Mechanical and Magnetic Load Effects in Nb_3Sn Cable-in-Conduit Conductors

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STRAIN EFFECTS IN CABLES

2 SOURCES

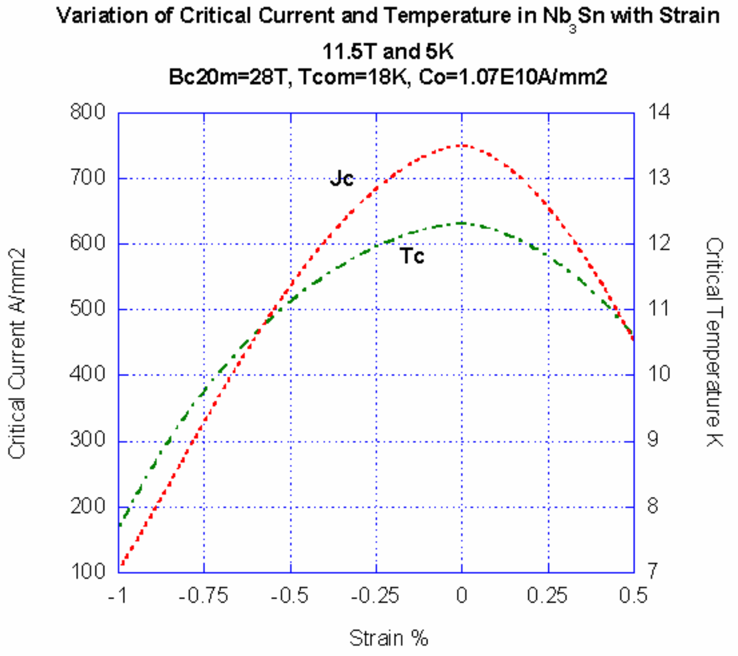
- Longitudinal compression by differential contraction from jacket
- Transverse loading by magnetic forces on strands (local and transmitted from other strands

MODELLING

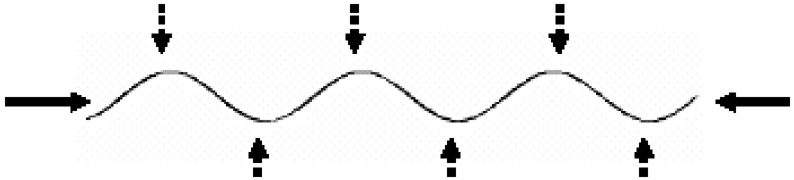
- Analytical approximation to longitudinal compression
- Analytical approximation to transverse loads
- Elasto-plastic finite element model of both

IMPACT

- Based on well-known strand behaviour



How to model strands in a cable?



Cable is a mess of curved strands, clamped at cross-over points

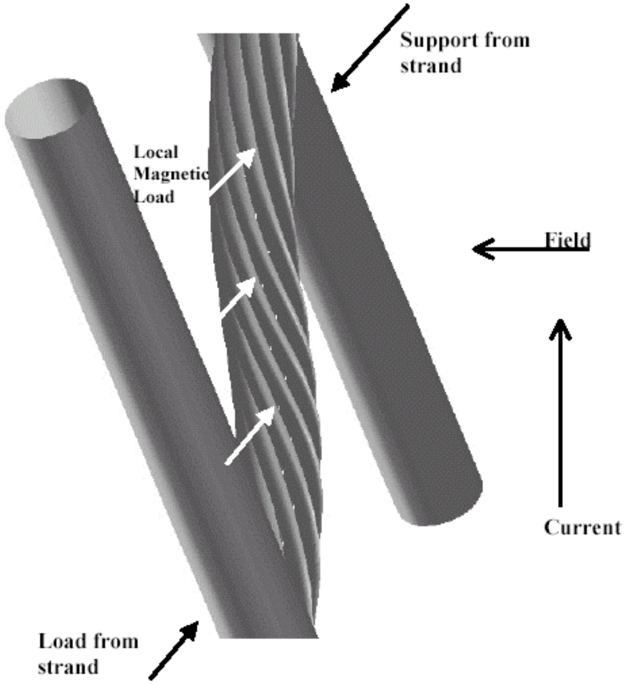
Strands have 'wavy' shape → allows bending to develop

Assume:

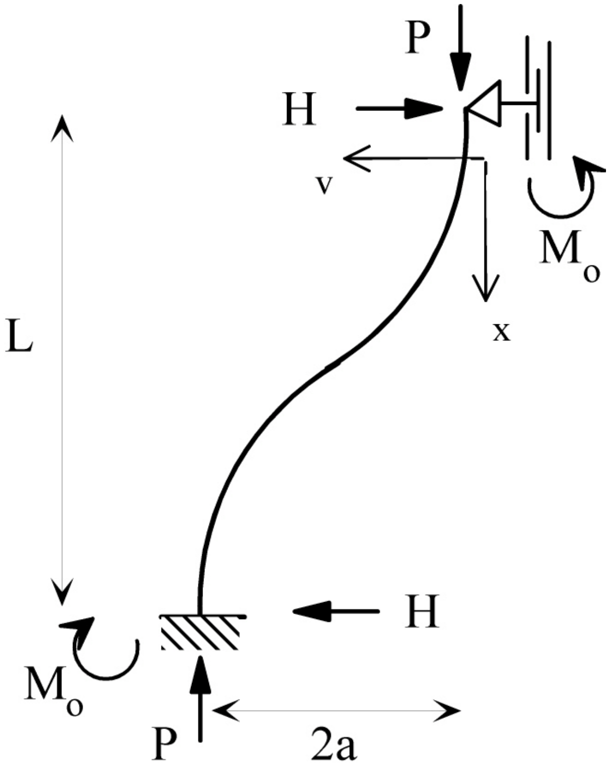
- One wavelength for waviness
- Strands clamped at change-over points

Because:

- Most conservative for bending (inflexible)
- Represents average over cable
- No bending transmission to other strands at cross-overs



ANALYTICAL MODEL OF A STRAND IN A CABLE COMPRESSED BY THE JACKET



Strand can be analysed as a 'strut' in compression

Strands in a cable have a 'wavey' shape

$$v_o = a\left(1-\cos\left(\frac{\pi x}{L}\right)\right)$$

which is related to the average cable angle θ

$$a = \frac{L}{2}\left[\left(\frac{1}{(\cos\theta)^2}\right)-1\right]^{1/2}$$

$$v = A\cos kx + B\sin kx - a - \frac{k^2 a\cos(\pi x/L)}{(k^2-(\pi/L)^2)} - \frac{M_o}{k^2 EI} - \frac{Hx}{k^2 EI}$$

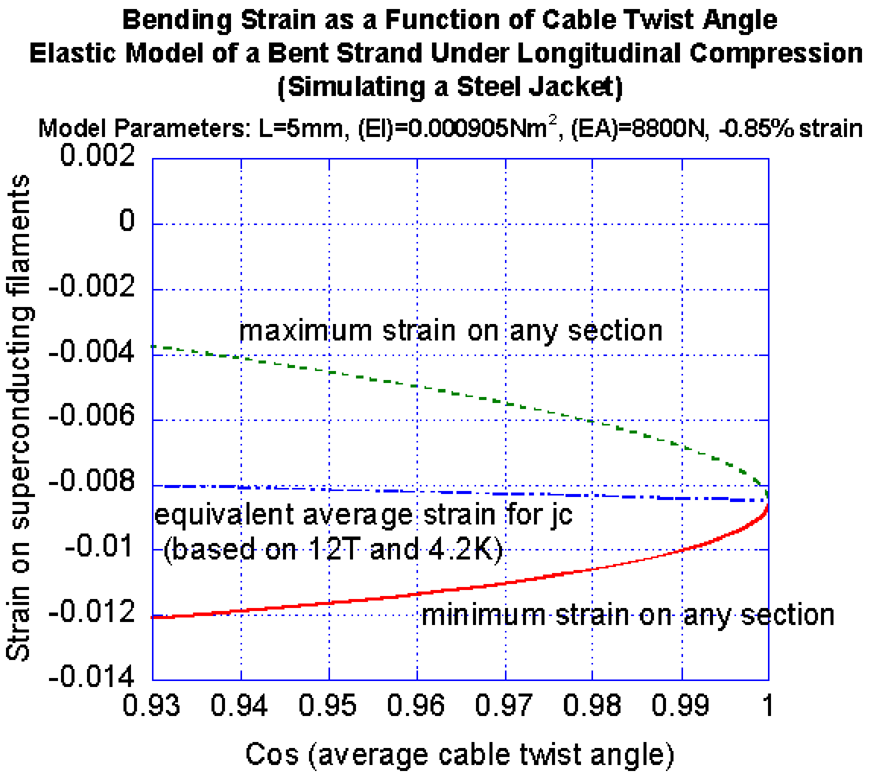
with unknowns *A, B, H, M$_o$* which are determined from the boundary conditions at x=0,L

$$v = 0, \frac{dv}{dx} = 0$$

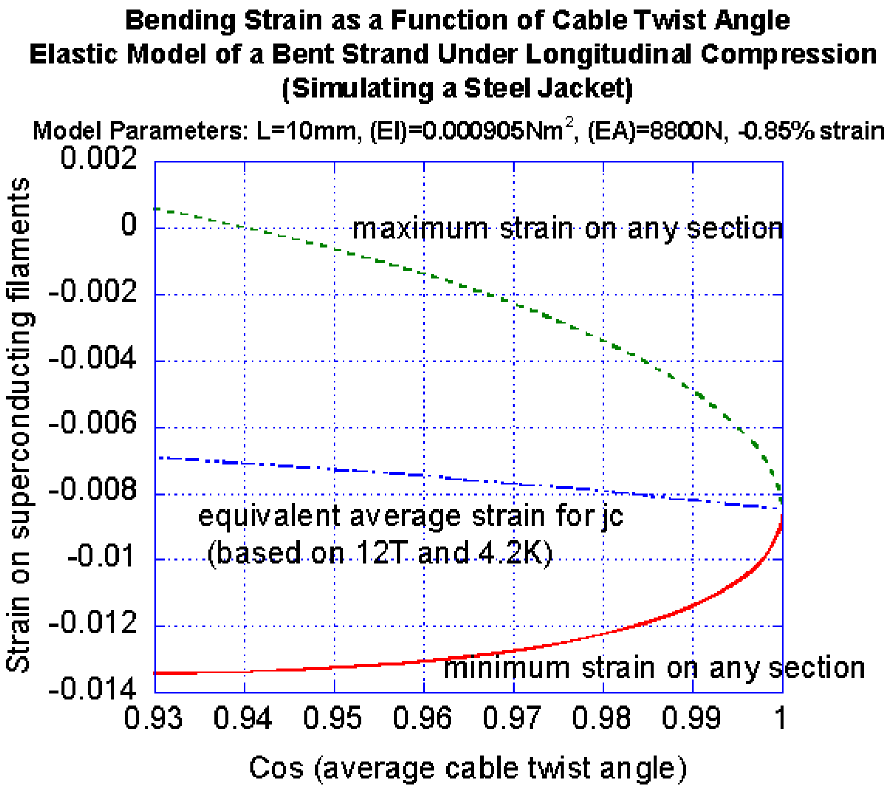
the end displacement δ is found from $\frac{1}{2}P\delta = \frac{1}{2}\frac{P^2 L}{AE} + \int_0^L \frac{M(x)^2}{2EI}dx$ **which can only be solved implicitly**

BENDING STRAIN ON FILAMENTS IN STRANDS COMPRESSED BY A STEEL JACKET – ANALYTICAL ELASTIC MODEL

- strands are compressed by displacement at crossovers
- allows bending of free length
- direct compression reduced but +/- bending strain created
- as the cable angle increases, so does the bending
- average compression strain on filaments reduced


Bending Strain as a Function of Cable Twist Angle
Elastic Model of a Bent Strand Under Longitudinal Compression
(Simulating a Steel Jacket)

Model Parameters: L=5mm, (EI)=0.000905Nm^2, (EA)=8800N, -0.85% strain


Bending Strain as a Function of Cable Twist Angle
Elastic Model of a Bent Strand Under Longitudinal Compression
(Simulating a Steel Jacket)

Model Parameters: L=10mm, (EI)=0.000905Nm^2, (EA)=8800N, -0.85% strain

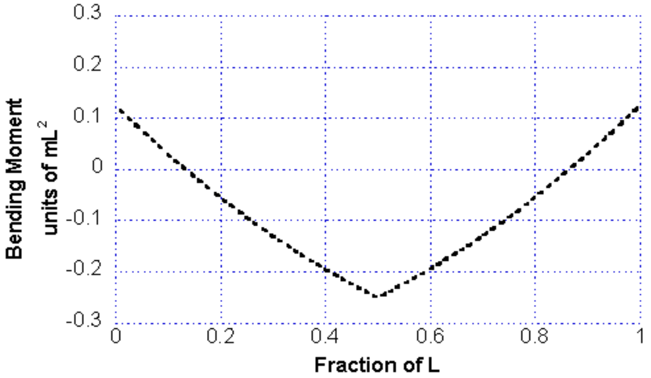
BENDING STRAIN ON FILAMENTS IN STRANDS DUE TO TRANVERSE MAGNETIC LOADS – ANALYTICAL ELASTIC MODEL

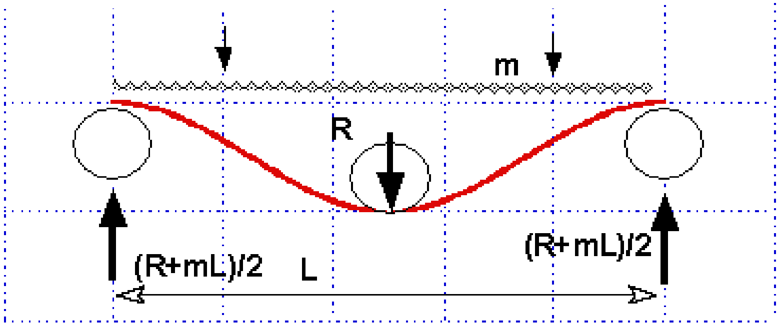
- Strands, twisted with a pitch of 10-20mm, contain filaments
- In the cable the strands rest on other strands
- The transverse magnetic loads cause strand bending
- ‘Wavelength’ of bending 4-6mm

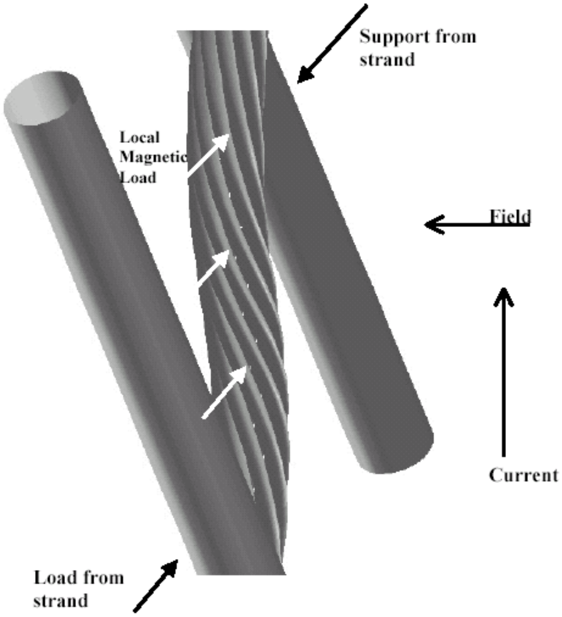
Filament Tc, Jc vary due to bending strain

➔Possible current transfer between filaments

➔Low n in strands in cable







Linear Elastic Strand Strain Assessment

Take the case of 1 strand loaded by the magnetic forces and supporting 1 other strand

Load accumulation through the cable can give bending strains several times larger than this

Maximum bending moment

$$M = \left({}^{RL}\!/_{4} \right) + {}^{BIL^2}\!/_{8N}$$

Maximum filament strain

$$\varepsilon_m = {}^{\left(RL + {}^{L^2 BI}\!/_{2N}\right) r_f}\!/_{\pi r^4 E}$$

B=12T, I=50kA, N=1152, r_f=0.3mm, E=150GPa and (L/2)=5mm ==> ε_m = 0.13%

Average transverse strain based on 'strand space' of $(\pi r^2/(1-v))^{1/2}$

$$\varepsilon_t = {}^{RL^3}\!/_{192 E I_a} \left(1 - {}^{v}\!/_{\pi r^2} \right)^{1/2}$$

Cable Transverse Elasticity Assessment

All the strands are assumed to have an identical sinusoidal bending moment pattern
$M = M_o \sin 2\pi x/L$

Mechanical energy E_M due to the bending stored in a length L $$E_M = M_o^2 L \big/ \pi E r^4$$

Cable stored energy density E_D $$E_D = M_o^2 (1-v) \big/ \pi^2 E r^6$$

Effective transverse modulus of the cable is E_{eff} then also
(average presure is BI/2b) $$E_D = B^2 I^2 \big/ 8 E_{eff} b^2$$

Combining E_D

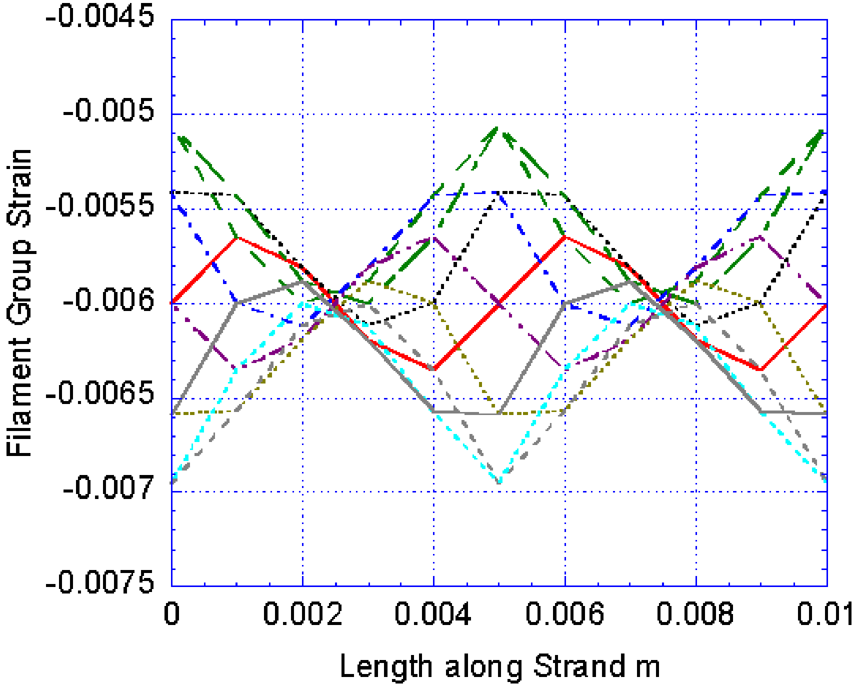
$$M_o = \pi B I r^3 \big/ 2b \left(E \big/ 2E_{eff}(1-v) \right)^{1/2} \qquad \varepsilon_m = \left(2BIr_f \big/ rbE \right) \left(E \big/ 2E_{eff}(1-v) \right)^{1/2}$$

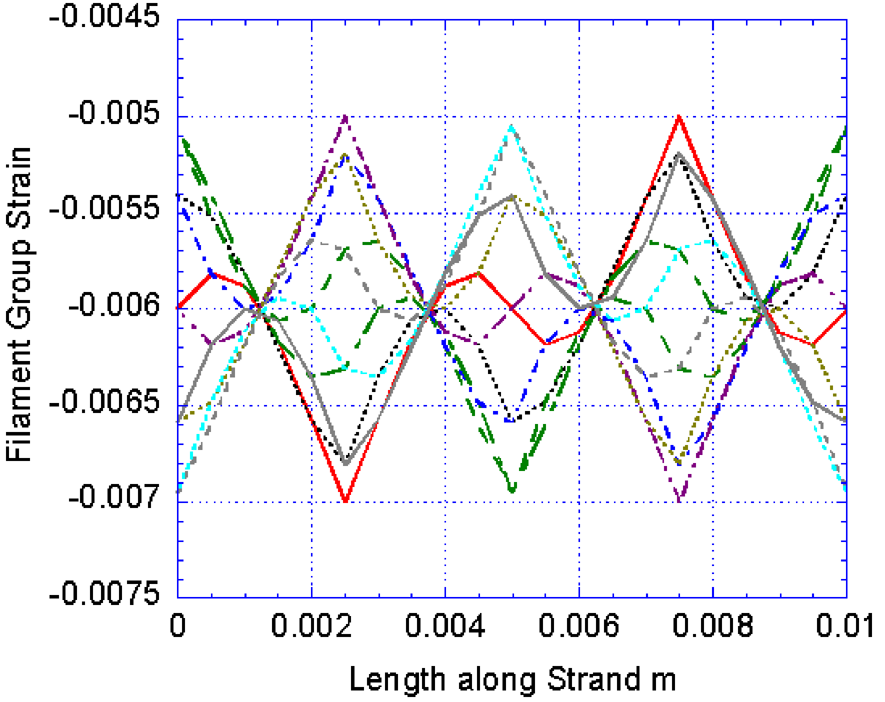
Taking as parameters B=12T, I=50kA, b=0.038m, v=0.36, r_f=0.3mm, E=150GPa and E_{eff} in the range 2-15GPa (measured) gives ε_m in the range 0.05 to 0.12% ➔ on average each strand supports one other

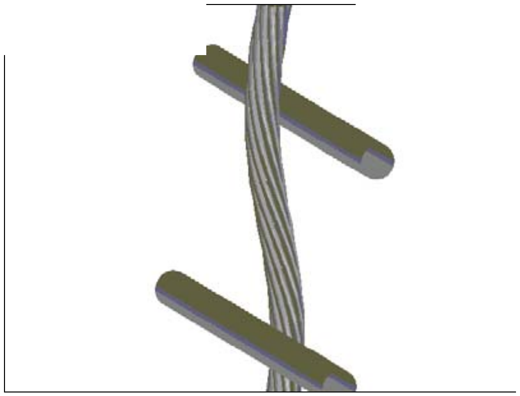
Allow a peaking factor of 2 (linear pressure variation) and ε_m is in the range 0.1 to 0.24%

How Does Strand Bending Affect Superconductor Performance?

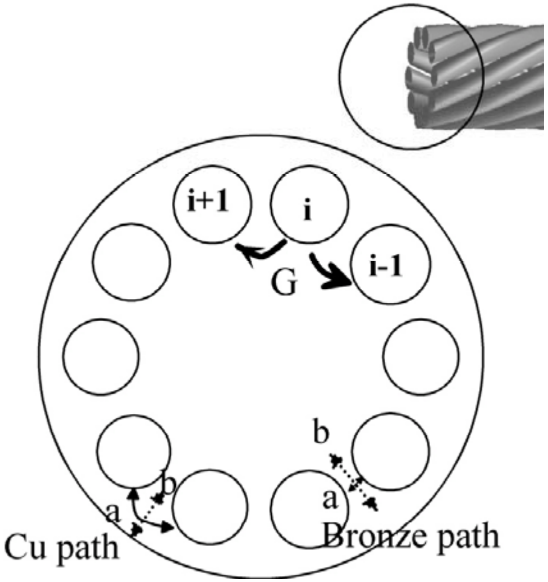
Filament strain distributions used in analysis of current redistribution

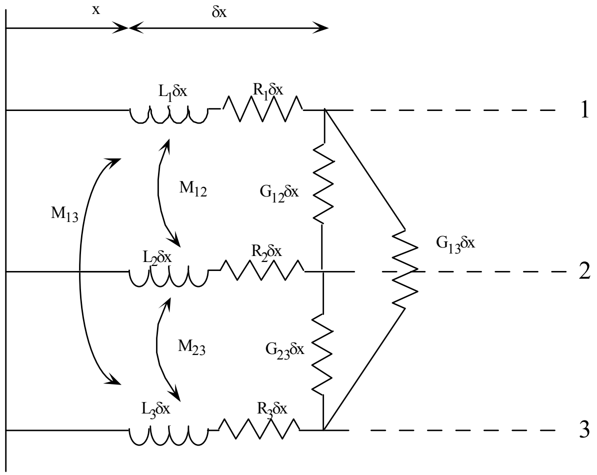

Strain Variation along 10 Filament Groups in a Strand with 10mm twist pitch and 5mm supports Mean strain -0.006, strain amplitude .001


Strain Variation along 10 Filament Groups in a Strand with 10mm twist pitch and 2.5mm supports Mean strain -0.006, strain amplitude .001



Electrical Model for Filament Non-Uniform Current Distribution



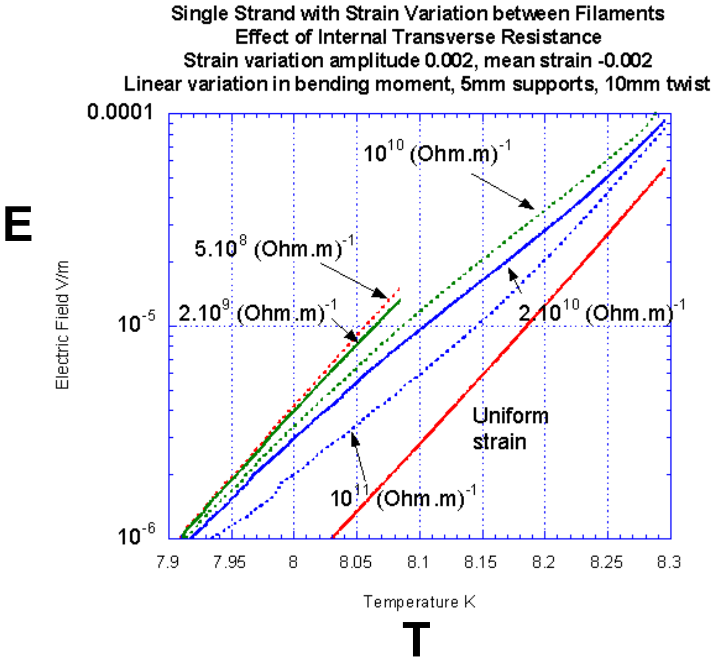


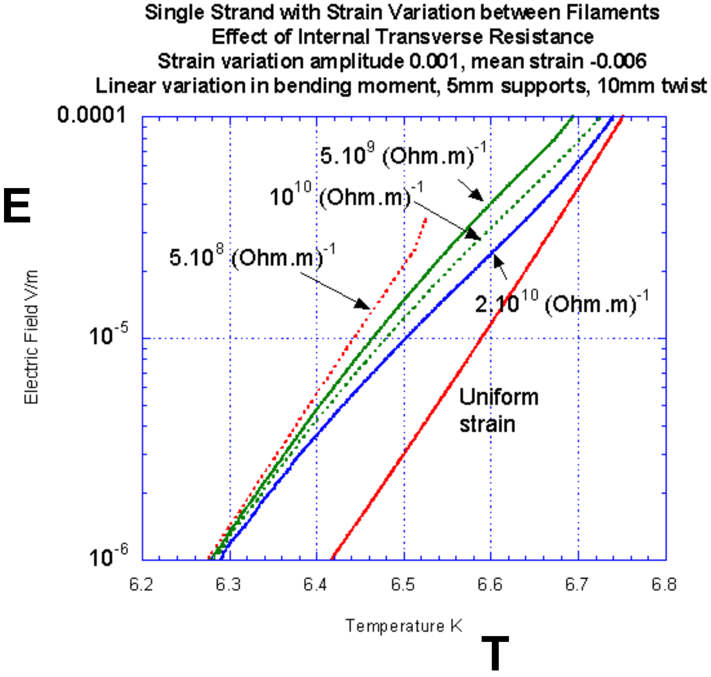
10 filament groups
Each group contains about 500-1000 physical filaments
Each group is electrically connected only to the group on each side
Current path can be through bronze or copper, conductivity G depends on strand structure

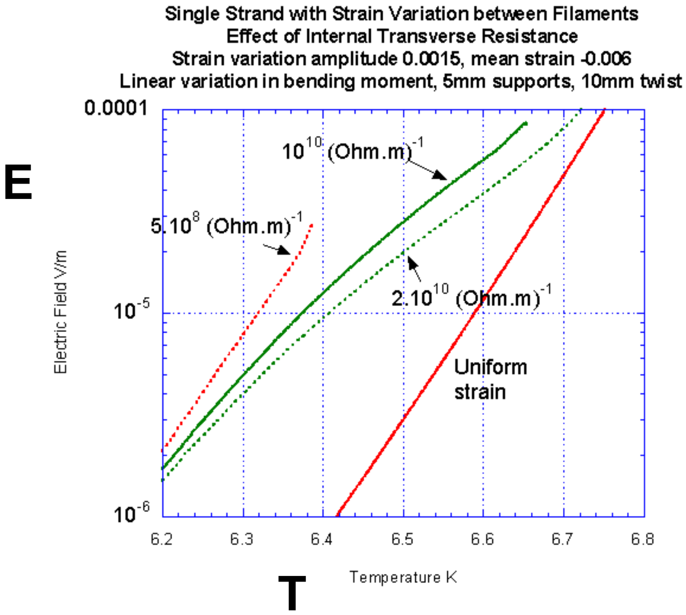
Expect G in the range 10^9 to 10^{10} $(\text{Ohm.m})^{-1}$ (not identical to conductivity of bronze or conductivity from time constant)

Use 10mm length of strand, symmetric boundary conditions at ends (i.e. infinitely repeated bending)

Impact of Strain Magnitude and Strand Transverse Conductance on V-T Curve


Single Strand with Strain Variation between Filaments
Effect of Internal Transverse Resistance
Strain variation amplitude 0.002, mean strain -0.002
Linear variation in bending moment, 5mm supports, 10mm twist


Single Strand with Strain Variation between Filaments
Effect of Internal Transverse Resistance
Strain variation amplitude 0.001, mean strain -0.006
Linear variation in bending moment, 5mm supports, 10mm twist


Single Strand with Strain Variation between Filaments
Effect of Internal Transverse Resistance
Strain variation amplitude 0.0015, mean strain -0.006
Linear variation in bending moment, 5mm supports, 10mm twist

(i) There is clearly a transverse conductance 'window' when the strand n is reduced.

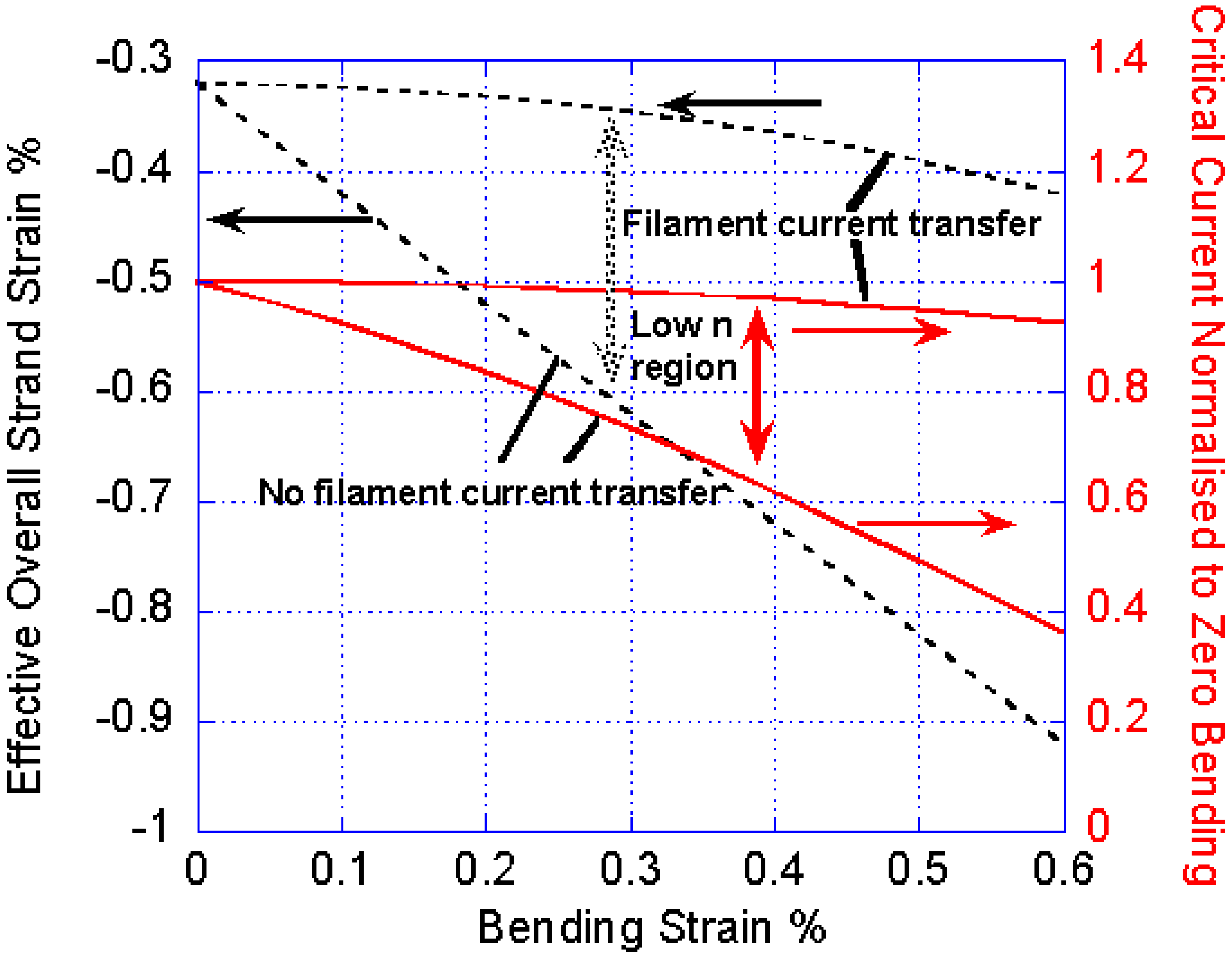
(ii) The reduction in strand n depends on the bending strain and transverse conductance but can easily be from 30 to under 10

Bending Strain Effects Both Critical Current and 'n'

Effect depends on

- wavelength of bending
- magnitude of bending
- strand internal resistance
- strand twist
- current transfer between strands

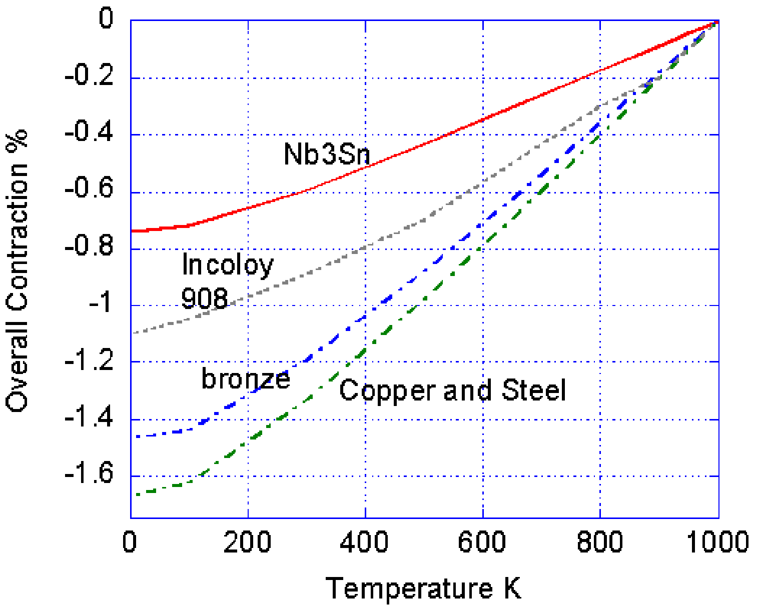
Effect of Filament Current Transfer on Overall Strand S/C Performance with Applied Bending Strain
Mean strain -0.32%, Strand jc 556A/mm2 at 12T and 4.2K



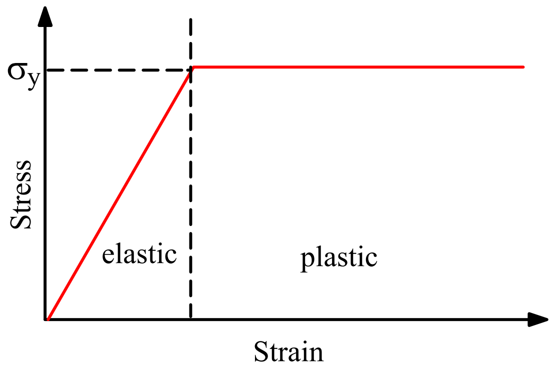
Elasto-Plastic Modelling of Strand Mechanical Behaviour

Differential expansion between strand/conductor components from 1000K heat treatment to 4K creates complex strand stress system

Thermal Contraction Coefficients from 1000K to 5K for Nb3Sn, Bronze, Copper, Steel and Incoloy



Traditional (dating back ¼ century) model is the 'fully bonded' one



$$\varepsilon_f = \frac{A_c E_c\left(\frac{\Delta l}{l_c} - \frac{\Delta l}{l_f}\right) - \sigma_{Ybz} A_{bz} - \sigma_{Ycu} A_{cu}}{A_c E_c + A_f E_f}$$

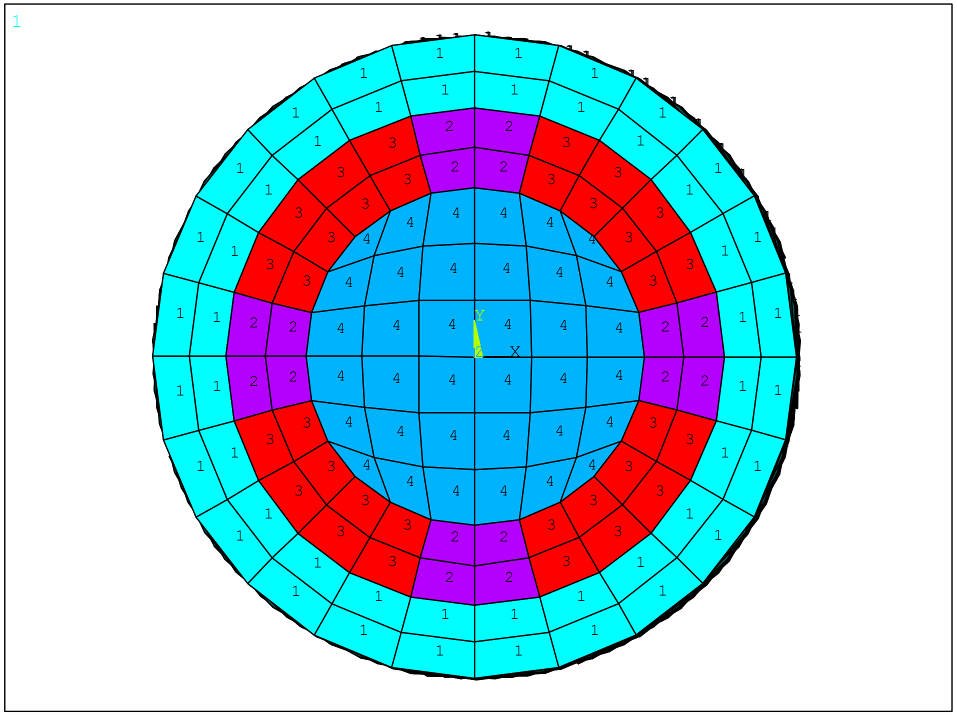
f: filament
c:conduit

This model is very approximate, neglecting work hardening and only 1D. Strands in CIC conductors are also loaded by transverse magnetic loads and can bend.
➔ object of the FE model is to develop a better strand mechanical model (but still approximate)

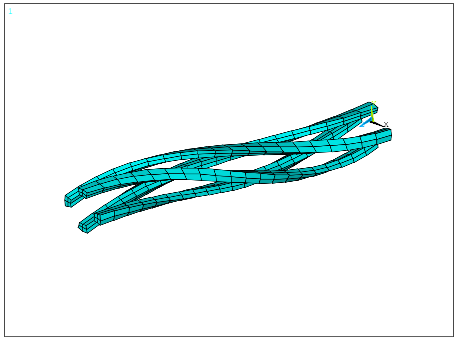
FINITE ELEMENT MODEL OF SINGLE STRAND

FEATURES

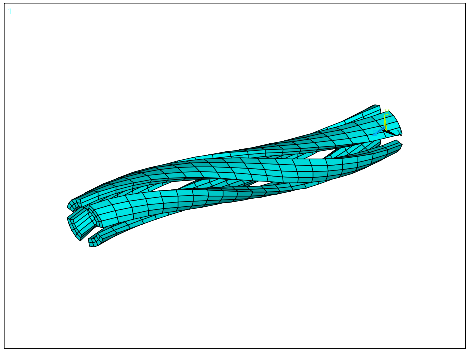
- Curved to model cabling
- 4 Components
- Includes twist for strands in cable



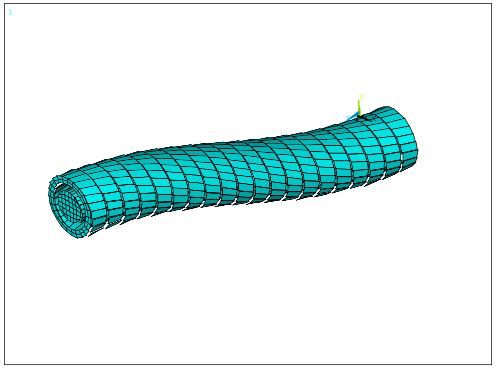
Filaments



Hard Bronze



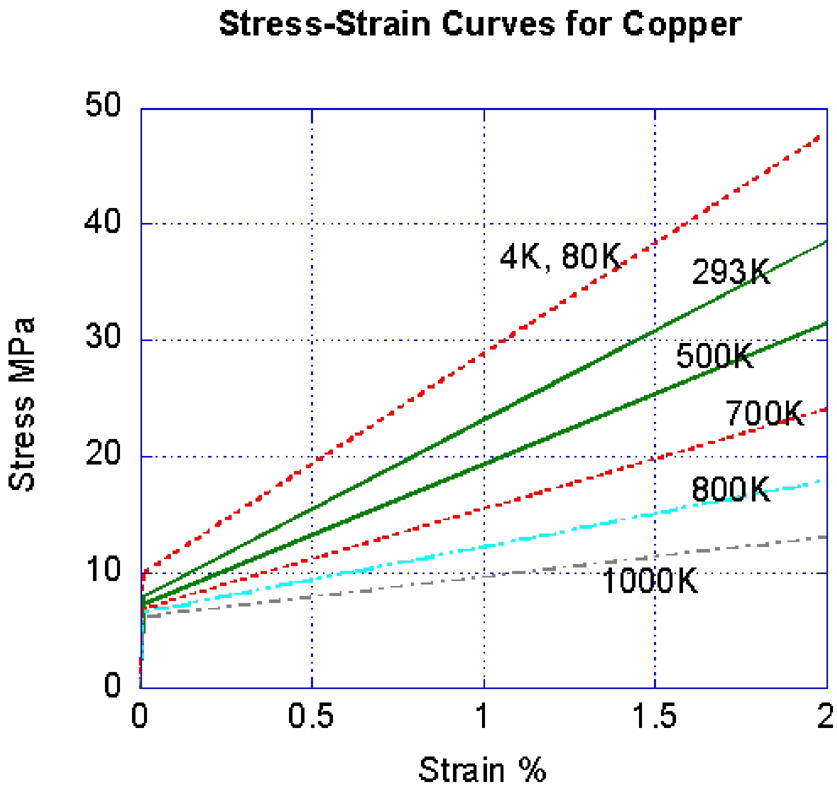
Copper and Soft Bronze

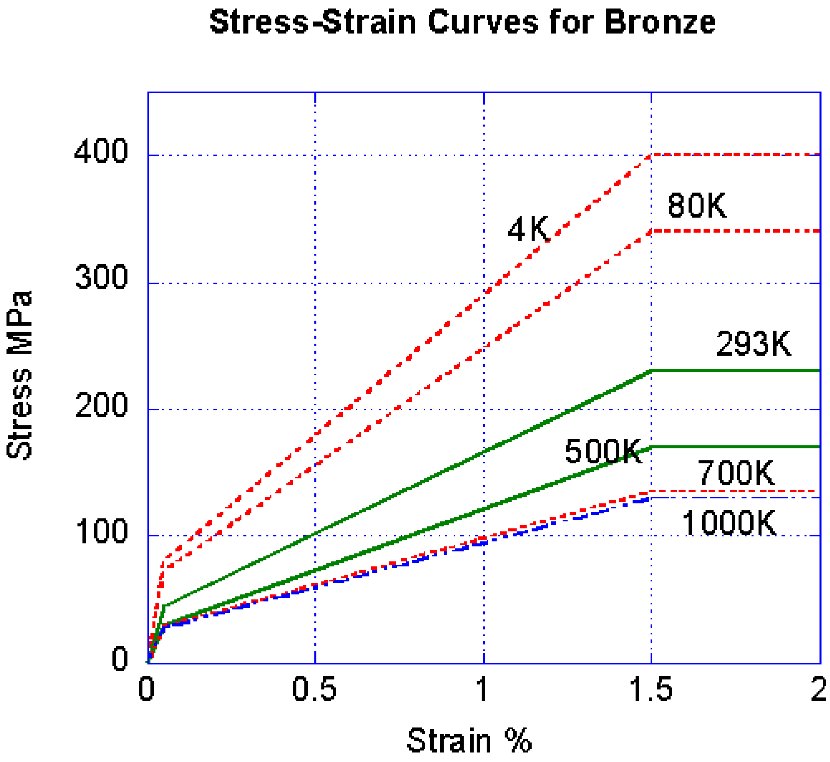


Working out the mechanical properties of the components ($\sigma-\varepsilon$) at temperatures 1000-4K is difficult, very little data

Approximate strand build with 4 components

- Nb_3Sn filaments
- Copper
- Soft 'bronze'
- Hard 'bronze'

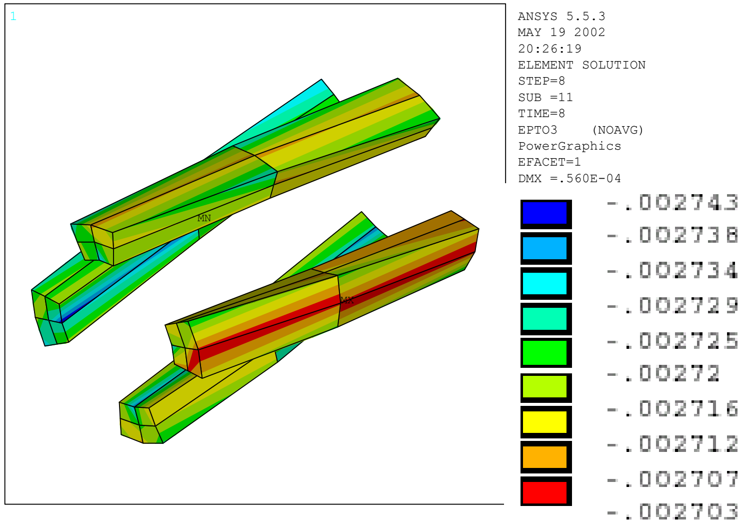

Stress-Strain Curves for Copper


Stress-Strain Curves for Bronze

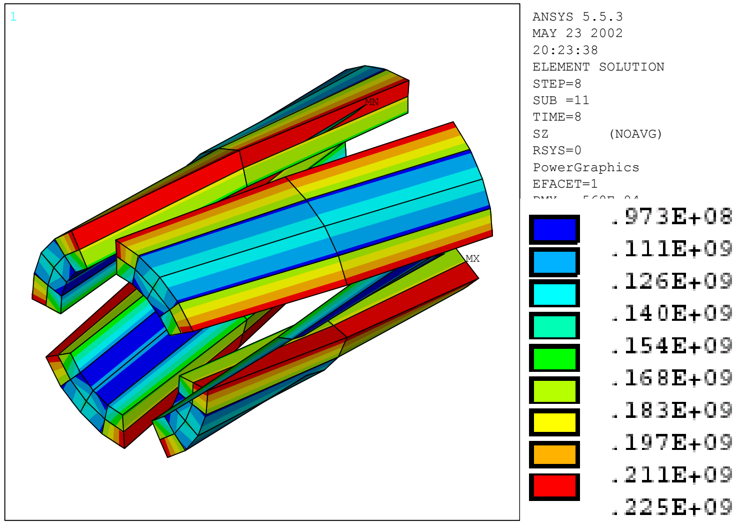
Nb3Sn filaments assumed elastic over whole temperature range, with E=160GPa.

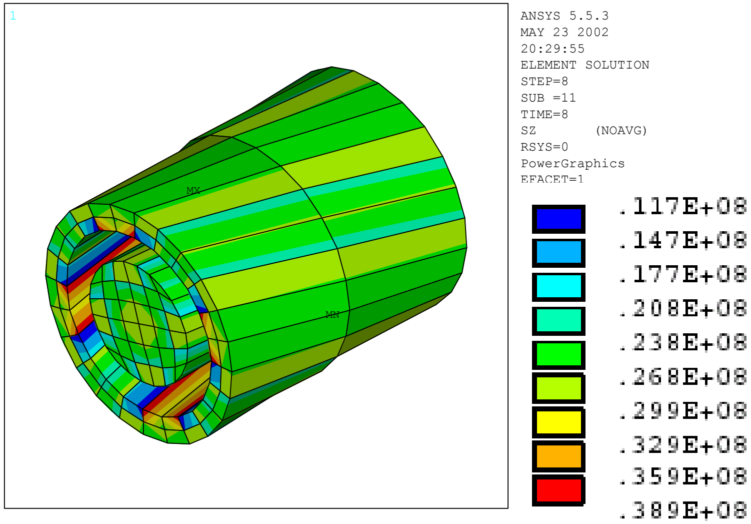
COOLDOWN OF AN ISOLATED STRAIGHT STRAND

Filament Strain (principal, along strand) in Isolated Straight Strand after Cooldown



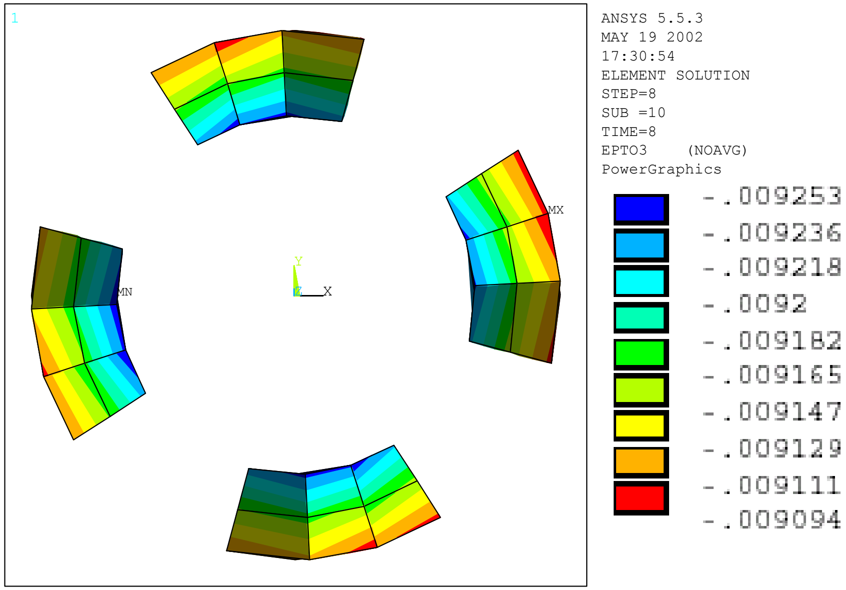
Bronze Stress (in global z direction along strand) in Isolated Straight Strand after Cooldown





Copper Stress (in global z direction along strand) in Isolated Straight Strand after Cooldown

COOLDOWN OF A STRAIGHT STRAND IN A STEEL JACKET 'FULLY BONDED' SIMULATION



Detail of the Filament Strain (principal, along strand) in a Straight Strand with a Steel Jacket after Cooldown

compared to an isolated strand

- **bronze and copper strain reduced in magnitude**
- **Nb3Sn filament strain increased**

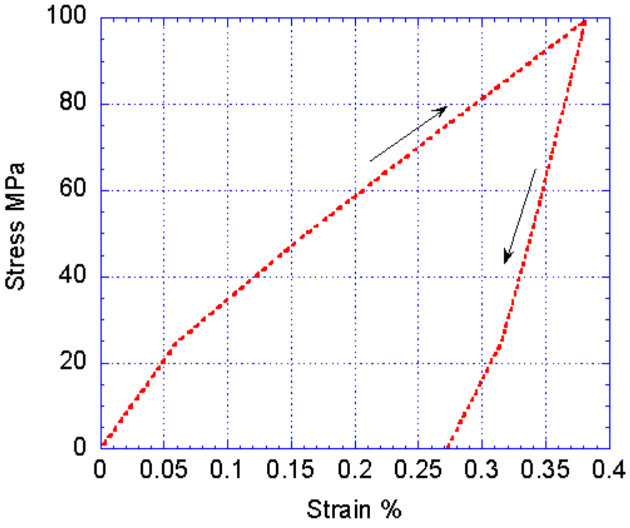
- **due to lower work hardening the 'strand in steel' is softer in transverse bending than the isolated strand**

Strain along strand	Isolated Strand	Steel Jacket
Nb3Sn	-0.27%	-0.92%
Hard Bronze	0.46%	-0.23%
Copper	0.67%	0

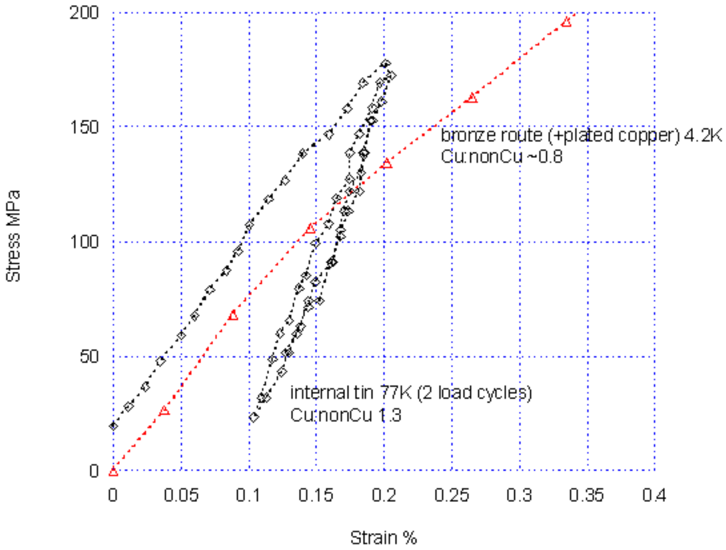
SIMULATION OF MECHANICAL TESTS AND MODEL VERIFICATION

Tensile Test at 4K

- Qualitative match between simulation and measurement
- Different strands
- $\sigma-\varepsilon$ cycles very sensitive to strand internal properties



Simulation of tensile test at 4K



Stress-Strain Curves for Reacted Nb3Sn Strands (0.8mm diameter).

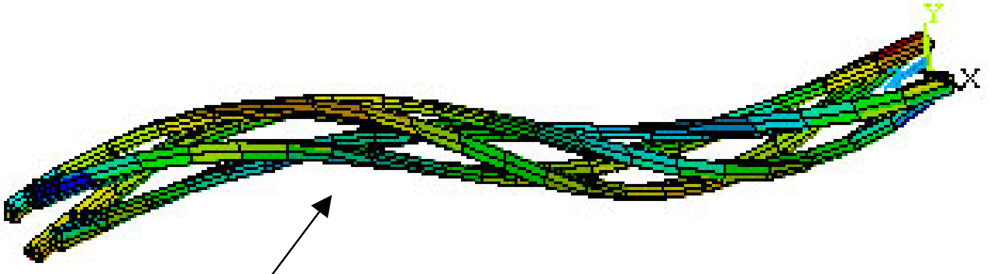
Material properties taken from literature but not usually self-consistent. Thermal contraction and $\sigma-\varepsilon$ measured on different compositions

Effect of Thermal Cycling on Overall Strand Contraction

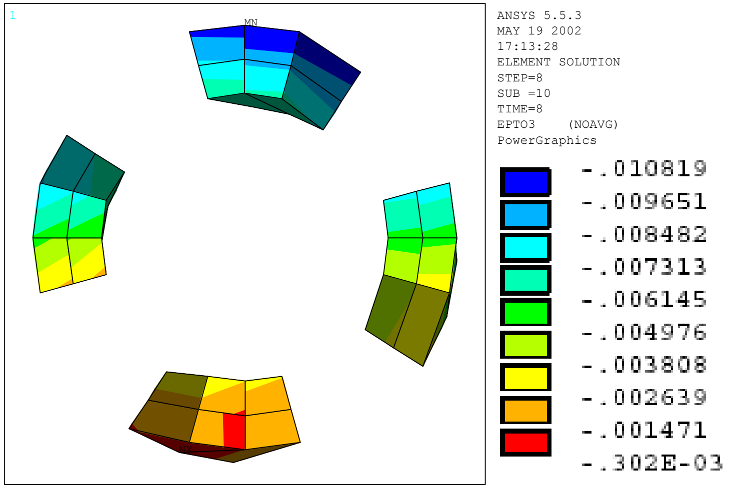
- contraction 300-4K changes after first cooldown
- due to work hardening and plastic yielding
- easy to test, possible model verification route

	First Cooldown	After warm up to 300K and cooldown again
1000->4K	-1.0%	-1.0%
300->4K	-0.23%	-0.3%

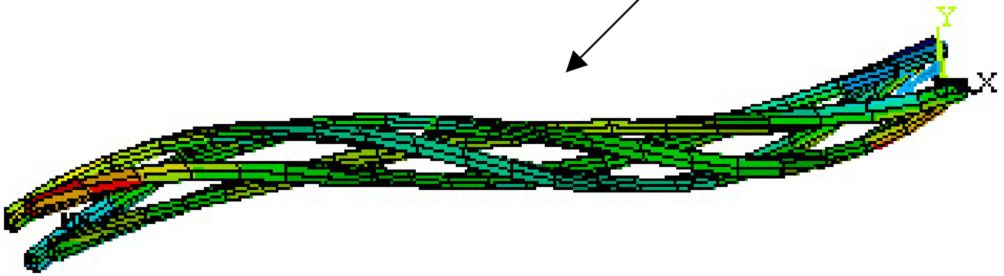
COOLDOWN OF A CURVED STRAND IN A CABLE IN A JACKET USING ELASTO-PLASTIC FE MODEL

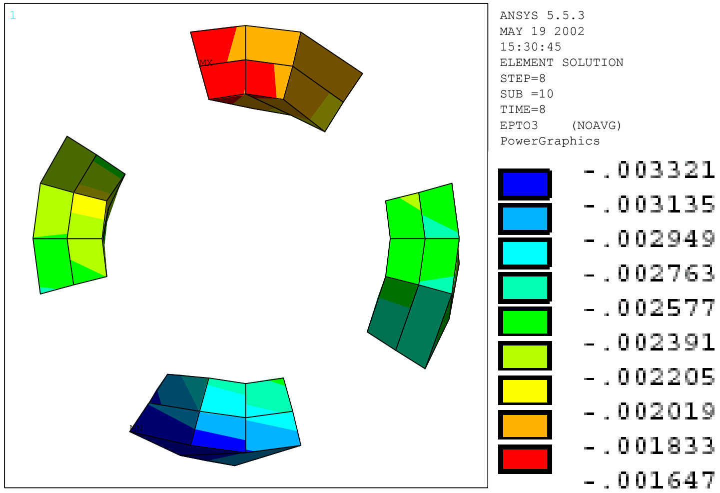


Steel Jacket



Incoloy Jacket





Filament Strain (principal, along strand) in Curved Strand after Cooldown
(left:overall, right: at left end)

Changes in Strand Strain after Cooldown Caused by Bending

Axial Strain %

Cooldown of Curved Strand

	Incoloy	**Steel**
Nb3Sn	-0.27 to -0.23 (mid) -0.33 to -0.16 (end)	-0.62 to -0.44 (mid) -1.08 to -0.03 (end)
Hard Bronze	0.39 to 0.58 (end)	-0.35 to 0.95 (end)
Copper	0.12 to 1.09 (end)	-0.69 to 1.63 (end)

Cooldown of Straight Strand

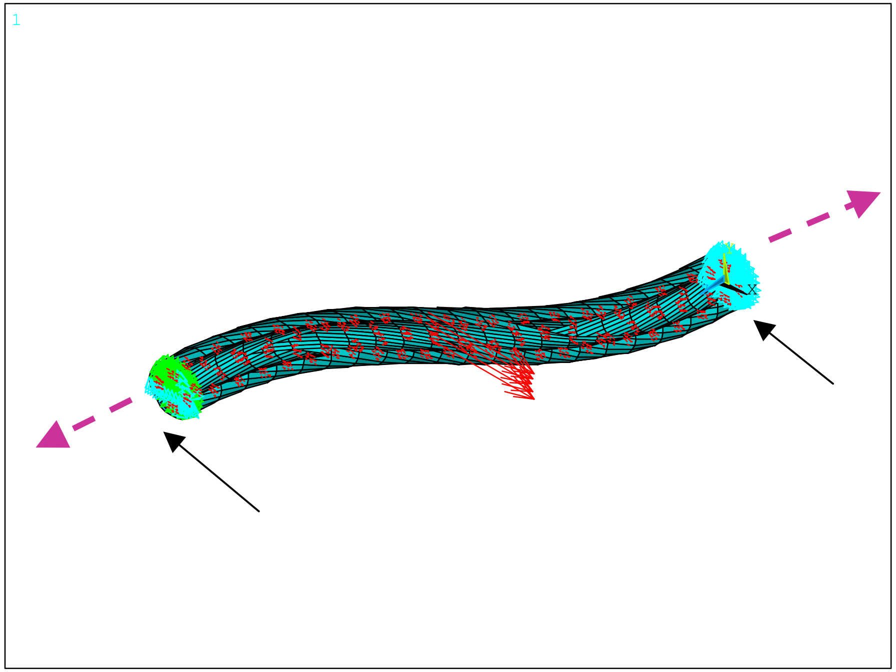
Strain along strand	Isolated Strand	Steel Jacket
Nb3Sn	-0.27%	-0.92%
Hard Bronze	0.46%	-0.23%
Copper	0.67%	0

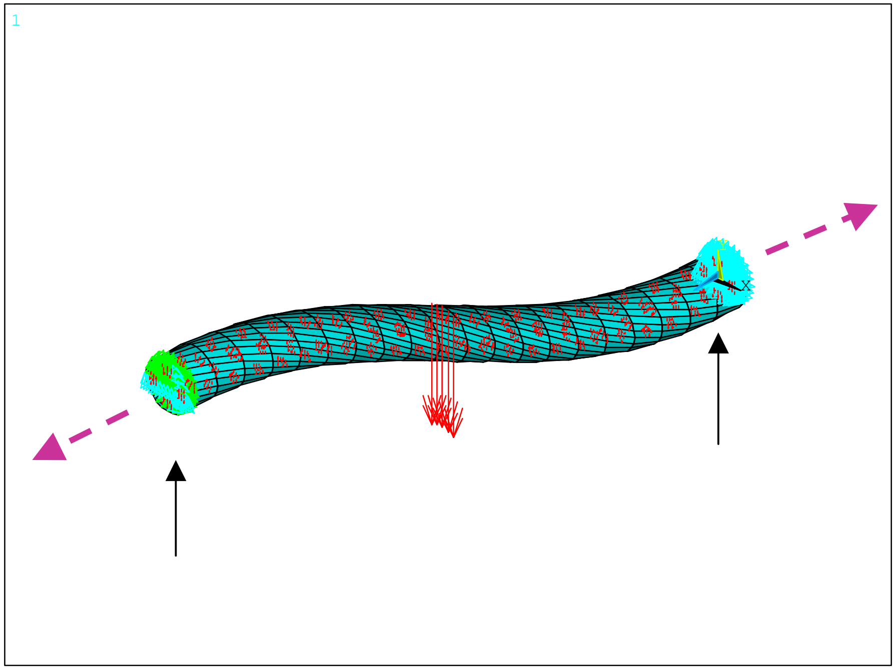
In steel conductors:

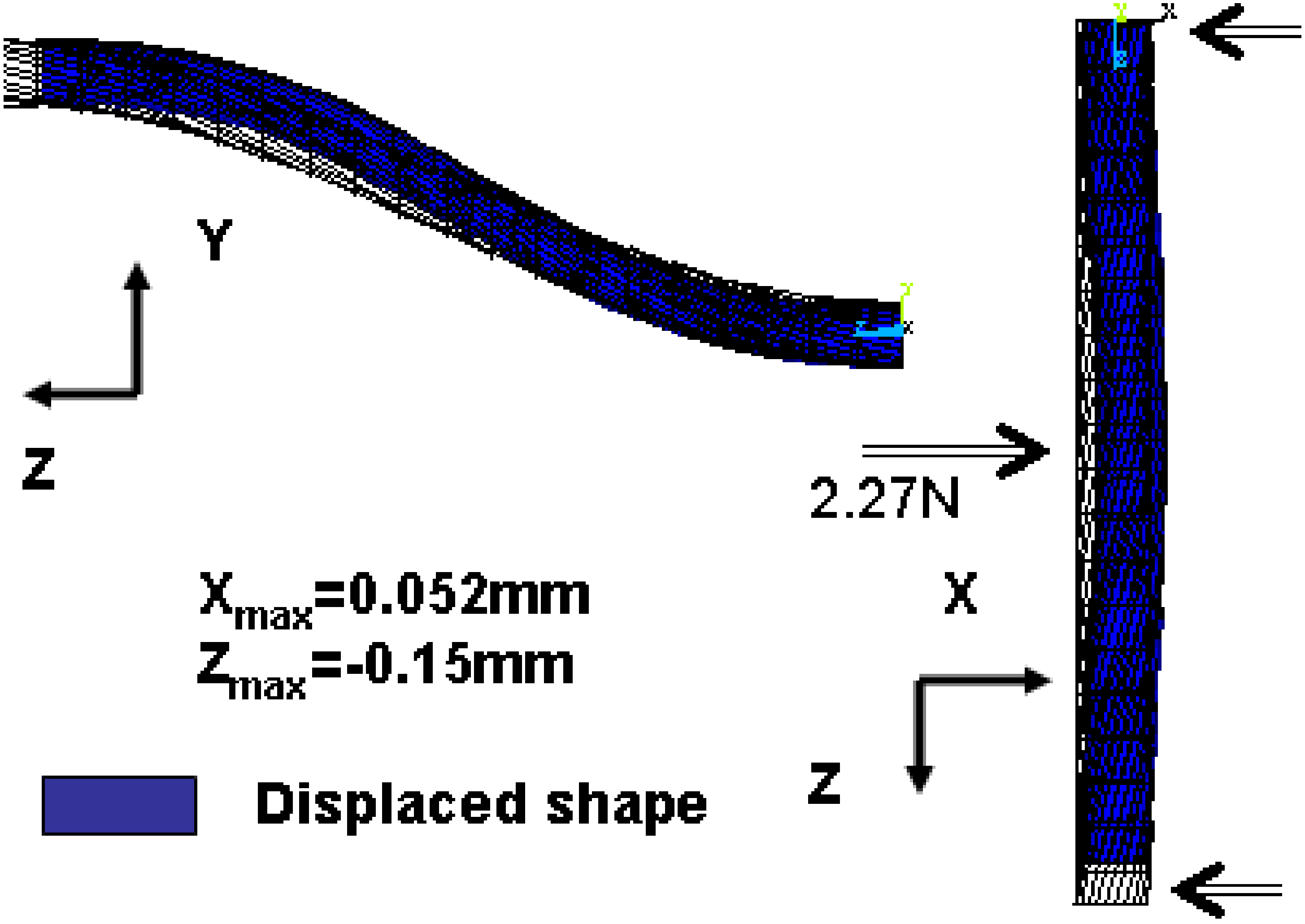
- **bending dominates**
- **due to work hardening, bending + plasticity completely changes strand mechanical properties**

MAGNETIC LOADING ON CURVED STRANDS IN CABLES

- Magnetic loading is applied either in or at 90° to the curvature plane
- Distributed force load corresponding to 35A and 13T on Nb3Sn
- Transmitted force load corresponding to magnetic load on half length at middle
- Reaction through supports at ends
- Tensile displacement at ends of +0.15%





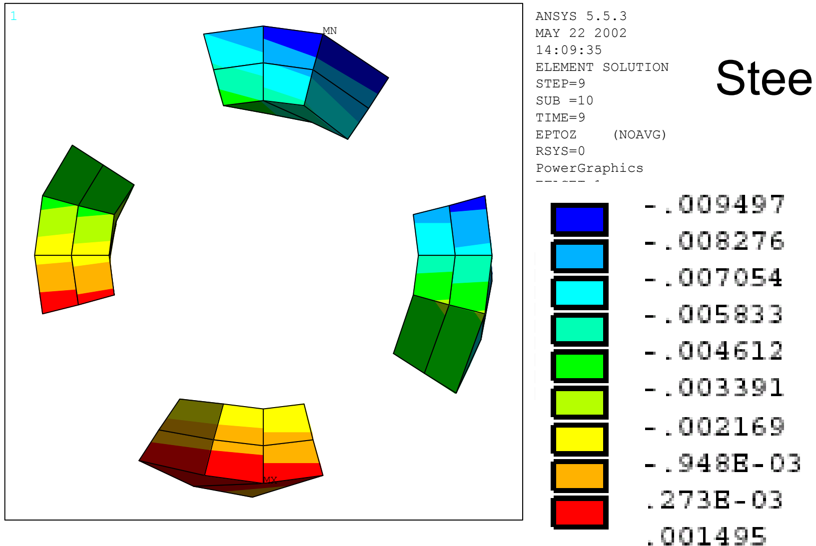


X_{max}=0.052mm

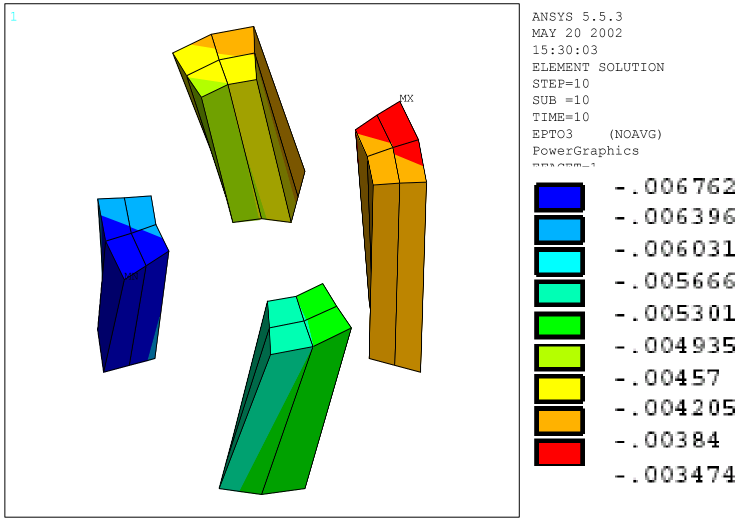
Z_{max}=-0.15mm

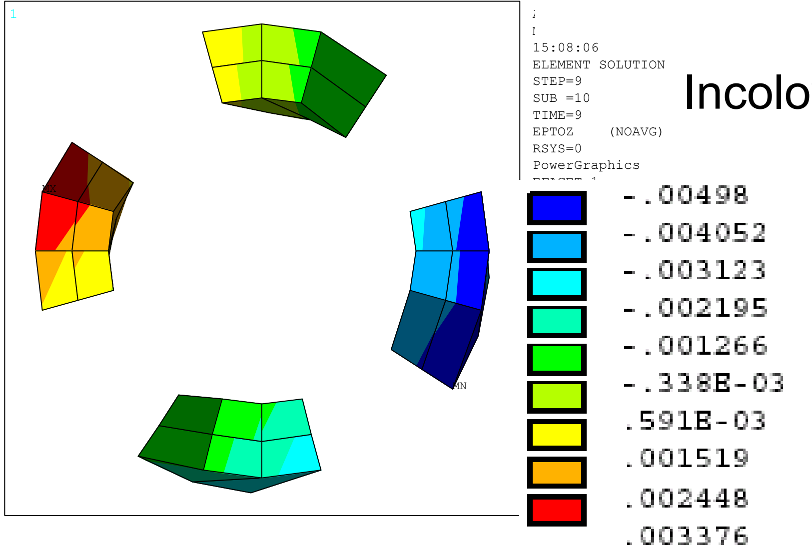
🟦 **Displaced shape**

STRESSES AND STRAINS WITHIN STRANDS IN A CURRENT-CARRYING CABLE

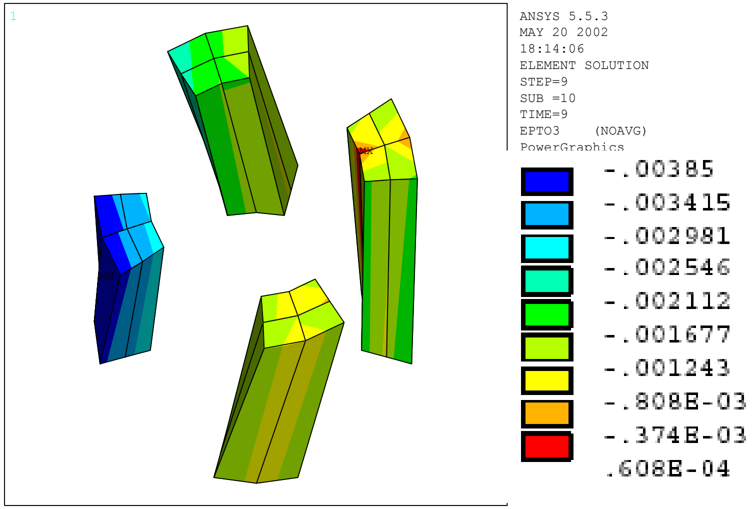


Steel Jacket





Incoloy Jacket



Filament Strain along curved strand after cooldown, with full magnetic loads 90° to curvature plane: left end and middle

SUMMARY OF STRAINS WITHIN CURVED STRANDS IN A CURRENT-CARRYING CABLE

Axial Strain %

Magnetic Loads
(at 90°)
10mm strand

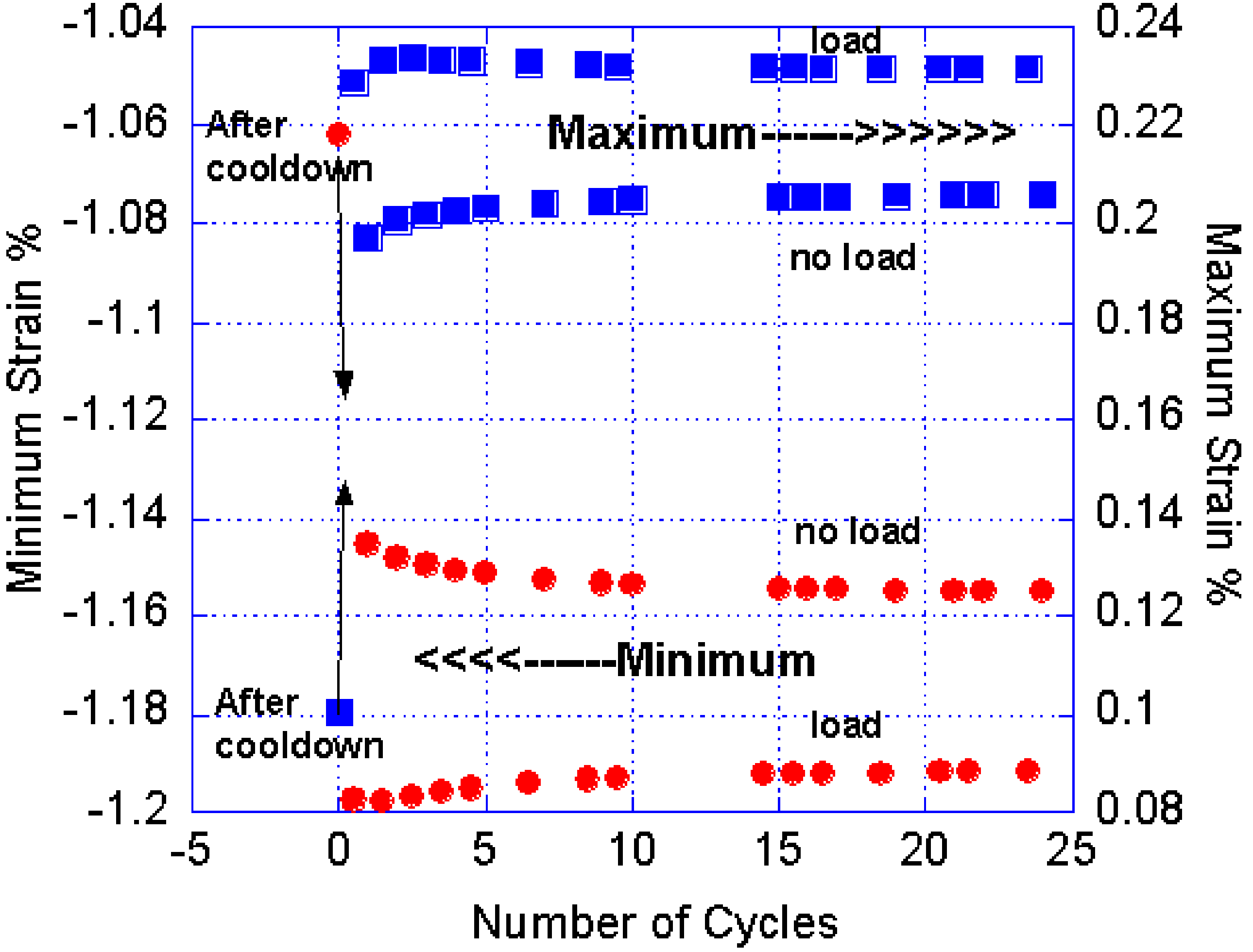
	Incoloy	Steel
Nb3Sn	-0.39 to 0.01 (mid) -0.50 to 0.34 (end)	-0.68 to -0.35 (mid) -0.95 to 0.15 (end)
Copper	0.09 to 1.52 (end)	-0.56 to 1.53 (end)
Bronze	0.23 to 1.16 (end)	-0.28 to 0.90 (end)
Nb3Sn after Magnetic Load Removal (compared to before loading)	-0.39 to 0.0 (end) (compare -0.33 to -0.16 (end))	-1.17 to 0.23 (end) (compare -1.08 to -0.03 (end))

Displacements under Magnetic Loads mm

displacements at
90° to curvature
plane

	Incoloy	Steel
5mm Strand	0.017	0.011
10mm Strand	0.064	0.052
10mm Strand after unloading	0.040	0.027

Effect of Load Cycling on Filament Strain



- ➢Most plastic deformation occurs on first cycle
- ➢Most deformation is permanent
- ➢Small cyclic component

Cyclic Stress Adjustment Under Transverse Magnetic Loads, Steel Jacket with Curved Strand, Loads at 90° to Curvature Plane, Maximum and Minimum Filament Strain at End

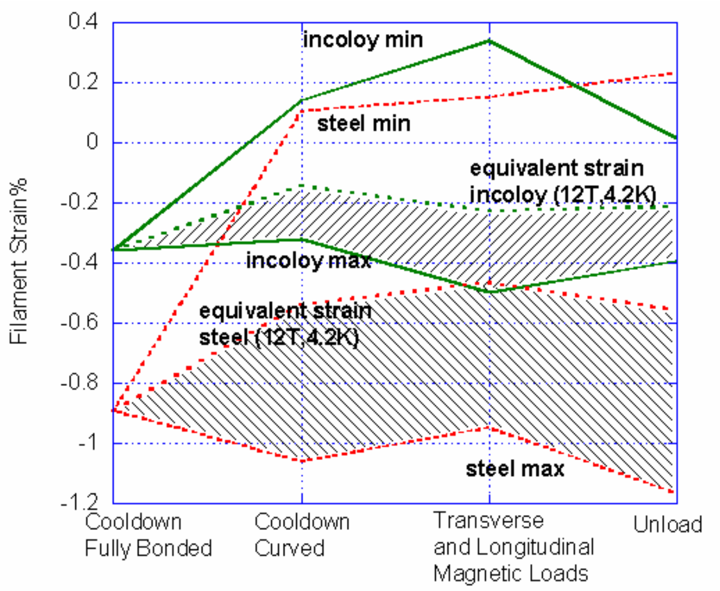
PREDICTED SUPERCONDUCTING PERFORMANCE OF STRANDS IN CABLES IN COILS: LARGE CABLE

Predicted Performance

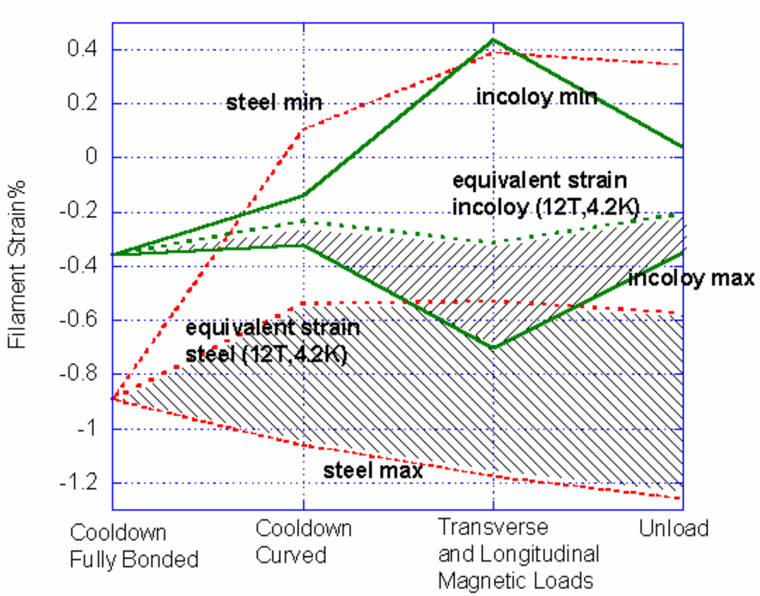
- assume current transfer between filaments
- assume no current transfer between strands in cable
- assume 13T and 4.2K
- average jc over filament region assuming linear stress variation min-max to give 'effective' thermal strain

	Incoloy		Steel	
	90^{o} Loads	In-plane Loads	90^{o} Loads	In-plane Loads
Fully Bonded Thermal Strain	-0.36%		-0.89%	
Thermal strain on cooldown	-0.24%		-0.54%	
Derived Effective Thermal Strain in operation (corrected by -0.15%)	-0.38%	-0.47%	-0.62%	-0.68%

Curvature at 90^{o} to Load



Curvature in plane of Load



Filament Strain Distribution with Curved Strands and Steel/Incoloy Jackets.

CONCLUSIONS 1

Effect of Jacket Material

Curvature due to cabling has dominant effect on strand strain even with small cable angle

STEEL JACKETS

Large strand bending→ high work hardening of copper, bronze
Some relaxation of thermal compression (by up to 0.2% from 'fully bonded' value), strands stiffened against transverse loads

INCOLOY JACKETS

Small strand bending→ copper, bronze stay soft, deflect more under transverse loads

CONCLUSIONS 2

Cycling and Permanent Deformation

With both STEEL and INCOLOY more than 50% of the bronze and copper strain is plastic, so strand deformation after first magnetic load is at least half permanent.

Cycling equilibrium reached after a few cycles, typically 66% of deformation due to magnetic loads is permanent

Longer term cycling effects seen in measurements probably due to friction based strand position adjustment in cable

CONCLUSIONS 3

Effect of Transverse Loads

Transverse loads cause extra strain in cable, simulations suggest up to -0.2 to -0.3% with Incoloy, -0.1 to -0.2% with steel. Fits observations on model coils

STEEL JACKETS

Stiff strands→smaller deflection under magnetic loads→smaller effect on filament strain compared to thermal bending strain

INCOLOY JACKETS

Soft strands→larger deflection under magnetic loads→resulting magnetic bending strains dominate over thermal bending strains→apparent extra degradation

CABLE SIZE

Smaller cables show smaller effect as load accumulation, cable angle less

CONCLUSIONS 4

Filament Damage and Fatigue

With both STEEL and INCOLOY tensile strains on the filaments exceed +0.2% when local damage could occur on some filaments

Due to copper/bronze work hardening, cyclic loads are small➔no fatigue damage effect