The orbifold path to neutral naturalness

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Thanks in part to collaborators Kiel Howe, Christoph Englert, Matthew McCullough, Simon Knapen, Pietro Longhi, Andrey Katz, Matt Strassler, Raman Sundrum, David Curtin, Tim Cohen, Tim Lou, and David Pinner.

Thanks also to Roni, Chacko, and Gustavo.

CERN-TH Institute on “Neutral Naturalness”

“Is neutral naturalness the beautiful reason we haven’t seen anything, or the last desperate hope of theorists?”

–Gian (more or less)

We don’t know yet; now is the time to explore and figure out what “neutral naturalness” can offer.

Perhaps you think neutral naturalness is not minimal.

I do not particularly care.

I do not think nature cares much, either.

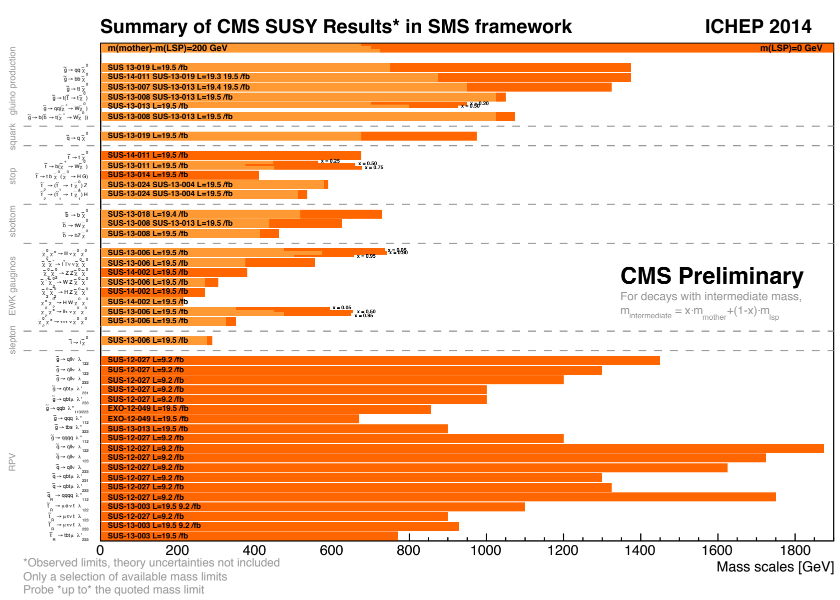
We had 30 years to write down the theory of physics beyond the weak scale, and so far it does not look like we have done very well using *minimality* alongside naturalness as our guiding principle.

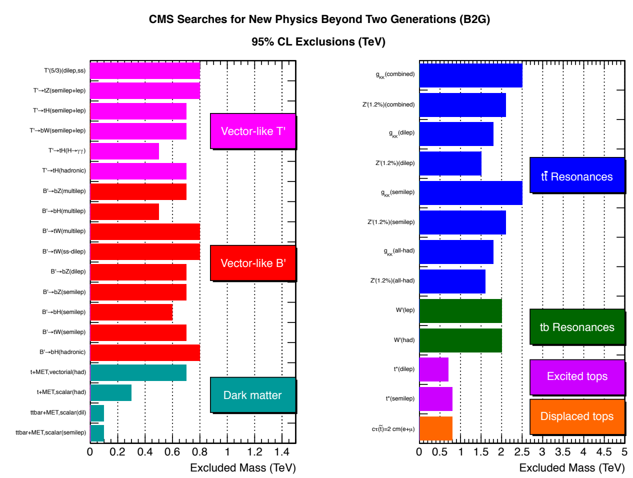
Neutral naturalness

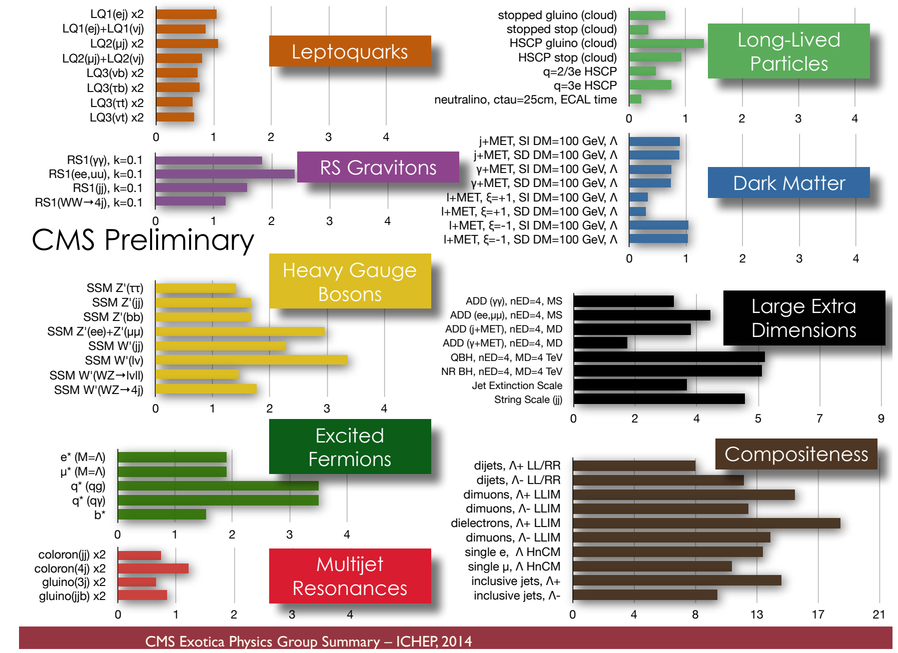
Partner quantum #s	Global dim-6 mixing	SUSY dim-8 mixing
QCD x EWK	CHM, Little Higgs	MSSM
Neutral x EWK	Quirky Little Higgs	Folded SUSY
Neutral x Neutral	Twin Higgs	????

*Table borrowed from David Curtin

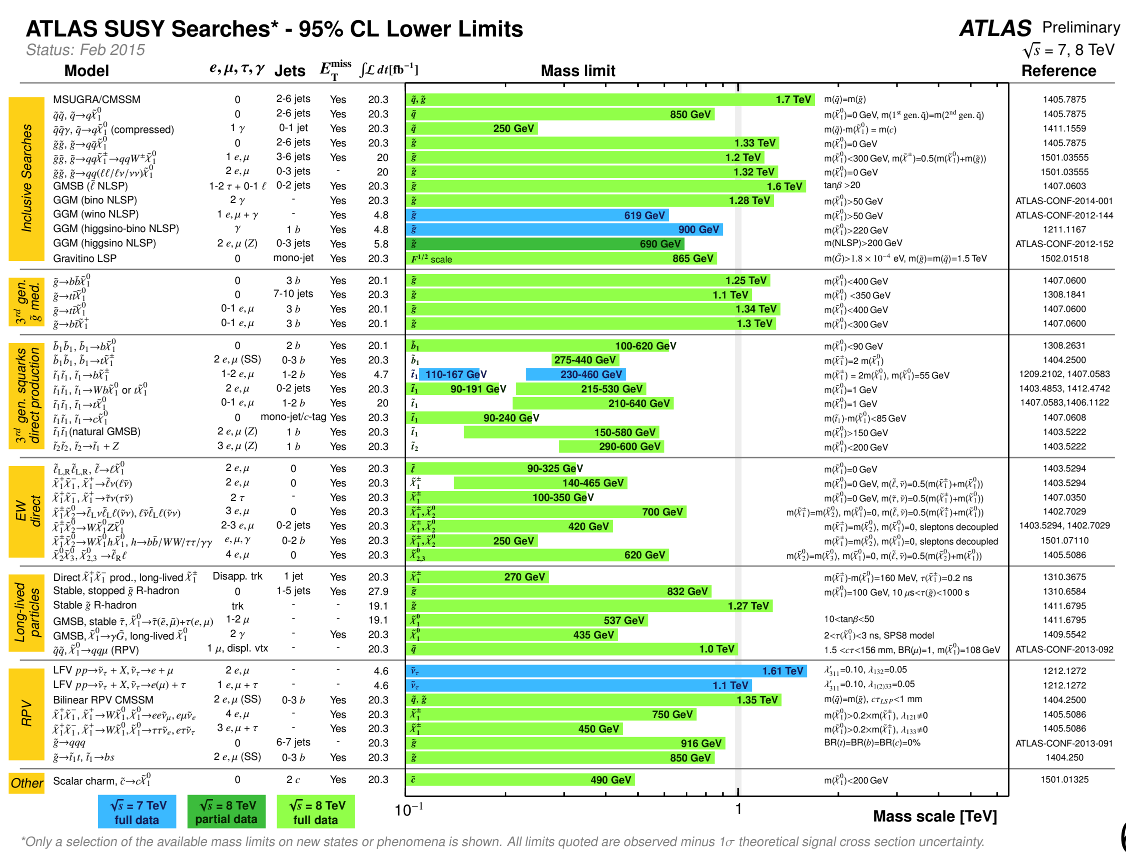
Naturalness: physics driver @ LHC

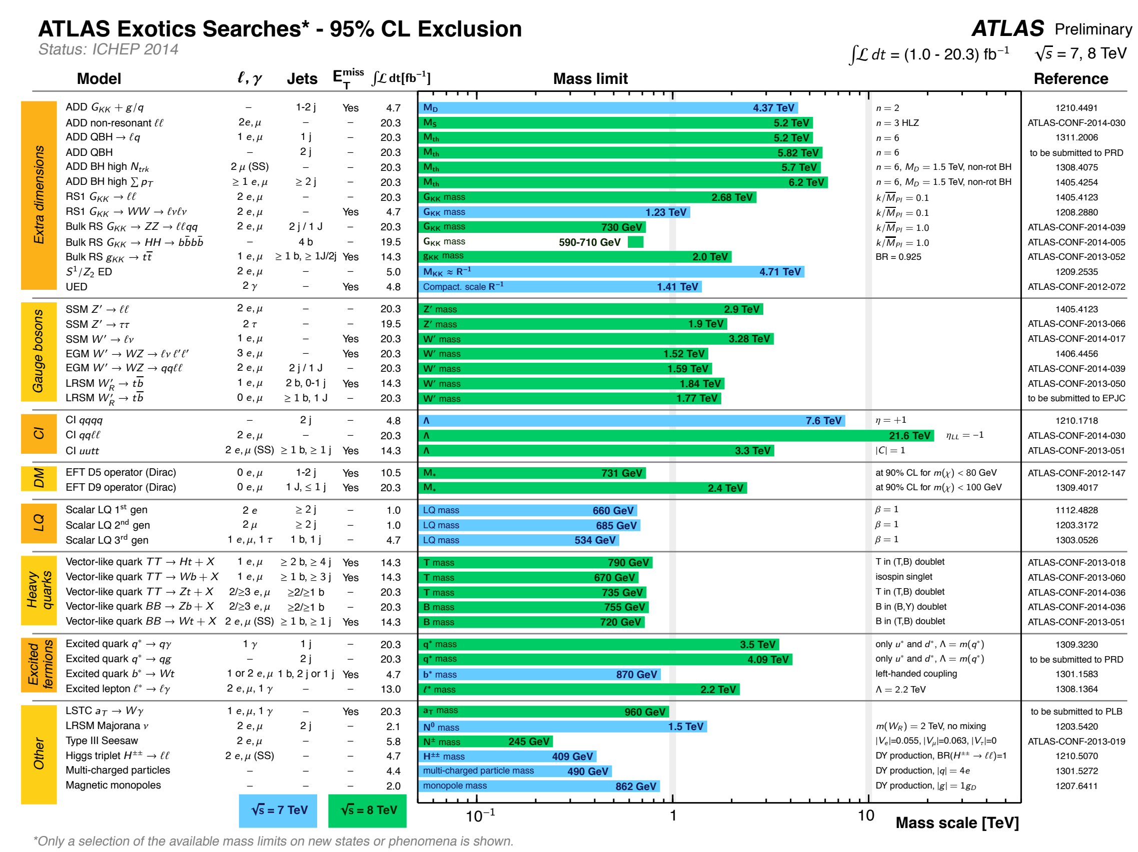






170 of these 226 channels tied to naturalness





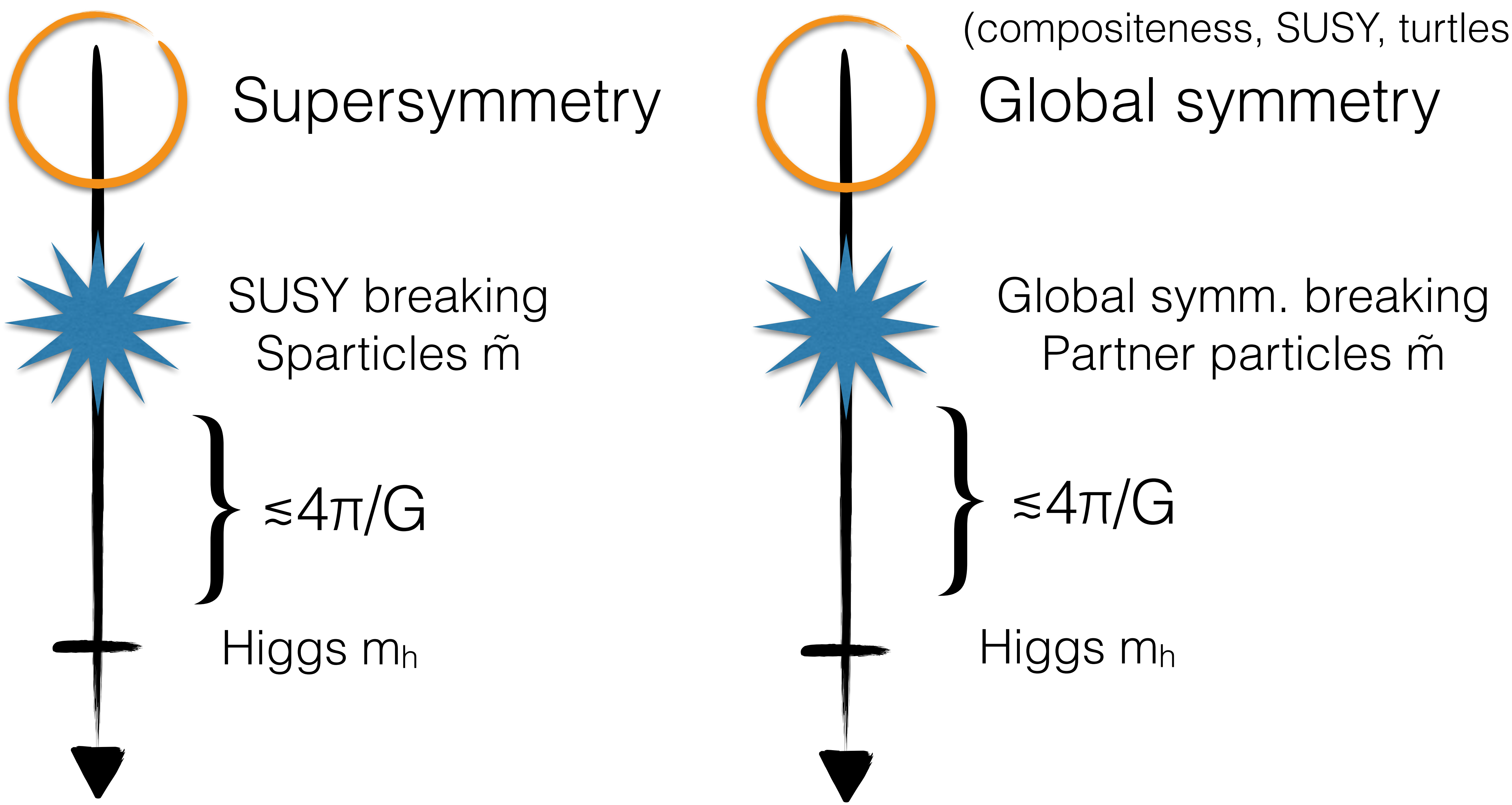
Signs of neutral naturalness

- [Chacko's talk] Higgs invisible width [mirror Twin Higgs]
- [Chacko's talk] Tree-level Higgs coupling deviations [Twin Higgs]
- [Burdman's, Farina's talks] Loop-level $h\gamma\gamma$, hZZ coupling deviations [folded SUSY]
- [Burdman's, Curtin's, Katz's talks] Displaced Higgs decays [folded SUSY, fraternal Twin Higgs]
- [Howe's talk?] Heavy higgs with reduced couplings, invisible width [Twin Higgs]
- [Burdman's, Curtin's talks] $W\gamma$, hh, displaced 4b resonances [folded SUSY]
- …

So far, so good…

What is the organizing principle?

Conventional symmetries



Continuous symmetries → partner states w/ SM quantum #s

$$m_h^2 \sim \frac{3y_t^2}{4\pi^2}\tilde{m}^2 \log(\Lambda^2/\tilde{m}^2)$$

Totally natural: $\tilde{m} \lesssim 200\,\mathrm{GeV}$

Orbifold correspondence

[Kachru, Silverstein ’98, Bershadsky, Johansen ’98, Schmaltz ’99,…]

Parent theory $\xrightarrow{\text{orbifold}}$ Daughter theory

- In the large N limit, correlation functions (two-point functions!) of the parent and daughter theory are identical up to an overall rescaling.
- Daughter theory does not possess all the irreps of parent theory, but still reflects (up to 1/N) symmetries of parent theory.
- Given a continuous symmetry solution to hierarchy problem, orbifold probably solves it too.
- Provides an organizing principle for neutral naturalness.

Caution!

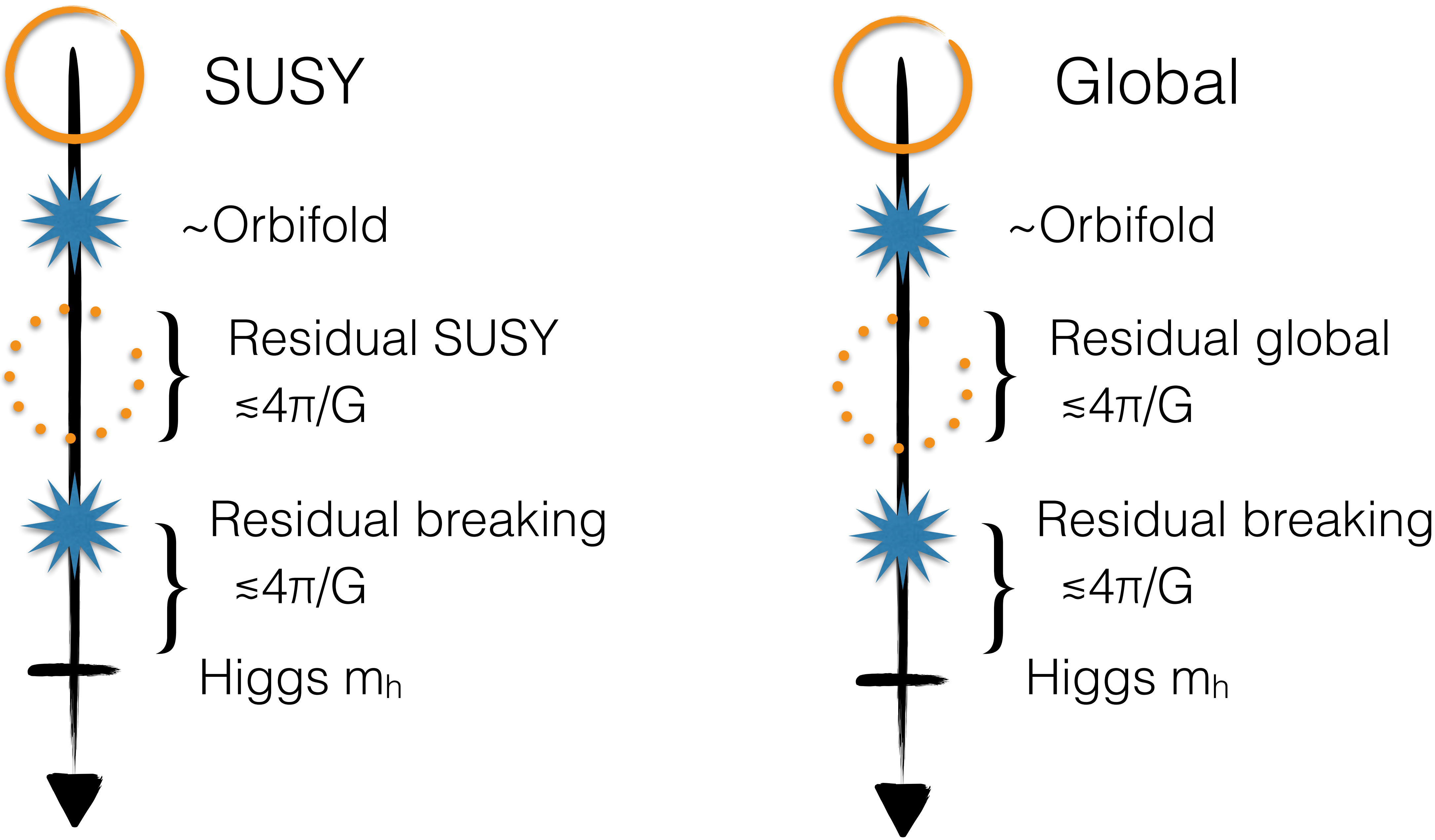
Orbifold equivalence
is really…

Theory A		Theory B
Planar diagrams around	$\Leftrightarrow$	Planar diagrams around
Vac_A		$\text{Im}(\text{Vac}_A)$

In general, $\text{Vac}_B \neq \text{Im}(\text{Vac}_A)$ for global symmetries.

1. We have not yet actually tried to exploit this correspondence directly.
2. *Proceeding with caution, leveraging orbifold correspondence more completely may lead to new lessons.*

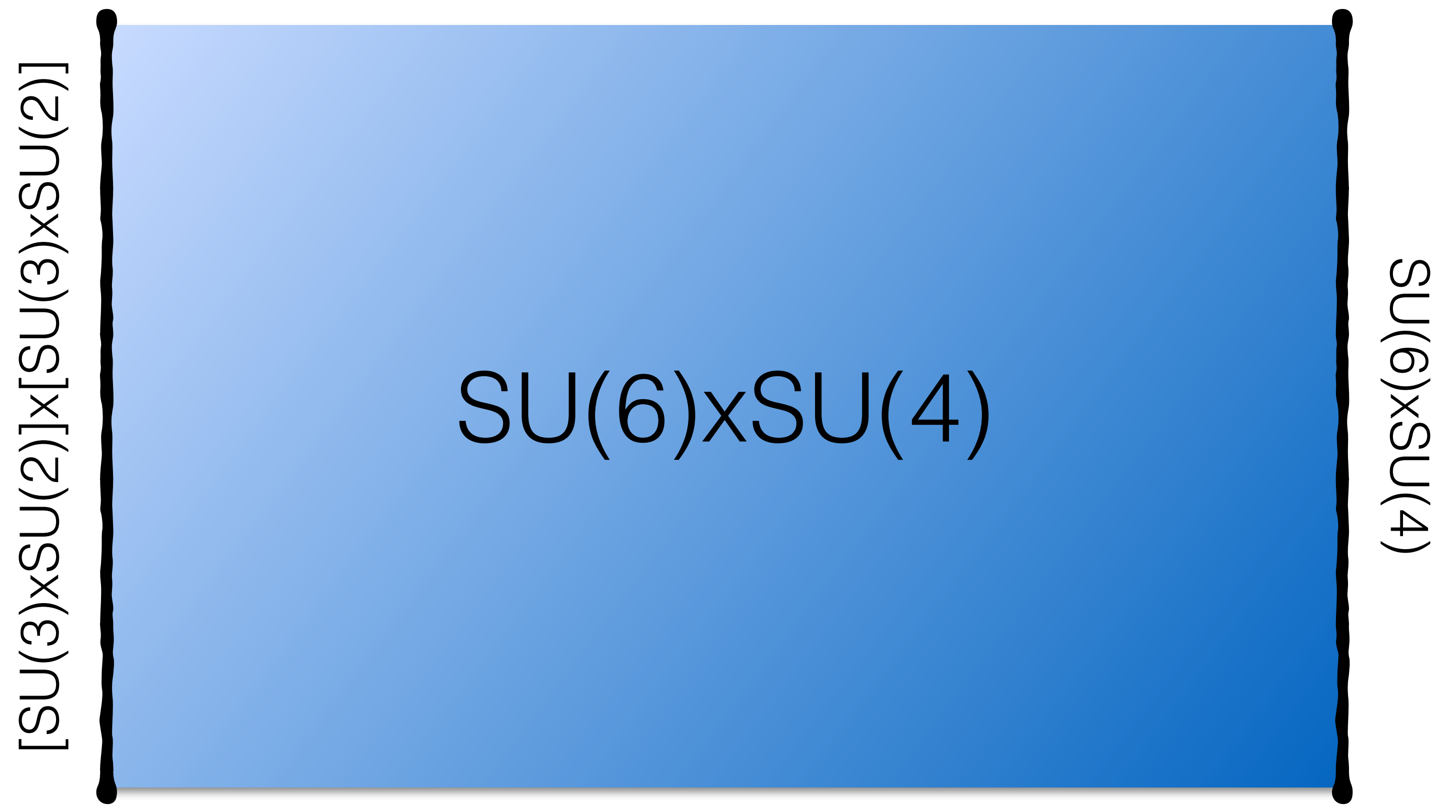
Unconventional symmetries



Residual symmetries → partner states without SM quantum #s

$$m_h^2 \sim \frac{3y_t^2}{4\pi^2}\tilde{m}^2 \log(\Lambda^2/\tilde{m}^2)$$ Totally natural: $\tilde{m} \lesssim 200\,\mathrm{GeV}$

Twin Higgs*

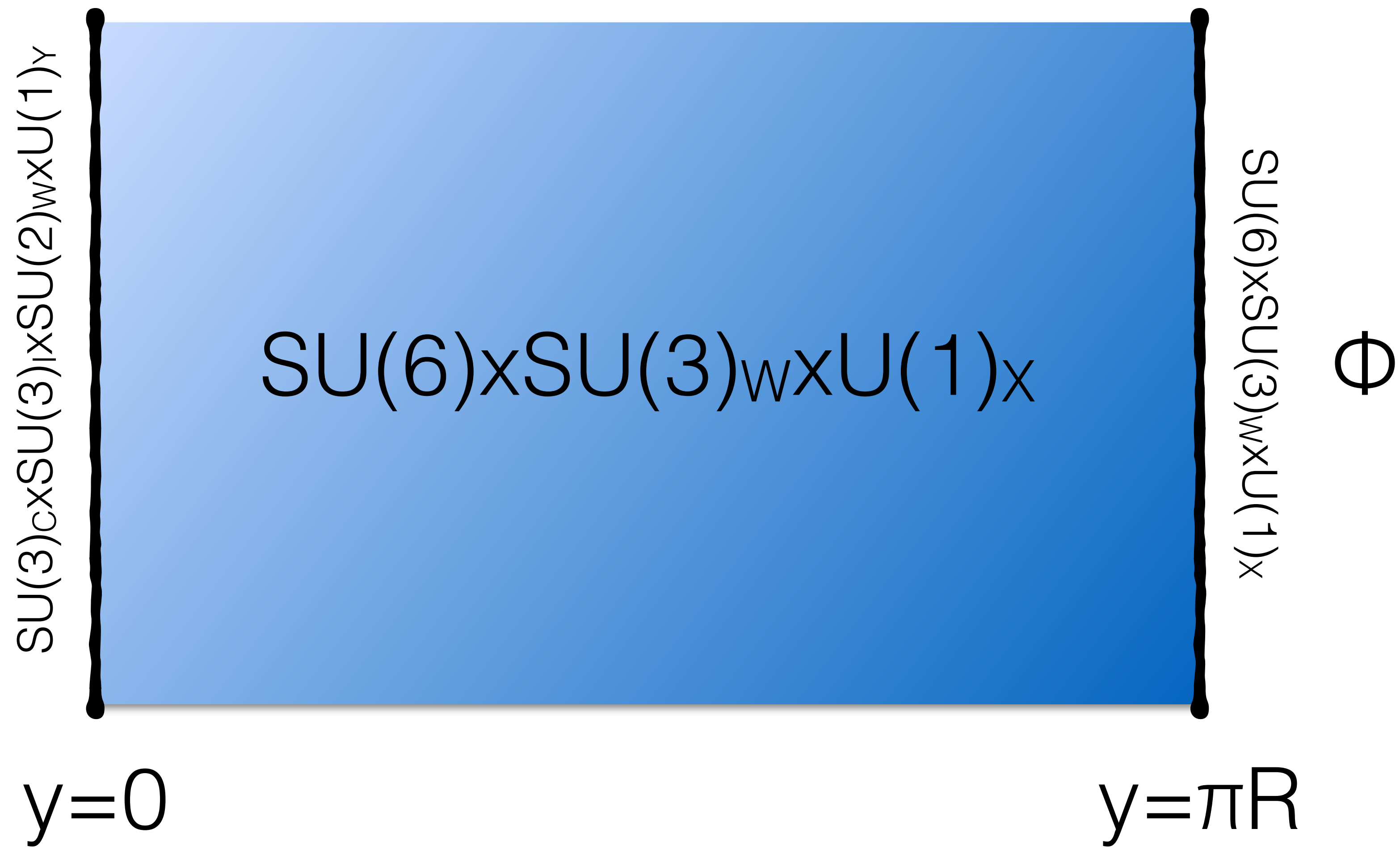


Zero mode spectrum: $SU(3)_A x SU(3)_B x SU(2)_A x SU(2)_B x [U(1)_Y]$

Fermionic top partners charged under $SU(3)_B x SU(2)_B$

*Mirror twin Higgs doesn't *need* to be an orbifold; bottom-up Z_2 symmetry sufficient

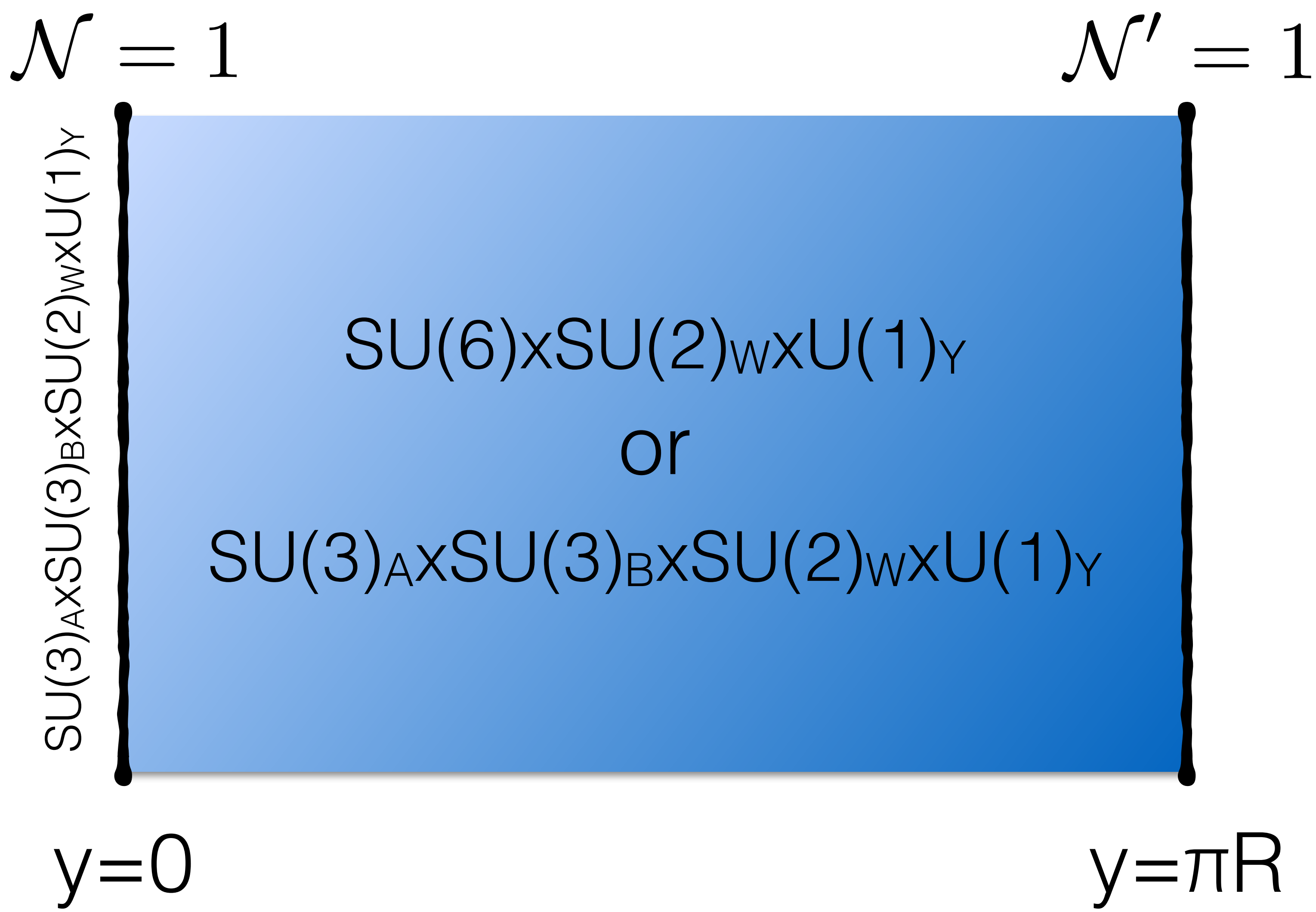
Quirky Little Higgs



Zero mode spectrum: $SU(3)_C \times SU(3)_I \times SU(2)_W \times U(1)_Y$

Fermionic top partners charged under $SU(3)_I \times SU(2)_W \times U(1)_Y$

Folded SUSY



Zero mode spectrum: $SU(3)_A x SU(3)_B x SU(2)_W x U(1)_Y$

Scalar top partners charged under $SU(3)_B x SU(2)_W x U(1)_Y$

When do you *need* an orbifold?

- Can construct Z_2 Twin Higgs (or even Z_N Twin Higgs) without recourse to orbifolds, purely from the infrared: just impose parity and assume quartic; accidental quadratic global symmetry follows.

- *Or* you can get the parity as an accidental symmetry from an orbifold. I.e., $SU(4)/Z_2$ gives $SU(2)xSU(2)xS_2$. No big deal?

- But orbifolds can also give you approximate symmetries that cannot be written as exact parities of IR theory. I.e., incomplete multiplets for Z_2 Twin Higgs, or $SU(12)/S_3 \to SU(2)xSU(2)xSU(4)xS_2$

- Also need orbifolds whenever SUSY is nontrivially involved in IR.

Thinking more about orbifolds will lead us to new theories.

Where to go
from here?

1. Further explore orbifolds: both field theory and geometry
2. Fill the last box: totally neutral scalar top partners?
3. Outmaneuver Rattazzi: do we always hit color at two loops?
4. Explore four dimensions: deconstruct everything.
5. Embrace accidents: Accidental symmetries without symmetries.

Orbifold away…

Much more room for orbifolds, both field theory orbifolds & geometric completions. E.g., $Z_N x Z_M$, combinations of global/gauge/R-symmetries, irregular embeddings, etc.

Also more exotic reduction of symmetries possible:

Z_2 inner automorphism

G	H	restrictions
SU(p+q)	SU(p)×SU(q)×U(1)	
SO(p+q)	SO(p)×SO(q)	p or q even
SO(2n)	SU(n)×U(1)	
E_6	SU(6)×SU(2)	
E_6	SO(10)×U(1)	

[Hebecker & March-Russell]

Z_2 outer automorphism

G	H	action or restrictions
U(1)	1	$q \to -q$
SU(n)	SO(n)	$R \to \overline{R}$
SO(p+q)	SO(p)×SO(q)	$R \to \overline{R}$, p, q odd $p + q = 4n + 2$
SO(p+q)	SO(p)×SO(q)	$S \to S'$, p, q odd $p + q = 4n$
SU(2n)	Sp(n)	$R \to \overline{R}$
E_6	Sp(4)	$R \to \overline{R}$
E_6	F_4	$R \to \overline{R}$

Rank-preserving

Rank-breaking

Totally neutral scalars?

SU(6)xSU(2)$_W$xU(1)$_Y$

or

SU(3)$_A$xSU(3)$_B$xSU(2)$_W$xU(1)$_Y$

In folded, take a common SU(2) factor:

$$\mathcal{L} \supset \lambda_t H_u q_3^A u_3^A + \lambda_t^2 |H_u \cdot \tilde{q}_3^B|^2 + \lambda_t^2 |H_u|^2 |\tilde{u}_3^B|^2$$

so F-stops are electroweak states;
bounds from Drell-Yan production.

Is it possible to arrange for completely neutral top partners?

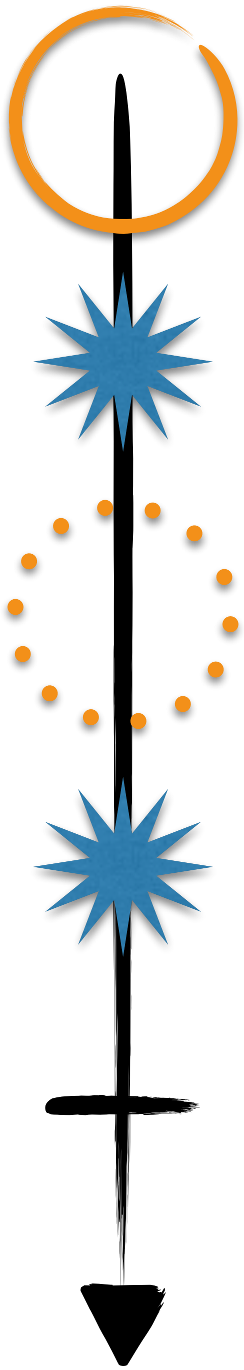
$$\lambda_t H_u q_3^A u_3^A + \lambda_t^2 |H_u|^2 |\tilde{q}_3^B|^2 + \lambda_t^2 |H_u|^2 |\tilde{u}_3^B|^2$$

Need to get the scalar potential without an SU(2) contraction

$$\begin{pmatrix} H_u & \cdot \end{pmatrix} \begin{pmatrix} \cdot & X \\ \cdot & \cdot \end{pmatrix} \begin{pmatrix} \cdot \\ \tilde{q}_3^B \end{pmatrix}$$

Not impossible; suggests
larger parent symmetry and
judicious orbifold projection.

Outmaneuver Rattazzi



$$\boxed{m_H^2 \sim \frac{3y_t^2}{8\pi^2}\frac{\lambda_t^2}{g_*^2} m_{\text{colored}}^2}$$

Does *UV-complete* neutral naturalness always give SM-charged states at two loops?

E.g., full SUSY or compositeness enters at two loops above Higgs mass.

True of all current UV-complete examples.

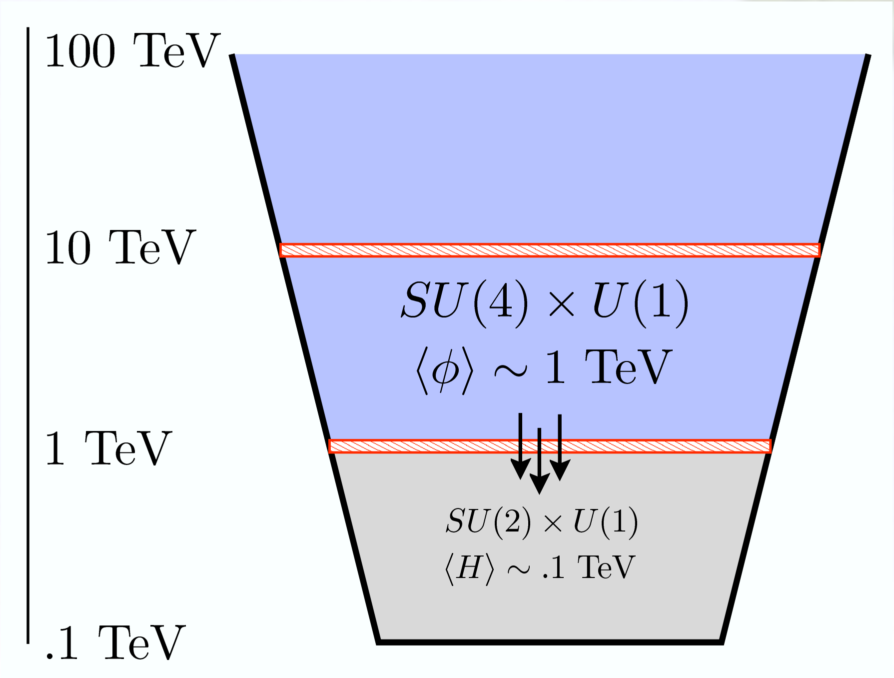
If true: Essentially a no-lose theorem for 100 TeV collider, bonanza for FCC-hh/SppC physics case.

Turtles?

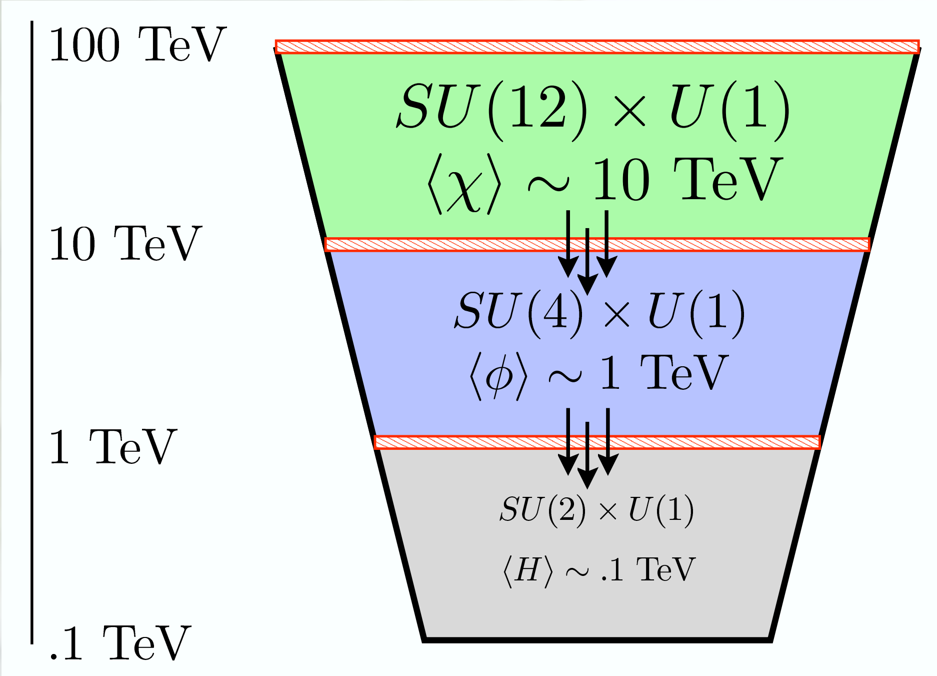
[Batra, Kaplan]

Is there an alternative?

SUSY/CHM



Turtles



N grows with each level; couplings rapidly grow strong.

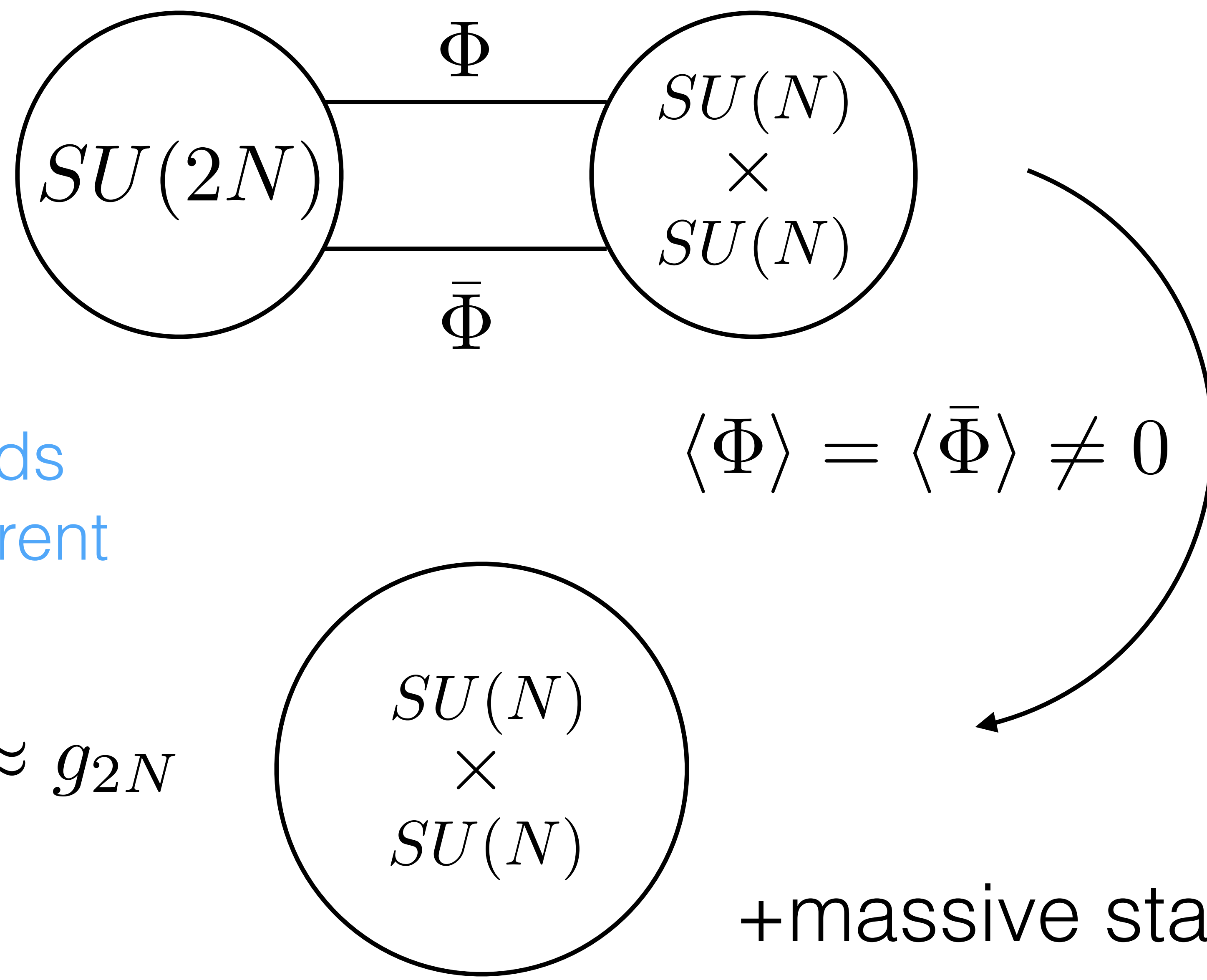
Unless theory has good large-*N* behavior, i.e. ’t Hooft couplings Ng^2, Ny^2, $N\lambda^2$ remain under control. Plausible, no examples yet…

Explore four dimensions

Deconstruction

Fairly simple to build 4d orbifolds for global symmetries

$g_{2N} \ll g_{N\times N}$



Geometry = fields localized on different gauge nodes

$g \approx g_{2N}$

+massive states

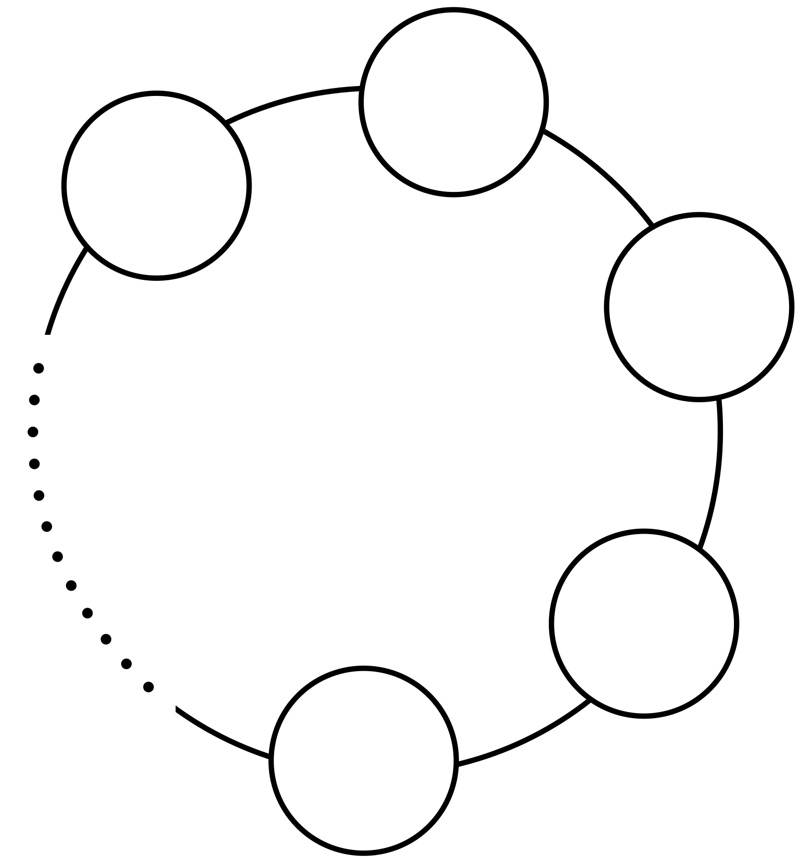
What about supersymmetry?

Twisted supersymmetry

[Arkani-Hamed, Cohen, Georgi]

Link-gaugino couplings:

$$\sum_{i=1}^{N} \sqrt{2}\phi_i^*(h_i\lambda_i - h'_{i-1}\lambda_{i-1})\psi_i + \text{h.c.}$$



SUSY: $h_i, h'_i = g$

Twist: $|h_i|, |h'_i| = g \qquad h_i, h'_i \neq g$

E.g. $h_i^*, h'_i = g e^{i\theta/N}$

Can always locally rephase, but global phase exists in circular quiver.

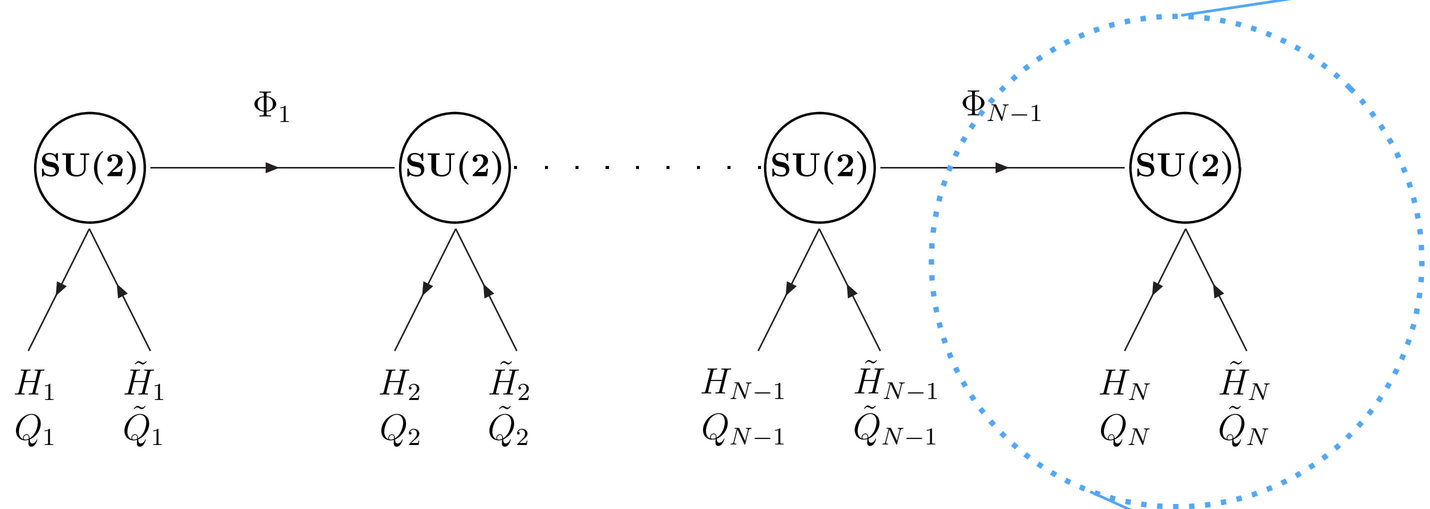
$$m_{B,n}^2 = \left(\frac{2}{a}\right)^2 \sin^2\left(\frac{na}{2R}\right)$$

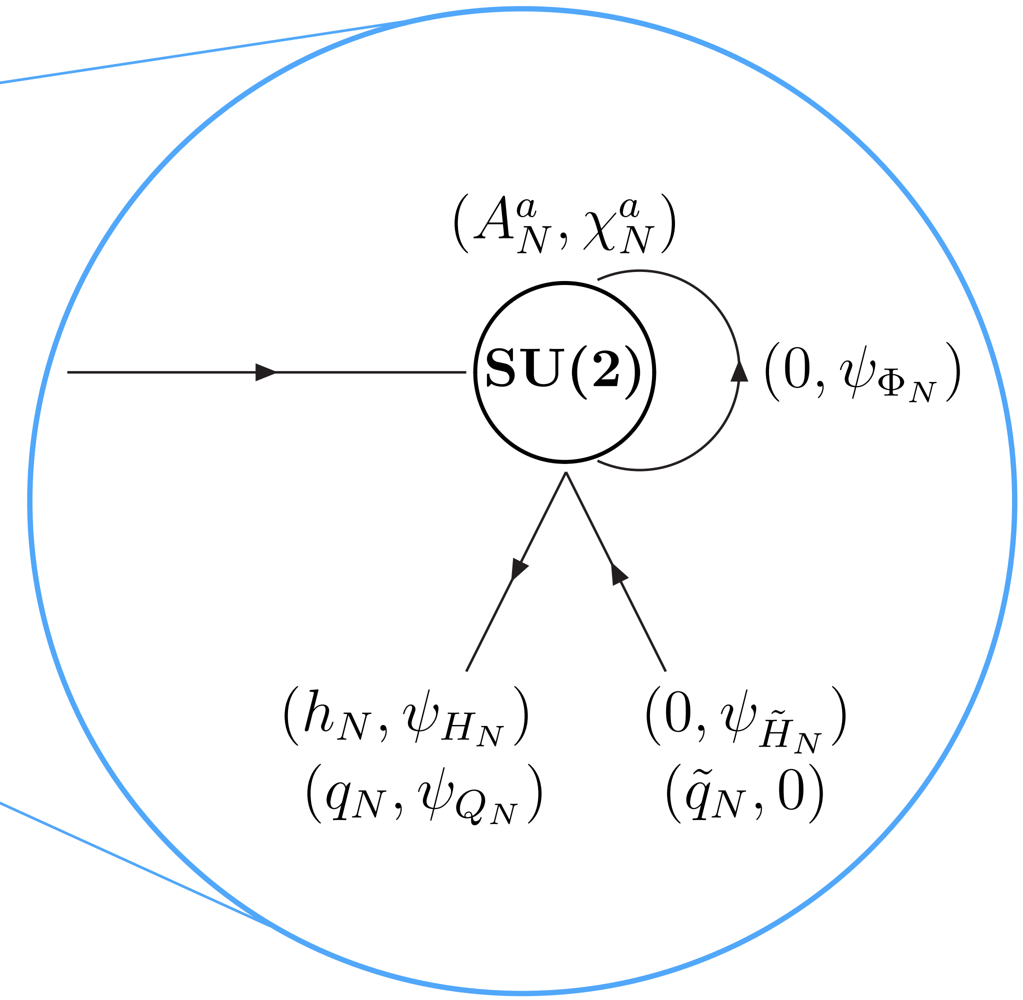
$$m_{F,n}^2 = \left(\frac{2}{a}\right)^2 \sin^2\left(\frac{na}{2R} + \theta\right)$$

1. Maximal twist only (gauginos dirac)
2. Incompatible with chiral fermions (S_1)

Soft ~~SUSY~~ from hard ~~SUSY~~

[Falkowski, Grojean, Pokorski]





	gauge and link	quark	Higgs
$m_{(0)} = 0$	$A^a_{(0)}$	$\psi_{Q_{(0)}}$	$h_{(0)}$
$m_{(n)} = 2g_0 v \sin\left(\frac{n\pi}{2N}\right)$	$A^a_{(n)},\ \phi^a_{(n)}$	$\begin{pmatrix} \psi_{Q_{(n)}} \\ -i\sigma_2\, \psi^*_{\tilde{Q}_{(n)}} \end{pmatrix}$	$h_{(n)}, \tilde{h}_{(n)}$
$\tilde{m}_{(n)} = 2g_0 v \sin\left(\frac{(2n-1)\pi}{2(2N+1)}\right)$	$\begin{pmatrix} \psi_{\phi^a_{(n)}} \\ -\sigma_2\, \chi^{a\ *}_{(n)} \end{pmatrix}$	$q_{(n)}, \tilde{q}_{(n)}$	$\begin{pmatrix} \psi_{H_{(n)}} \\ -i\sigma_2\, \psi^*_{\tilde{H}_{(n)}} \end{pmatrix}$

Delete supermultiplet components on boundary

1. Maximal twist only (gauginos dirac).
2. Two loop quadratic divergences.

SS is radion mediation

[Marti & Pomarol; Kaplan & Weiner]

$$V \supset -i\bar{\theta}^2\theta\lambda_1 + i\theta^2\bar{\theta}\bar{\lambda}_1 \qquad \chi \supset \sqrt{2}\theta\lambda_2$$

Break SUSY with radion F-term

$$T \sim \frac{1}{R}(R + \theta^2 F_T) \qquad \mathcal{L}_{\cancel{\text{SUSY}}} \ni -\frac{F_T}{4R}\lambda_1^a\lambda_1^a + \frac{F_T^\dagger}{4R}\lambda_2^a\lambda_2^a$$

Write in SU(2)$_R$-symmetric form

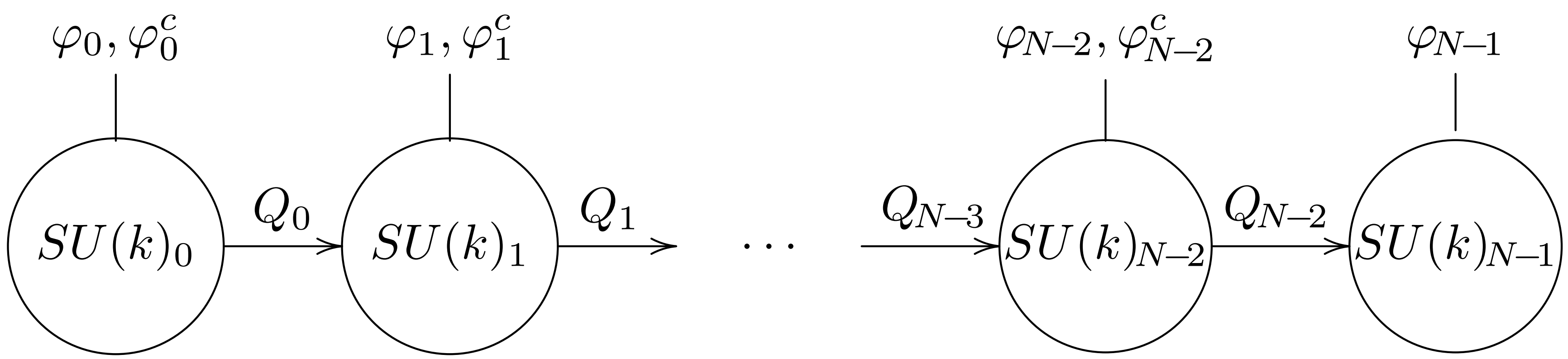
$$\mathcal{L} \ni i\boldsymbol{\lambda}_i^{a\dagger}\bar{\sigma}^\mu\partial_\mu\boldsymbol{\lambda}_i^a + \frac{1}{2R}\boldsymbol{\lambda}_i^a\epsilon_{ik}\left(\delta_{kj}\partial_\phi + if_{kj}\right)\boldsymbol{\lambda}_j^a \qquad f_{ij} = \frac{1}{2}\begin{pmatrix} 0 & -iF_T^\dagger \\ iF_T & 0 \end{pmatrix}$$

Rephase $\quad \lambda_i \to e^{i\phi f_{ij}}\lambda_j$

Removes majorana term, gives nontrivial winding in 5th dimension.

So deconstruct radion mediation instead

[NC, Tim Lou]



$$\int d^4\theta \sum_i Q_i^\dagger e^{V_i} Q_i e^{-V_{i+1}} \longrightarrow \int d^4\theta \, \frac{2}{T+T^\dagger} \sum_i^N Q_i^\dagger e^{V_i} Q_i e^{-V_{i+1}}$$

$$\int d^4\theta \sum_i \varphi_i^\dagger e^V \varphi_i \longrightarrow \int d^4\theta \, \frac{T+T^\dagger}{2} \sum_i \varphi_i^\dagger e^V \varphi_i$$

$$\int d^4\theta \sum_i \varphi_i^c e^{-V} \varphi_i^{c\dagger} \longrightarrow \int d^4\theta \, \frac{T+T^\dagger}{2} \sum_i \varphi_i^c e^{-V} \varphi_i^{c\dagger}$$

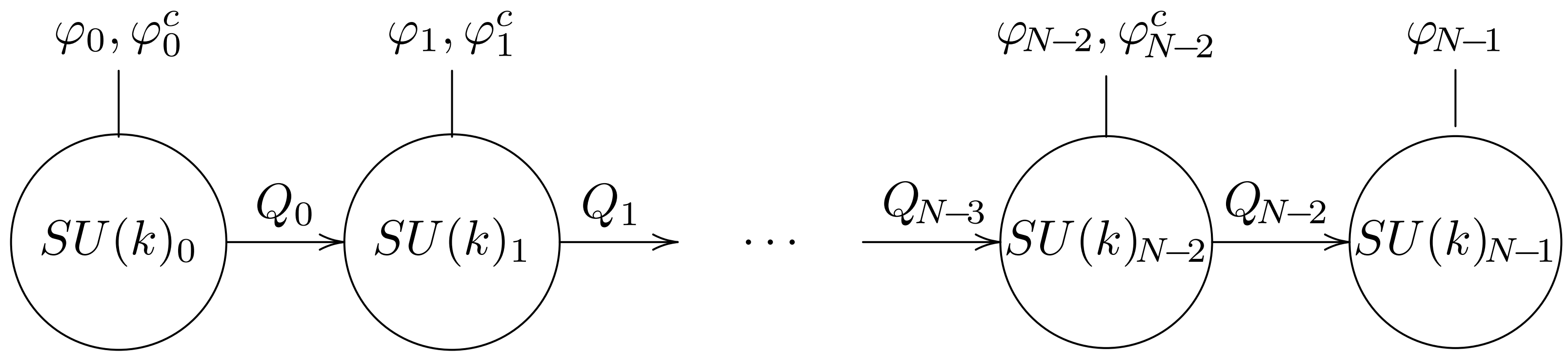
$$\frac{1}{2g^2} \int d^2\theta \sum_i \mathrm{tr}\, W_i^\alpha W_{i\alpha} \longrightarrow \frac{1}{2g^2} \int d^2\theta \, T \sum_i \mathrm{tr}\, W_i^\alpha W_{i\alpha}$$

$$T \sim \frac{1}{R}(R + \theta^2 F_T)$$

1. Deconstruct S_1/Z_2
2. Add radion-type couplings
3. Give “radion” an F-term

Manifestly soft

Mass spectrum



$$m_{\lambda_n^\pm} = \frac{2}{a}\left|\sin\left(\frac{na}{2R}\right) \pm \frac{a\alpha}{2R}\right| \simeq \frac{n}{R} \pm \frac{\alpha}{R} \qquad a = \frac{\sqrt{2}}{gv}$$

$$m_{\varphi_n^\pm} = \frac{2}{a}\left|\sin\left(\frac{na}{2R}\right) \pm \frac{a\alpha}{2R}\right| \simeq \frac{n}{R} \pm \frac{\alpha}{R} \qquad R = \frac{N}{\sqrt{2}\pi gv}$$

Reproduces Scherk-Schwarz spectrum; SUSY breaking is soft
Works to all loops, allows SS spectrum for arbitrary twist.

Full disclosure

Radiative corrections to zero modes:

$$\delta m^2 \sim N \frac{3}{4\pi^2} g^2 v^2 \ln\left(\frac{\Lambda}{2gv}\right)^2$$

Logarithmic dependence; same as soft mass in 5d for finite # of KK modes. Usual 5d result involves integrating over infinitely many modes.

Deconstruction of SS possible, but finite quiver loses ideal features of 5d picture.

Still opportunities, e.g. warping+SS?

Embrace accidents

Orbifolds provide organizing principle for neutral naturalness, but somewhat cumbersome.

What if necessary symmetries are accidental symmetries of IR fixed point?

E.g. accidental supersymmetry [Sundrum]

IR theory contains SM + scalars

$$\lambda_t H Q_3 \bar{u}_3 \qquad \lambda_\phi |H|^2 |\phi|^2$$

Strong dynamics drives

$$\lambda_\phi \sim \lambda_t^2$$

Conclusions

- We’re in the early days of exploring neutral naturalness; much to learn.
- Clearly an excellent signal generator; excellent motivation for new searches at LHC.
- Orbifolds of continuous symmetries provide a natural organizing principle for generalizations.
- Many opportunities for exploration; likely we’ll discover more surprising and delightful ideas.

Thank you!