Analysis of the NHMFL 45-T Hybrid Magnet Thermal Behavior

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Outline

- Background
- Model description
- Numerical algorithm
- Model results and analysis
- Conclusions

Background

- NHMFL 45-T hybrid magnet outsert wound with CICC and cooled by static He II
- AC losses due to current ramp up produce regions of normal fluid (He I) within the coils
- Index heating that remains after completion of the ramp may cause thermal runaway
- Develop model to understand coil thermal behavior during and after current ramping
- Use model to explain observed coil behavior, before and after unprotected quench

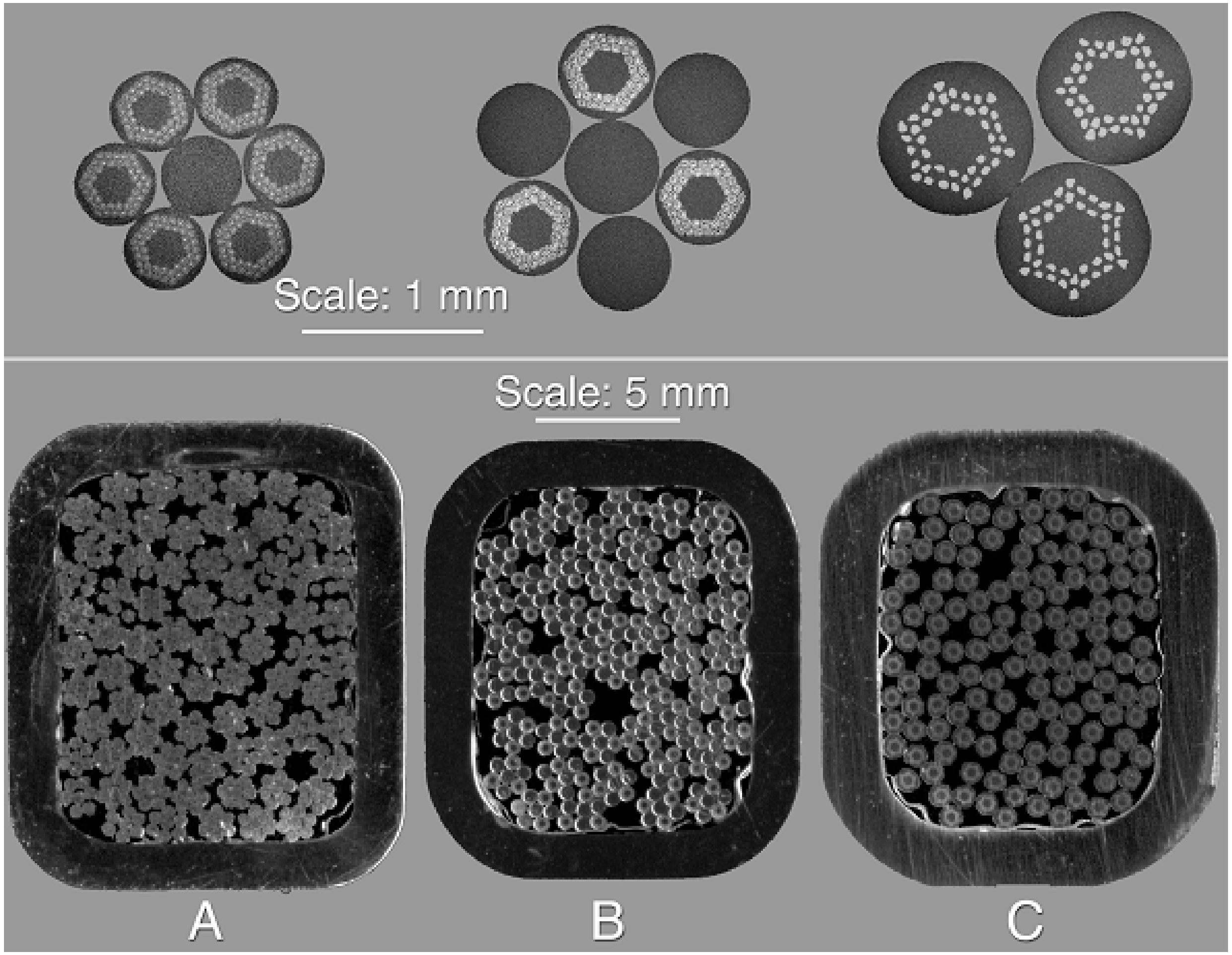
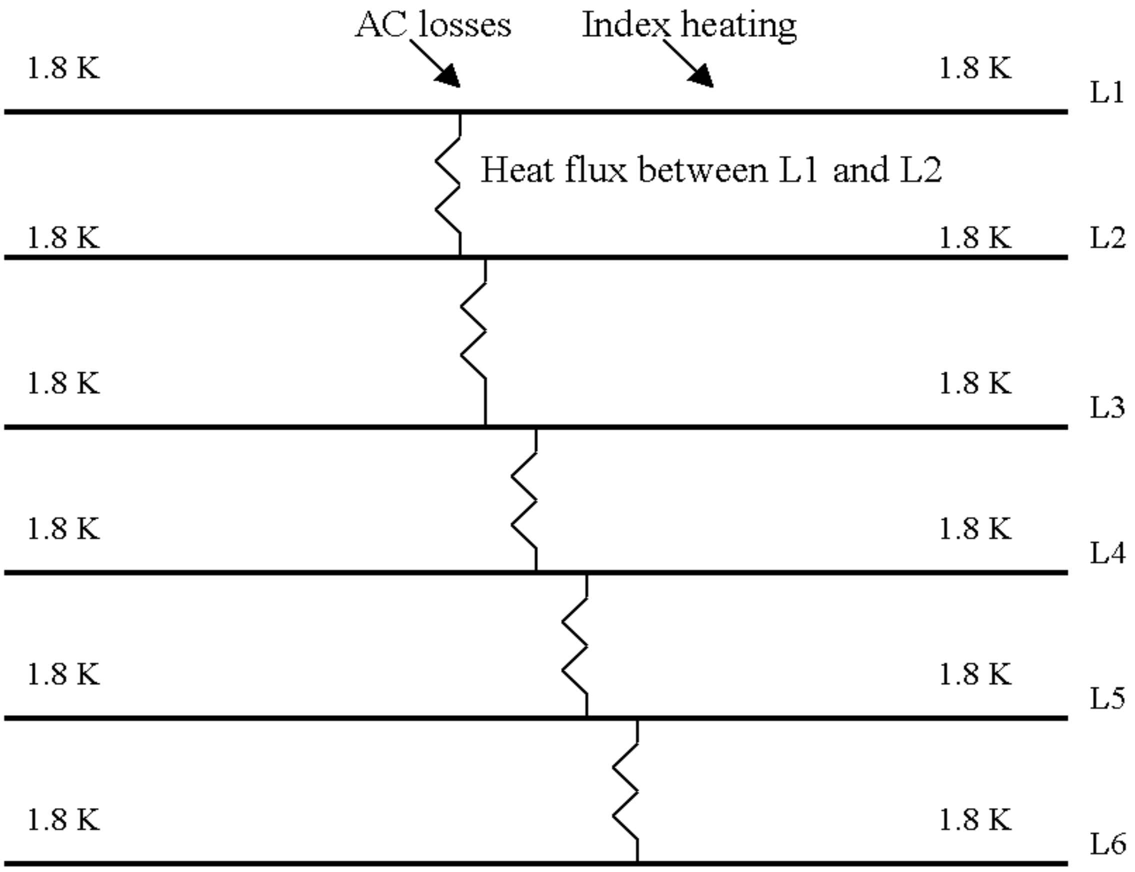


Figure 1. Cross sectional views of conductors used in 45-T system

Model Assumptions
(Heat Balance for Each Coil Layer)

- Thermal conduction along the metal in the CICC is neglected in the He II region
- Only helium heat capacity is considered in the He II region
- Channel (layer) end-temperature is clamped at 1.8K
- Helium, strands, and jacket are at the same temperature
- Liquid helium is static
- Helium thermal conductivity in the He I region is neglected, but copper thermal conductivity is included
- Layer-to-layer heat transfer is included (two steel jackets, one layer-to-layer epoxy insulation, Kapitza resistance for each interface)
- First and last layer adiabatic to the outside

Schematic of the one-and-a-half dimensional model



Governing Equation in the He II Region

- Gorter-Mellink Law:

$$q_x = -K(dT/dx)^{1/3} \qquad K = (f^{-1}(T,p))^{1/3}$$

- Layer heat balance:

$$\rho_{He} c_{He} \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[K\left(\frac{\partial T}{\partial x}\right)^{1/3} \right] + q_{in} + q_{AC} + q_{Across}$$

q_{in} is the index heating,

q_{AC} represents the AC losses

q_{Across} is the layer-to-layer heat transfer

Governing Equation in the He I Region

$$A'\rho'c'\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[A_{Cu}K_{Cu}\frac{\partial T}{\partial x}\right] + q_{in} + q_{AC} + q_{Across}$$

K_{Cu} the thermal conductivity of copper

$A'\rho'c'$ the equivalent heat capacity

$$A'\rho'c' = A_{He}(\rho\, c)_{He} + A_{Cu}(\rho\, c)_{Cu}$$

Boundary and Initial Conditions

Boundary conditions at the end of each layer:

$$T(0,t) = T(l,t) = T_b$$

T_b Clamped helium bath temperature (1.8 K)

Second boundary condition at moving He I/He II interface:

$$T(s(t), t) = T_\lambda$$

$x = s(t)$ Location of lambda transition (moving front)

T_λ Lambda temperature (2.176 K)

Initial condition:

$$T(x,0) = T_b$$

Stefan condition
(heat balance on the interface)

$$\dot{q}'' + K_{Cu}\left(\frac{\partial T}{\partial x}\right) = K\left(\frac{\partial T}{\partial x}\right)^{\frac{1}{3}}$$

$$\dot{q}'' = \rho_{He}\Delta c_{He}\frac{d\,s(t)}{d\,t}$$

We assume there are no heat sources at the interface. The first term on the left hand side is net heat flux from He I region. The right-hand side is the heat flux in the He-II region. Stefan condition is used to determine the position of the HeI/HeII interface (moving front), x=s(t)

Index heating

$$q_{in} = \frac{E_0 I}{A}\left(\frac{J_{op}}{J_c}\right)^n$$

J_{op} Operating current density

J_c Critical current density

E_0 Critical electric field =50 μV

$n \approx 15$ "index" number

AC Losses

AC losses =Hysteresis losses+cable coupling losses, with the corresponding coefficients benchmarked by design parameters and data during commissioning.

Coupling losses:

$$p_c = \frac{2\tau}{\mu_0}\dot{B}^2 A_{st}$$

τ The conductor effective time constant

A_{st} Total strands cross section in the cable

μ_0 The permeability of free space

AC Losses (cont'd)

Hysteresis losses* are given by piece-wise formulae:

$$p_h = \frac{2}{3\pi} J_c \dot{B} D_{eff} A_{s/c} \qquad B > B_p = \mu_0 J_c D_{eff}$$

$$p_h = \frac{2}{3\pi} J_c \dot{B} D_{eff} A_{s/c} \frac{3B^2}{B_p^2} \qquad B < B_p = \mu_0 J_c D_{eff}$$

D_{eff}	The superconductor filament effective diameter
$A_{s/c}$	Total superconductor cross section in the cable
J_c	Superconductor critical current density

* Hysteresis losses are the dominant mechanism for this coil

Equivalent heat transfer coefficient
(Layer-to-Layer)

The equivalent heat transfer coefficient is given by:

$$\frac{1}{h_{eq}} = \frac{\ell_{ss}}{k_{ss}} + \frac{\ell_{glass-epoxy}}{k_{glass-epoxy}} + \sum_i \frac{1}{h_k}$$

k_{ss}	Thermal conductivity of stainless steel
$k_{glass-epoxy}$	Thermal conductivity of glass-epoxy insulation
h_k	The Kapitza conductance at each interface

There are eight Kapitza resistances between two adjacent layers

Layer-to-layer heat transfer

Layer-to-layer heat flux is given by:

$$\dot{q}''_L = -h_{eq}\left(T_{L+1} - T_L\right) \quad W/m^2$$

Heat source term to layer L due to layer-to-layer heat transfer:

$$q_{cross,L} = \left(\dot{q}''_{L-1} - \dot{q}''_L\right)/S_{cond,r} \quad W/m^3$$

$\dot{q}''_L$	The heat flux between L and L+1 layer
$q_{cross,L}$	Net heat flow to Layer L
$S_{cond,r}$	Cross section width of cable in the radial direction

We assume that L1 only exchanges heat with L2, and L6 only with L5 (adiabatic sides)

Final Remarks on the Model

- One and a half dimensional model
- Helium properties evaluated with HEPACK
- Stefan condition is used to evaluate the moving boundary position (He I/He II clamped at T-lambda)
- Discontinuity at the HeI/HeII front determines necessary space/time increments

Numerical Solution of the Governing Equations

MacCormack scheme (He II region)

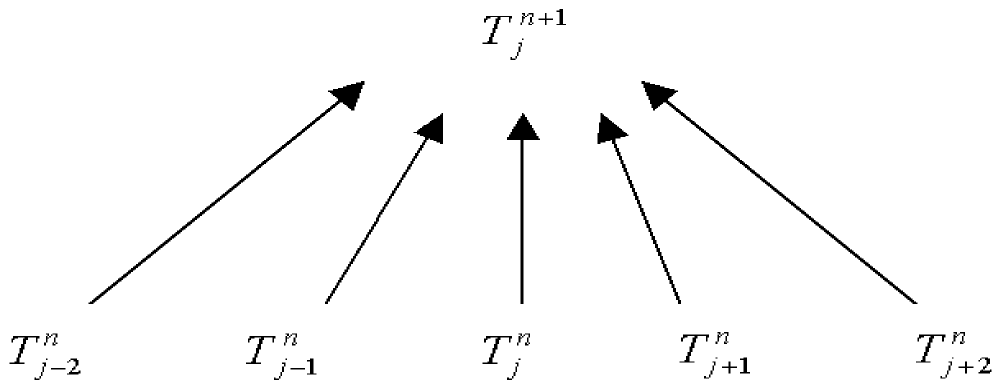
MacCormack explicit scheme to deal with He-II regime:

$$q_i^n = q\left(T_i^n\right) = \frac{K\left(T_{i-1}^n\right) + K\left(T_i^n\right)}{2\left(\Delta x\right)^{\frac{1}{3}}}\left(T_i^n - T_{i-1}^n\right)^{\frac{1}{3}}$$

Where K is counterflow thermal conductivity, which depends on temperature

MacCormack scheme consists of one prediction process and one correction process (two forward and two backward)

MacCormack scheme (stencil)



MacCormack scheme five-point stencil

MacCormack scheme
(implementation)

Prediction process:

$$\widetilde{T}_i = T_i^n + \frac{1}{(\rho c)_i^n} \frac{\Delta t}{\Delta x}\left(q_{i+1}^n - q_i^n\right) + f_i^n \Delta t$$

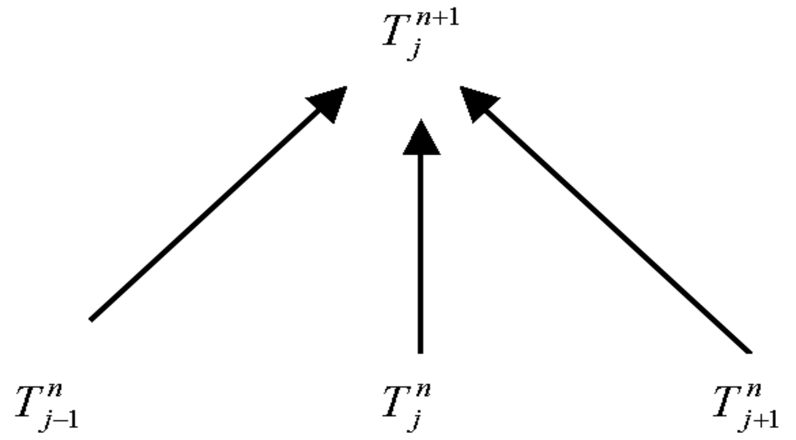
Correction process:

$$\widetilde{q}_i = q\left(\widetilde{T}_i\right) = \frac{K\left(\widetilde{T}_i\right) + K\left(\widetilde{T}_{i+1}\right)}{2(\Delta x)^{\frac{1}{3}}}\left(\widetilde{T}_{i+1} - \widetilde{T}_i\right)^{\frac{1}{3}}$$

Hence,

$$T_i^{n+1} = T_i^n + \frac{1}{(\rho c)_i^n} \frac{\Delta t}{2\Delta x}\left(q_{i+1}^n - q_i^n + \widetilde{q}_i - \widetilde{q}_{i-1}\right) + f_i^n \Delta t$$

Semi-implicit (Crank-Nicolson) scheme (He I region)



Semi-implicit scheme three-point stencil

Semi-implicit scheme
(implementation)

Semi-implicit (Crank-Nicolson) scheme is used to solve He-I regime:

$$\gamma_j^n\left(T_j^{n+1}-T_j^n\right)=0.5\beta A_{Cu}\left[\delta\left(K_{cu}\delta T\right)\Big|_j^{n+1}+\delta\left(K_{cu}\delta T\right)\Big|_j^n\right]+f_j^n$$

Where

$$\gamma_j^n=\left(A'\rho'c'\right)_j^n \qquad \beta=\frac{\Delta t}{\left(\Delta x\right)^2}$$

$$(\delta T)_j=T_{j+\frac{1}{2}}-T_{j-\frac{1}{2}} \qquad \left(\delta^2 T\right)_j=T_{j+1}-2T_j+T_{j-1}$$

$$f_j^n=(q_{in}+q_{AC}+q_{Across})_j^n$$

Semi-implicit scheme
(coefficient matrix)

Rearranging the finite difference equations, we get a linear system:

$$\mathbf{A}\,[\mathbf{T}]=[\mathbf{H}]$$

Where coefficient matrix A is given by:

$$\begin{bmatrix} d^n_{j+1} & u^n_{j+2} & 0 & \cdots & 0 & 0 \\ s^n_{j+1} & d^n_{j+2} & u^n_{j+3} & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & s^n_{\ell-1} & d^n_{\ell} \end{bmatrix}$$

Semi-implicit scheme (tri-diagonal)

[T], [H] are the variable (temperature) vector and source term vector:

$$[\mathbf{T}] = \left[T_{j+1}^{n+1}, T_{j+2}^{n+1}, \cdots, T_{\ell-1}^{n+1}, T_{\ell}^{n+1}\right]^T$$

$$[\mathbf{H}] = \left[H_{j+1}^{n}, H_{j+2}^{n}, \cdots, H_{\ell-1}^{n}, H_{\ell}^{n}\right]^T$$

This is a typical tri-diagonal linear algebra system, which can be easily solved by Gauss elimination method

Boundary Conditions

MacCormack scheme needs five point stencil, hence two additional points (ghost points) are needed at each end :

$$T_{-1}^{n} = T_b \qquad T_{-2}^{n} = T_b$$

On the moving boundary (He I/He II interface):

$$T_{front}^{n} = T_{\lambda}$$

We need a procedure to track the evolution of the moving front

Moving boundary procedure

We adopt front-tracking method and fixed finite-difference grid to deal with He II /He I front problem.

In the proximity of the front only two grid points are considered

Three points are used to construct Lagrangian base functions.

The third point is the front position (unequal grid).

The form of the general function is given by:

$$T(x) = \sum_{j=0}^{2} \ell_j(x) T(x_j)$$

Front-tracking method

$\ell(x_j)$ is a Lagrangian base function:

$$\ell_j(x) = \frac{p_2(x)}{(x - x_j)\, p_2'(x)}$$

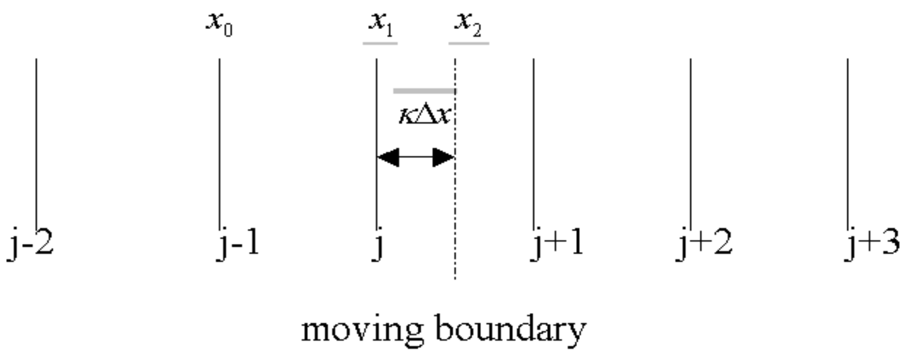
$$p_2(x) = (x - x_0)(x - x_1)(x - x_2)$$

where $p_2'(x_j)$ is its derivative with respect to x at the j-node

It follows that:

$$\frac{dT(x)}{dx} = \sum_{j=0}^{2} \ell'_j(x) T(x_j)$$

Front-tracking method (Cont'd)



Fixed grid nodes, front is between j and j+1 nodes.

To calculate the derivative at j-node three points j-1, j, and the front are used (in the left side of the front)

To calculate the derivative at j+1 node three points j+2, j+1,and the front are used (in the right side of the front)

Front-tracking method (Cont'd)

Then for x<s(t) we have

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_j^n = \frac{2}{(\Delta x)^2}\left(\frac{T_{j-1}^n}{\kappa+1} - \frac{T_j^n}{\kappa} + \frac{T_\lambda^n}{\kappa(\kappa+1)}\right)$$

$$\left(\frac{\partial T}{\partial x}\right)^n = \frac{2}{(\Delta x)}\left(\frac{\kappa T_{j-1}^n}{\kappa+1} - \frac{(\kappa+1)T_j^n}{\kappa} + \frac{(2\kappa+1)T_\lambda^n}{\kappa(\kappa+1)}\right)$$

Similarly we get the second-order derivative when $x > s(t)$ and the first-order derivative of temperature at the front

Finally, put these expressions into the Stefan condition (heat balance on the interface)

Final remarks on the numerical implementation

- In the He-II region the same process is used, except for the presence of a 1/3 exponent
- All equations are solved in non-dimensional form
- Time step varies between 10^-4 to 10^-2 sec
- Spatial mesh includes 200 elements per layer
- MacCormack scheme is second-order accurate in space and first-order in time, Crank-Nicolson scheme is second-order accurate in space and time

Model Results

- Effect of index heating and influence of n-value (pre and post-quench)
- Effect of layer-to-layer heat transfer (adiabatic vs. non-adiabatic layers)
- Influence of the resistive insert
- Thermal recovery following a ramp

Record of high-current operations, superconducting outsert

Date	Max. Current	Ramp time	Hold time	Result
9 Dec. 1999	10 kA	80 min.	40 min.	Dump to test protection circuit.
10 Dec. 1999	10 kA	100 min.	2 hr.	Combined test with RI, 44.2 T. Normal ramp down.
11 Dec. 1999	10 kA	80 min.	2 hr.	Normal ramp down.
12 Dec. 1999	10 kA	100 min.	7 hr. 30 min.	User service, normal ramp down.
12 Dec. 1999	10 kA	80 min.	17 min.	Quench, RI reversed.
22 June 2000	10 kA	120 min.	8 min.	Crowbar, VCL over voltage.
26 June 2000	10 kA	30 min.	2 hr. 30 min.	Combined test with RI, 45.2 T. Normal ramp down.
3 July 2000	10 kA	30 min.	50 min.	Normal ramp down.
6 July 2000	10 kA	30 min.	8 hr.	User service, normal ramp down.
7 July 2000	10 kA	30 min.	7 min.	Dump following RI trip. Quench?
10 July 2000	10 kA	30 min.	5 min.	Unprotected quench.
2 Aug. 2000	9.5 kA		0 min.	Slow ramp to assess damage. Quench.
4 Aug. 2000	9 kA	7 hr.	2 hr.	Slow ramp. Assess new operating margins. Normal ramp down.
11 Aug. 2000	8 kA	80 min.	20 min.	Normal ramp down.
21 Aug. 2000	8 kA	3 hr.	6 hr. 30 min.	Normal ramp down.
22 Aug. 2000	8 kA	70 min.	0 min.	Normal ramp down.
25 Aug. 2000	8 kA	70 min.	0 min.	Normal ramp down.
29 Aug. 2000	7.5 kA	60 min.	0 min.	Normal ramp down.
30 Aug. 2000	8 kA	70 min.	3 hr.	User service, normal ramp down.
1 Sept. 2000	8 kA	70 min.	0 min.	Normal ramp down.
13 Sept. 2000	8 kA	70 min.	2 hr.	Dump following RI trip. Quench?

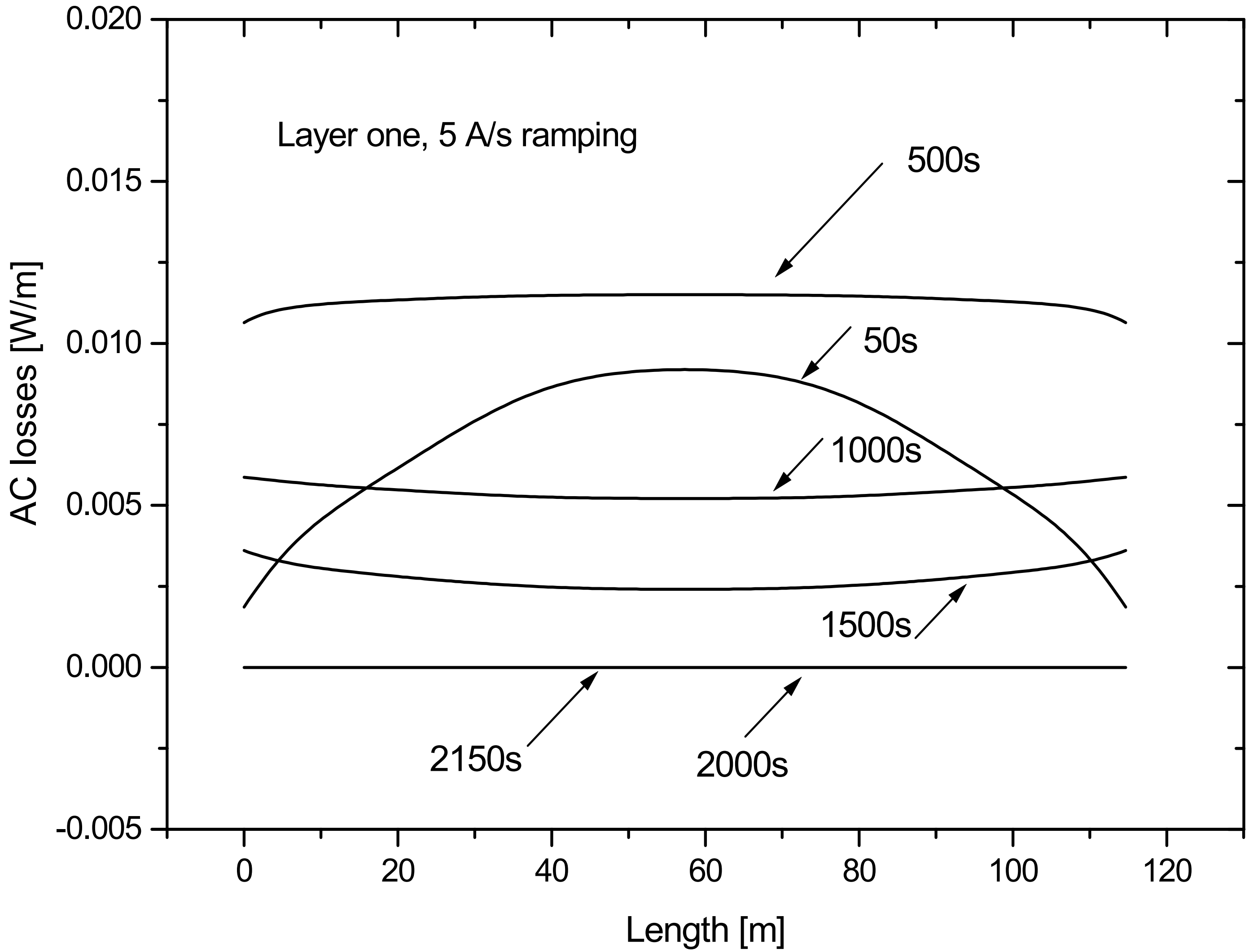


Figure 3. Evolution of total AC losses in layer1/ coil A (5 A/s current ramp)

Adiabatic condition cases
(Layer 1/Coil A)

- Corresponding to case of epoxy debonding between layers (post-quench scenario)
- 5 A/s ramp rate (2000s ramping time) was considered and compared with 2.5 A/s ramp
- Index n=15 and n=5 comparison (undamaged vs. damaged conductor)
- Insert coil is turned off
- To the end of the ramp the index heat is small but significant (index heating alone is enough to create a thermal runaway in few minutes)
- 2.5 A/s ramp rate (4000s ramping time) is a safe
- These results are consistent with the observation

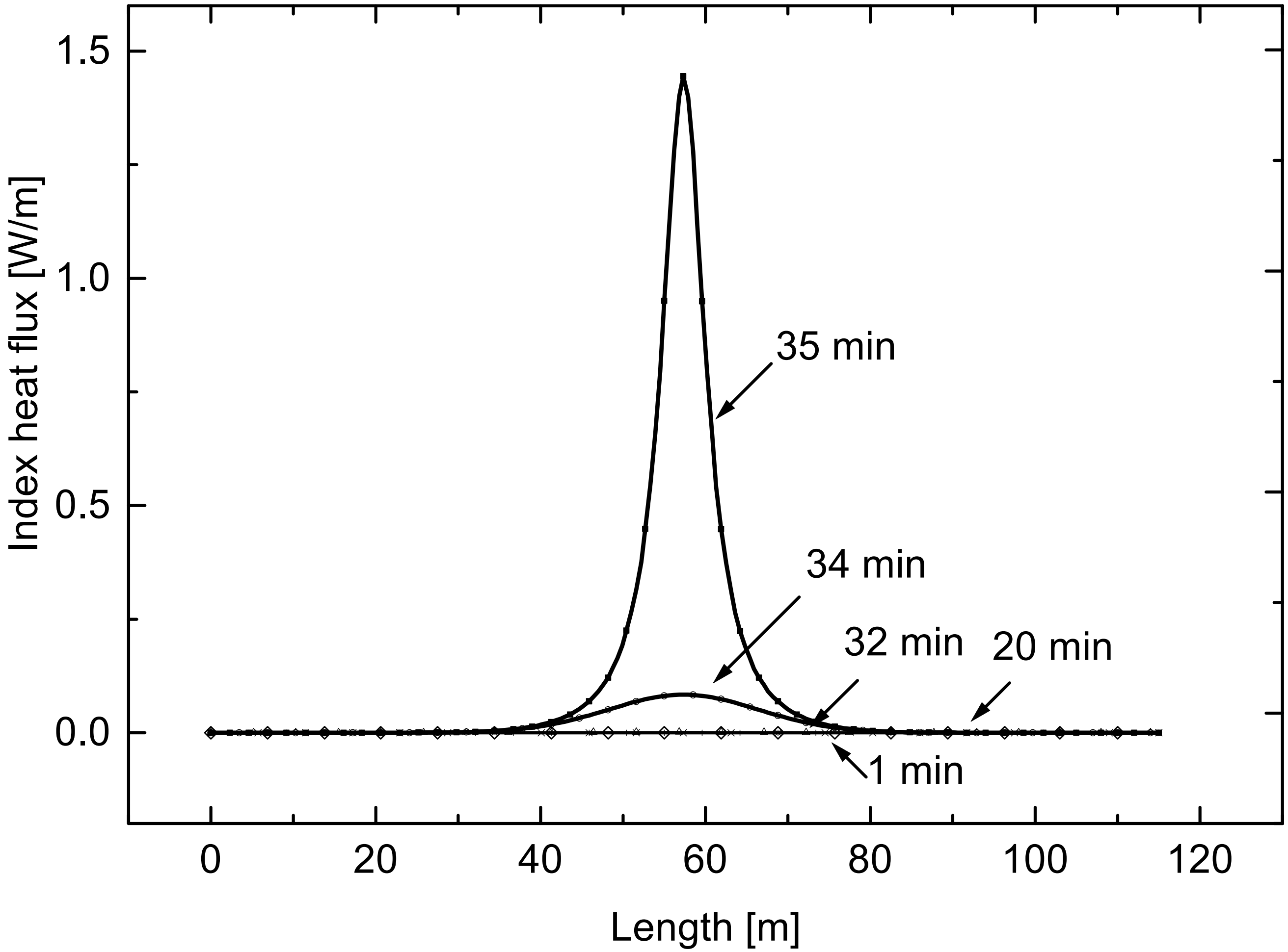


Figure 4. Evolution history of index heating in layer 1/ coil A (5 A/s ramp and n=5)

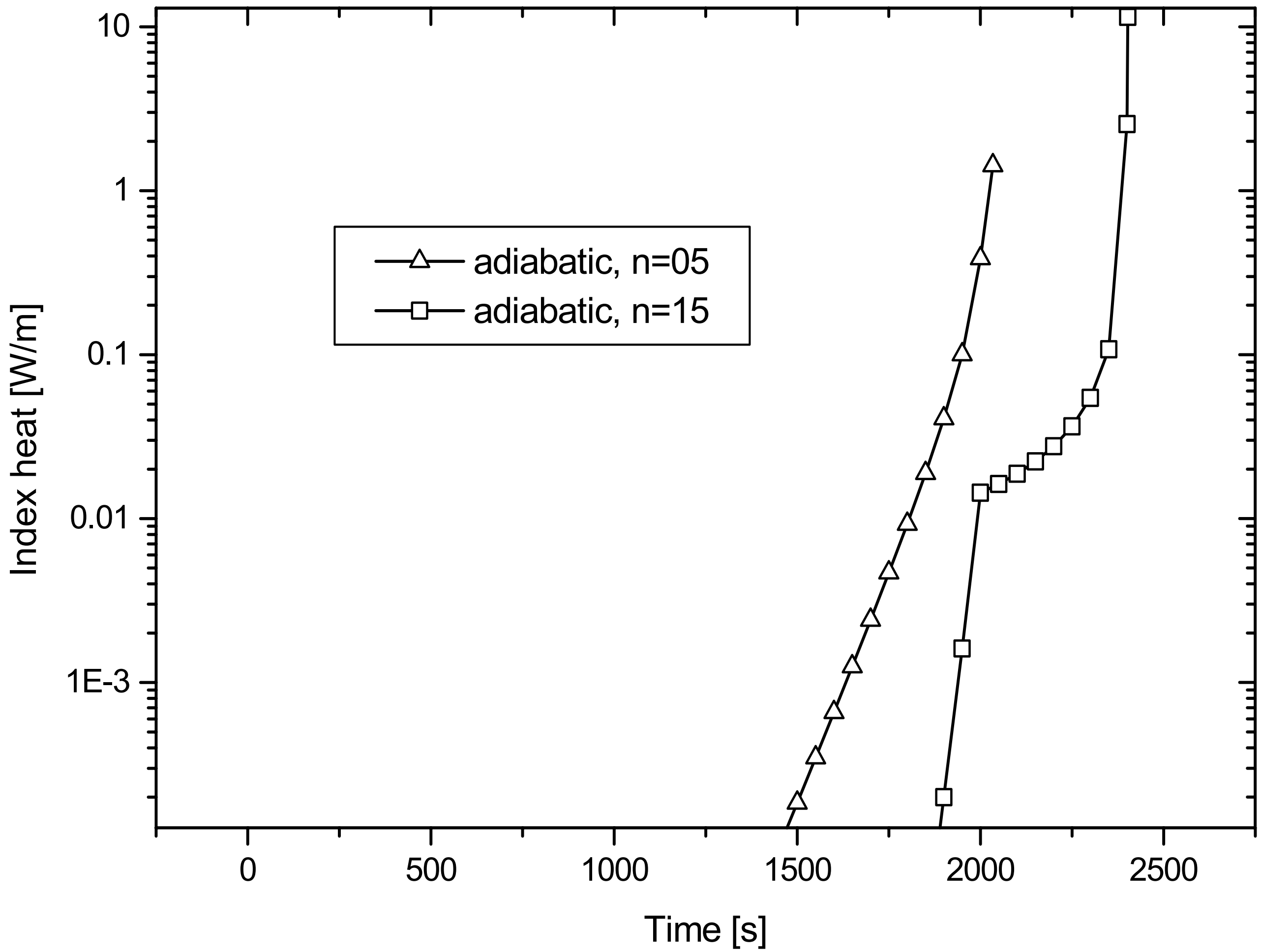


Figure 5. Index heating for L1/Coil A (5 A/s ramp, layer adiabatic, insert off)

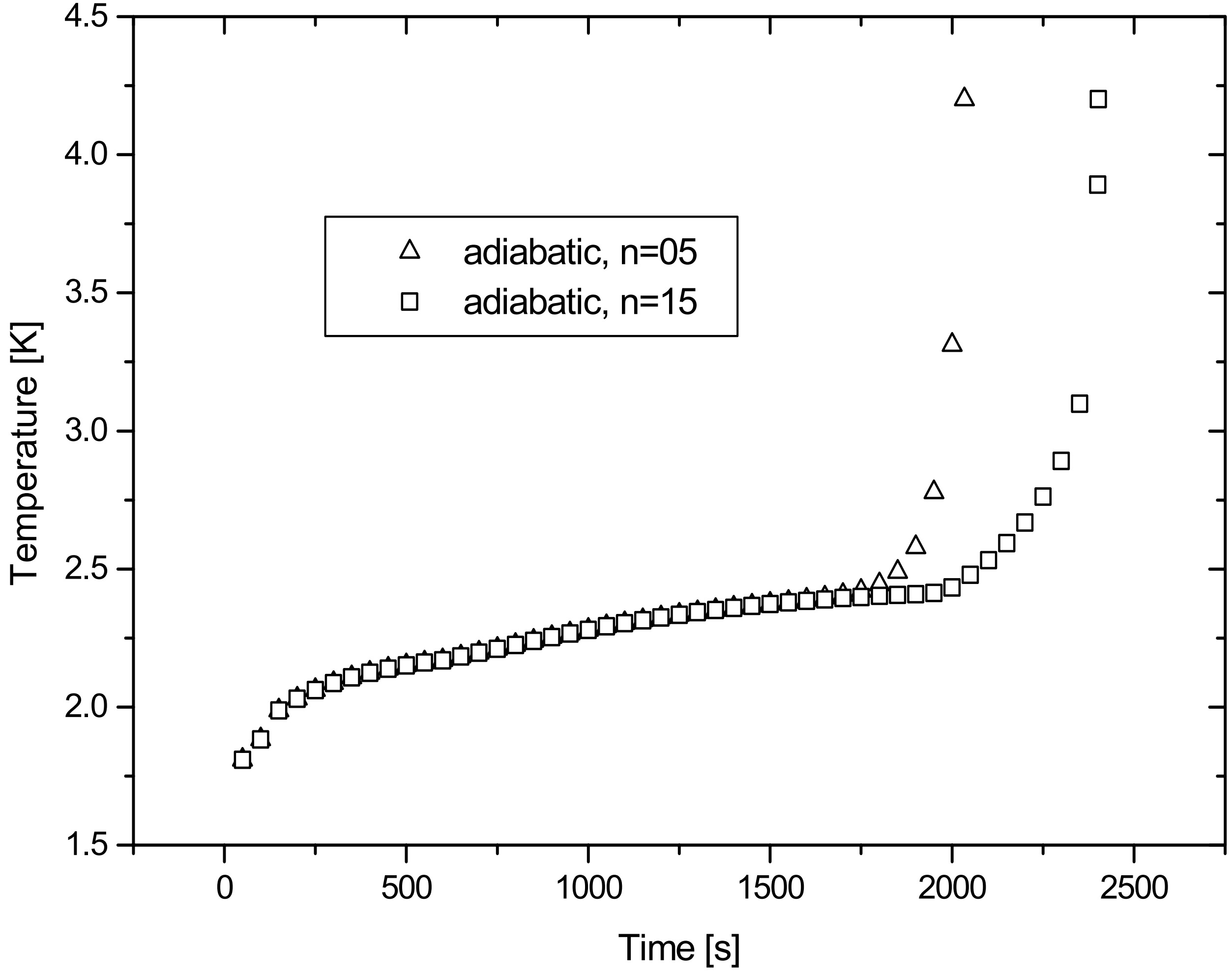


Figure 6. Peak temperature in L1/ Coil A during 5 A/s ramp (adiabatic, insert off)

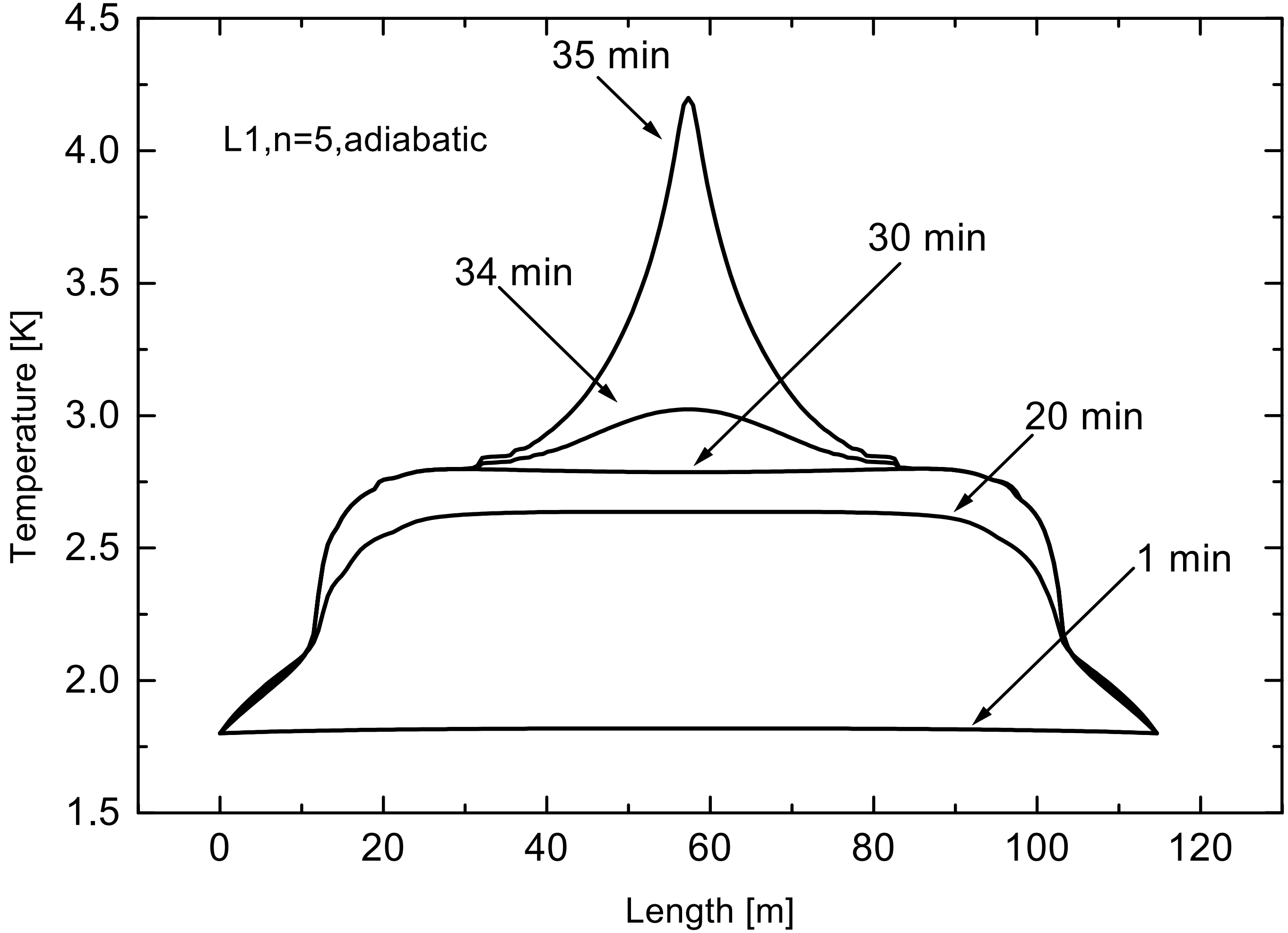


Figure 7. Temperature evolution of L1/ Coil A (5 A/s ramp, adiabatic, insert off)

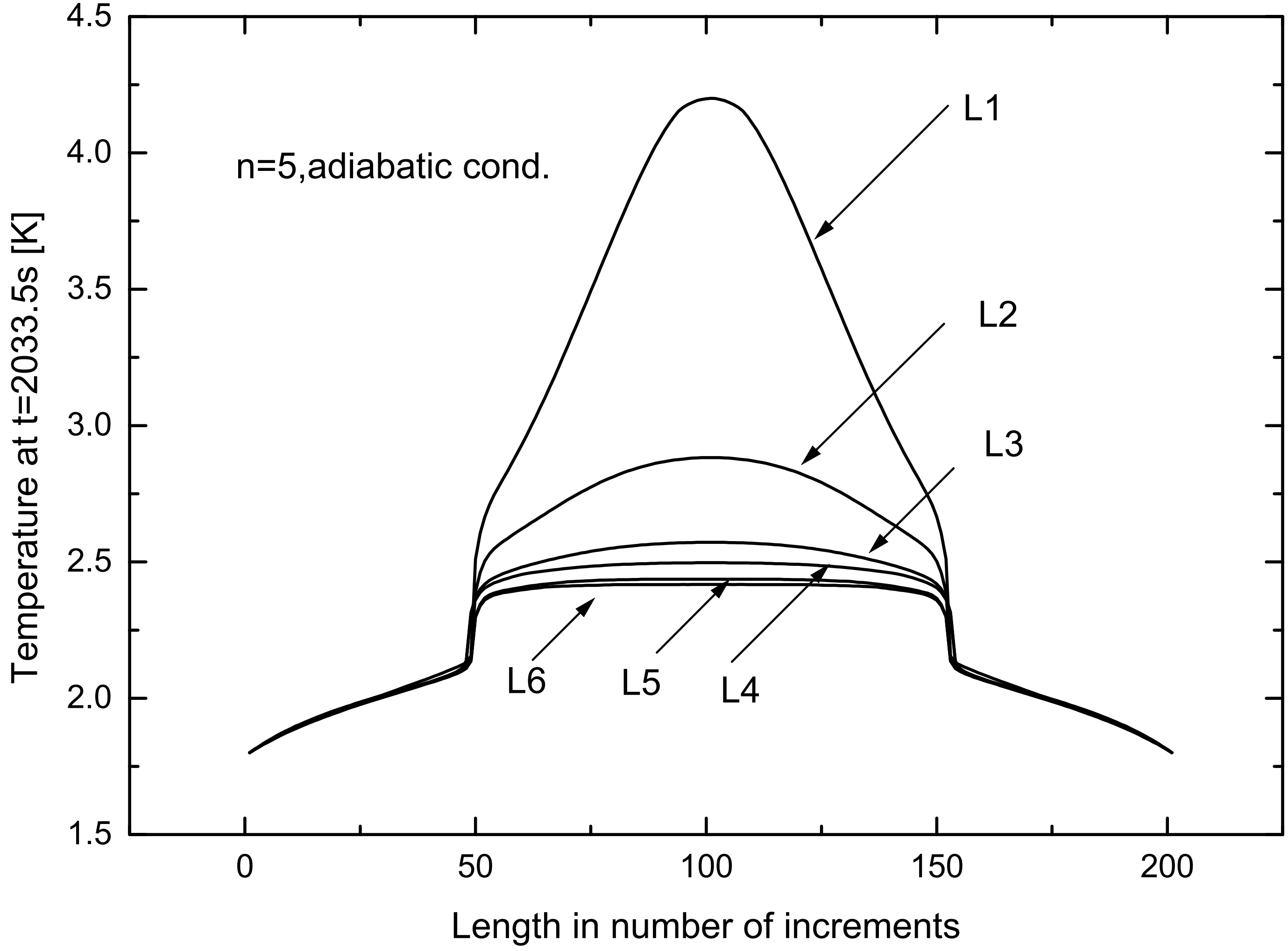


Figure 8. Temperature distribution in 6 layers of Coil A (5 A/s ramp, adiabatic conditions, The maximum temperature difference is 2.2 K.

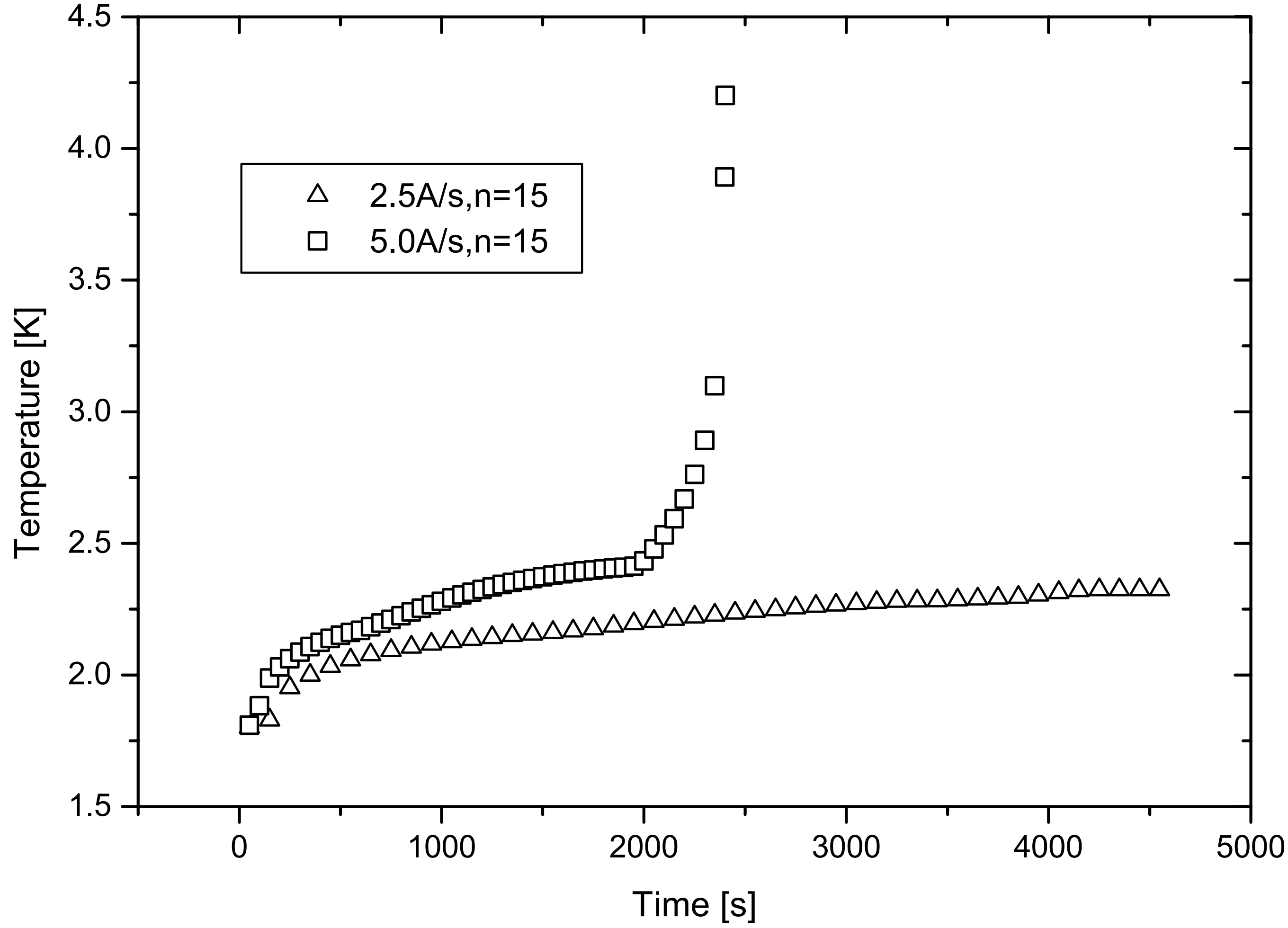


Figure 9. Peak temperature in L1 for different current ramp rates (adiabatic , insert off)

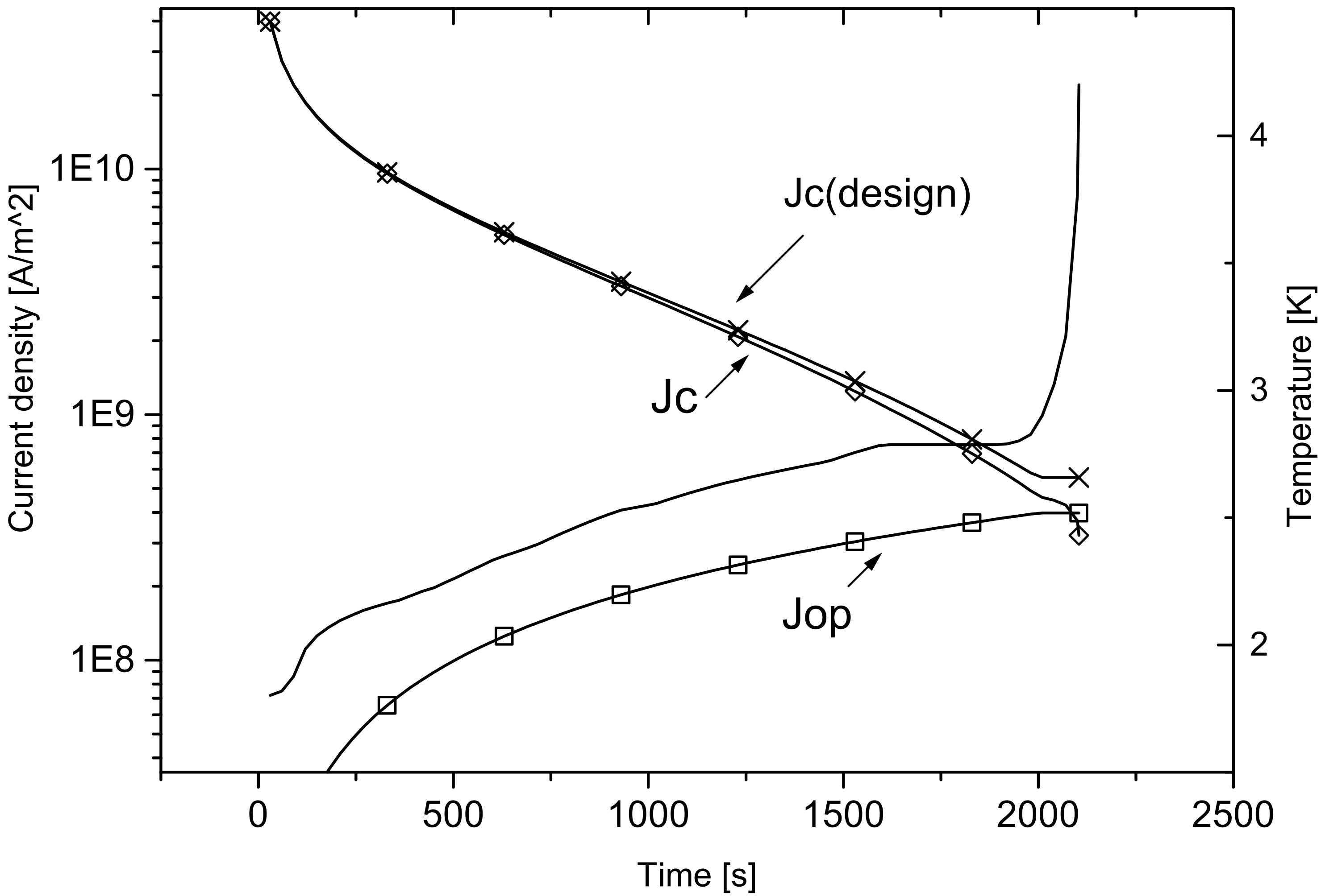


Figure 10. Temperature, operating current, critical current density at the center of L1 (5 A/s ramp). The temperature has exceeded the lambda value and leading to a thermal runaway due to index heat

Non-adiabatic condition cases
(layer-to-layer heat transfer)

- Prior to unprotected quench there is good inter-layer heat transfer
- After unprotected quench (epoxy delamination) layer-to-layer heat transfer is reduced or eliminated (adiabatic layer)
- Compare both cases
  - Is index heating still relevant?
  - What is the temperature difference between layers in the case of good thermal contact?

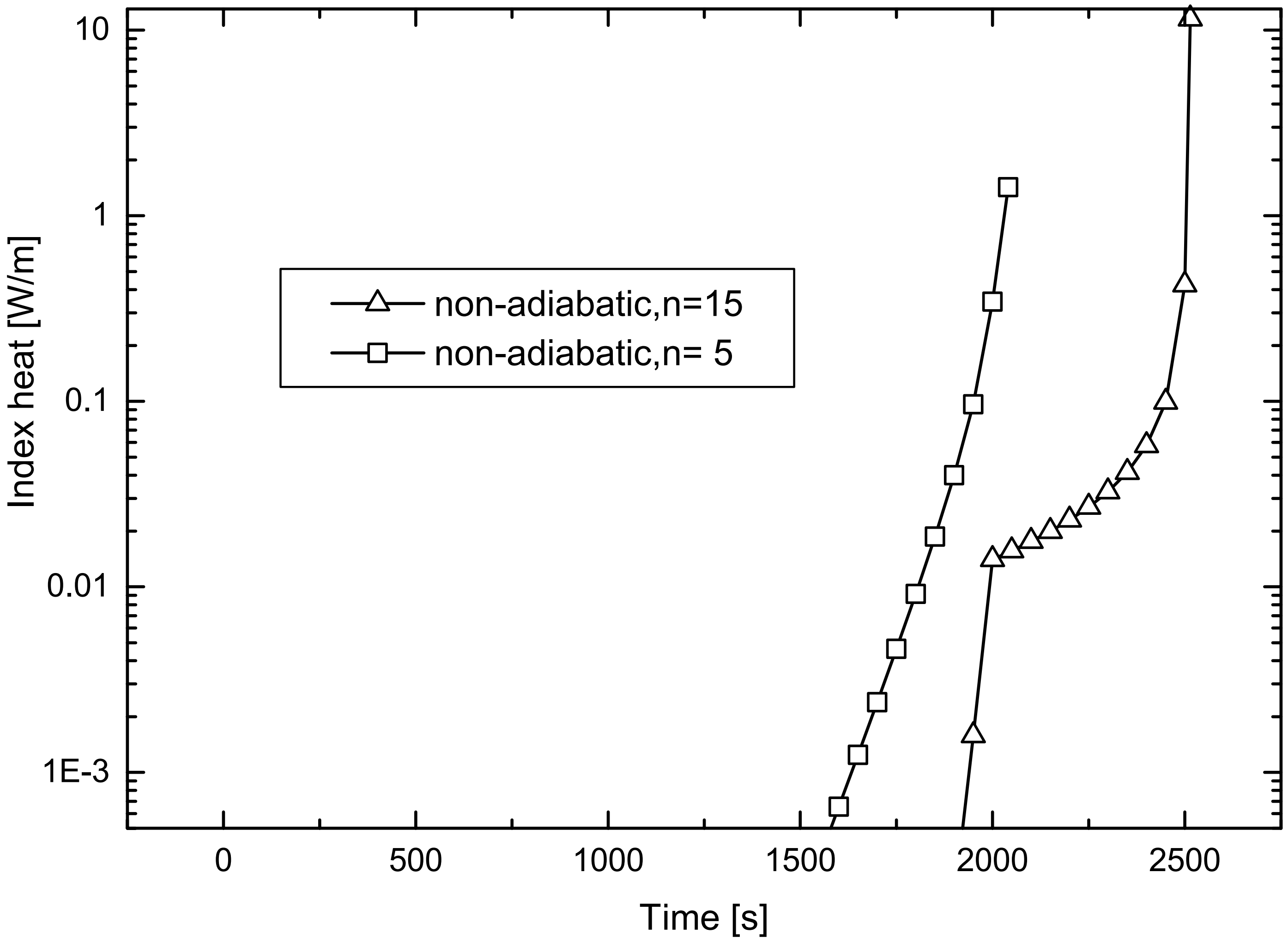


Figure 11. Index heating for L1 (5 A/s ramp, insert off), even if heat transfer between layers is included, a 5 A/s tamp leads to thermal runaway due to index heating

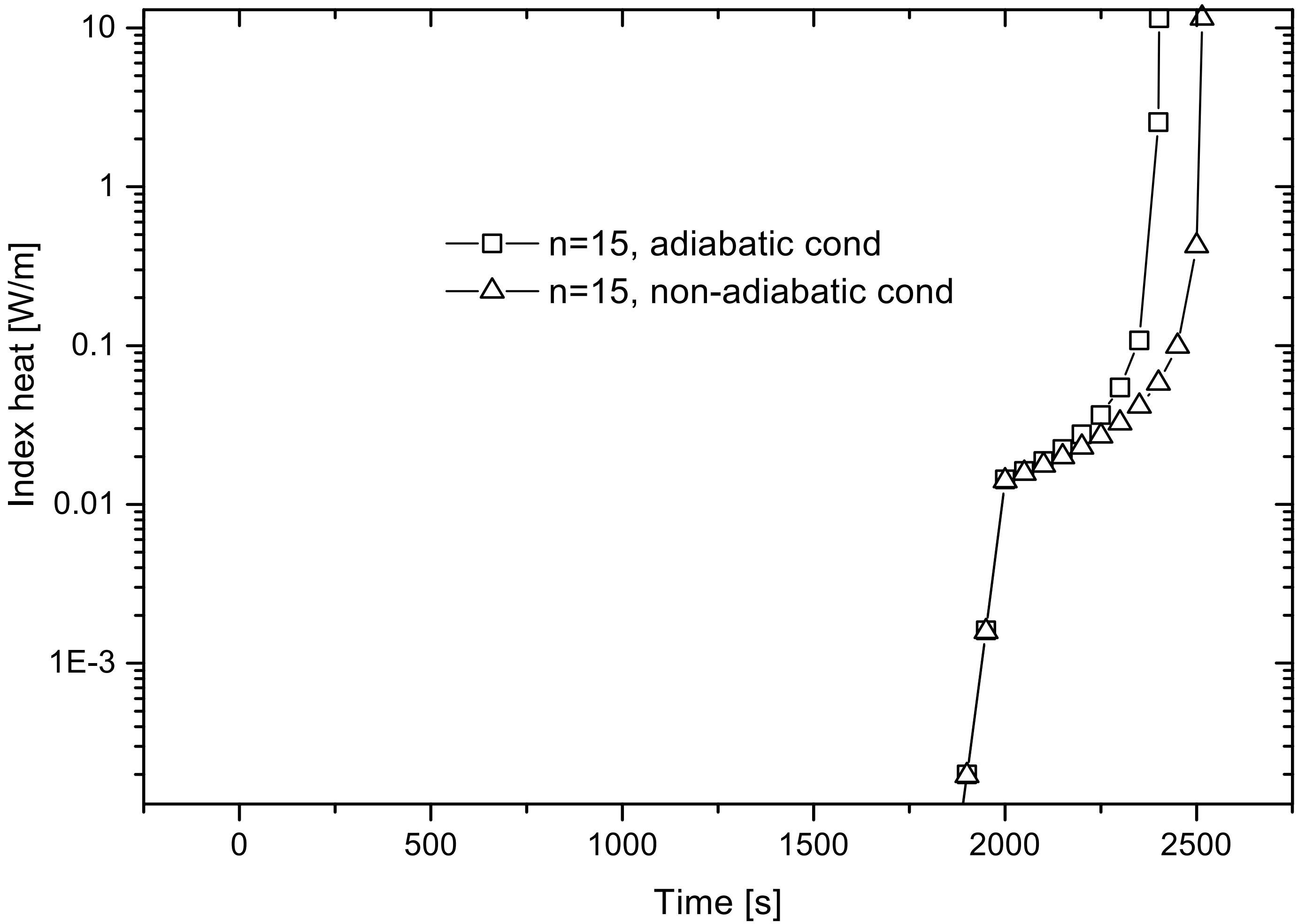


Figure 12. Comparison of index heating in L1 (adiabatic Vs non-adiabatic, 5 A/s ramp, insert off)

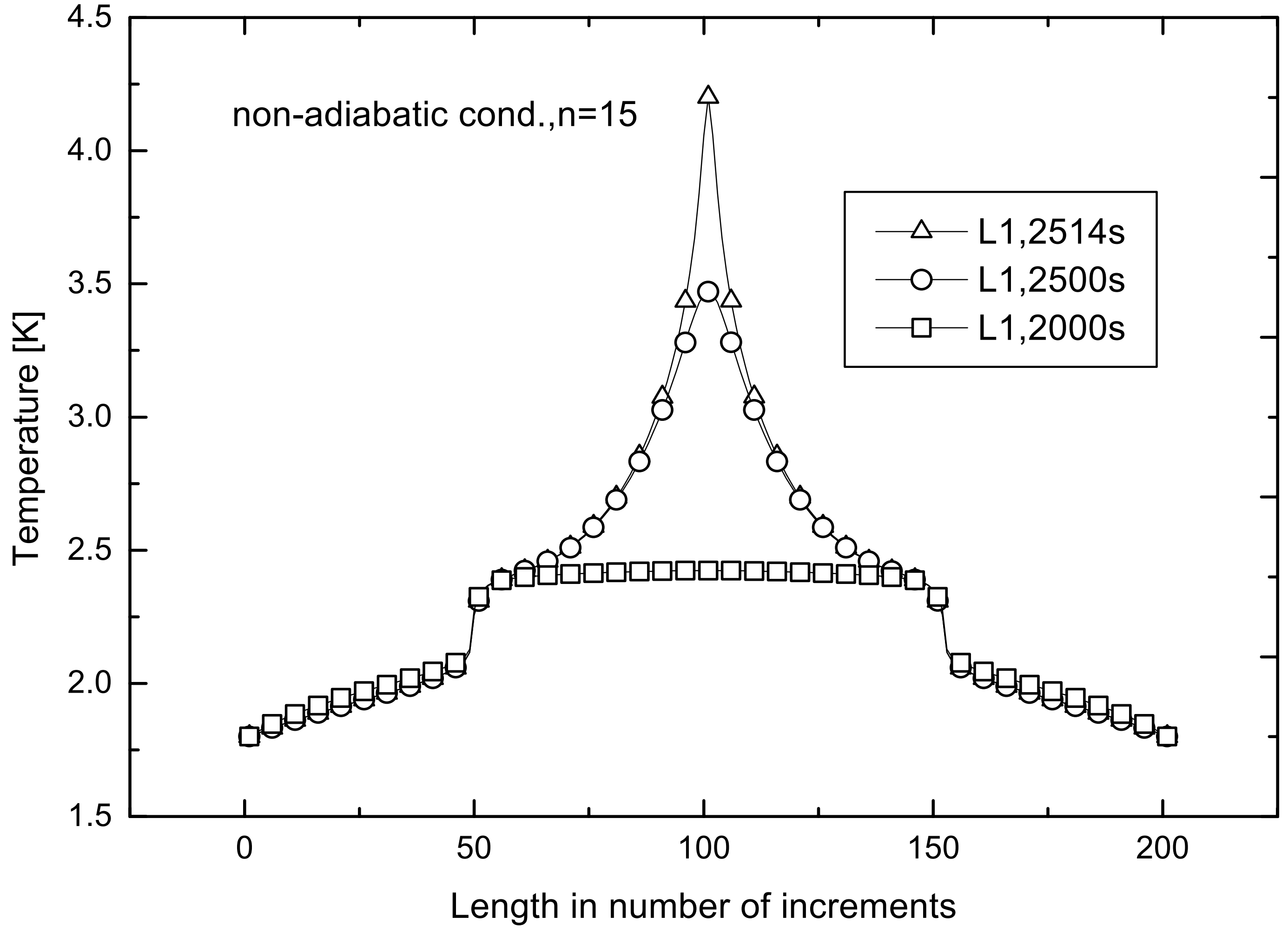


Figure 13. Temperature evolution of L1 of Coil A (5 A/s ramp, non-adiabatic conditions, insert coil is turned off)

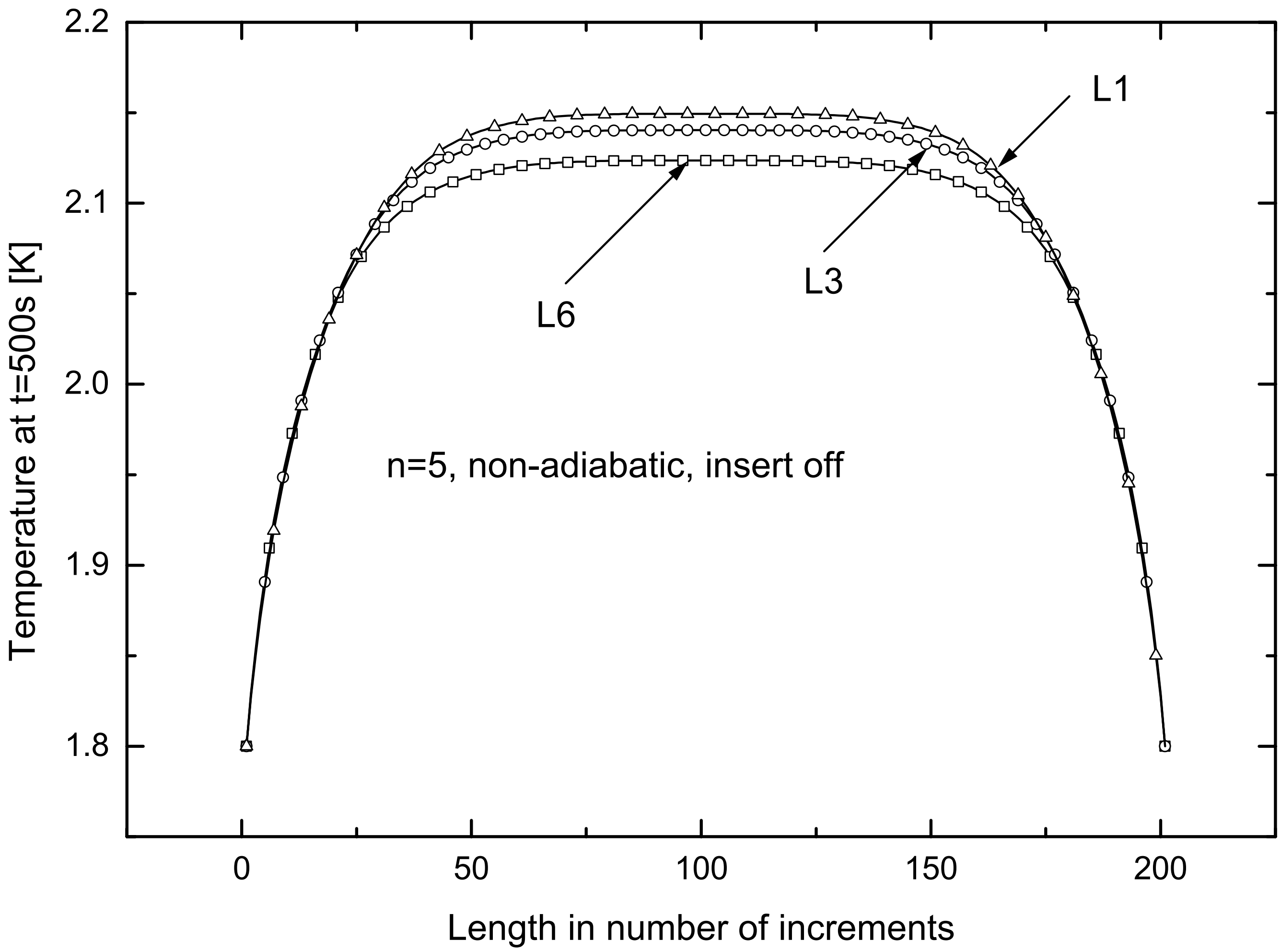


Figure 14. Temperature distribution in different layer of Coil A at t=500s (5 A/s ramp, insert off, non-adiabatic conditions between adjacent layers.

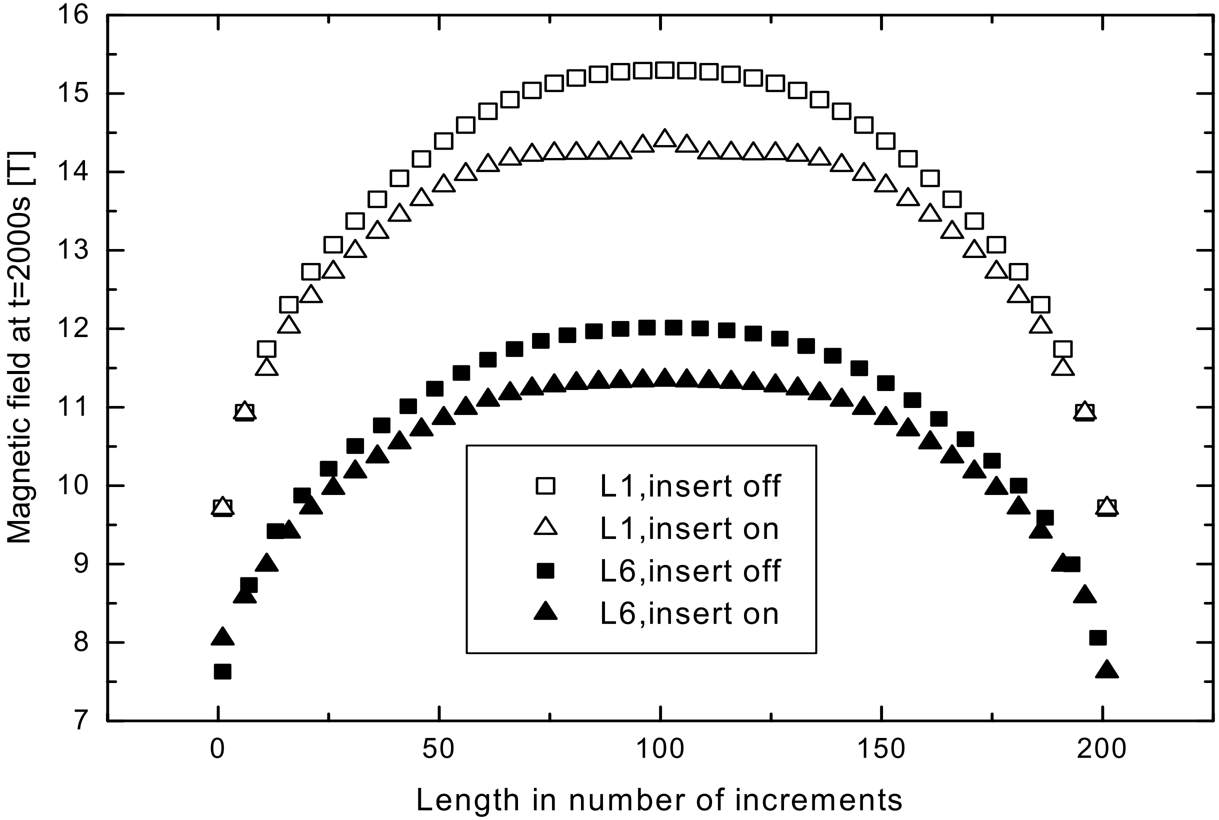
Non-adiabatic condition cases
(Conclusions)

- For a 5 A/s ramp rate index heating is still significant and will lead to thermal runaway even if layers are in good thermal contact (with a slight delay over the adiabatic case)

- Thermal time constants are short compared to ramp time, so all six layers attain a high degree of temperature uniformity

Influence of insert coil
(resistive insert turned on)

- When the insert is energized, the total magnetic field on the outsert is decreased (critical current increases)



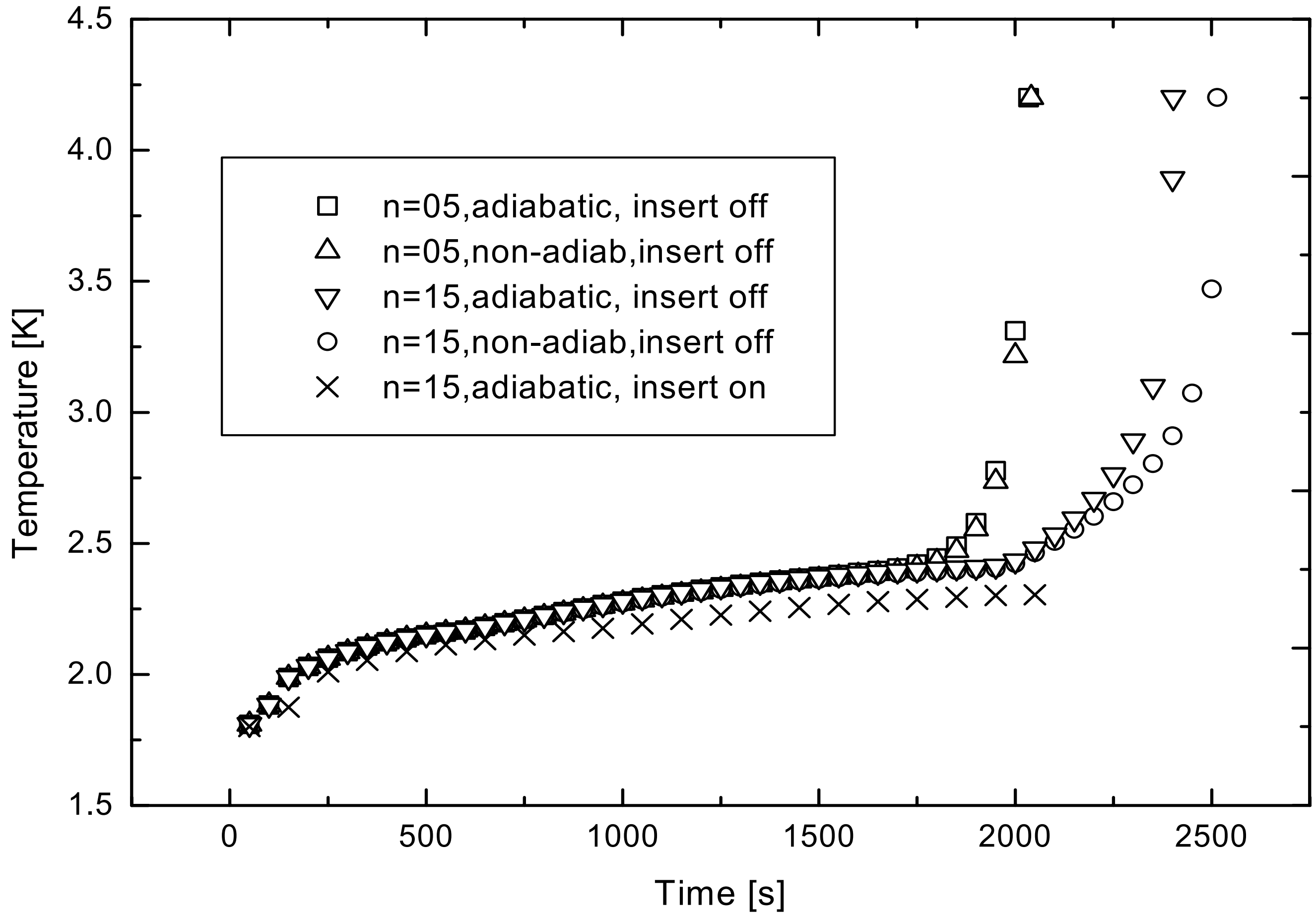


Figure 16. Peak temperature in L1 during a 5 A/s current ramp showing the influence of index number n, and layer-to-layer heat transfer. When insert coil is turned on there is no thermal runaway due to index heating.

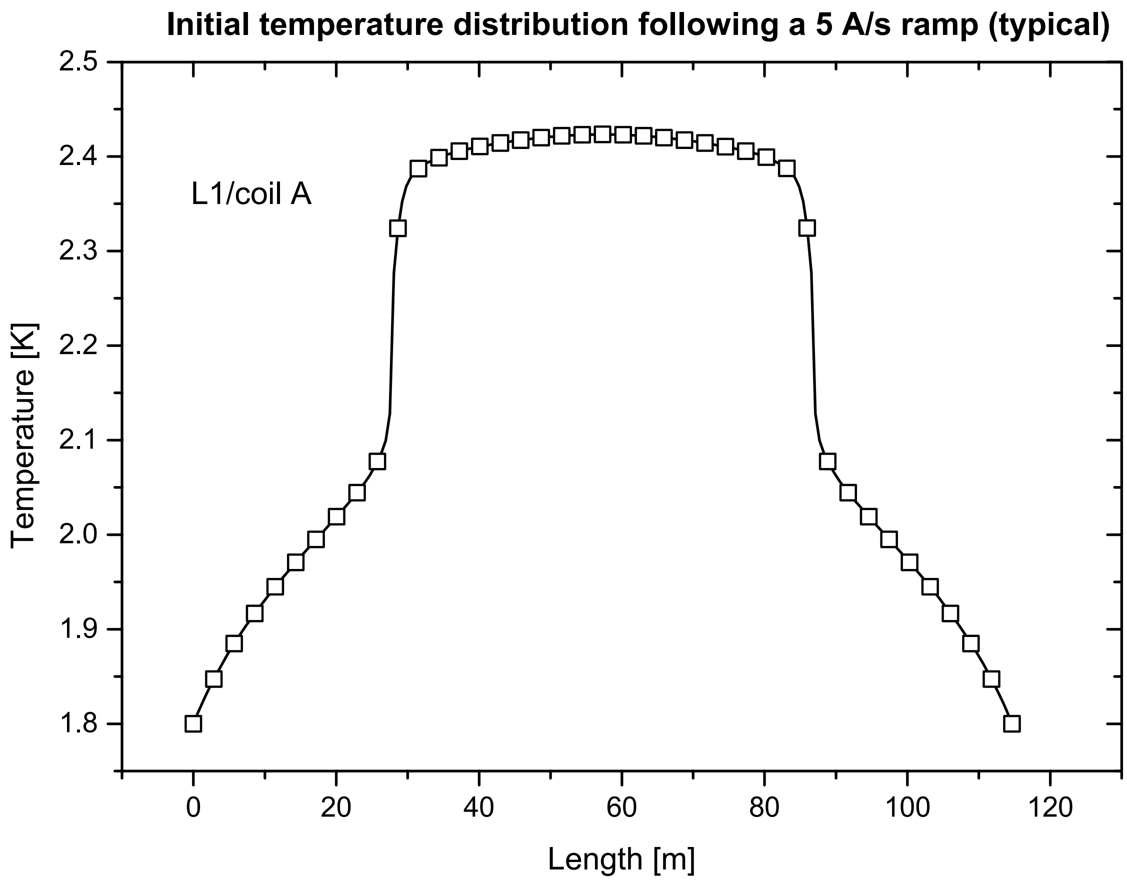
Influence of insert coil
(Conclusions)

- There is no thermal runaway since index heating is negligible when the insert is turned on
  - Future: Investigate scenario as to when to turn on the insert (towards the end of the outsert ramp)

- Following a 5 A/s ramp rate the coil would reach a maximum temperature 2.25 K

Recovery of outsert coil temperature

- To assess the thermal recovery time of the outsert coil following a 5 A/s ramp rate. Use model to track He I/He II front 'recovery'



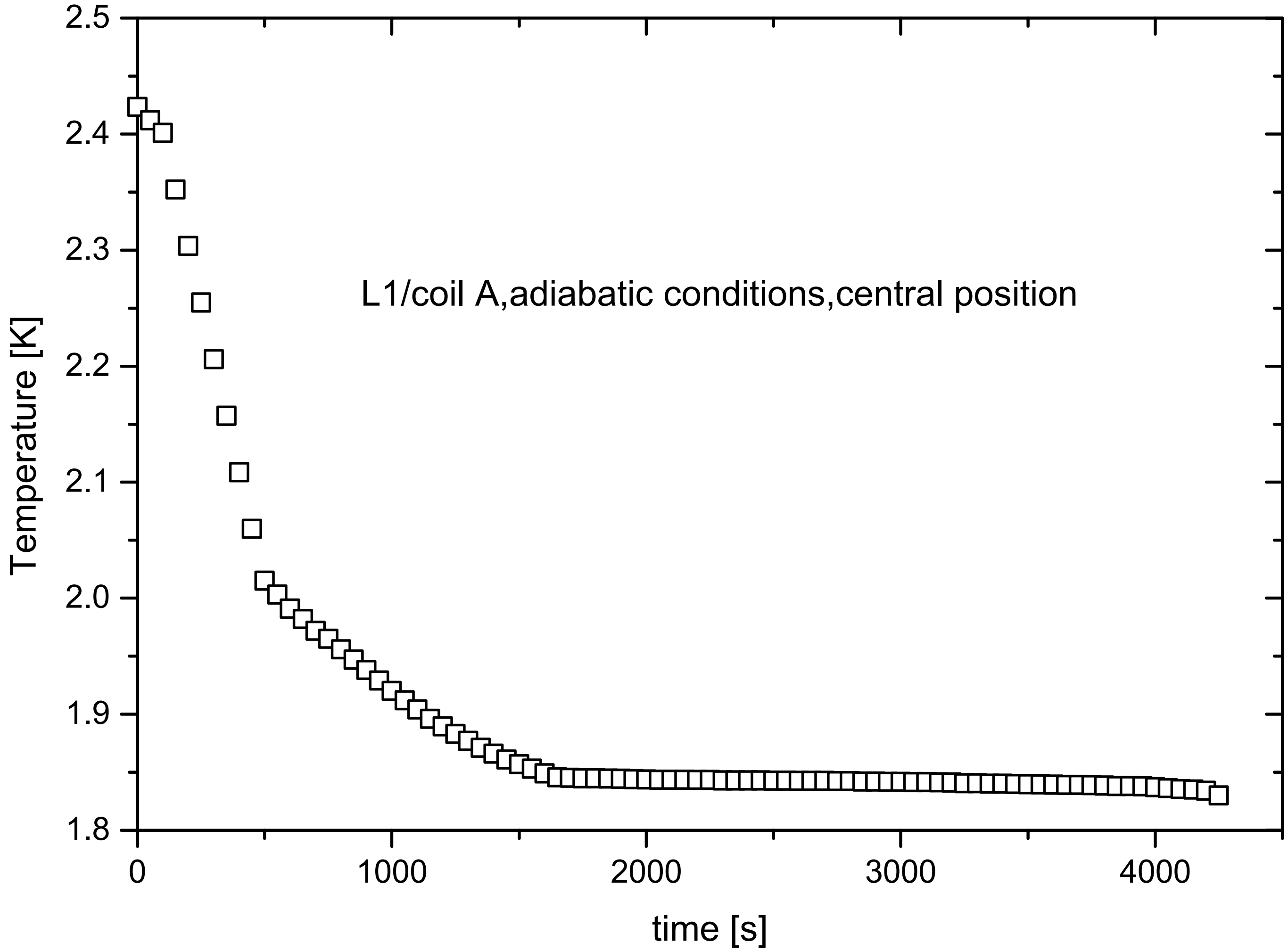


Figure 19. The thermal recovery time of L1/Coil A under adiabatic conditions is approximately 1 hr. (due to end-layer cooling through a column of superfluid helium)

Conclusions

- A transient heat transfer model has been implemented to study the quench behavior of NHMFL 45-T hybrid magnet
  - Phase 1 considered one adiabatic layer
  - Phase 2 includes all six layers and heat transfer
- The simulation shows that index heating will induce a thermal runaway under adiabatic condition with a 5 A/s current ramp rate (few minutes after ramp completion)
- Layer-to-layer heat transfer slightly delays runaway
- With insert turned on index heating is eliminated
- Corresponds to observed behavior (pre and post-quench)
- Recovery time by end-cooling in He II is about 1 hr