O(8) and Collective Symmetry Breaking in Twin Higgs

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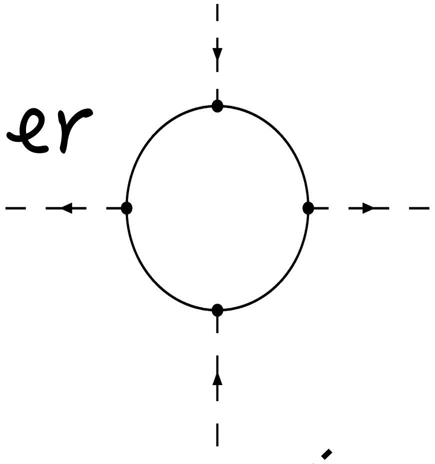
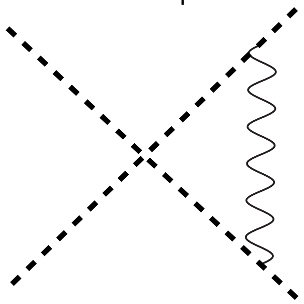
Appendices of Chacko, Goh, RH hep-ph/0512088 (The LR Twin Model)
Chacko, RH, Kiel Howe - in progress

U(4) Breaking Effects

* In the TH model the Goldstone mass come from global symmetry breaking effects.
* In the linear U(4) model this arrises at order $g^4/(16\pi^2)$ and $y_t^4/(16\pi^2)$.



* It could have arisen at order $g^2\lambda/(16\pi^2)$. But it does not!



* There is a deeper reason for this which is the topic of this talk.

Outline

* Effects of order g^2 from the UV.
* Effects of order g^2 in the linear model. Two loops. (Appendix A)
* O(8) and collective sym. breaking. (Appendix B)
* Thoughts-
  - On custodial symmetry
  - The top sector.

U(4) Breaking

* Consider the linear sigma model with the addition of Higher dim operators (and keep track of the strong coupling limit, $\lambda \to 16\pi^2$).

* U(4) is broken explicitly by gauging only the $SU(2)_A \times SU(2)_B$ subgroup.

* We can account for this by considering the D_μ of SU(4) as a **spurion**

$$D_\mu = \partial_\mu + ig W_\mu^a \tau^a$$

where a runs **only** over $SU(2)_A \times SU(2)_B$ generators.

U(4) Breaking

* Consider a T-like operator in the NDA limit:

$$(4\pi)^2 \frac{(H^\dagger D_\mu H)\,(H^\dagger D^\mu H)}{\Lambda^2}$$

(might come from integrating out a heavy U(4) resonance at Λ, but lets stay within the EFT)

* Expanding:

$$\frac{(4\pi)^2}{\Lambda^2} g^2 \left[(H_A^\dagger H_A)^2 W_A^2 + (H_B^\dagger H_B)^2 W_B^2 \right] + \text{cross terms.}$$

U(4) Breaking

* Loop-off the W's. Assuming the loop is cut off at Λ, we get

$$\sim g^2 \left[(H_A^\dagger H_A)^2 + (H_B^\dagger H_B)^2 \right]$$

* Recall:

$$H_A^\dagger H_A = f^2 \sin^2(h/f) = |h|^2 + \ldots$$
$$H_B^\dagger H_B = f^2 \cos^2(h/f) = f^2 - |h|^2 + \ldots$$

so the Higgs potential is corrected by

$$V(h) \sim g^2 f^2 |h|^2$$

Possibilities

* There are many ways to avoid this problem.
   - A weakly coupled model, e.g. SUSY UV Completion (Cheng, Hall, and Weiner), or (Craig and Howe)
   - If SU(4) is restored below the cutoff, the W loop will be suppressed.
   - For example, if U(4) is broken non-locally in 5D?
* But can we avoid this problem in strongly coupled models.

Custodial Symmetry

* The operator is related to the SM T-operator. Can (and should!) be forbidden by custodial sym.

* In the SM we assume a large global symmetry and embed H into a custodial doublet

$$\mathcal{H} = (\varepsilon H^*, H) = \begin{pmatrix} H_2^* & H_1 \\ -H_1^* & H_2 \end{pmatrix}$$

so the "custodial partner" of H_1 is H_2^*. (will be relevant later)

or, equivalently, assume $SU(2)_L \times SU(2) = SO(4)$.

O(8)

* In the twin Higgs case we can also assume a larger O(8) symmetry.

* SM custodial sym is a subgroup. By itself a good reason to impose O(8).

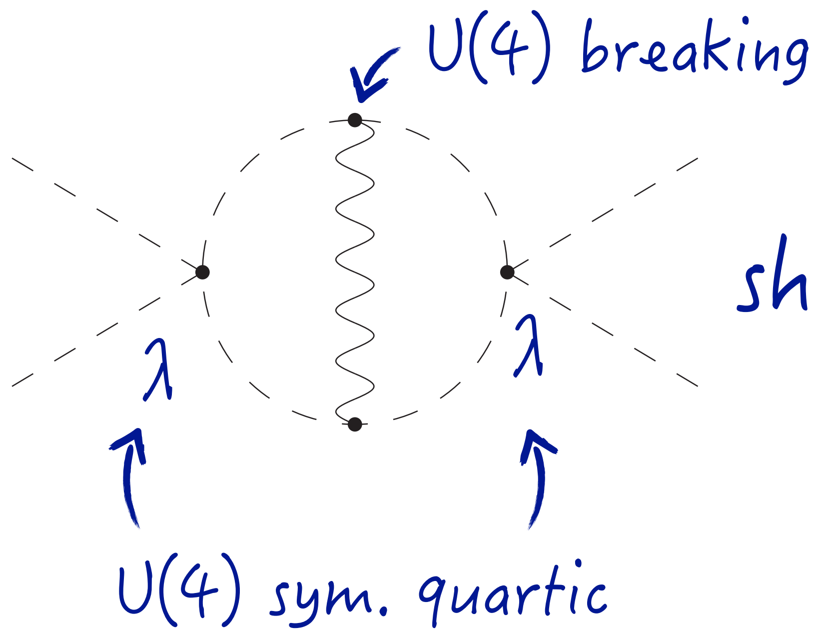
* This forbids $(H^\dagger D_\mu H)\,(H^\dagger D^\mu H)$

* But, are there other, perhaps higher dim operators that generate $g^2\left[(H_A^\dagger H_A)^2 + (H_B^\dagger H_B)^2\right]$ at loop level?

Before proving O(8) protects the Higgs,
I'll tell you about Appendix A.

Linear Model

* Barbieri, Gregiore, and Hall pointed out the "$g^2(H_A{}^4+H_B{}^4)$ problem" exists in the linear model (hep-ph/0509252).

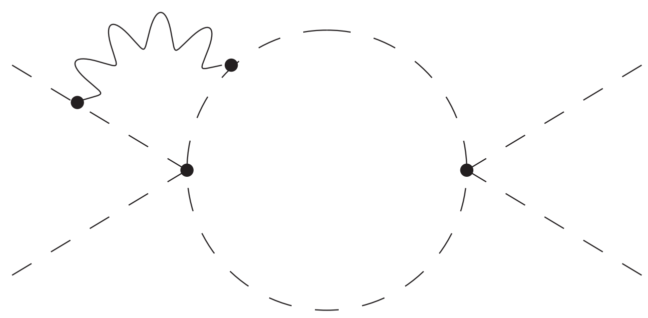


should give $\dfrac{\lambda^2 g^2}{(16\pi^2)^2}(H_A{}^4 + H_B{}^4)$

NDA limit: $\lambda \sim (4\pi)^2$

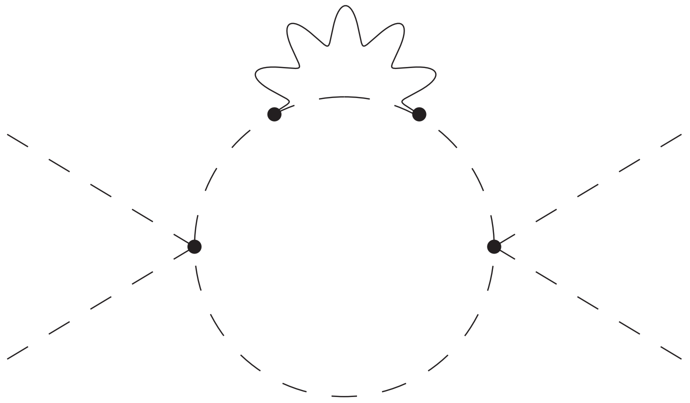
Linear Model

* Appendix A:



(b)

$\propto \mathrm{Tr}[\tau^a]=0$, sum to zero.

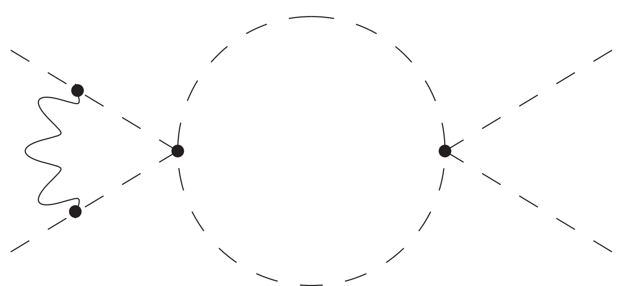


(c)

A correction to the scalar propagator.
Is $U(4)$ symmetric due to the Z_2.

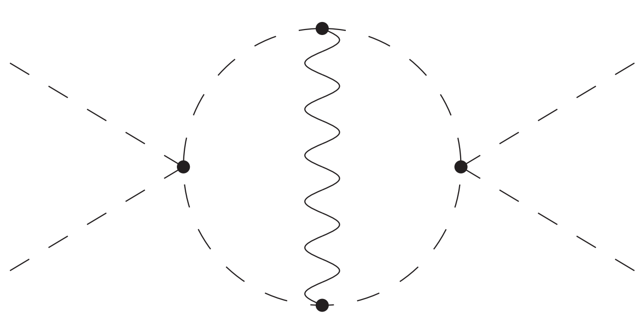
Linear Model

* And more diagrams:



(a)

A renormalization of λ, the U(4) quartic.
Turns out the $H_A^4 + H_B^4$ and $2H_A^2 H_B^2$ renormalize in the same way. (4-2=2)
(In fact, this cancelation happens already at one loop).



(d)

$H_A^4 + H_B^4 \propto 15\ \lambda^2 g^2$

$H_A^2 H_B^2 \propto 30\ \lambda^2 g^2$

Linear Model

* Note: The "large" term in the linear model are all **accidentally O(8) symmetric.**

$$\mathcal{L} \supset |\partial_\mu H|^2 - m^2 |H|^2 + \lambda |H|^4 + \ldots$$

* What is O(8) doing for us? That leads us to "Appendix B", a collective symmetry argument.

* I'll show a more elegant version here based on spurions.

The Claim

In a model with O(8) symmetry which is broken by gauging $SU(2)_A \times SU(2)_B \times Z_2$ no goldstone mass is generated at order g^2.

As Andrea showed, Barbieri et al have recently showed the same thing in two site models.
We showed this within the low energy EFT.

The proofs are sufficiently different, making both interesting.

O(8) Spurions

* The field H live in an 8 or O(8). Lets pick the embedding

$$H = \begin{pmatrix} H^1_{Ar} \\ H^1_{Ai} \\ \vdots \\ H^2_{Br} \\ H^2_{Bi} \end{pmatrix}$$

* Gauge only some of the O(8) generators and promote them to spurions. Which ones?

O(8) Spurions

* We will need to keep track of the diagonal SU(2) generators (the others can be made diagonal by a gauge transformation).
* In our O(8) embedding

$$\tau^3_A = \begin{pmatrix} i\sigma_2 & & & \\ & -i\sigma_2 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}_{8\times 8} \qquad \tau^3_B = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & i\sigma_2 & \\ & & & -i\sigma_2 \end{pmatrix}_{8\times 8}$$

These are our spurions.

The ± basis

* The symmetry is most transparent in the basis (dropping Lorentz indices)

$$W_+ = \frac{1}{2}[\,W_A + W_B\,] \qquad W_- = \frac{1}{2}[\,W_A - W_B\,]$$

and in this basis

$$\tau^3_+ = \begin{pmatrix} i\sigma_2 & & & \\ & -i\sigma_2 & & \\ & & i\sigma_2 & \\ & & & -i\sigma_2 \end{pmatrix}_{8\times 8} \qquad \tau^3_- = \begin{pmatrix} i\sigma_2 & & & \\ & -i\sigma_2 & & \\ & & -i\sigma_2 & \\ & & & i\sigma_2 \end{pmatrix}_{8\times 8}$$

* Thanks to the Z_2: W_+ and W_- do not mix.

A U(4) symmetry

* What do these spurions break O(8) into?
  Claim: Each breaks it to a U(4)

* To see this, consider τ^3_+.
  Perform the following O(8) transformation

$$U_+ = \mathrm{diag}(1,1,1,-1,\ 1,1,1,-1)$$
$$= (\text{a } 180^\circ \text{ rotation in the 4-8 plane})$$

$$\tau^3_+ = \begin{pmatrix} i\sigma_2 & & & \\ & -i\sigma_2 & & \\ & & i\sigma_2 & \\ & & & -i\sigma_2 \end{pmatrix}_{8\times 8} \xrightarrow[U_+]{} \begin{pmatrix} i\sigma_2 & & & \\ & i\sigma_2 & & \\ & & i\sigma_2 & \\ & & & i\sigma_2 \end{pmatrix}_{8\times 8}$$

A U(4) symmetry

* It is easy to show that the matrix

$$\begin{pmatrix} i\sigma_2 & & & \\ & i\sigma_2 & & \\ & & i\sigma_2 & \\ & & & i\sigma_2 \end{pmatrix}_{8\times 8} = I_{4\times 4} \otimes i\sigma_2$$

respects a U(4) symmetry (which acts on the blocks)

A U(4) symmetry

* U_+ has flipped the sign of the imaginary part of H_{A2} and H_{B2}. Or,

$$H_{A2} \leftrightarrow H_{A2}^* \qquad\qquad H_{B2} \leftrightarrow H_{B2}^*$$

and not surprisingly $\tau_+^3 = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}_{4\times4} \rightarrow \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}_{4\times4}$

* The field H_+ will transform as a fundamental, w/

$$H_+ \equiv \begin{pmatrix} H_{A1} \\ H_{A2}^* \\ H_{B1} \\ H_{B2}^* \end{pmatrix}$$

Note: $H_+^\dagger H_+ = H^\dagger H$

This U(4) protects the Higgs.

Another U(4)

* For τ^3_-, we can do a different O(8) rotation

$$U_- = \mathrm{diag}(1,1,1,-1,1,-1,1,1)$$

$$\tau^3_- \xrightarrow[U_-]{} I_{4\times4} \otimes i\sigma_2$$, a different U(4) subgroup is preserved.

$$H_- \equiv \begin{pmatrix} H_{A1} \\ H^*_{A2} \\ H^*_{B1} \\ H_{B2} \end{pmatrix}$$

$$H_-^\dagger H_- = H^\dagger H$$

This U(4) also protects the Higgs.

Collective Symmetry Breaking

* You can repeat this for off diagonal τ's (by doing a gauge transformation into a diagonal τ).
* The conclusions are:
  - The various generators do not mix.
  - No gauge generator can harm the goldstone by itself.

No goldstone potential at order g^2.

To give the Higgs a potential toy need at least two generators and another $g^2/(16\pi^2)$.

Thoughts on Custodial Symm.

* We enlarged $U(4)$ to $O(8)$ to include custodial symmetry.

* Custodial symmetry "made an appearance":

$$H_+ \equiv \begin{pmatrix} H_{A1} \\ H_{A2}^* \\ H_{B1} \\ H_{B2}^* \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} H \\ H \end{matrix}}\right\} \text{a custodial doublet of the A sector} \\ \left.\vphantom{\begin{matrix} H \\ H \end{matrix}}\right\} \text{a custodial doublet of the B sector} \end{matrix}$$

* Question: Is custodial symmetry enough to guarantee the safety of the Higgs?
  Or do we need the whole $O(8)$?

Custodial Symmetry

We suspect the answer is
"No, custodial symmetry is not enough.
You need more.
But we would like to understand
under what conditions it is."

In progress.

Custodial Symmetry

* To do harm we need operators w/ 4 Higges and two gauge bosons.

* Considering only $SU(2)_A \times SU(2)_B \times Z_2$, We can write other operators that can give the goldstone a mass, but give not T parameter.

Examples*:

$$|H_A|^2 |D_\mu H_A|^2 + |H_B|^2 |D_\mu H_B|^2$$

– or –

$$|H_B|^2 |D_\mu H_A|^2 + |H_A|^2 |D_\mu H_B|^2$$

*In the usual SILH story such operators are shifted away by $H \to H + H^3/f^2$. But for us such a shift breaks $U(4)$.

Custodial Symmetry

* To do harm we need operators w/ 4 Higgses and two gauge bosons.

* Considering only $SU(2)_A \times SU(2)_B \times Z_2$, We can write other operators that can give the goldstone a mass, but give not T parameter.

Examples*: $|H_A|^2 |D_\mu H_A|^2 + |H_B|^2 |D_\mu H_B|^2$

– or –

$|H_A|^2|D$ ...

These do not arise in O(8), but might arise in other models.

*In the usual SILH ...

$H \rightarrow H + H^3/f$ ... for us such a shift breaks U(4).

Sp(4)xSU(2)

Batra, Chacko (08), for the LR model.

* Sp(4) is can also host the twin Higgs mechanism.

$$Sp(4) \underset{7 \text{ goldstones}}{\longrightarrow} SU(2)$$

* Sp(4) is compatible with a SM-like custodial extension.

$$\hat{H} = (JH^*, H)_{2\times 4}$$

$$J = \begin{pmatrix} \varepsilon & \\ & \varepsilon \end{pmatrix}_{4\times 4}$$

the Sp(4) invariant tensor.

Sp(4)xSU(2)

* Again, we break Sp(4) by gauging the $SU(2)_A \times SU(2)_B$ subgroup, etc, etc.

* The $H_\pm$ multiplets do not transform under a full Sp(4).

* Indeed, the operator

$$g^2 \mathrm{Tr}[\,(\hat{H}^T \varepsilon \hat{H}\, J\, \tau^a)^2\,]$$

(J acts in Sp(4) space.
ε acts in SU(2) space)

leads to $g^2[(H_A^\dagger H_A)^2 + (H_B^\dagger H_B)^2]$.

Investigation in progress.

Top Sector

* Generically, there is no analogous problem with the top sector.

* U(4) or O(8) breaking:

$y_A H_A Q_A U_A + y_B H_B Q_B U_B$ can be "spurionized"

* The spurion y accompanies a single power of H.

* A quartic arrises at order y^4.

Parametrically similar for U(4), Sp(4), or O(8).

Conclusions

* In Twin Higgs models care should be taken to avoid a large goldstone mass at order g^2.
* This is easily achieved in models with either
  - Weakly coupled UV completions.
  - An extended O(8) symmetry, exhibiting collective symmetry breaking.
* Custodial symmetry plays a role, but is probably not sufficient.

Deleted Scenes