Cone vs. k_t , FastJet, and UE/MB jet corrections in k_t -clustering

Matteo Cacciari and Gavin Salam

LPTHE, Universities of Paris VI and VII and CNRS

MC4LHC - CERN, 20 July 2006

Outline

- ► Cone vs. k_t
 - (Since Joey "announced" it. He might not like what I'll say, though...)
 - Overview of iterative cone algorithms (& what's wrong with them)
 - Clustering algorithms
 - How they work
 - ▶ Where they've been criticised (speed, underlying-event (UE) sensitivity)
- lacktriangle How to solve the speed problem. Fast algorithm for k_t clustering:

A brief presentation of the $\mathcal{O}(N \ln N)$ algorithm

Underlying event and minimum bias/pile-up subtraction using FastJet and jet areas

Some preliminary plots and results

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What is **needed** of a jet algorithm

▶ Must be infrared and collinear (IRC) safe

soft emissions shouldn't change jets collinear splitting shouldn't change jets

Must be identical procedure at parton level, hadron-level So that theory calculations can be compared to experimental measurements

What is *nice* for a jet algorithm

- ► Shouldn't be too sensitive to hadronisation, underlying event, pileup

 Because we can only barely model them
- ► Should be realistically applicable at detector level

Not too slow, not too complex to correct

Should behave 'sensibly'

e.g. don't want it to spuriously ignore large energy deposits

Mainstream jet-algorithms

▶ Iterative cone algorithms (JetClu, ILCA/Midpoint, ...)

Searches for cones centred on regions of energy flow Dominant at hadron colliders

Sequential recombination algorithms (k_t , Cambridge/Aachen, Jade)

Recombine closest pair of particles, next closest, etc.

Dominant at e^+e^- and ep colliders

Other approaches

▶ 'Optimal Jet Finder', Deterministic Annealing

Fit jet axes (and #) so as to minimise a weight function [forms of 'k-means' clustering]

...

As LHC startup approaches it's important for the choice of jet algorithm to be well-motivated.

First 'cone algorithm' dates back to Sterman and Weinberg (1977) — the original infrared-safe cross section:

To study jets, we consider the partial cross section $\sigma(E,\theta,\Omega,\epsilon,\delta) \text{ for } e^+e^- \text{ hadron production events, in which all but}$ a fraction $\epsilon <<1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta <<1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 <<\Omega <<1$) at an angle θ to the e^+e^- beam line. We expect this to be measur-

$$\sigma(E,\theta,\Omega,\epsilon,\delta) = (d\sigma/d\Omega)_0 \Omega \left[1 - (g_E^2/3\pi^2)\left\{3\ln\delta + 4\ln\delta \ln 2\epsilon + \frac{\pi^3}{3} - \frac{5}{2}\right\}\right]$$

Modern cones address various issues

Where do you put the cones?

- ▶ Place a cone at some trial location
- ▶ Sum four-momenta of particles in cone find corresponding axis
- ▶ Use that axis as a new trial location, and *iterate*
- ► Stop when you reach a stable axis [or when you get bored]

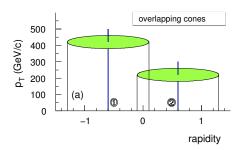
What are the initial trial locations?

► 'Seedless' — i.e. everywhere

But too slow on computer

Use locations with energy flow above some threshold as seeds Issue: is seed threshold = parton energy, hadron energy (collinear unsafe)? Or calorimeter tower energy (experiment and η-dependent)?

Jets can overlap



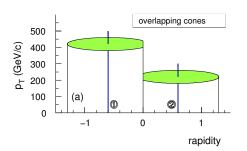
They are either *split* if the overlapping energy is

 $E_{
m overlap} < f_{
m overlap} \, E_{
m softer-jet}$ otherwise they are *merged*.

NB: $f_{
m overlap}$ is parameter of cone-algo

NB: when many jets overlap, procedure for merging/splitting must be specified (e.g. wrt order in which jets are treated).

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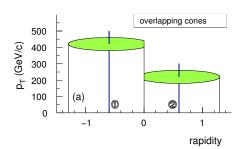
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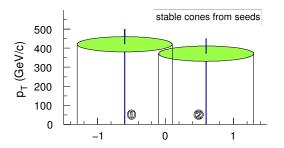


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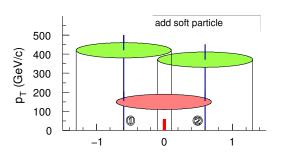
This is IR unsafe.

Kilgore & Giele '9

Solution: add extra seeds at midpoints of all pairs, triplets, . . . of stable cones.

Seymour '97 (?

NB: only in past 1-2 years has this fix appeared in CDF and D0 analyses. . .



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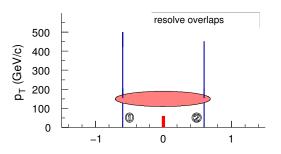
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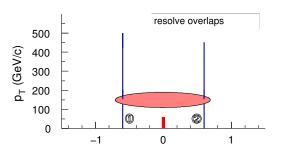
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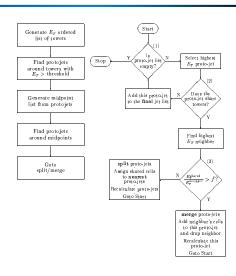
All of these considerations led recommendation of the *Improved Legacy Cone Algorithm* (ILCA), a.k.a. *Midpoint* algorithm.

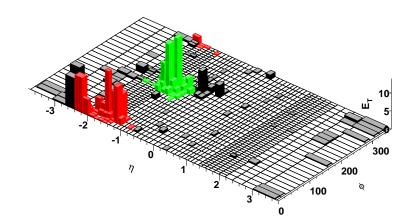
 $\mathsf{hep\text{-}ex}/0005012$

Quite complex and has several parameters:

cone radius (R)seed threshold (E_0) f_{overlap}

Only one of these is remotely physical: *R*.

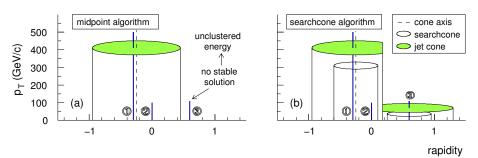




Considerable energy can be left out of jets ≡ Dark Towers

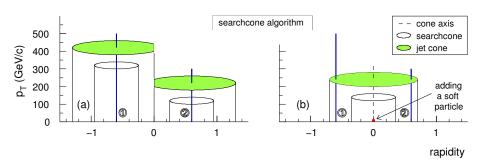
S. Ellis, Huston & Tönnesmann '01

Dark towers are consequence of particles that are never in stable cones:

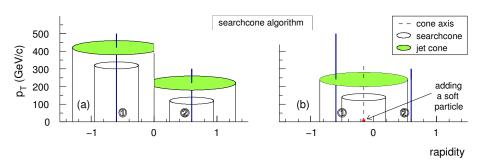


Ellis, Huston and Tönnesmann suggest *iterating a smaller 'search-cone'* and then drawing final cone around it.

Searchcone adopted by CDF (to confuse issue they still call it 'midpoint'...) ${\sf hep-ex/0505013,\ hep-ex/0512020}$



Whether you see 1 or 2 jets depends on presence and position of a soft gluon — this is *IR unsafe (and unphysical)*. Wobisch, '06



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- Cone algorithms are complicated beasts.
- ► So much so, it's often not clear *which* cone algorithm is being used!
- ▶ They often behave in unforeseen ways.
- ▶ Patching them makes them more complex and error-prone.

Didn't even mention the hacks people put into cone theory calculations to 'tune' them to hadron level: (cf. R_{sep} , which breaks the NLO jet X-section).

LHC experiments should be wary of cone algorithms

Best known is kt algorithm:

1. Calculate (or update) distances between all particles *i* and *j*, and between *i* and beam:

$$d_{ij} = \min(k_{ti}^2, k_{tj}^2) \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = k_{ti}^2, \qquad \Delta R_{ij}^2 = \Delta y_{ij}^2 + \Delta \phi_{ij}^2$$

- 2. Find smallest of d_{ij} and d_{iB}
 - ▶ If d_{ij} is smallest, recombine i and j (add result to particle list, remove i, j)
 - if d_{iB} is smallest call i a jet (remove it from list of particles)
- 3. If any particles are left, repeat from step 1.

Catani, Dokshitzer, Olsson, Turnock, Seymour & Webber '91–93

S. Ellis & Soper, '93

Variant: Cambridge / Aachen algorithm. Like k_t with but $d_{ij} = \Delta R_{ij}^2/R^2$ and $d_{iB} = 1$. Dokshitzer, Leder, Moretti & Webber '97; Wobisch '00

 k_t distance measures

$$d_{ij} = \min(k_{ti}^2, k_{tj}^2) \frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = k_{ti}^2$$

are closely related to structure of divergences for QCD emissions

$$[dk_j]|M_{\mathbf{g}\to\mathbf{g}_i\mathbf{g}_j}^2(k_j)| \sim \frac{\alpha_s C_A}{2\pi} \frac{dk_{tj}}{\min(k_{ti}, k_{tj})} \frac{d\Delta R_{ij}}{\Delta R_{ij}}, \qquad (k_{tj} \ll k_{ti}, \ \Delta R_{ij} \ll 1)$$

and

$$[dk_i]|M^2_{\mathsf{Beam}\to\mathsf{Beam}+\mathsf{g}_i}(k_i)| \sim \frac{\alpha_{\mathsf{s}}\,\mathsf{C}_{\mathsf{A}}}{\pi}\frac{dk_{ti}}{k_{ti}}\,d\eta_i\,,\qquad (k_{ti}^2\ll\{\hat{\mathsf{s}},\hat{\mathsf{t}},\hat{u}\})$$

 k_t algorithm attempts approximate inversion of branching process

One parameter: R (like cone radius), whose natural value is 1 Optional second parameter: stopping scale d_{cut} 'exclusive' k_t algorithm

k_t algorithm seems better than cone

- ▶ it's simpler, safer and better-defined (IRC safe to all orders)
- exclusive variant is more flexible (allows cuts on momentum scales)
- ▶ less sensitive to hadronization
- ▶ In MC studies k_t alg. is systematically as good as, or better than cone algorithms for typical reconstruction tasks

 Seymour '94

Butterworth, Cox & Forshaw '02

Benedetti et al (Les Houches) '06

But seldom used at Tevatron. Why?

- 1. Because it's slow?
- 2. Because it includes more underlying event?
- 3. Because it's harder to understand/correct for detector effects/noise?

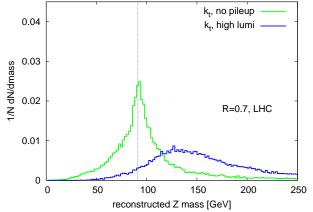
But all LEP and HERA experiments managed fine

And as of '05, CDF too

k_t v. cone: Z mass (uncorrected)

Try reconstructing M_Z from $Z \rightarrow 2$ jets [Use inv. mass of two hardest jets]

On same events, compare uncorrected k_t v. ILCA (midpoint) cone



 k_t allegedly more sensitive to min-bias.

Is this true?

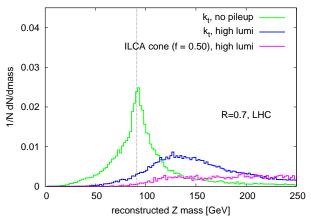
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ILCA with modified params. is no better than k_t .

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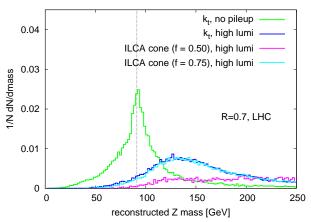
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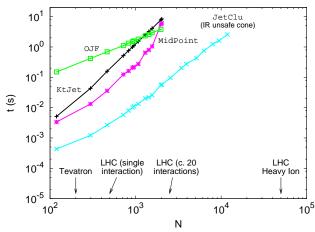
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Time to cluster *N* particles



Standard C++ (and fortran) k_t -clustering takes time $\sim N^3$.

a Pb-Pb event takes 1 day!

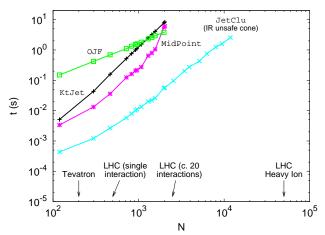
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Jet-clustering speed is an issue for high-luminosity pp ($\sim 10^8$ events) and Pb-Pb ($\sim 10^7$ events) collisions at LHC.

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- 1. Given the initial set of particles, construct a table of all the d_{ij} , d_{iB} .

 [$\mathcal{O}(N^2)$ operations, done once]
- 2. Scan the table to find the minimal value d_{\min} of the d_{ij} , d_{iB} . [$\mathcal{O}(N^2)$ operations, done N times]
- 3. Merge or remove the particles corresponding to d_{min} as appropriate.

 $[\mathcal{O}(1) \text{ operations, done } N \text{ times}]$

4. Update the table of d_{ij} , d_{iB} to take into account the merging or removal, and if any particles are left go to step 2.

 $[\mathcal{O}(N)]$ operations, done N times

This is the "brute-force" or "naive" method

There are N(N-1)/2 distances d_{ij} — surely we have to calculate them all in order to find smallest?

k_t distance measure is partly geometrical:

- ► Consider smallest $d_{ij} = \min(k_{ti}^2, k_{tj}^2) R_{ij}^2$
- ▶ Suppose $k_{ti} < k_{tj}$
- ▶ Then: $R_{ij} <= R_{i\ell}$ for any $\ell \neq j$. [If $\exists \ \ell$ s.t. $R_{i\ell} < R_{ij}$ then $d_{i\ell} < d_{ij}$]

In words: if i, j form smallest d_{ij} then j is geometrical nearest neighbour (GNN) of i.

 k_t distance need only be calculated between GNNs

Each point has 1 GNN ightarrow need only calculate N d_{ij} 's

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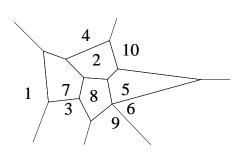
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Finding Geom Nearest Neighbours



Given a set of vertices on plane (1...10) a *Voronoi diagram* partitions plane into cells containing all points closest to each vertex

Dirichlet '1850, Voronoi '1908

A vertex's nearest other vertex is always in an adjacent cell.

E.g. GNN of point 7 will be found among 1,4,2,8,3 (it turns out to be 3)

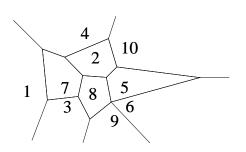
Construction of Voronoi diagram for *N* points: *N* In *N* time

Fortune '88

Update of 1 point in Voronoi diagram: In *N* time

Convenient C++ package available: CGAL http://www.cgal.org

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Devillers '99 [+ related work by other authors]

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Assembling fast k_t clustering

The FastJet algorithm:

Construct the Voronoi diagram of the N particles with CGAL $\mathcal{O}(N \ln N)$

Find the GNN of each of the N particles, calculate d_{ij} store result in a priority queue (C++ map) $O(N \ln N)$

Repeat following steps **N** times:

- ▶ Find smallest d_{ij} , merge/eliminate i, j
- Update Voronoi diagram and distance map

 $N \times \mathcal{O}(1)$

 $N \times \mathcal{O} (\ln N)$

Overall an $\mathcal{O}(N \ln N)$ algorithm

MC & GPS, hep-ph/0512210

http://www.lpthe.jussieu.fr/~salam/fastjet/

Results **identical** to standard N^3 implementations: this is **NOT** a new k_t jet-finder

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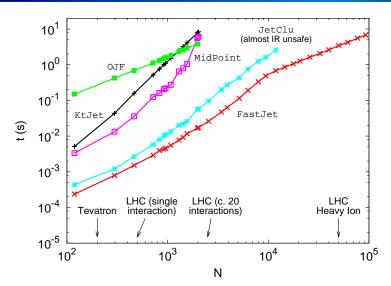
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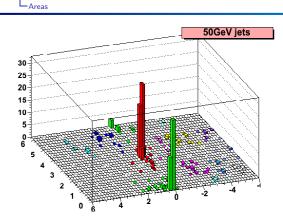
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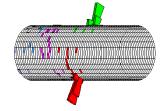
NB: for ${\it N} < 10^4$, FastJet switches to a related geometrical ${\it N}^2$ alg.

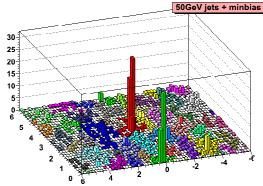


'Standard hard' event Two well isolated jets

50GeV jets

 \sim 200 particles Easy even with old methods

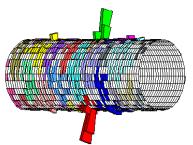




Add 10 min-bias events (moderately high lumi)

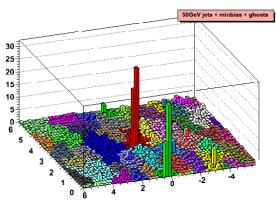
 \sim 2000 particles Clustering takes $\mathcal{O}\left(10s\right)$ with old methods.

20ms with FastJet.



L Areas

What is speed good for?

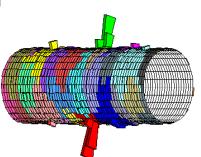


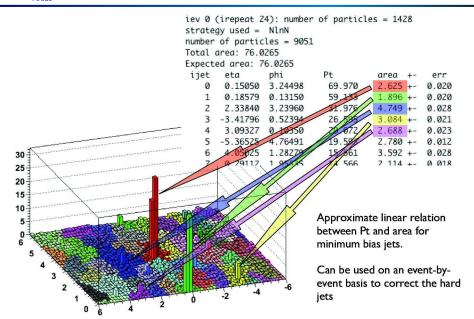
Add dense coverage of infinitely soft "ghosts"

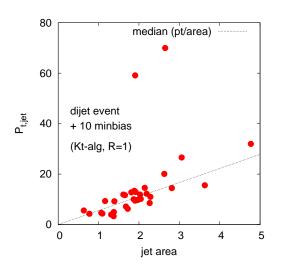
See how many end up in jet to measure jet area

 ~ 10000 particles Clustering takes ~ 20 minutes with old methods.

0.6s with FastJet.







Jet areas in k_t algorithm are quite varied

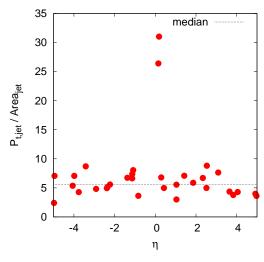
Because k_t -alg adapts to the jet structure

▶ Contamination from min-bias ~ area

Complicates corrections: minbias subtraction is different for each jet.

Cone supposedly simpler Area = πR^2 ?

Subtraction using areas



Key observation: p_T /area is quite uniform, except for the hard jets

Correction procedure:

Measure area *A* of each jet using ghost particles

Find median $p_t/A = Q_0$

Subtract $\Delta p_t = A \times Q_0$ from each jet.

NB. This is an event-by-event correction

NB: cone much harder to correct this way — too slow to add 10^4 ghosts

Examples of UE/MB subtraction using FastJet and area method

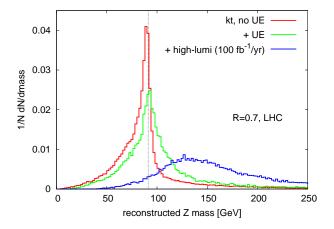
Preliminary results (MC & GPS) for

- High-lumi LHC
 - Z production
 - ightharpoonup Z' (mass = 2 TeV)
 - W bosons in $t\bar{t}$ events
 - **.**..
- Heavy ion collisions
 - inclusive jet distribution in Pb-Pb collisions

NB. Value of these results can only be judged by comparing to similar subtractions done with other algorithms/techniques....

Use jet areas to correct jet kinematics

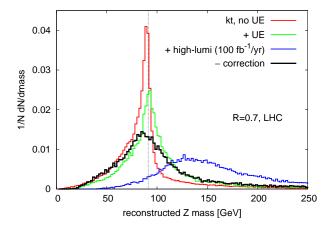
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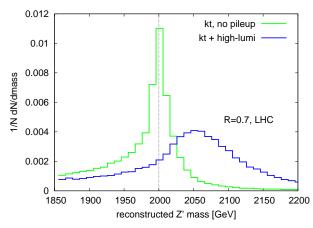
Some loss in resolution, but good value for the Z mass

Use jet areas to correct jet kinematics

Try reconstructing M_Z from $Z \rightarrow 2$ jets, with subtraction of UE/MB



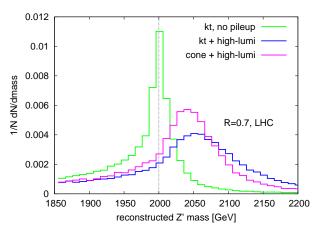
Some loss in resolution, but good value for the Z mass



Uncorrected cone better than k_* .

Cam is intermediate $(\langle A_{cam} \rangle \simeq \langle A_{cone} \rangle$, but fluctuations larger)

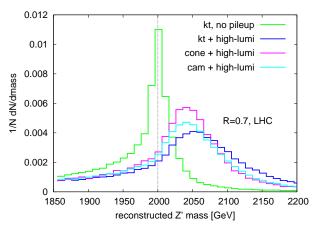
Corrected Cam (and k_t is best.



Uncorrected cone better than k_t .

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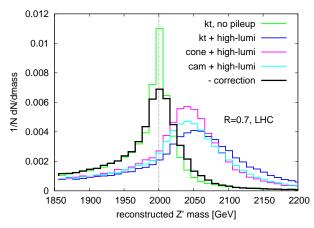
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Corrected Cam (and k_t is best.

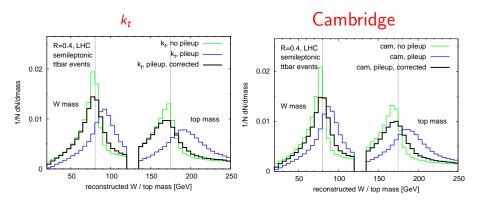


Uncorrected cone better than k_t .

Cam is intermediate $(\langle A_{cam} \rangle \simeq \langle A_{cone} \rangle$, but fluctuations larger)

Corrected Cam (and k_t) is best.

 $t\bar{t}$ production in high-lumi pp collisions at LHC W mass reconstruction via dijet mass in semileptonic decay with b-tagging

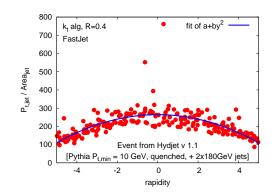


At LHC one expects \sim 30000 particles per Pb-Pb collisions

Very few will be hard (e.g. a dijet event), most will be very soft (10 GeV or less).

Easy way of decluttering the event: a minimum p_T cut. However, this is not an infrared safe procedure, and the result must then be artificially corrected back to the 'real' one.

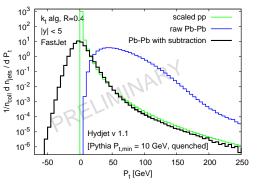
Alternative: same kind of subtraction used in high-lumi pp events



NB: the simulation of a heavy ions collision suggests a parabolic fit of the background

Inclusive jets in Pb-Pb collisions

The subtraction procedure allows one to recover the *pp* single inclusive jet distribution from Pb-Pb collisions



Good agreement with 'hard' distribution after subtraction of huge background

The jets can apparently be measured down to low p_T . Interesting for studying quenching effects.

- ▶ k_t alg. can be fast key observation is geometrical reformulation
 Get code from http://www.lpthe.jussieu.fr/~salam/fastjet
- ightharpoonup Jet areas (ightharpoonup min. bias. contributions) do fluctuate Some aspects of areas amenable to analytical calculations
- But areas can (should) be measured and used for correction on jet-by-jet basis.
 Preliminary studies seem promising

Next version of FastJet will include the subtraction

 \blacktriangleright k_t is part of a class of algorithms — other example deserving more attention is Cambridge/Aachen alg. It too can be made fast

http://hepforge.cedar.ac.uk/hepjet/

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hepjet

HepJet will be a general jet-finding package implementing a range of jet algorithms, starting with the Kt algorithm as implemented in KtJet and FastJet, as well as ConeJet.

It is currently at the design and conception stage. More information is to be found on the tracker/wiki pages.

You can contact the developers at the HepJet developers' mailing list.