Synthetic Analysis of Electrical Coupling Parameters in SC Cables

L. Bottura(❄), M. Breschi (❋), C. Rosso(❊)

(❄)CERN, Switzerland

(❋)University of Bologna, Italy

(❊)CryoSoft, France

CHATS, FzK, Karlsruhe, September 2002

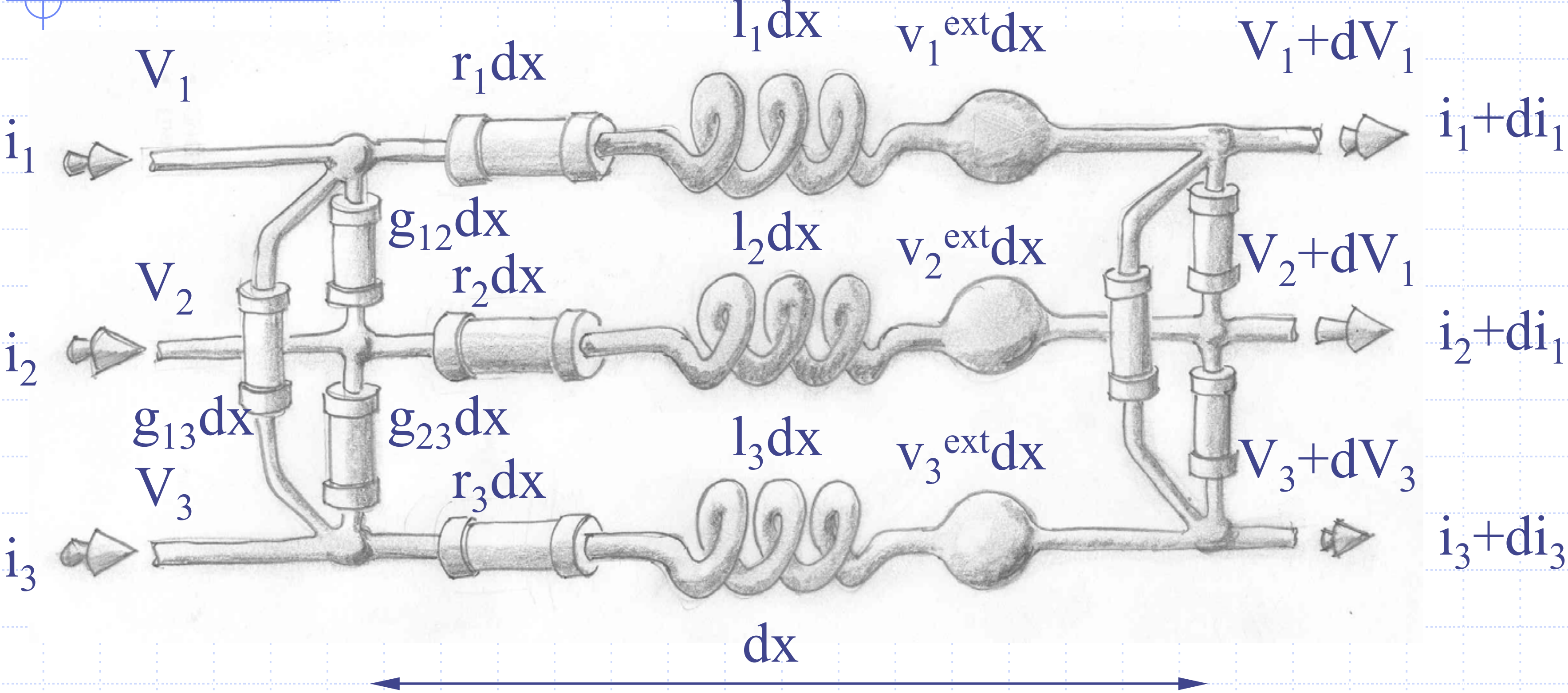
Overview

- Motivation
- Cable geometry generation and modeling
- Electrical coupling parameters
  - inductance matrix calculation
  - conductance matrix calculation
- Validation of the model
- Examples and applications
- Conclusions and perspective

Motivation - 1

- The electro-dynamic behavior of SC cables is governed by the electrical characteristics of the assembled strands
  - strand inductance matrix
  - interstrand conductance
  - parallel resistance *(V-I characteristic)*
- knowledge of above parameters is mandatory to understand current distribution and re-distribution

Motivation - 2



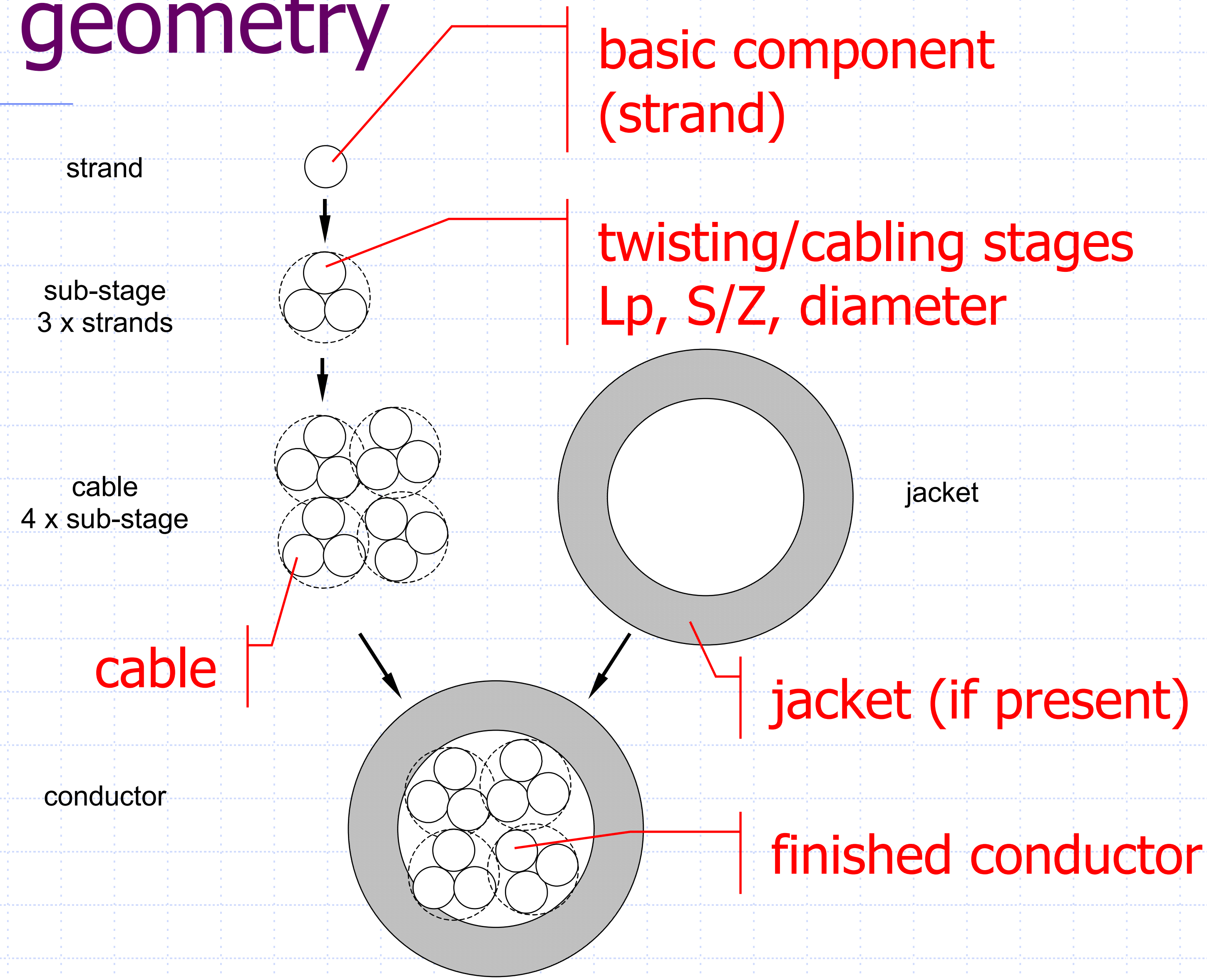
inductance

inter-strand conductance

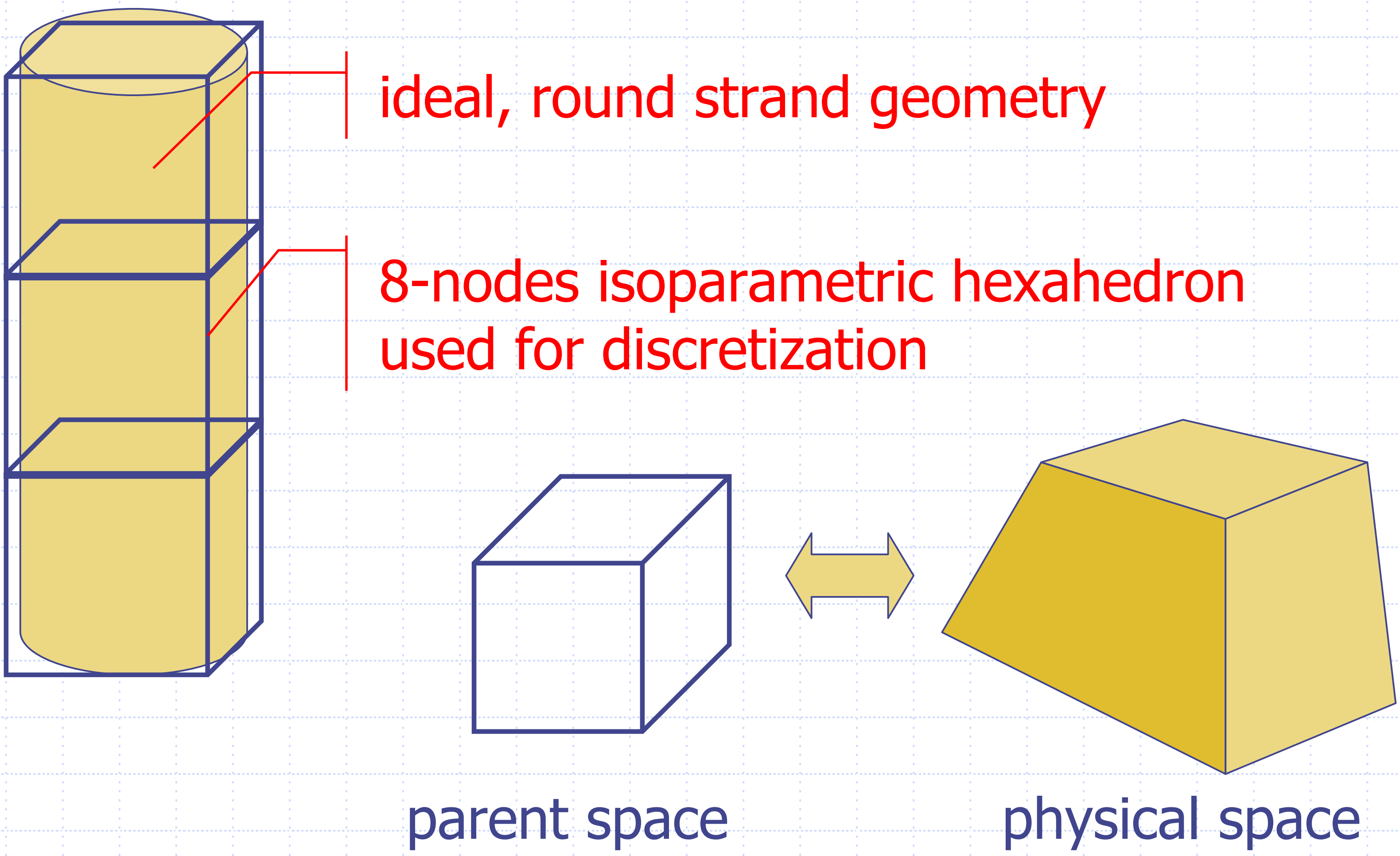
$$\mathbf{gl}\frac{\partial\,\mathbf{i}}{\partial\,t}+\frac{\partial^2\mathbf{i}}{\partial\,x^2}+\mathbf{gri}-\mathbf{gv}^{ext}=0$$

parallel resistance

Cable geometry

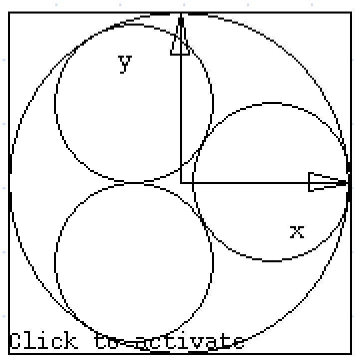


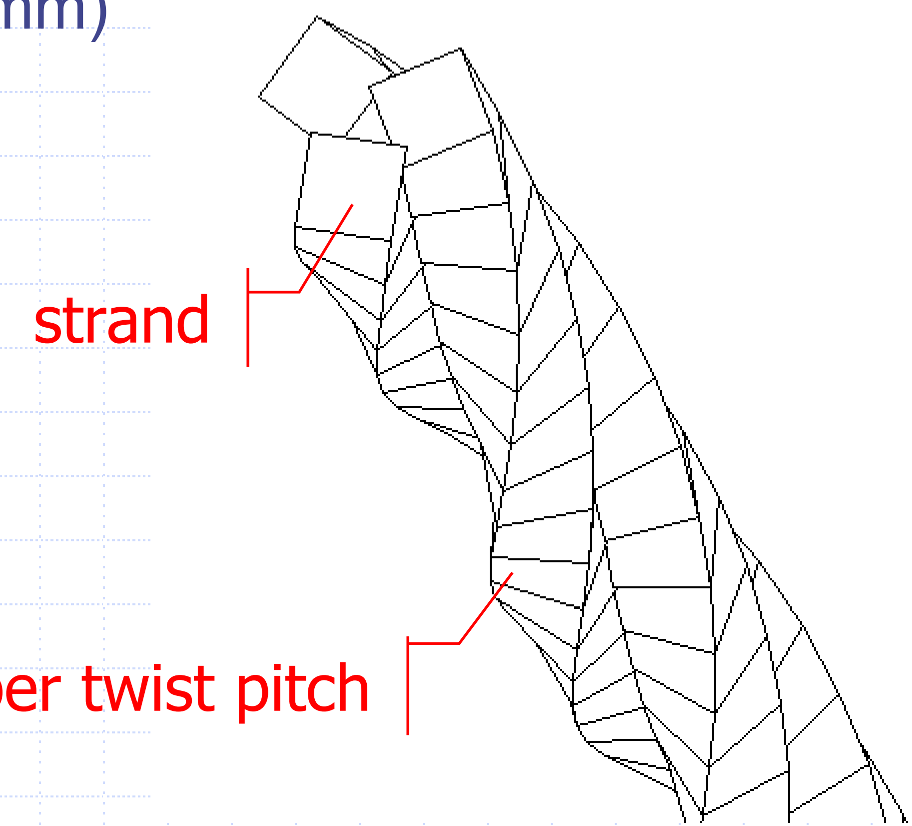
General Current Elements



Cable model example - 1

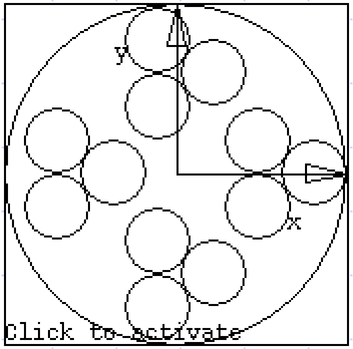
- model of a triplet (ITER strand)
  - D_{strand} = 0.81 (mm)
  - L_p = 25 (mm)

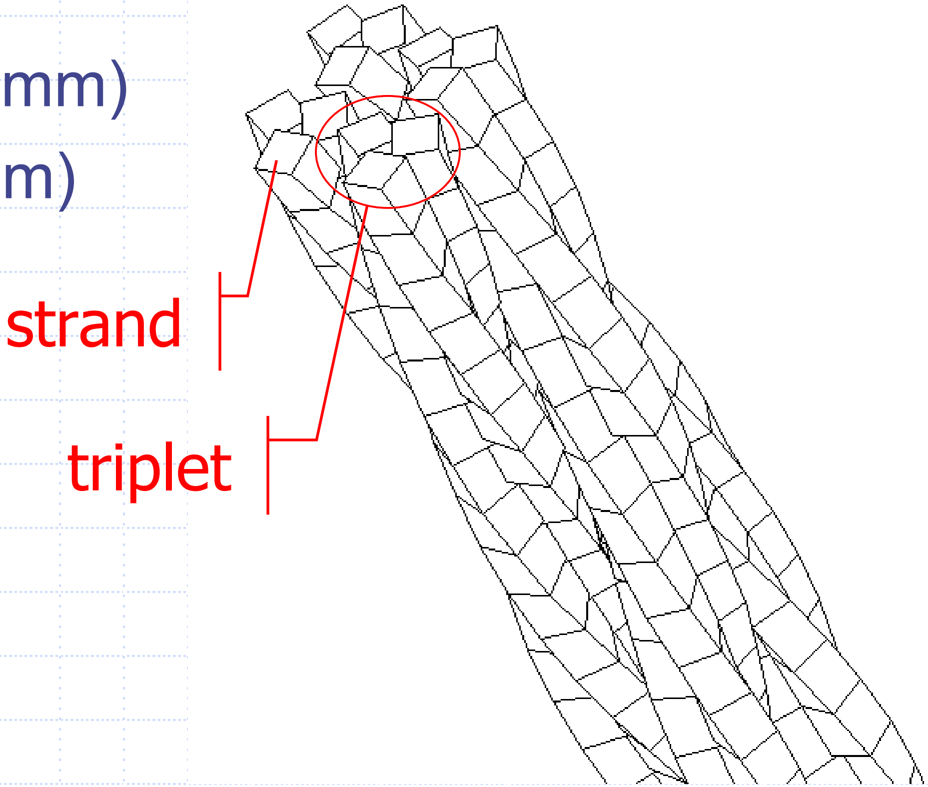




Cable model example - 2

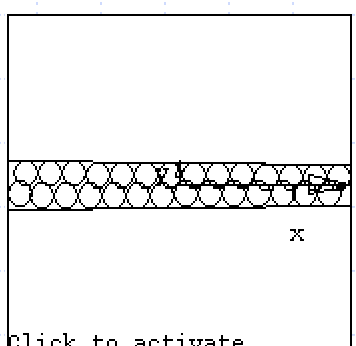
- model of a 3 x 4 cable (first two stages of an ITER cable)
  - D_{strand} = 0.81 (mm)
  - L_p = 25, 54 (mm)

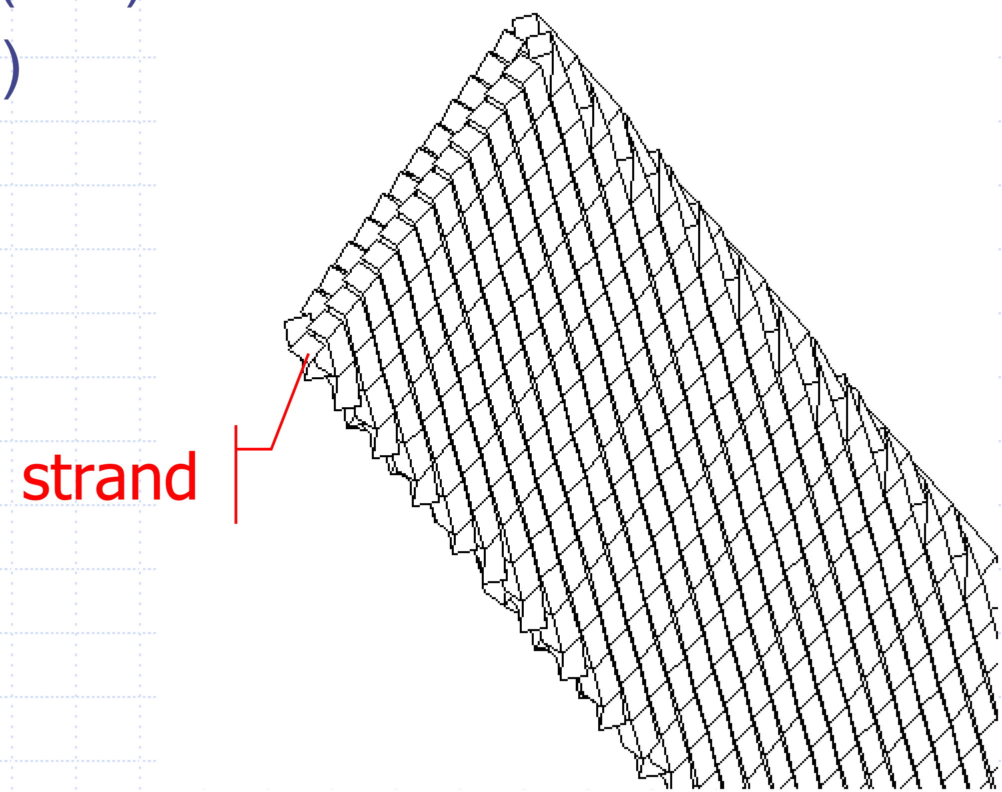




Cable model example - 3

- model of an inner LHC Rutherford cable
  - D_{strand} = 1.06 (mm)
  - L_p = 115 (mm)
  - w = 15 mm
  - t_I = 1.72 mm
  - t_o = 2.05 mm





Inductance calculation - 1

- definition of the inductance coefficient L_{ij}:

$$L_{ij} = \frac{\mu_0}{4\pi} \int_{Vi} \int_{Vj} \frac{\mathbf{j}_i \bullet \mathbf{j}_j}{\left|\mathbf{r}_{PQ}\right|} dV_j dV_i$$

j: current density such that strand current is 1 A

- using the scalar vector potential A_j:

$$\mathbf{A}_j = \frac{\mu_0}{4\pi} \int_{Vj} \frac{\mathbf{j}_j}{\left|\mathbf{r}_{PQ}\right|} dV_j$$

- the inductance is given by:

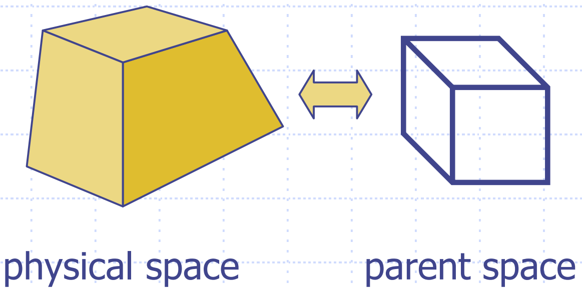
$$L_{ij} = \int_{Vi} \mathbf{j}_i \bullet \mathbf{A}_j dV_i$$

Inductance calculation - 2

- the volume integral is performed over all GCE's:

$$L_{ij} = \int_{Vi} \mathbf{j}_i \bullet \mathbf{A}_j dV_i = \sum_{Ni} \int_{V_{Ni}} \mathbf{j}_i \bullet \mathbf{A}_j dV_{Ni}$$

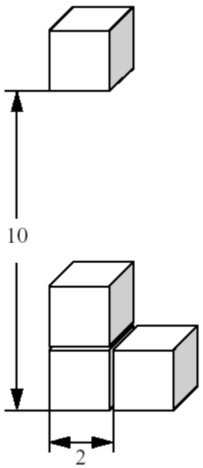
- using Gaussian integration in parent space



$$\int_{V_{Ni}} \mathbf{j}_i \bullet \mathbf{A}_j dV_{Ni} = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} (\mathbf{j}_i \bullet \mathbf{A}_j) D \, d\xi \, d\eta \, d\zeta$$

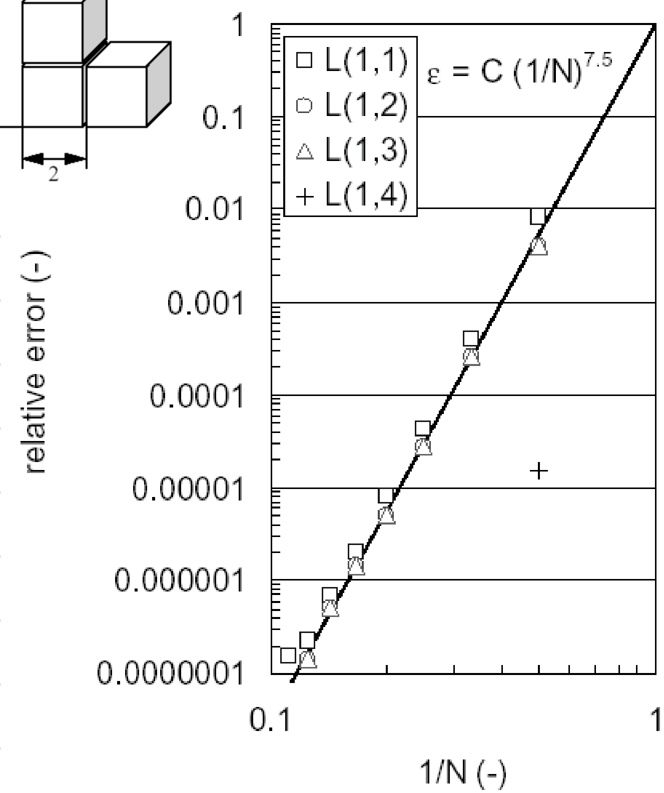
Inductance calculation - 3

- the number of Gauss points N used for numerical integration is chosen adaptively, based on the error estimate
- numerical convergence is accelerated through extrapolation to N→∞ (Richardson *deferred approach to the limit*)



$$u_N^\infty \approx u_N - C\left(\frac{1}{N}\right)^R$$

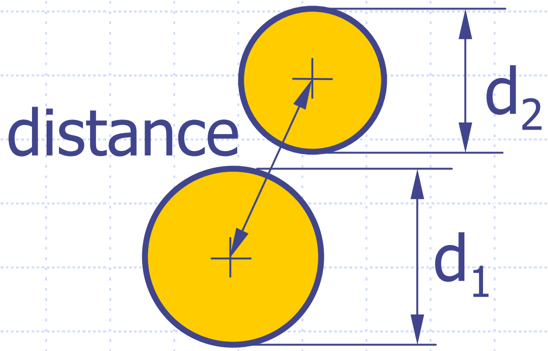
R ≈ 7...8



Conductance calculation - contact

- simple contact model for neighbouring strands. Contact is defined when:

$$\text{distance} < FF*(d_1+d_2)/2$$



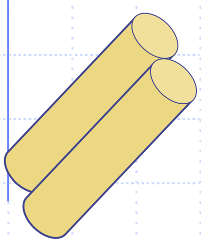
- FF is a *fudge factor* ≈ 1 :

Line and cross contacts

- different types of contacts are possible:
  - line contact (R_{line}) if both:

    $d_{upper} < FF*(d_1+d_2)/2$

    $d_{lower} < FF*(d_1+d_2)/2$



  - cross contact (R_{cross}) if either:

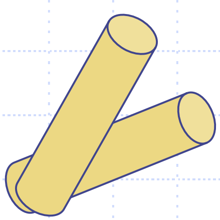
    $d_{upper} > FF*(d_1+d_2)/2$

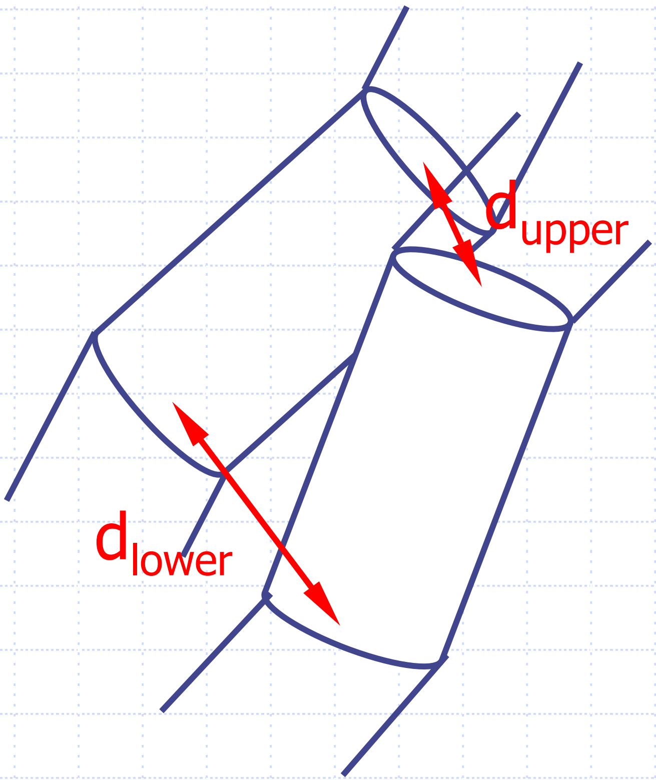
    $d_{lower} < FF*(d_1+d_2)/2$

    or:

    $d_{upper} < FF*(d_1+d_2)/2$

    $d_{lower} > FF*(d_1+d_2)/2$





Conductance matrix calculation

- total conductance matrix is given by:

$$g_{ij} = \frac{1}{L}\left(N_{cross}G_{cross} + N_{line}g_{line}\frac{L}{N_z}\right)$$

number of *cross* contacts

number of *line* contacts

entry in g matrix (S/m)

GCE length

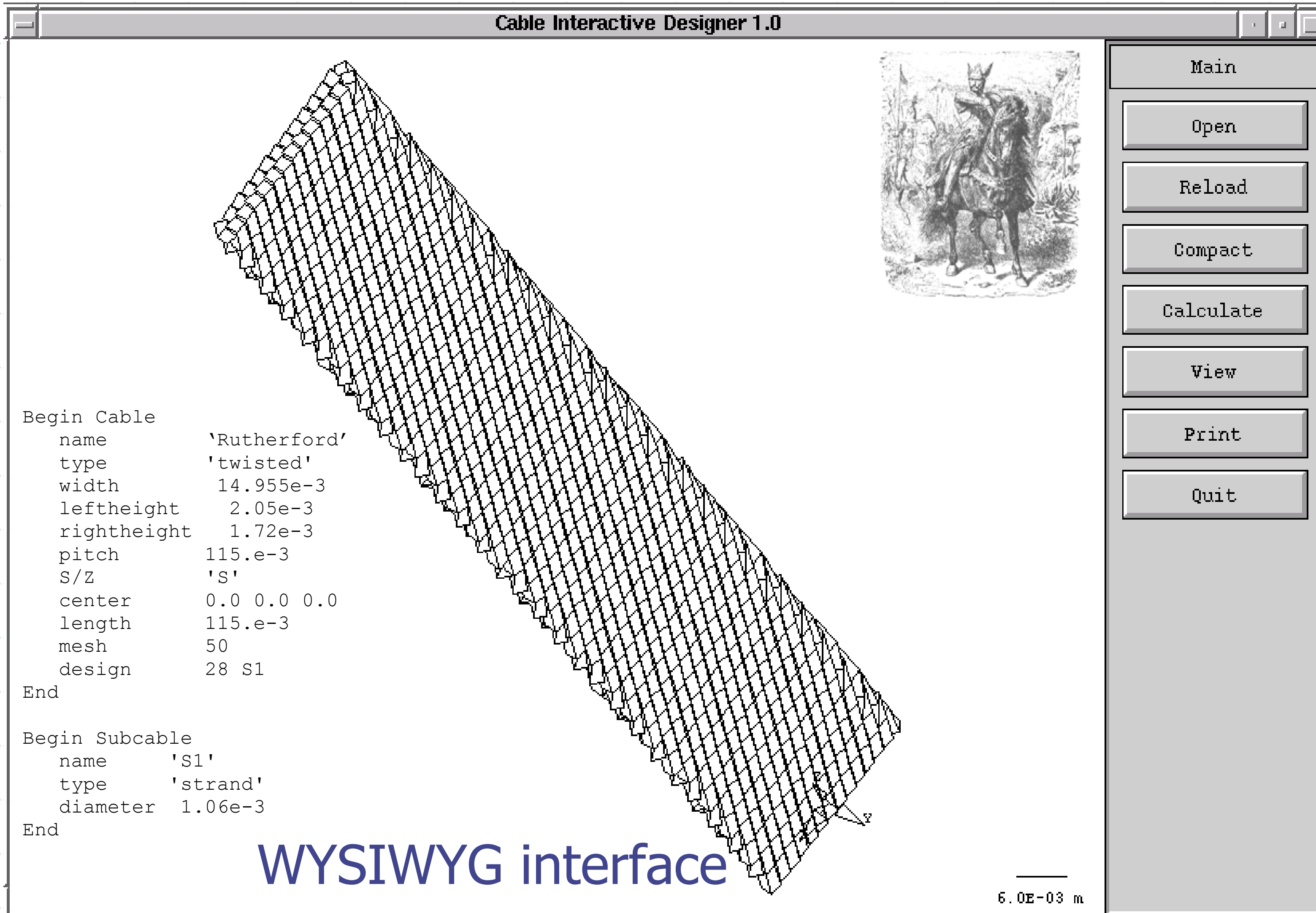
cable length

cross resistance per contact (S)

line conductance per unit length (S/m)

Model (CID v1.0)

event-driven GUI for:
- model loading
- display
- calculations



WYSIWYG interface

Model validation - 1

- comparison of numerical calculations to inductance calculation performed via a 6-D adaptive integration (Gauss method):

$$L_{ij} = \frac{\mu_0}{4\pi} \int_{Vi} \int_{Vj} \frac{\mathbf{j}_i \bullet \mathbf{j}_j}{\left|\mathbf{r}_{PQ}\right|} dV_j dV_i$$

$$L_{ii} = \frac{\mu_0}{4\pi} \int_{Vi} \int_{Vj} \frac{\mathbf{j}_i \bullet \mathbf{j}_j}{\left|\mathbf{r}_{PQ}\right| + \varepsilon} dV_i dV_i$$

regularization parameter for self inductance
$\varepsilon < 10^{-9}$

- and it can be shown that:

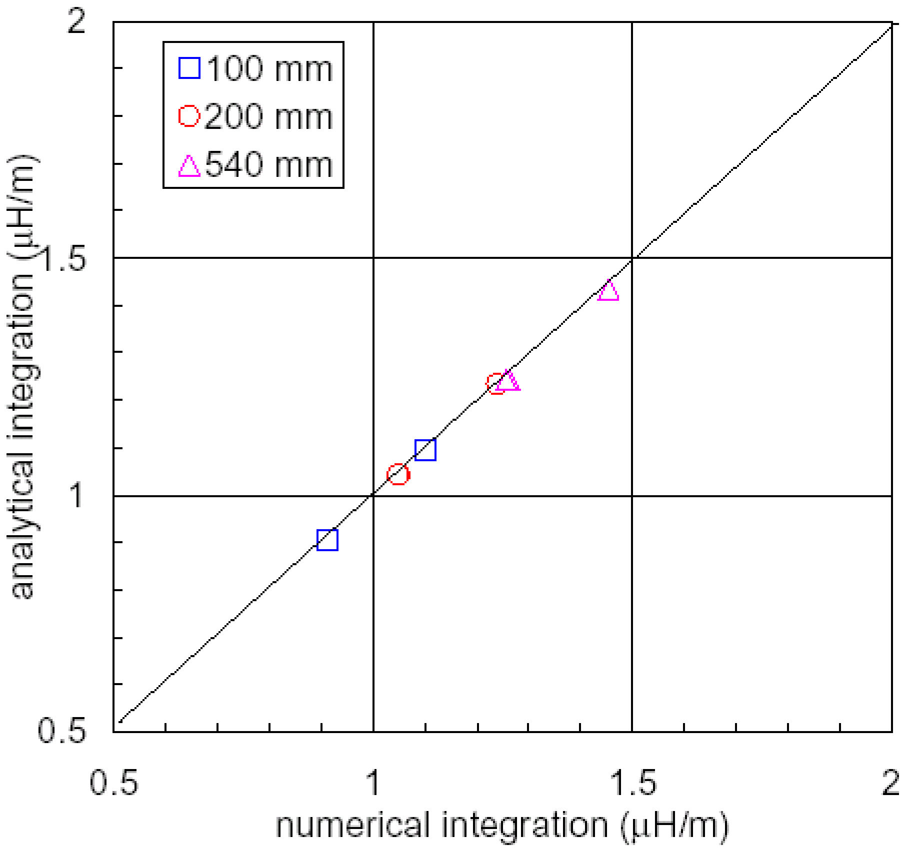
$$\lim_{\varepsilon \to 0} \int_{Vi} \int_{Vj} \frac{\mathbf{j}_i \bullet \mathbf{j}_j}{\left|\mathbf{r}_{PQ}\right| + \varepsilon} dV_i dV_i = \int_{Vi} \int_{Vj} \frac{\mathbf{j}_i \bullet \mathbf{j}_j}{\left|\mathbf{r}_{PQ}\right|} dV_i dV_i$$

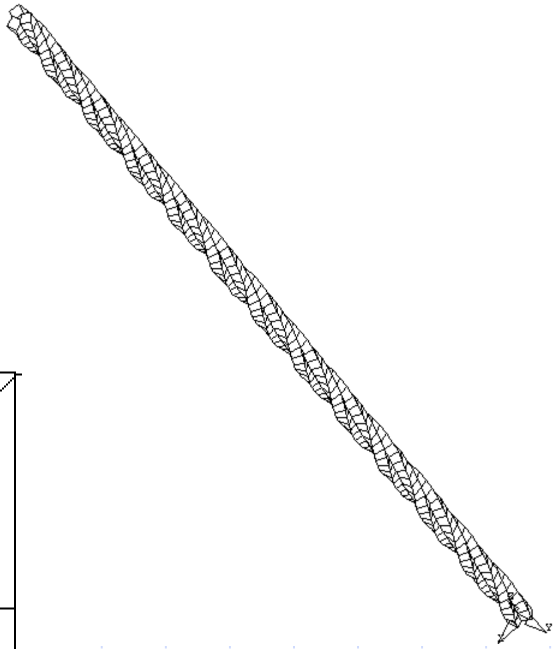
Model validation - 2

- *analytical* calculation of strand geometry and coordinate transformation Jacobians
- strand have *round cross section*

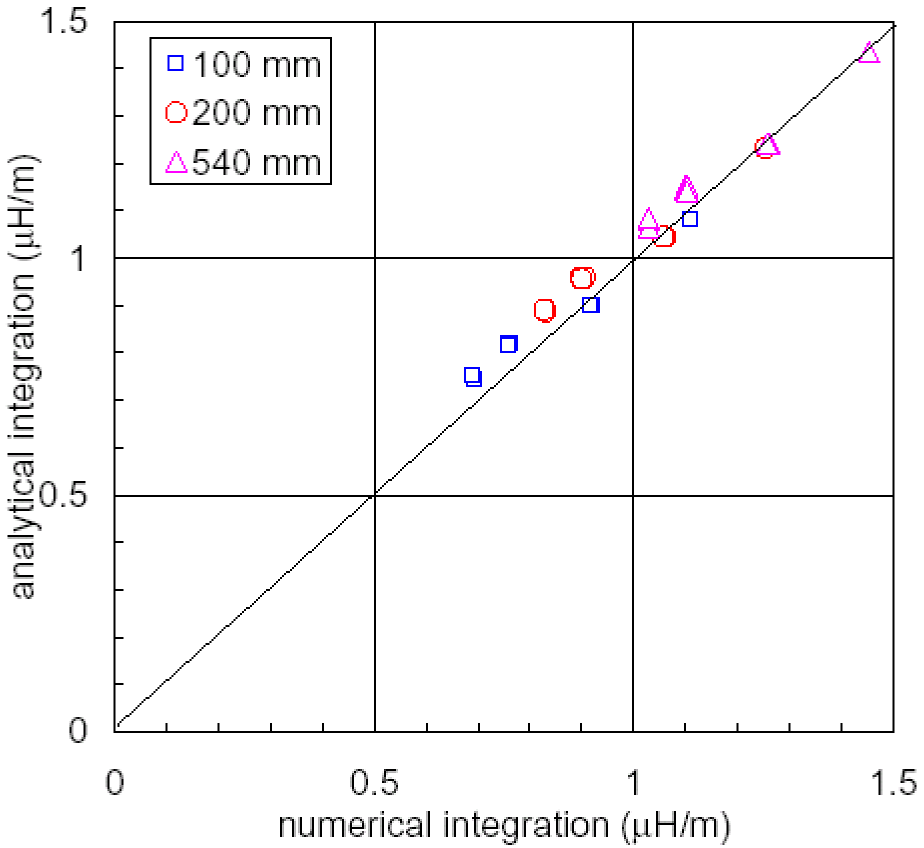
- see for details:
  - M. Breschi, “Current Distribution in Multistrand Superconducting Cables”, Ph.D. Thesis, University of Bologna, Italy.
  - M Fabbri, P.L. Ribani, “A priori error bounds on potentials, fields, and energies evaluated with a modified kernel”, IEEE Transactions on Magnetics, 37, 4 (2) , pp. 2970 –2976, 2001.

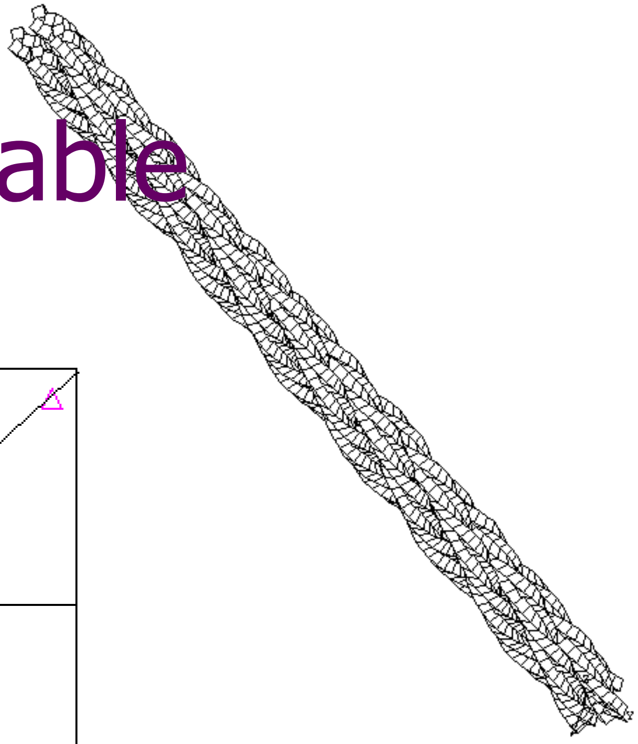
An ITER-like triplet



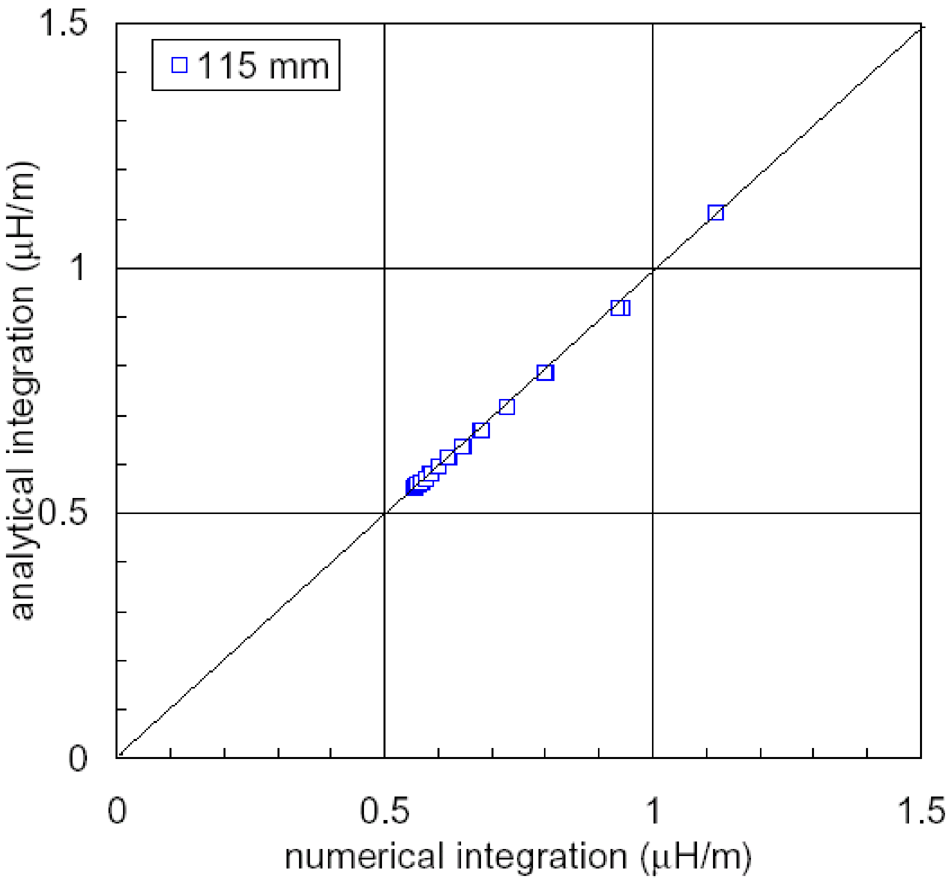


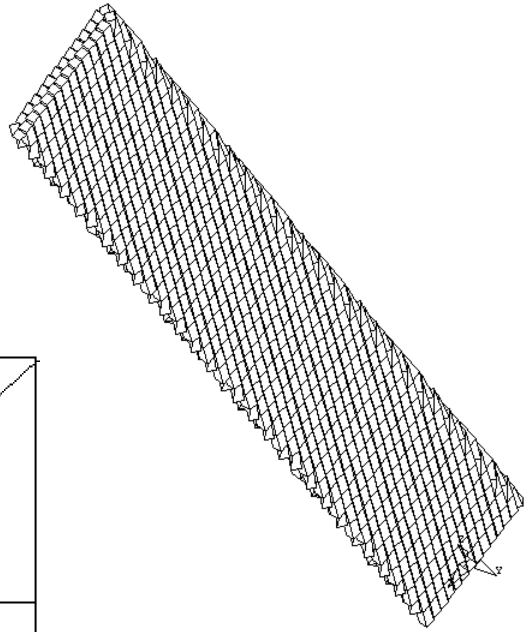
An ITER-like 3x4 subcable



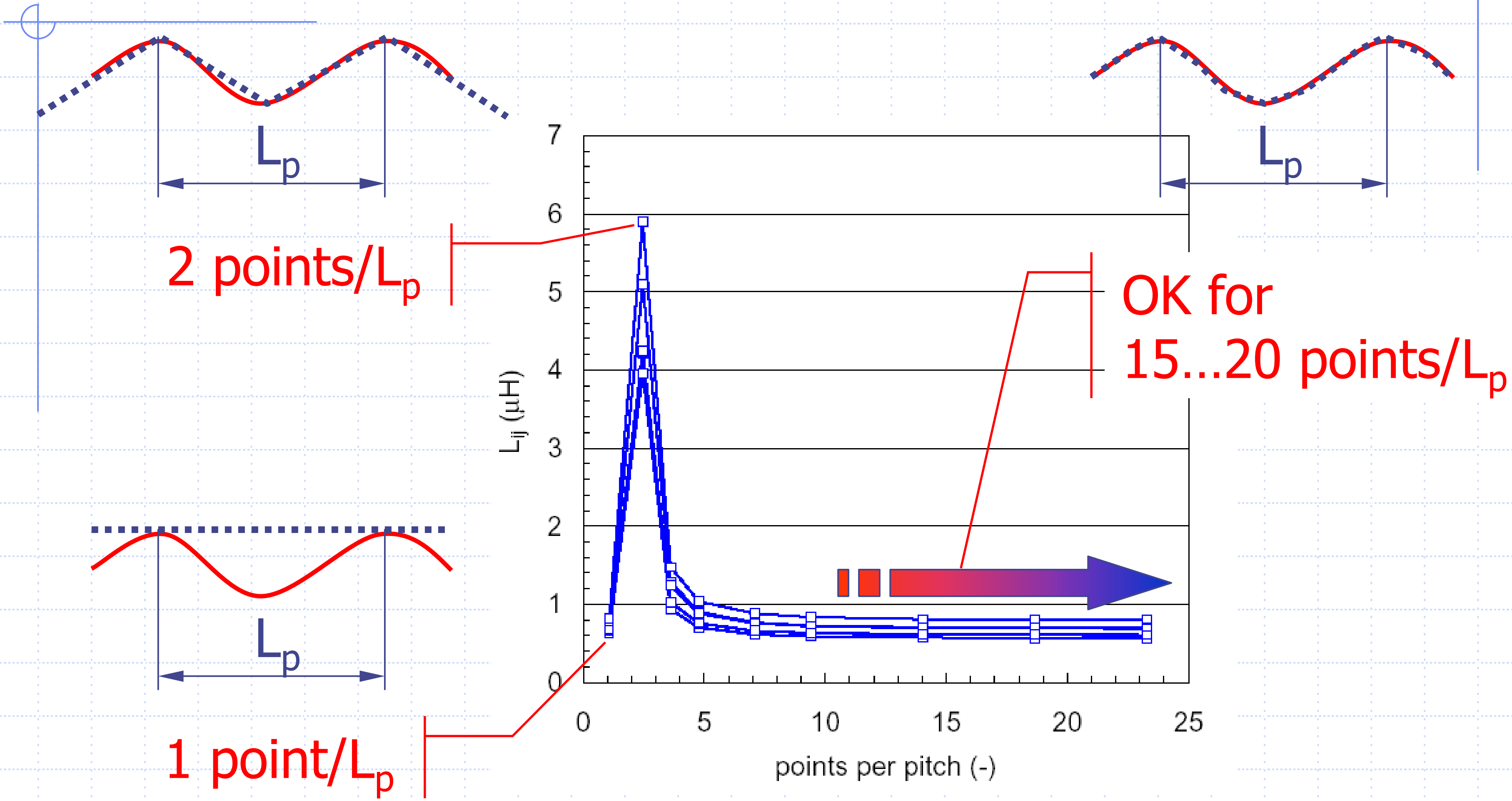


An LHC Inner Cable





Discretization error



More than ≈ 15 points/L_p are necessary to achieve good accuracy

Return line inductance λ_{ij}

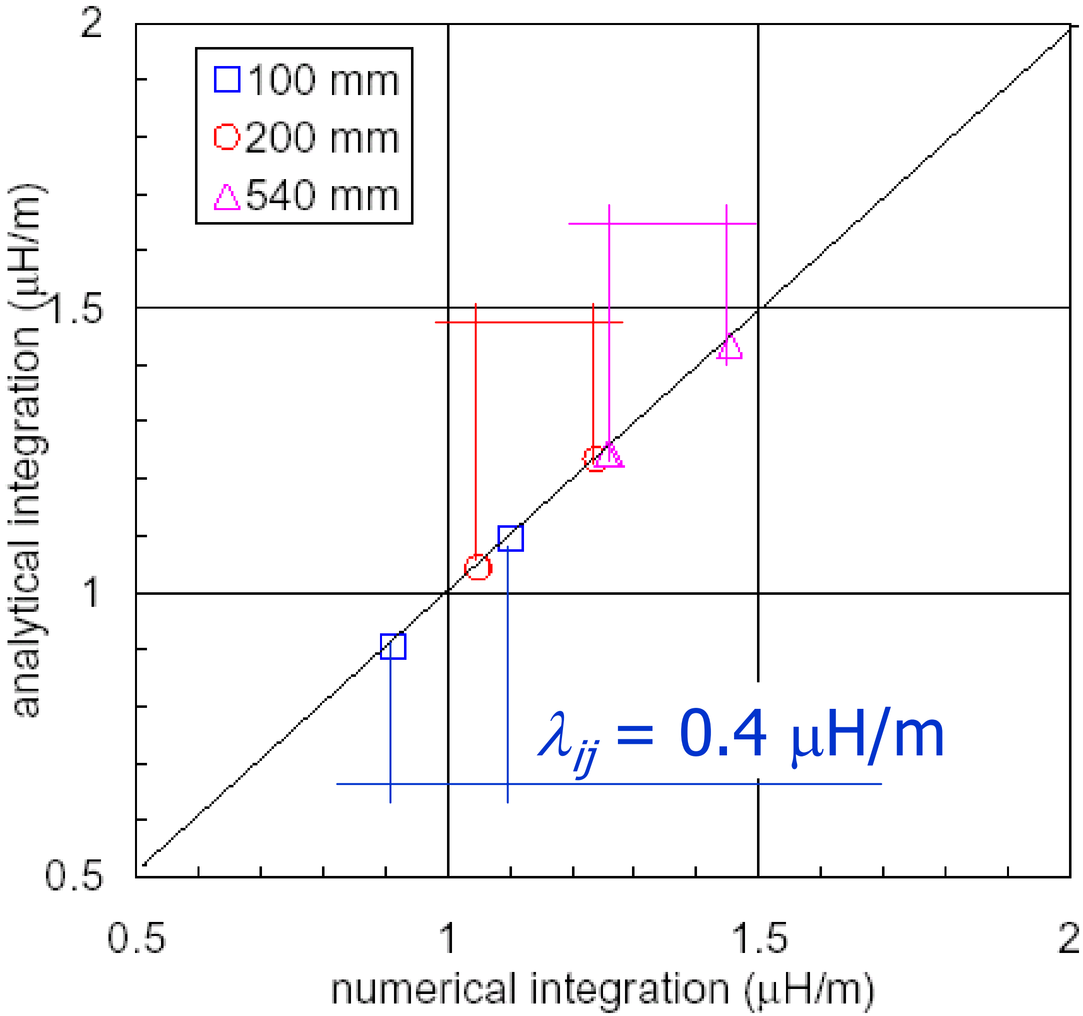
- the cable characteristics are determined by the difference of inductances:

$$\lambda_{ij} = 2\left(l_{ii} - l_{ij}\right)$$

- λ_{ij} is the *return line inductance* for the couple of strands $i \ldots j$, and governs the cable time constant(s):

$$\tau = N\left(l - m\right)g\left(\frac{L}{\pi}\right)^2$$

Accurate calculation of λ_{ij}

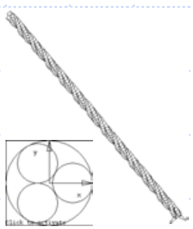


the calculation of few (2…3) L_p is already enough to characterize the cable return line inductance

Multi-stage cables λ_{ij}

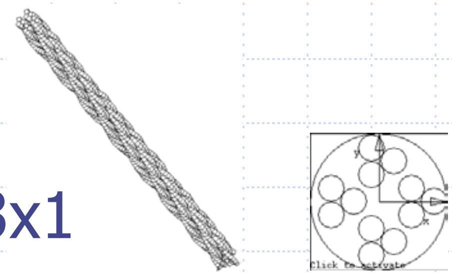
in multi-stage cables λ_{ij} grows steadily with the number of strands: careful with estimates !

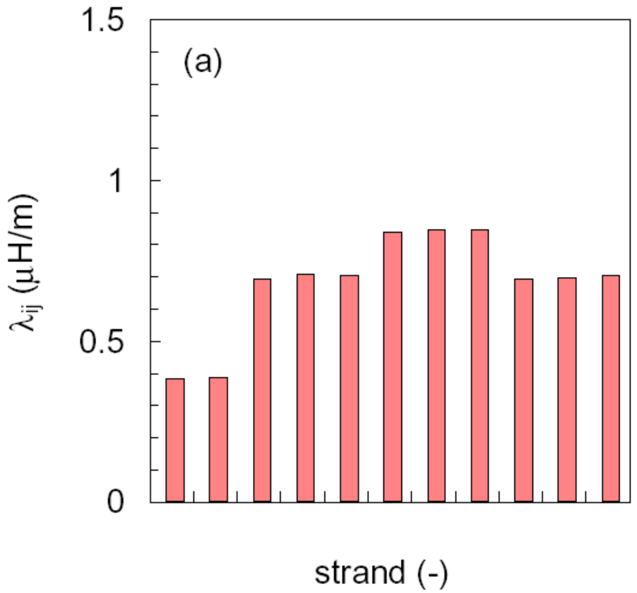
3x1



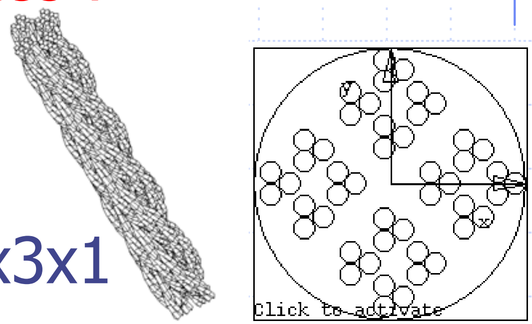
λ_{ij} = 0.4 μH/m

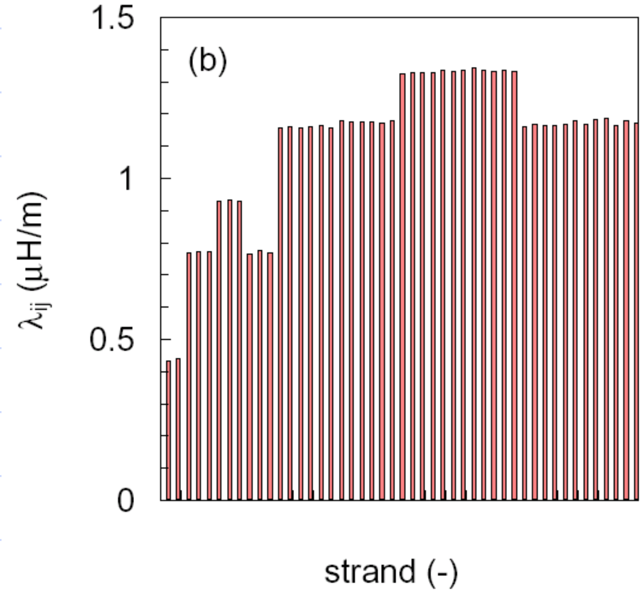
4x3x1



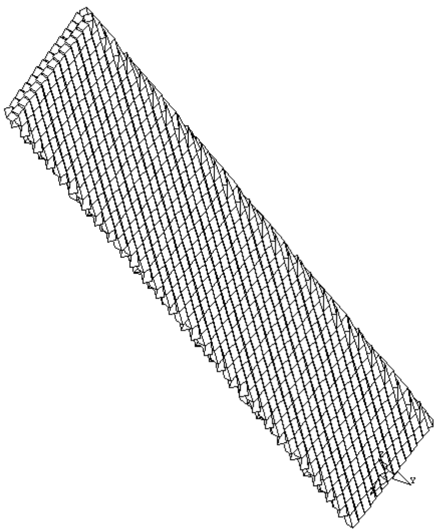


4x4x3x1

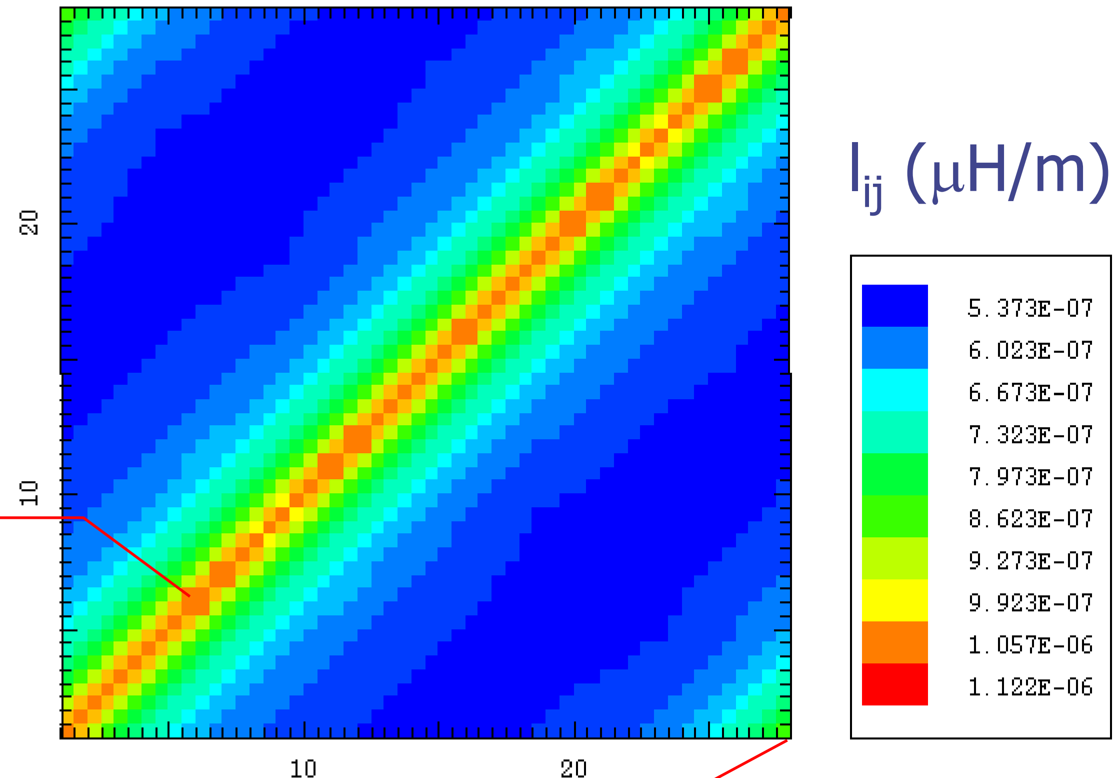




Rutherford cables – l_{ij} matrix



inductance matrix l_{ij}



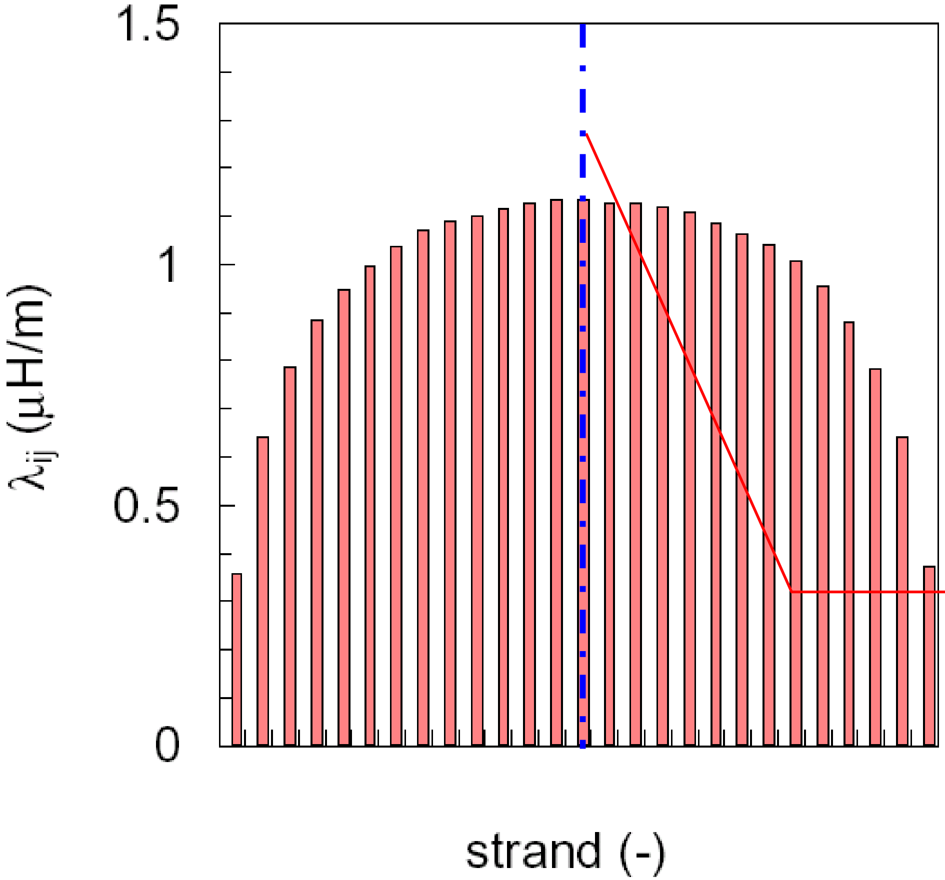
l_{ij} (μH/m)

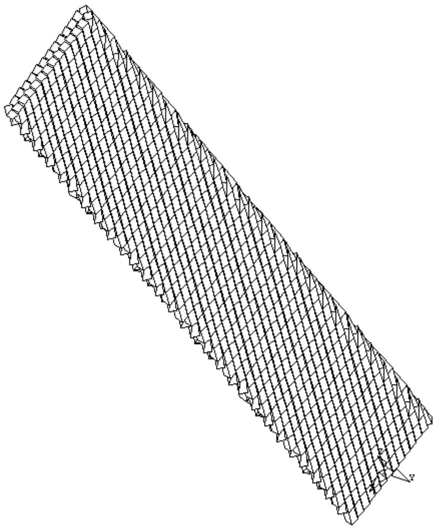
self inductance

periodicity in the mutual inductances

Rutherford cables λ_{ij}

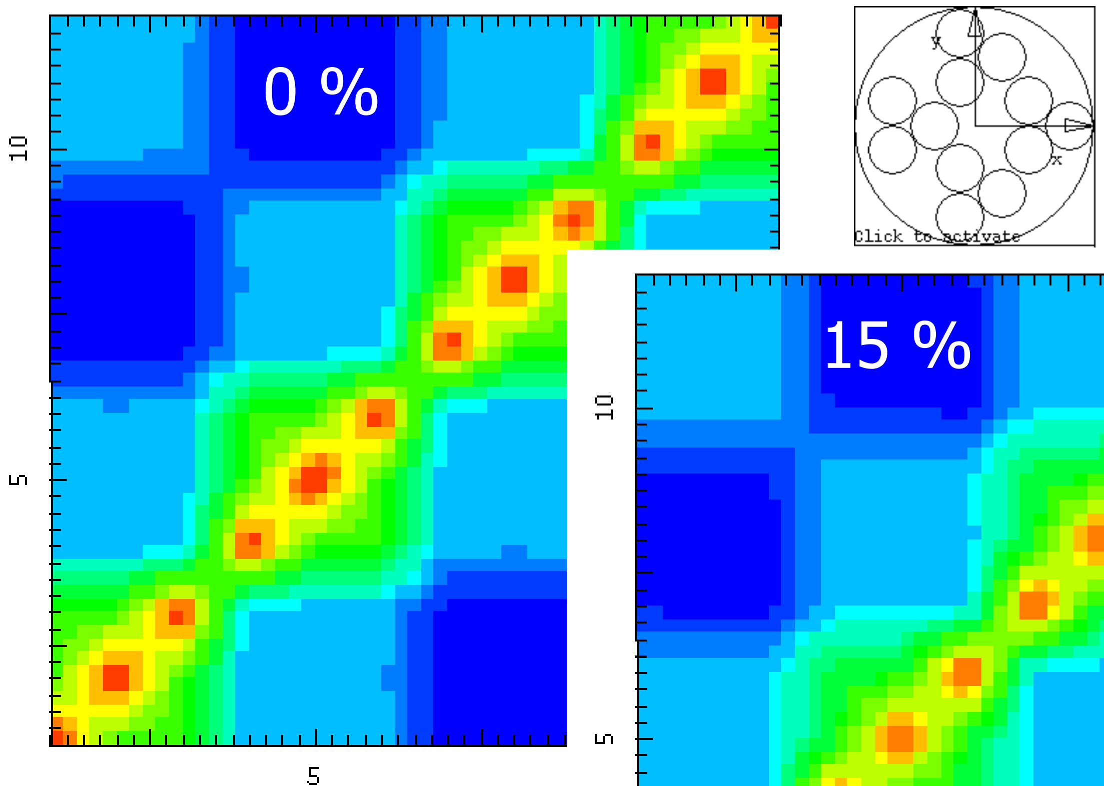
LHC inner cable has a large spectrum of λ_{ij} values are in the range of 1 μH/m

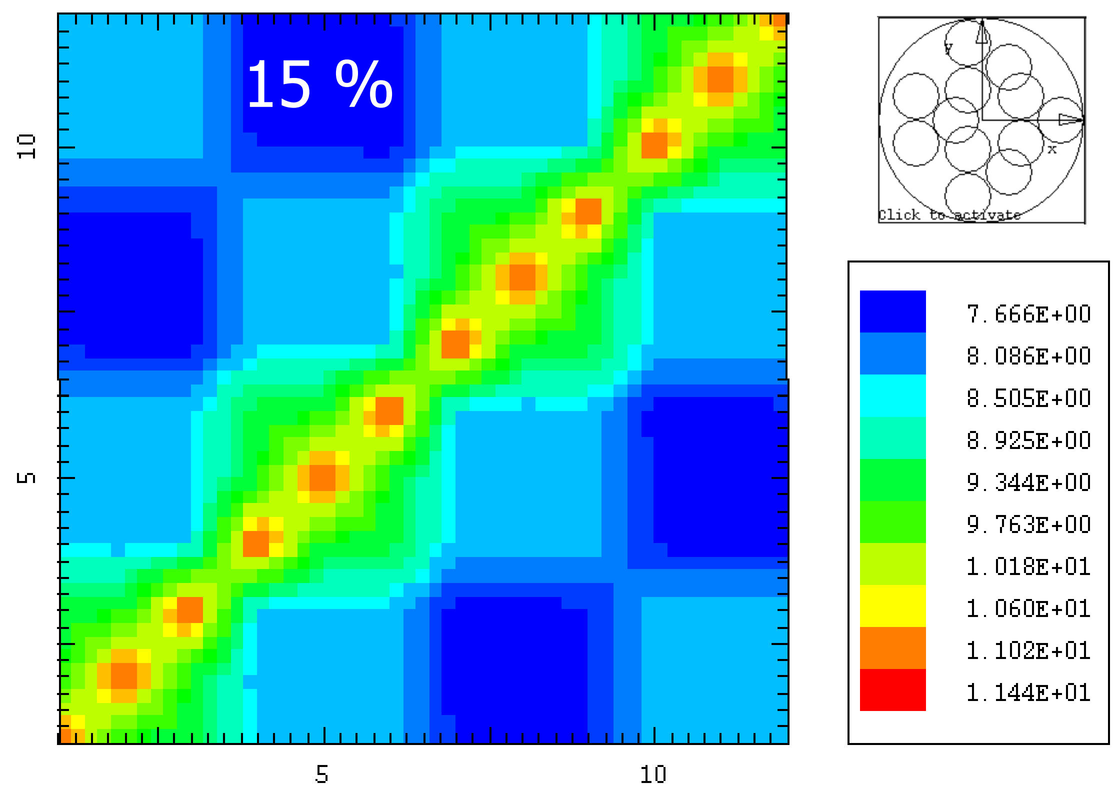




symmetry is associated with periodicity in the mutual inductances

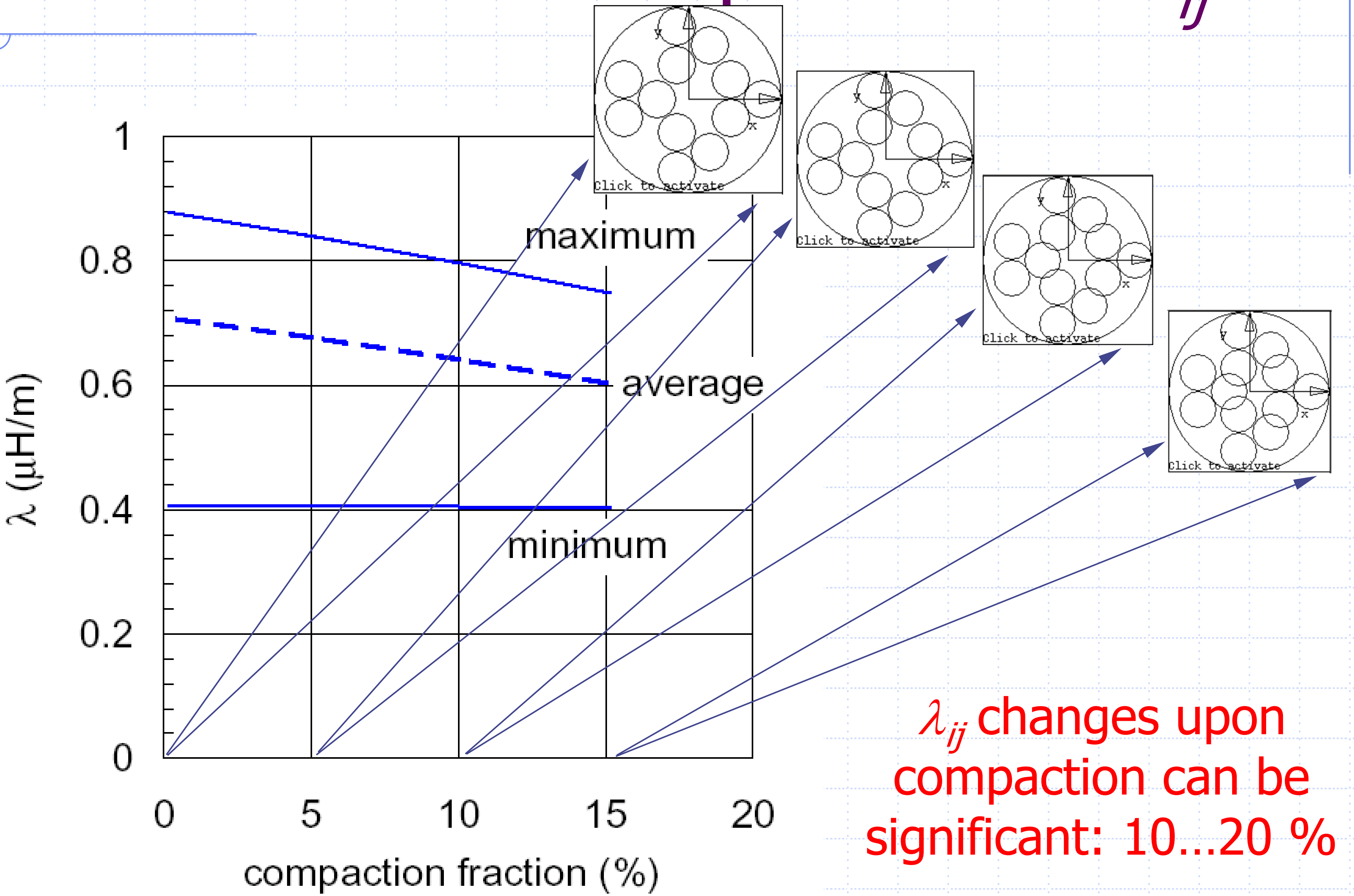
Effect of cable compaction on I_{ij}





no apparent effect…

Effect of cable compaction on λ_{ij}



λ_{ij} changes upon compaction can be significant: 10…20 %

Return line conductance γ_i

- total conductance for the current flowing in strand i *and returning in all other strands* :

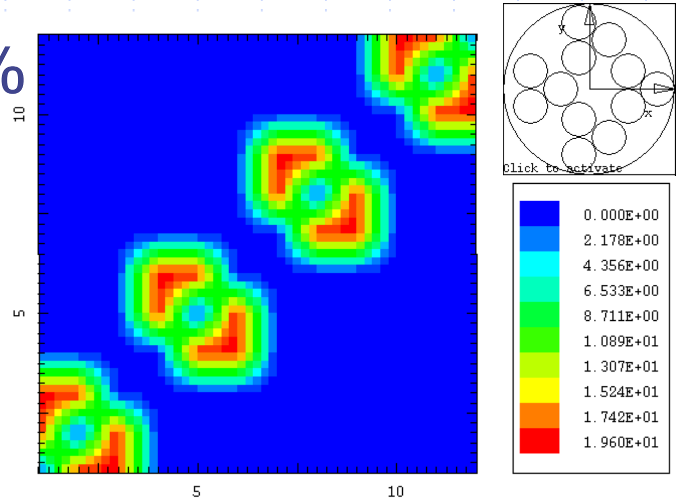
$$\gamma_i = \sum_{j=1}^{N_{strands}} g_{ij}$$

- governs the induced current intensity *and* cable time constant(s):

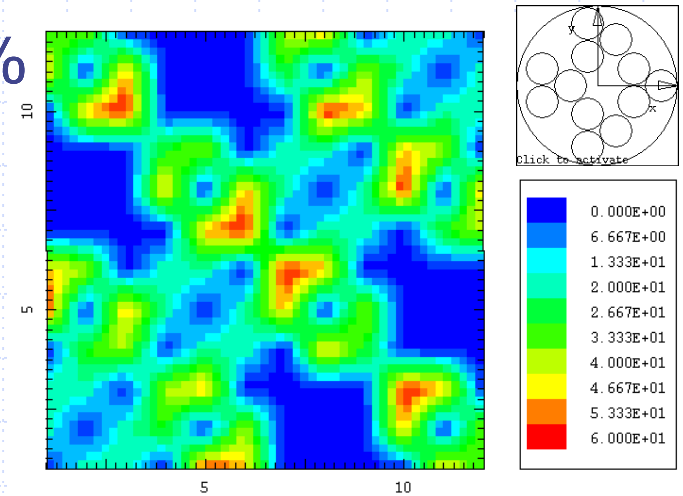
$$I = (N-1)\frac{wgV^{ext}}{2} \qquad \tau = N(l-m)g\left(\frac{L}{\pi}\right)^2$$

Effect of cable compaction – g_{ij}

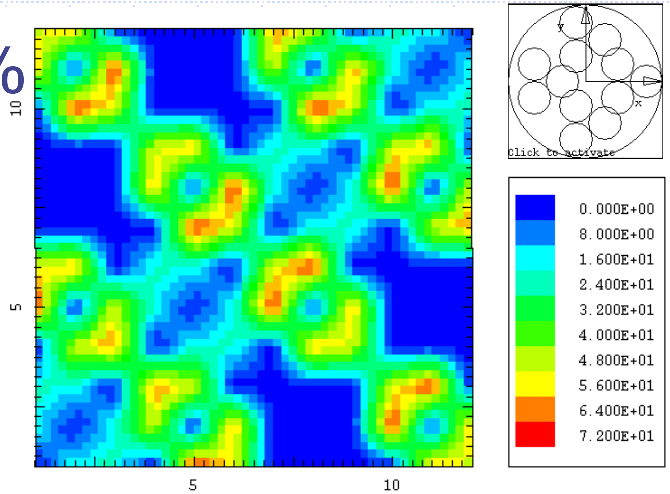
0 %



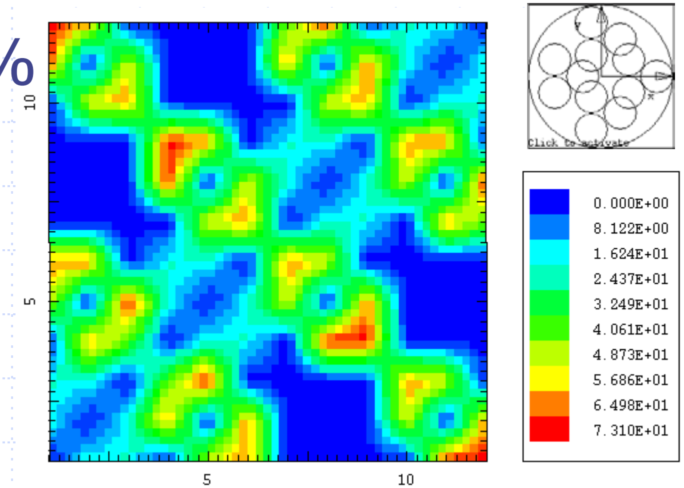
5 %



10 %



15 %

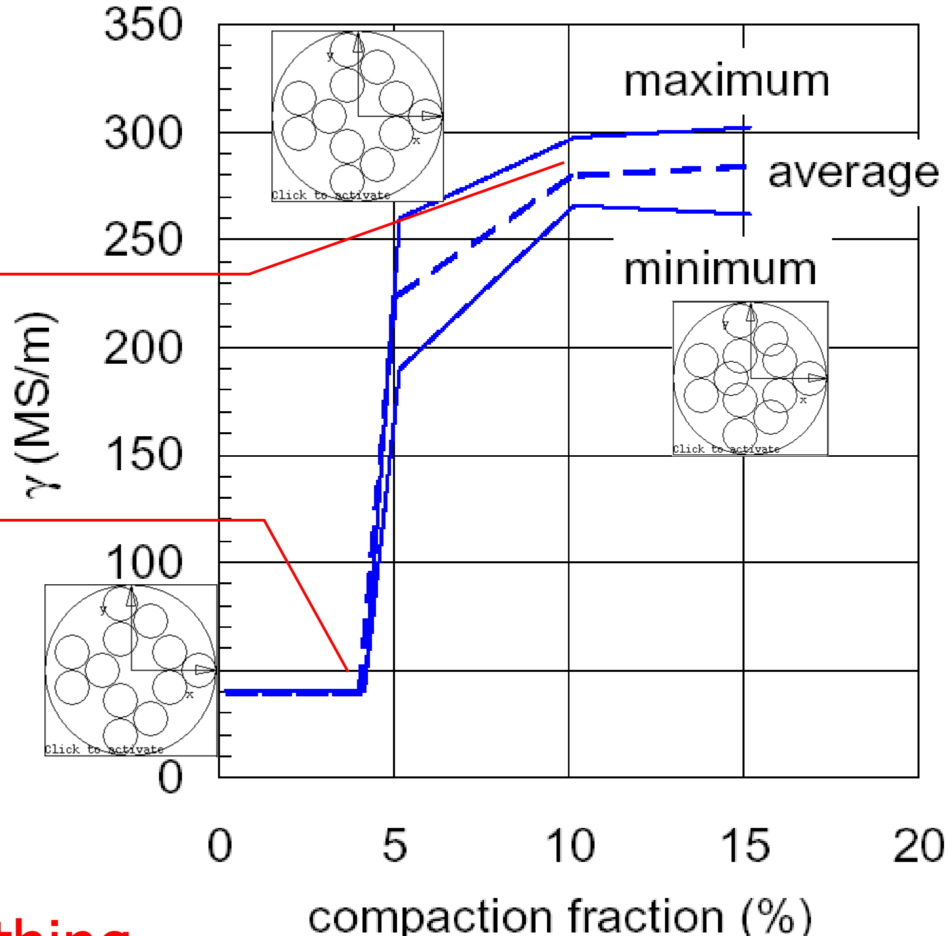


Effect of cable compaction – γ_i

- g_{line} = 20 MS/m
- G_{cross} = 1 MS

saturation at large compaction

large change at modest compaction



not necessarily physical, but already better than nothing…

Conclusions…

- model for the analysis of the electrical parameters in multi-stage cables with:
  - parametric, user's defined definition of the conductor
    - basic components (strands)
    - stage design
    - number of stages
    - twist pitch, twist direction (S/Z)
  - *practical* execution times (few seconds to few hours for the cases shown)
  - GUI for display of generated geometry

…more conclusions…

- useful to study the dependence of electrical parameters on the cable design…
  - cable size
  - cable compaction
  - overall differences among cable types (e.g. twisted vs. Rutherford)
- … and to compute *accurate* coupling parameters improving on available estimates

…and perspectives

- improve the cable compaction model
  - we are testing:
    - geometric compaction based on minimum interference
    - pseudo-mechanical model
- implement a current pattern reconstruction from measured signals (e.g. Hall plates, flux loops)
  - *influence matrix* calculation already available
- extend GUI for display of computed results