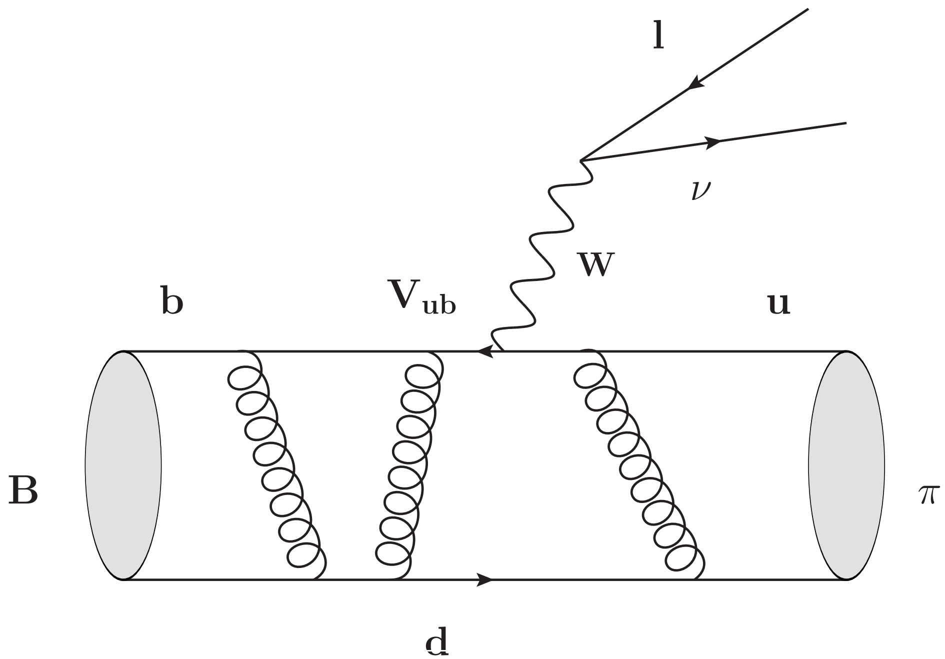
$$m_b/m_s = 52.90(44)$$

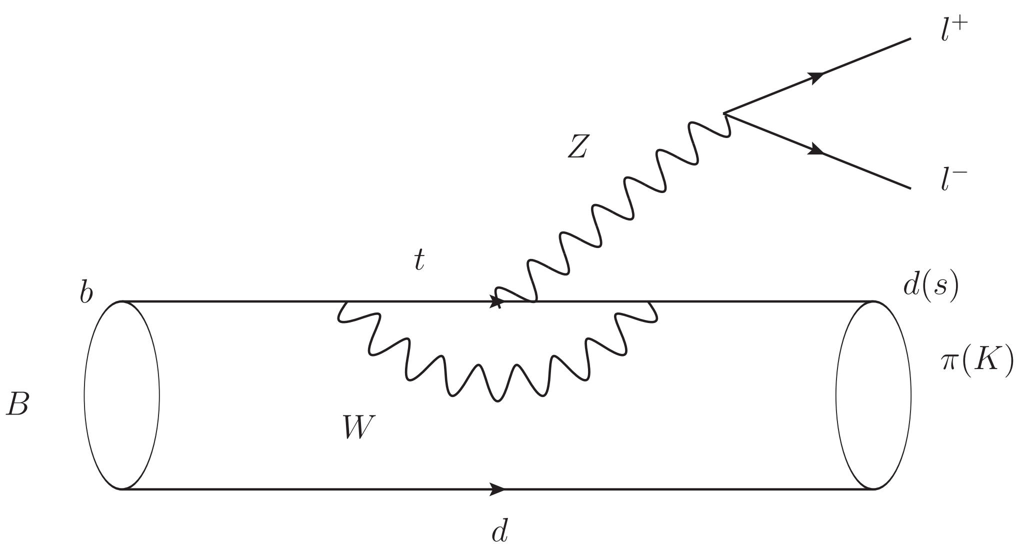
$$\neq 3m_\tau/m_\mu$$

$$m_c/m_s = 11.652(65)$$

allows 1% accuracy in m_s (94.0(6) MeV)

B semileptonic decays from Lattice QCD





Tree-level or loop processes - need hadronic form factors from lattice QCD to obtain SM rate

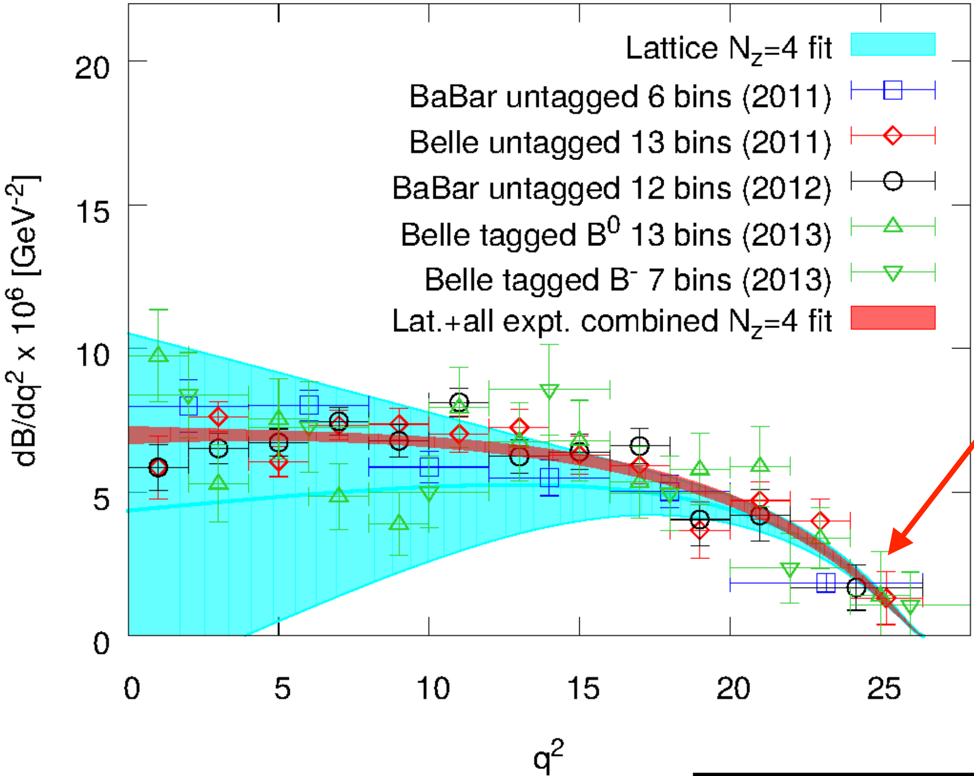
Tree-level: CKM elements, V_{ub} and V_{cb}

Loop: test for new physics

$B \to \pi l \nu$

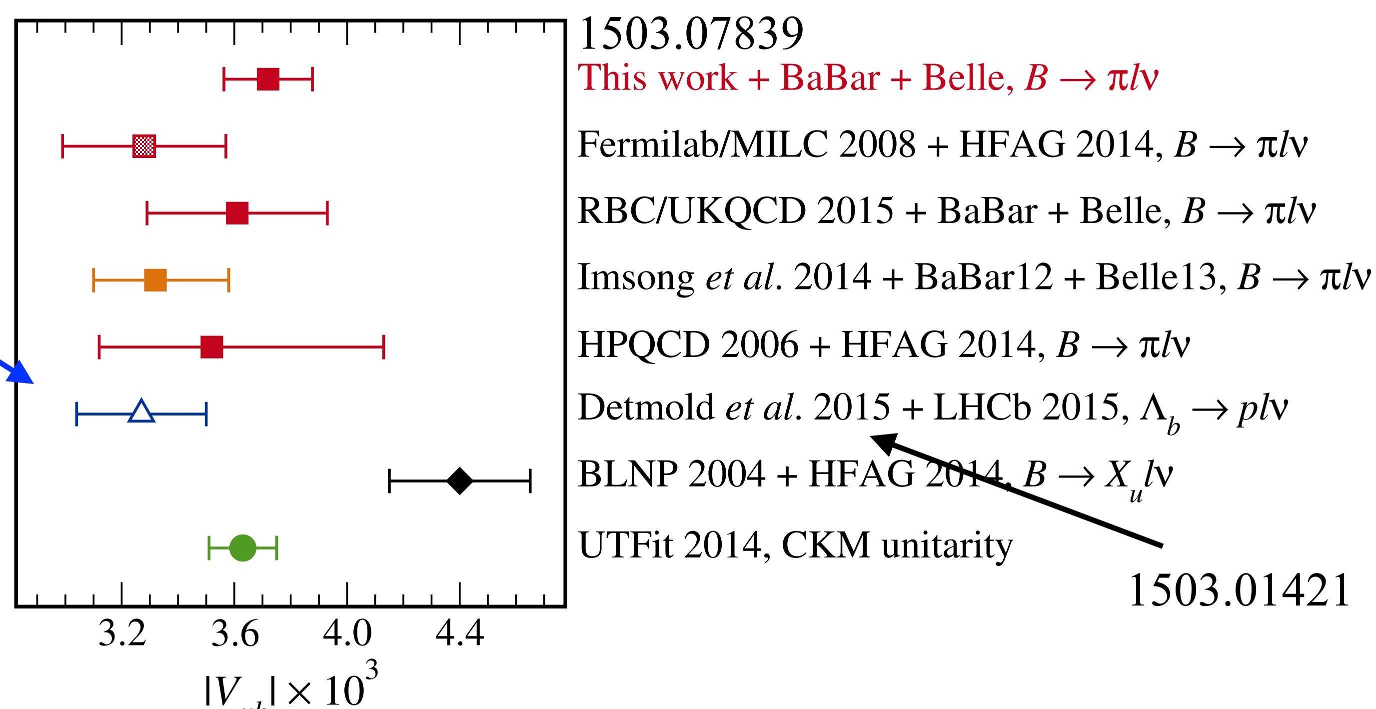
Fermilab/MILC
arXiv: 1503.07839

semileptonic decay



Combined fit to lattice and experimental results improves determination of V_{ub}

new result using baryon decay in lattice QCD

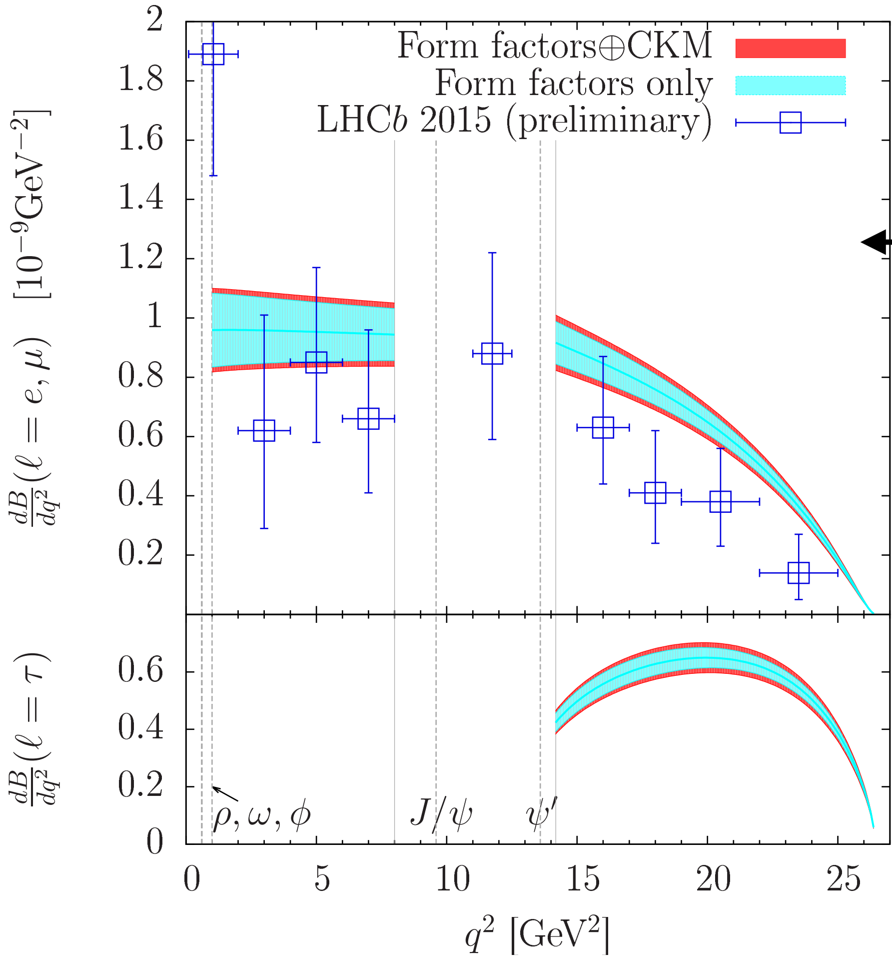


1503.01421

Conclude: tension between exclusive and inclusive methods remains

$B \to \pi(K) l^+ l^-$ semileptonic decay

Fermilab/MILC
arXiv: 1507.01618



potential sensitivity to new physics through loop effects

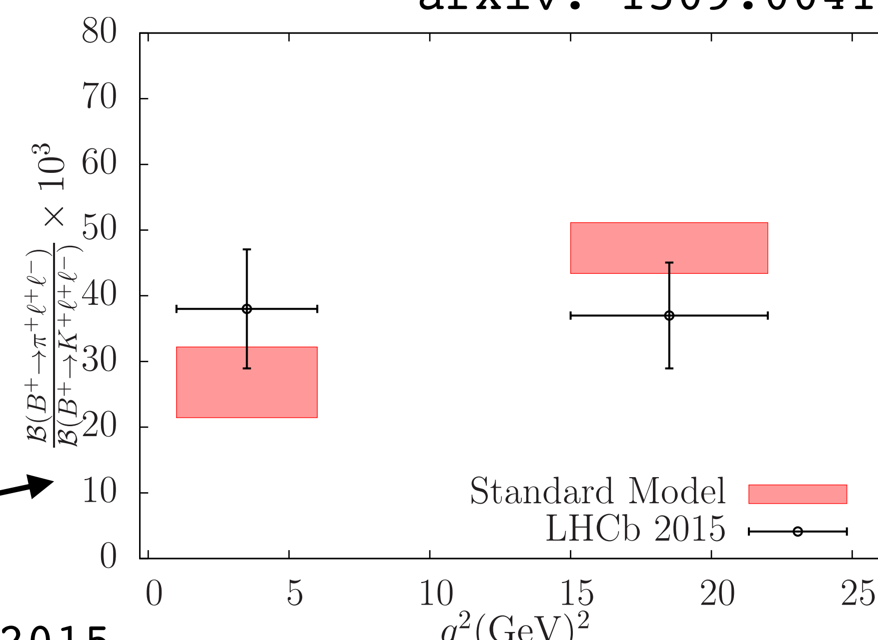
$B \to \pi \ell^+ \ell^-$

Lattice QCD gives SM rate:

$\mathrm{Br}(B^+ \to \pi^+ \mu^+ \mu^-) = 20.4(2.1) \times 10^{-9}$

LHCb : 18.3(2.5)x10^{-9}

arXiv: 1509.00414



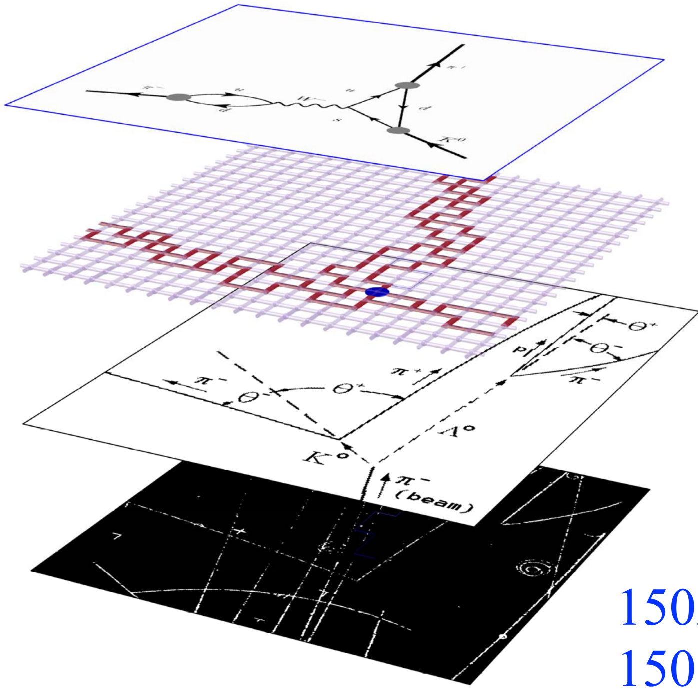
Ratio of π to K partially integrated rates

Ran Zhou: KITP2015

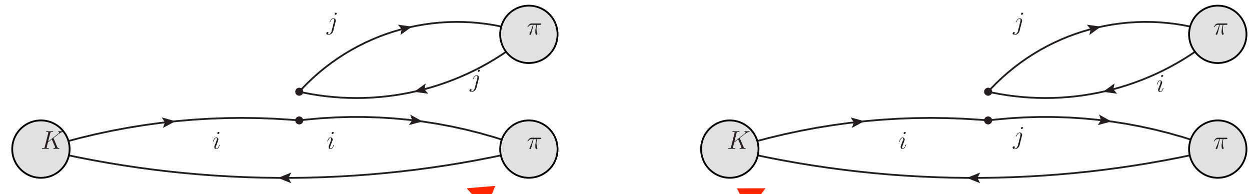
$K \to \pi\pi$

$\Delta I = 1/2$ rule

RBC/UKQCD

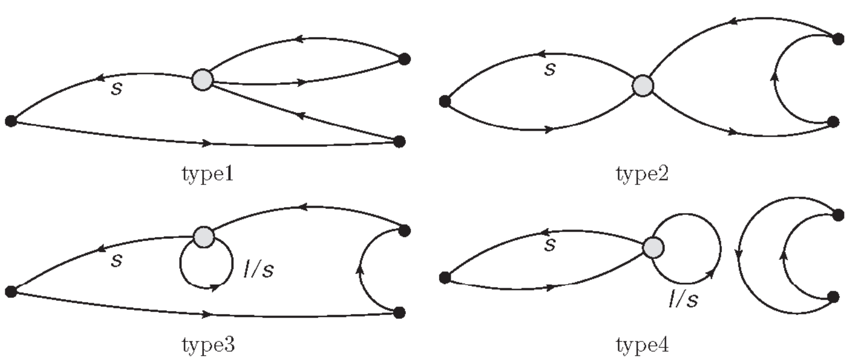


Decades-long puzzle of why $K \to \pi\pi(I=0)$ 450 x more likely than $K \to \pi\pi(I=2)$ resolved by lattice QCD.



unexpected cancellation of different colour charge contractions

1502.00263
1505.07863



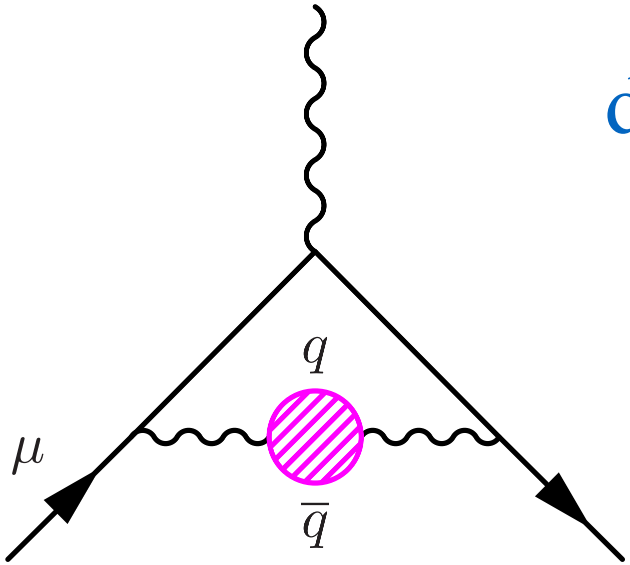
SM test of direct CP violation in $K \to \pi\pi$

First lattice QCD result for ε'/ε with physical kinematics

Technically challenging - future: significant accuracy gain

Hadronic vacuum polarisation contribution to anomalous magnetic moment of muon $(g-2)_\mu/2$

B.Chakraborty et al, HPQCD: 1403.1778



differs between expt and the SM by

$25(9) \times 10^{-10}$ *new physics*?

Uncertainty dominated by that from HVP contribution calculated from expt for $R_{e^+e^-}$

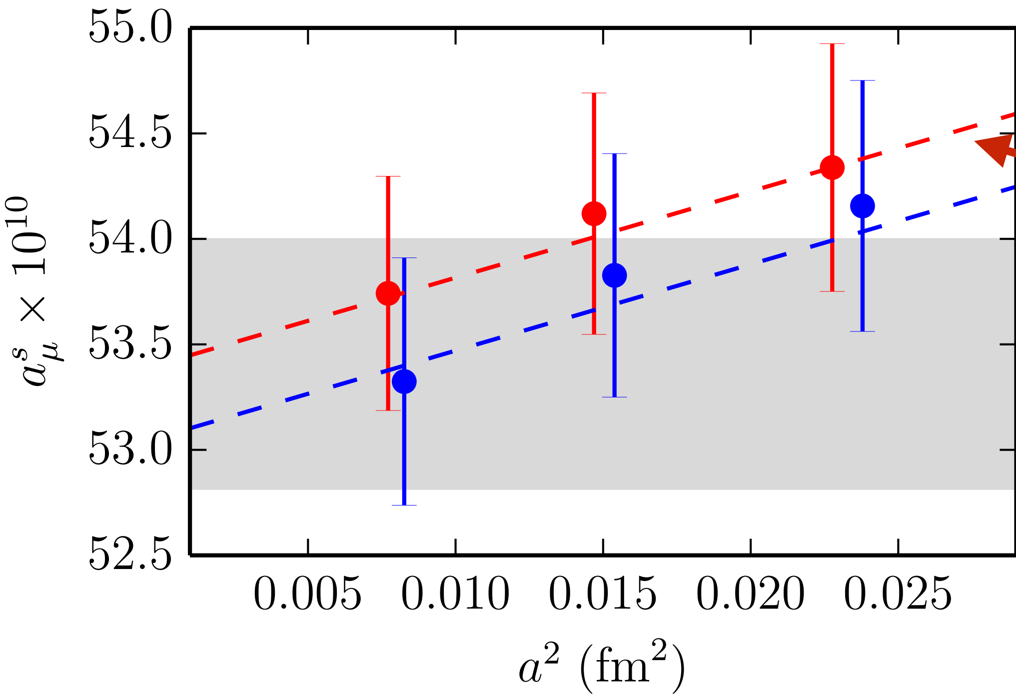
Can we improve ahead of E989 run?

On lattice, calculate :

$$a^{(\mathrm{f})}_{\mu,\mathrm{HVP}} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha Q_{\mathrm{f}}^2)\hat{\Pi}_{\mathrm{f}}(q^2)$$

very steep function, so small q^2 dominates

vacuum polarisation function



B.Chakraborty et al, HPQCD: 1403.1778

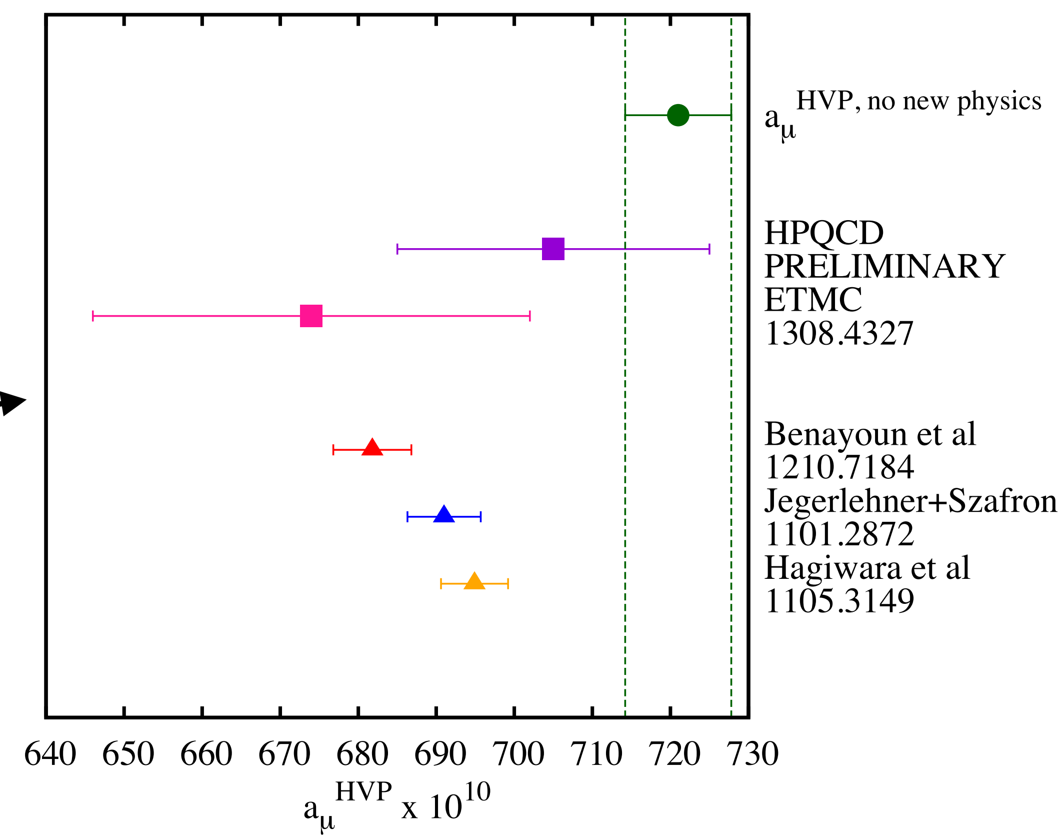
physical u/d in sea

New - accurate determination of s HVP from lattice QCD.

$$a_\mu^s = 53.41(59) \times 10^{-10}$$

1% accurate

Total HVP from lattice still not accurate enough, but significant progress …



Conclusion

- Lattice QCD results for gold-plated hadron masses and decay constants now providing stringent tests of QCD/SM.
- Gives QCD parameters and some CKM elements to 1%.
- Provides BSM constraints, tests of sum rules/HQET etc.

Future

- Lattice uncertainties being improved substantially e.g. with $m_{u,d}$ at physical value and finer lattices/higher statistics
- Analysis underway by several groups of effects from QED and $m_u \neq m_d$ e.g. BMW 1406.4088 m_n-m_p
- The range of methods and calculations is increasing "disconnected calculations", glueballs, unstable particles ..

Spares

Look at error budgets to see how things will improve in future ...

1302.2644: calculation of B, B_s masses and decay constants

errors divided into extrapolation and other systematics:

Error %	Φ_{B_s}/Φ_B	$M_{B_s} - M_B$	Φ_{B_s}	Φ_B
EM:	0.0	1.2	0.0	0.0
a dependence:	0.01	0.9	0.7	0.7
chiral:	0.01	0.2	0.05	0.05
g:	0.01	0.1	0.0	0.0
stat/scale:	0.30	1.2	1.1	1.1
operator:	0.0	0.0	1.4	1.4
relativistic:	0.5	0.5	1.0	1.0
total:	0.6	2.0	2.0	2.1

for different quantities different systematics are important