HELAC/PHEGAS- MC generator for arbitrary parton-level scattering process

Status Report

Costas G. Papadopoulos

MC4LHC, July 2006, CERN, Geneva

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  hep-ph/0012004 and Tokyo 2001,(CPP2001) Computational particle physics, p. 20-25

  T. Gleisberg, *et al.* Eur. Phys. J. C 34 (2004) 173

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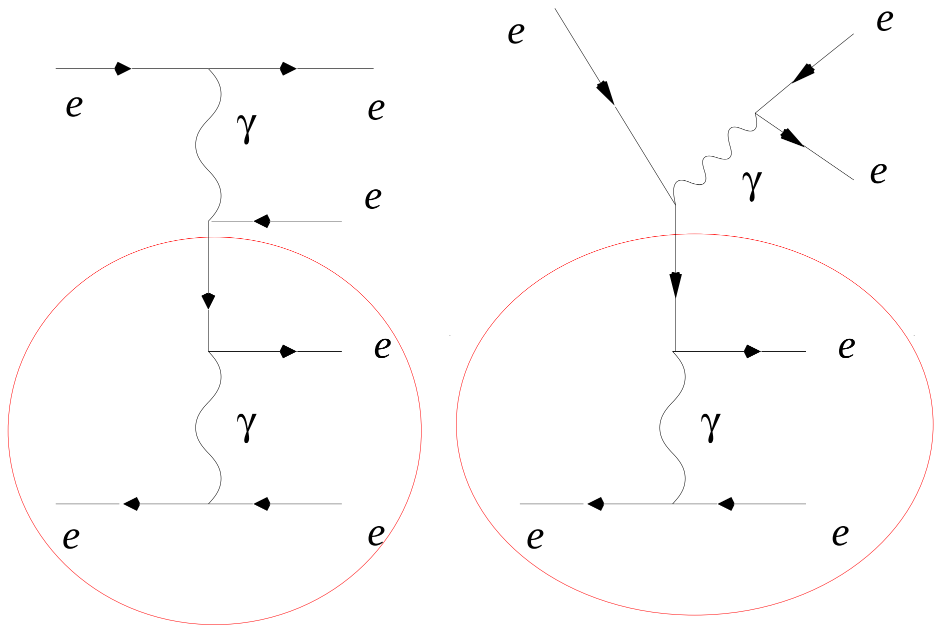
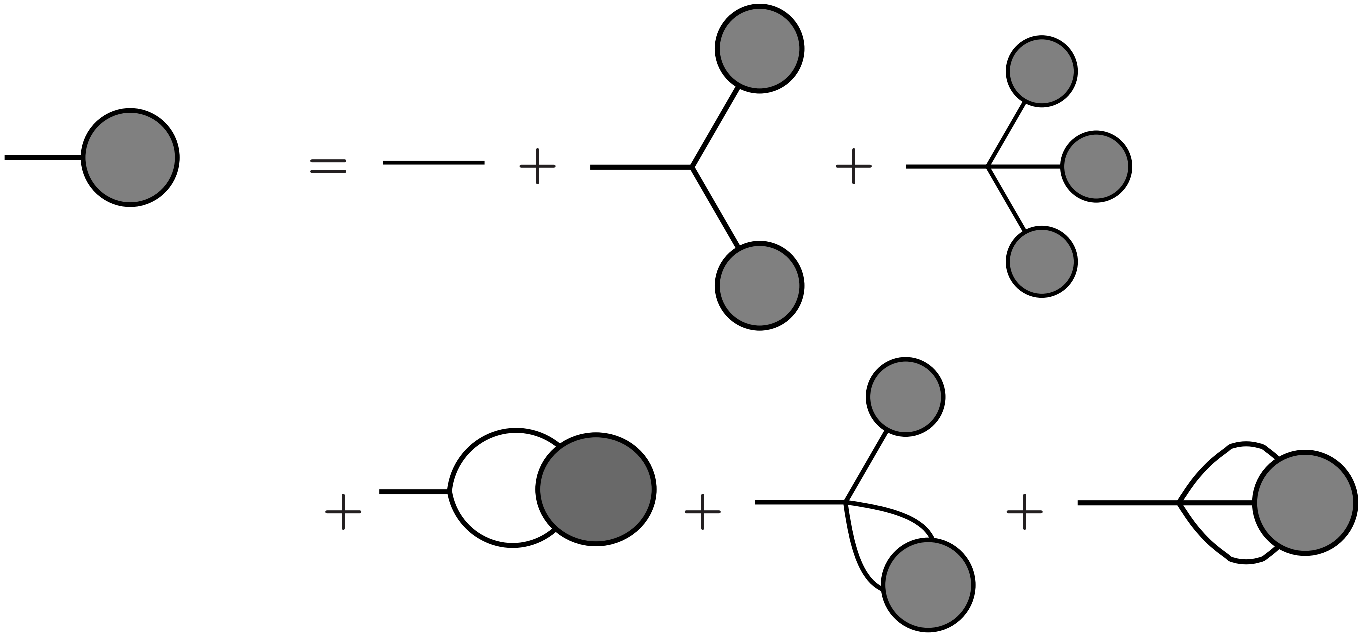
- *Example:* $e^- e^+ \rightarrow e^- e^+ e^- e^+$ in QED:

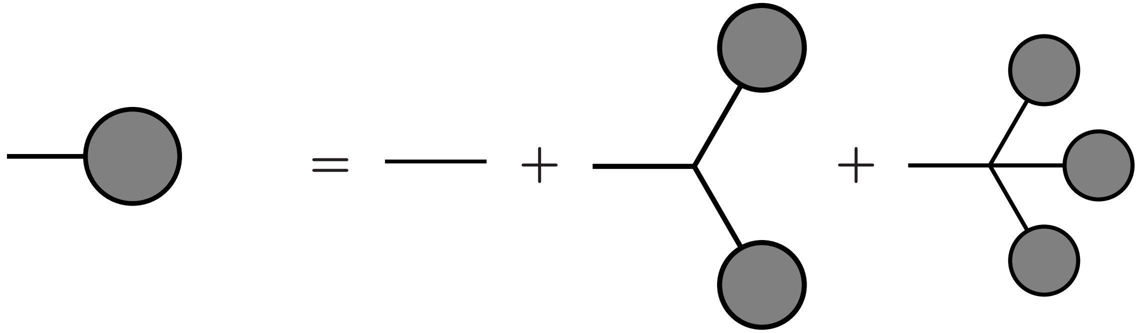


The Dyson-Schwinger recursion

- **Imagine a theory with 3- and 4- point vertices and just one field. Then it is straightforward to write an equation that gives the amplitude for $1 \rightarrow n$**



The Dyson-Schwinger recursion



$$a(n) = \delta_{n,1} + \sum \frac{n!}{n_1!n_2!} a(n_1)a(n_2)\delta_{n_1+n_2,n}$$
$$+ \frac{n!}{n_1!n_2!n_3!} \sum a(n_1)a(n_2)a(n_3)\delta_{n_1+n_2+n_3,n}$$

HELAC

- **Construction of the skeleton solution of the Dyson-Schwinger equations. At this stage only integer arithmetic is performed. This is part of the initialization phase.**
- **Dressing-up the skeleton with momenta, provided by `PHEGAS` and wave functions, propagators, n-point functions in general.**
- **Unitary and Feynman gauges implemented. Due to multi-precision arithmetic,tests of gauge invariance can be extended to arbitrary precision.**
- **All fermions masses can be non-zero.**
- **All Electroweak and QCD vertices are implemented, including Higgs and would-be Goldstone bosons.**

Colour Configuration - EWK$\bigoplus$QCD

Colour Configuration - EWK$\oplus$QCD

- **Ordinary approach $SU(N)$-type**

$$\mathcal{A}^{a_1 \ldots a_n} = \sum Tr(T^{a_{\sigma_1}} \ldots T^{a_{\sigma_n}}) \quad A(\sigma_1 \ldots \sigma_n)$$

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$$\mathcal{C}_{ij} = \sum Tr(T^{a_{\sigma_1}} \ldots T^{a_{\sigma_n}}) Tr(T^{a_{\sigma'_1}} \ldots T^{a_{\sigma'_n}})$$

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Quarks and gluons treated differently

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- **New approach $U(N)$-type**

Each color-configuration amplitude is proportional to

$$D_i = \delta_{1,\sigma_i(1)}\delta_{2,\sigma_i(2)}\dots\delta_{n,\sigma_i(n)}$$

where σ_i represents the i-th permutation of the set $1, 2, \dots, n$.

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⋆ quarks $1 \dots n$

⋆ antiquarks $\sigma_i(1 \dots n)$ and

⋆ gluons $= q\bar{q}$

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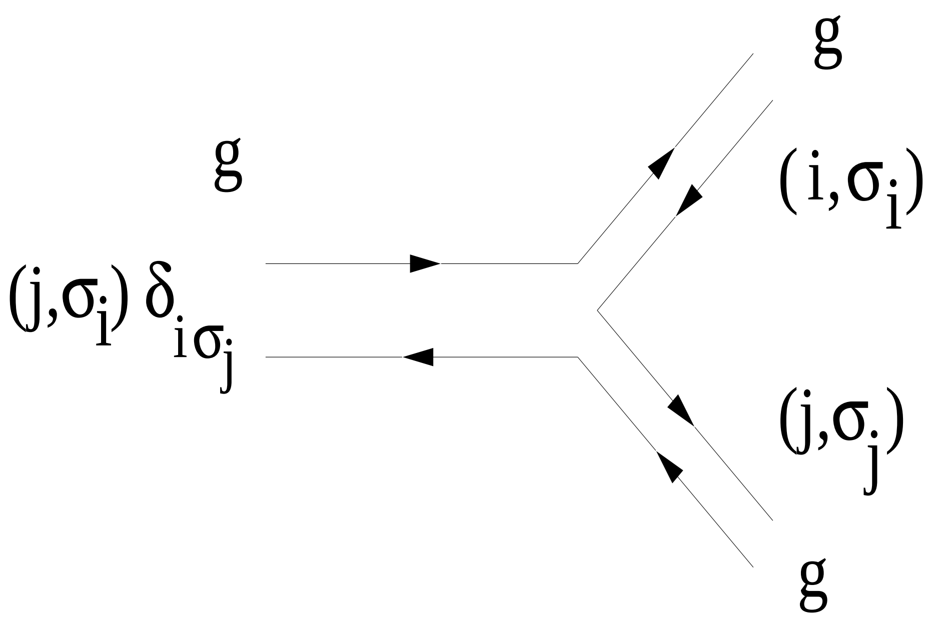
- ⋆ quarks $1\dots n$
- ⋆ antiquarks $\sigma_i(1\dots n)$ and
- ⋆ gluons $= q\bar{q}$

$$\mathcal{C}_{ij} = \sum D_i D_j = N_c^{\alpha} \;, \;\; \alpha = \langle\sigma_1, \sigma_2\rangle$$

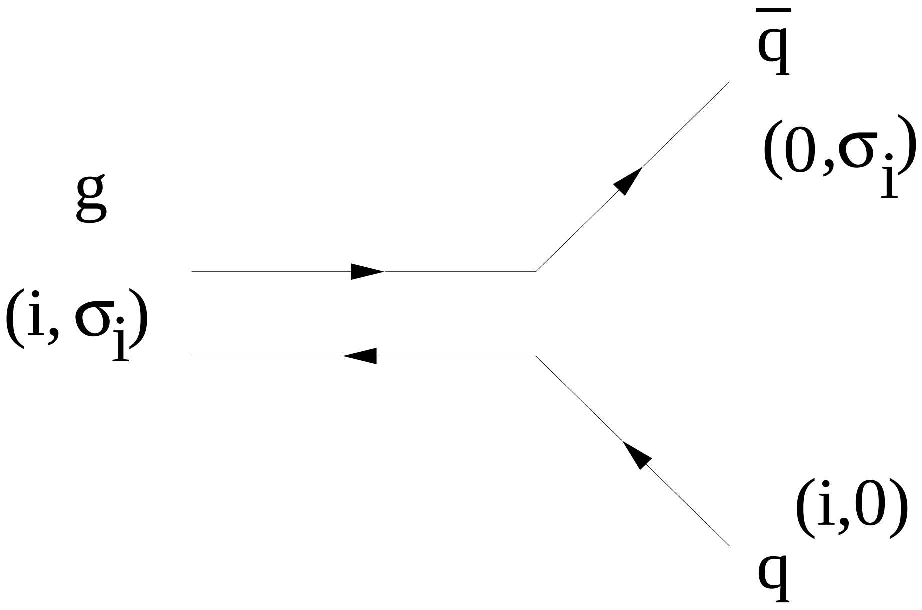
♠ exact color treatment ⇒ low color charge

Problem: number of colour connection configurations: $\sim n!$ where n is the number of gluons or $q\bar{q}$ pairs. ⇒ Monte-Carlo over continuous colour-space.



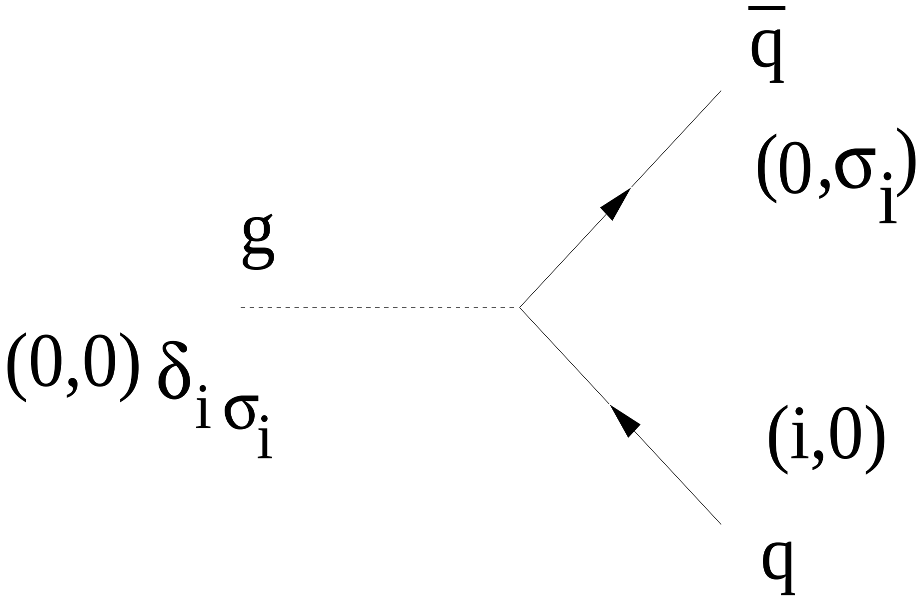
$$\sum f^{abc} t^a_{AB} t^b_{CD} t^c_{EF} = -\frac{i}{4}(\delta_{AD}\delta_{CF}\delta_{EB} - \delta_{AF}\delta_{CB}\delta_{ED})$$

$$\delta_{1\sigma_2}\delta_{2\sigma_3}\delta_{3\sigma_1}$$



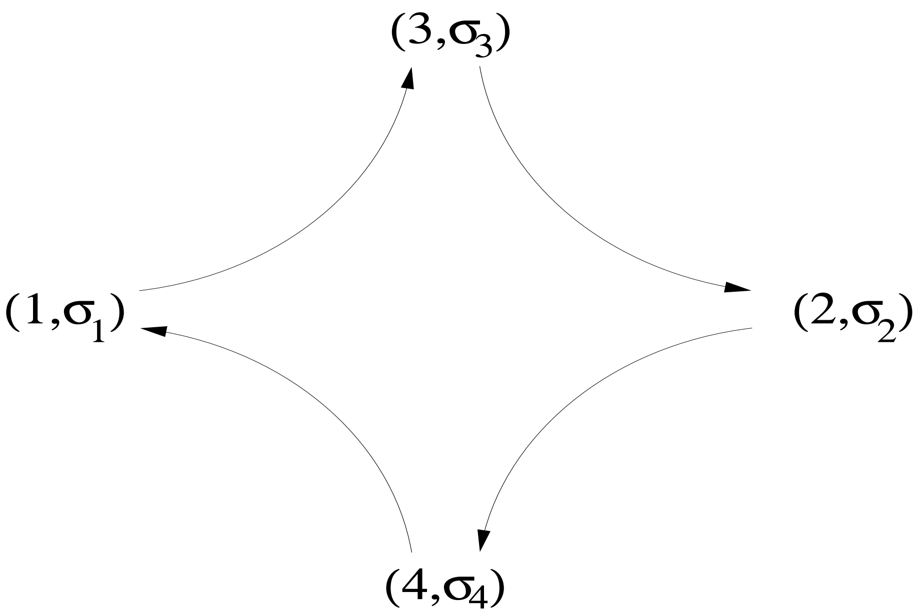
$$\sum t^{a}_{AB} t^{b}_{CD} = \frac{1}{2}(\delta_{AD}\delta_{CB} - \frac{1}{N_c}\delta_{AB}\delta_{AC})$$

$$\frac{1}{\sqrt{2}}$$



$$\sum t^a_{AB} t^b_{CD} = \frac{1}{2}(\delta_{AD}\delta_{CB} - \frac{1}{N_c}\delta_{AB}\delta_{AC})$$

$$\frac{1}{\sqrt{2N_c}}$$



$$\delta_{1\sigma_3}\delta_{3\sigma_2}\delta_{2\sigma_4}\delta_{4\sigma_1}$$

$$2g_{12}g_{34} - g_{13}g_{24} - g_{14}g_{23}$$

Summation/Integration over color

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$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n) \sim \sum_{P(2,\ldots,n)} Tr(t^{a_1} \ldots t^{a_n}) \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

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$$\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{I_i, J_i\}_1^n) \sim \sum_{P(2,\ldots,n)} \delta_{I_1,P(J_1)} \ldots \delta_{I_n,P(J_n)} \mathcal{A}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n)$$

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$$\sum_{\{a_i\}_1^n \{\varepsilon_i\}_1^n} |\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n)|^2 = g^{2n-4} \sum_{\varepsilon} \sum_{ij} \mathcal{A}_i \mathcal{C}_{ij} \mathcal{A}_j^*$$

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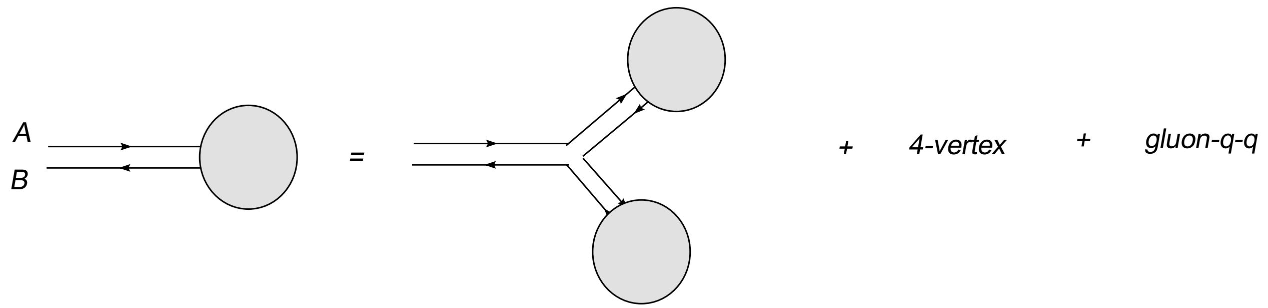
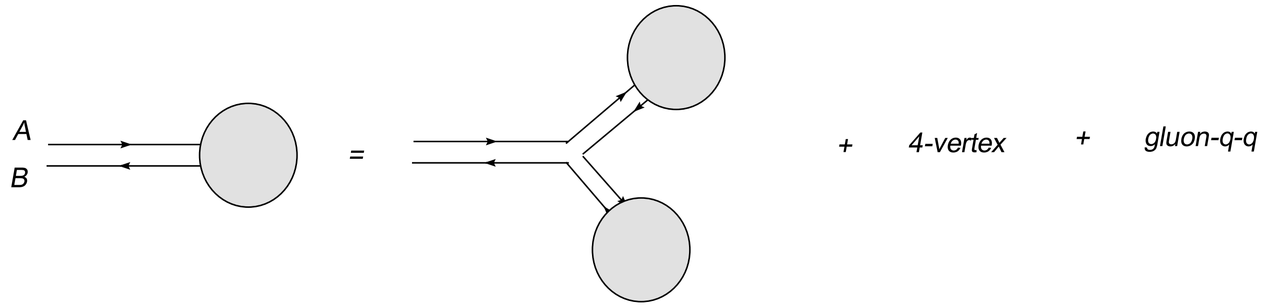
$$\sum_{P(2,\ldots,n)} \sim n!$$

$$\sum_{\{I_i,J_i\}_1^n} \sim 3^n \times 3^n$$


A
B
=
+
4-vertex
+
gluon-q-q



$$[A^\mu(P);(A,B)] = \sum_{i=1}^{n}[\ \delta(P-p_i)\ A^\mu(p_i);(A,B)_i]+$$

$$\sum[\ (ig)\ \Pi^\mu_\rho\ V^{\rho\nu\lambda}(P,p_1,p_2)A_\nu(p_1)A_\lambda(p_2)\sigma(p_1,p_2);(A,B)=(C,D)_1\otimes(E,F)_2]$$

$$-\sum[\ (g^2)\ \Pi^\mu_\sigma\ G^{\sigma\nu\lambda\rho}(P,p_1,p_2,p_3)A_\nu(p_1)A_\lambda(p_2)A_\rho(p_3)\sigma(p_1,p_2+p_3);$$

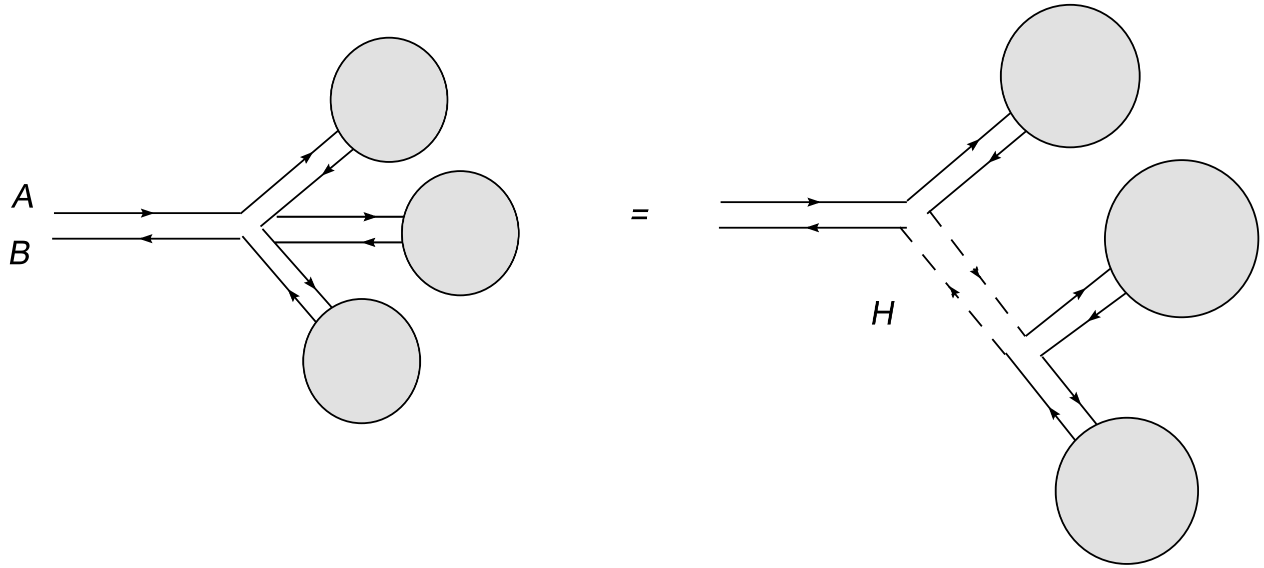
$$(A,B)=(C,D)_1\otimes(E,F)_2\otimes(G,H)_3]$$

$$+\sum_{P=p_1+p_2}[\ (ig)\ \Pi^\mu_\nu\ \bar{\psi}(p_1)\gamma^\nu\psi(p_2)\sigma(p_1,p_2);(A,B)=(0,D)_1\otimes(C,0)_2]$$

where $A,B,C,D,E,F,G,H=1,2,3.$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$\mathcal{L} = -\frac{1}{2} H^a_{\mu\nu} H^{\mu\nu a} + \frac{1}{4} H^a_{\mu\nu} F^{\mu\nu a}.$$



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$$+$$

$$[\ (ig)\ \Pi^\mu_\sigma\ (g^{\sigma\lambda}g^{\nu\rho}-g^{\nu\lambda}g^{\sigma\rho})\ A_\nu(p_1)H_{\lambda\rho}(p_2)\sigma(p_1,p_2);(A,B)=(C,D)_1\otimes(E,F)_2]$$

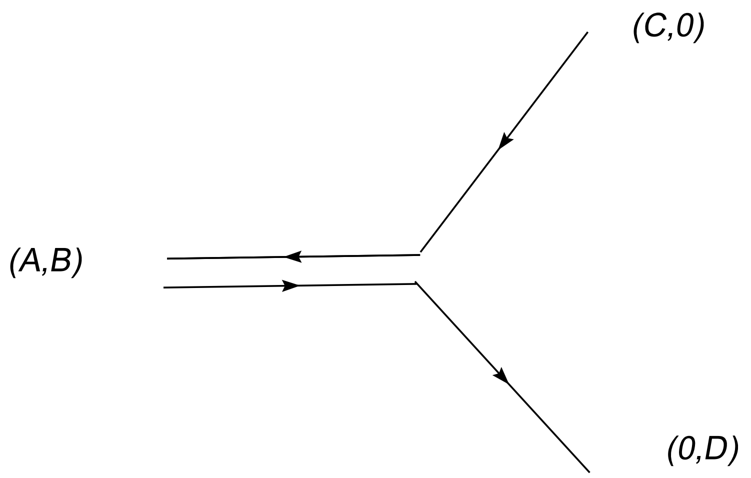
$$+$$

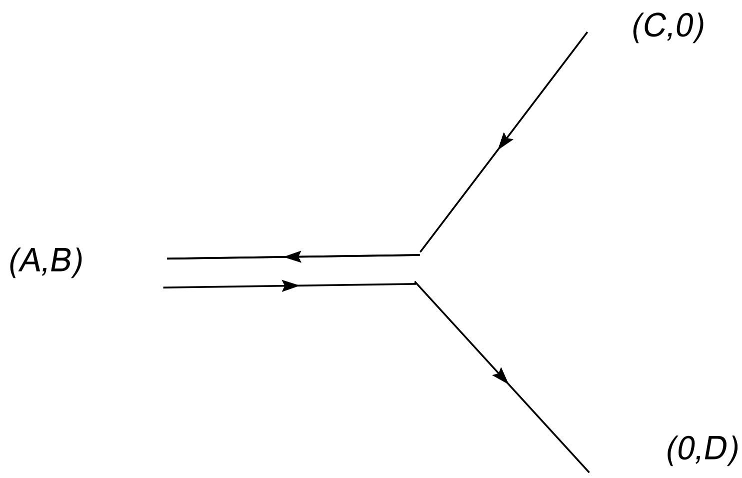
$$[\ (ig)\ \Pi^\mu_\nu\ \bar{\psi}(p_1)\gamma^\nu\psi(p_2)\sigma(p_1,p_2);(A,B)=(0,D)_1\otimes(C,0)_2]$$

and

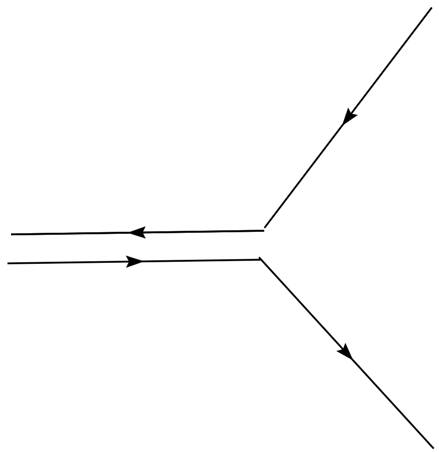
$$[H^{\mu\nu}(P);(A,B)] = \sum_{P=p_1+p_2}[\ (ig)\ (g^{\mu\lambda}g^{\nu\rho}-g^{\nu\lambda}g^{\mu\rho})\ A_\lambda(p_1)A_\rho(p_2)\sigma(p_1,p_2);$$

$$(A,B)=(C,D)_1\otimes(E,F)_2].$$


(C,0)
(A,B)
(0,D)



$$(A,B) = (C,0) \otimes (0,D) = (C,D)_{w=1}, \qquad \text{if } C \neq D.$$



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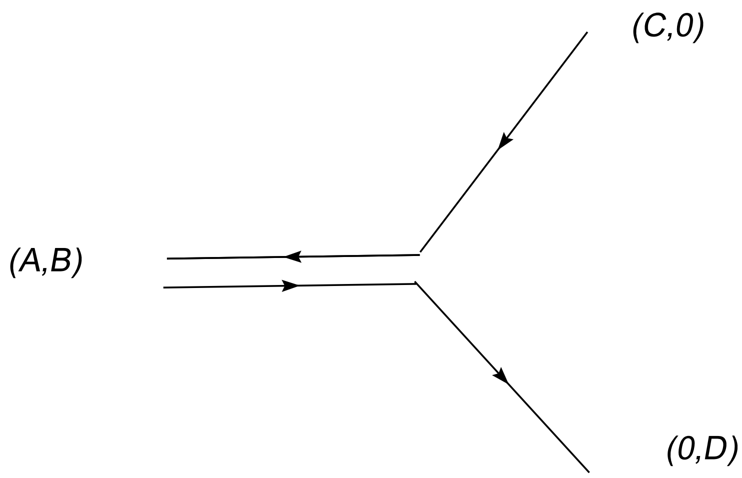
$$(\boldsymbol{A},\boldsymbol{B}) = (\boldsymbol{C},\boldsymbol{0}) \otimes (\boldsymbol{0},\boldsymbol{D}) = (\boldsymbol{1},\boldsymbol{1})_{\boldsymbol{w1}} \oplus (\boldsymbol{2},\boldsymbol{2})_{\boldsymbol{w2}} \oplus (\boldsymbol{3},\boldsymbol{3})_{\boldsymbol{w3}}, \qquad \text{if } \boldsymbol{C} = \boldsymbol{D}.$$



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$$(A,B) = (C,0) \otimes (0,D) = (1,1)_{w1} \oplus (2,2)_{w2} \oplus (3,3)_{w3}, \qquad \text{if } C = D.$$

$$(1,0) \otimes (0,1) = (1,1)_{2/3} \oplus (2,2)_{-1/3} \oplus (3,3)_{-1/3}$$

$$N_{\mathrm{CC}} = \sum_{A=0}^{n_q} \sum_{B=0}^{n_q - 1} \sum_{C=0}^{n_q - A - B} \left(\frac{n_q!}{A!B!C!} \right)^2 \delta(n_q = A + B + C)$$

$$N_{\rm CC} = \sum_{A=0}^{n_q} \sum_{B=0}^{n_q-1} \sum_{C=0}^{n_q-A-B} \left(\frac{n_q!}{A!B!C!} \right)^2 \delta(n_q = A + B + C)$$

Process	$\mathrm{N}_{\mathrm{CC}}^{\mathrm{ALL}}$	N_{CC}	$\mathrm{N}_{\mathrm{CC}}^{\mathrm{F}}$ (%)
$gg \to 2g$	6561	639	59.1
$gg \to 3g$	59049	4653	68.4
$gg \to 4g$	531441	35169	77.4
$gg \to 5g$	4782969	272835	85.0
$gg \to 6g$	43046721	2157759	90.4
$gg \to 7g$	387420489	17319837	94.0
$gg \to 8g$	3486784401	140668065	96.4

Process	N_{CC}^{ALL}	N_{CC}	N_{CC}^{F} (%)
$gg \rightarrow u\bar{u}$	729	93	93.5
$gg \rightarrow gu\bar{u}$	6561	639	91.6
$gg \rightarrow 2gu\bar{u}$	59049	4653	92.6
$gg \rightarrow 3gu\bar{u}$	531441	35169	94.6
$gg \rightarrow 4gu\bar{u}$	4782969	272835	96.4
$gg \rightarrow 5gu\bar{u}$	43046721	2157759	97.8
$gg \rightarrow 6gu\bar{u}$	387420489	17319837	98.6
$gg \rightarrow c\bar{c}c\bar{c}$	6561	639	99.1
$gg \rightarrow gc\bar{c}c\bar{c}$	59049	4653	98.8
$gg \rightarrow 2gc\bar{c}c\bar{c}$	531441	35169	99.0
$gg \rightarrow 3gc\bar{c}c\bar{c}$	4782969	272835	99.3
$gg \rightarrow 4gc\bar{c}c\bar{c}$	43046721	2157759	99.6

Process	$\sigma_{\text{MC}} \pm \varepsilon$ (nb)	ε (%)
$gg \rightarrow 7g$	$(0.53185 \pm 0.01149)\times 10^{-2}$	2.1
$gg \rightarrow 8g$	$(0.33330 \pm 0.00804)\times 10^{-3}$	2.4
$gg \rightarrow 9g$	$(0.17325 \pm 0.00838)\times 10^{-4}$	4.8
$gg \rightarrow 5gu\bar{u}$	$(0.38044 \pm 0.01096)\times 10^{-3}$	2.8
$gg \rightarrow 3gc\bar{c}c\bar{c}$	$(0.95109 \pm 0.02456)\times 10^{-5}$	2.6
$gg \rightarrow 4gc\bar{c}c\bar{c}$	$(0.81400 \pm 0.02583)\times 10^{-6}$	3.2

Process	$\sigma_{\mathrm{MC}} \pm \varepsilon$ (nb)	ε (%)
$gg \to Zu\bar{u}gg$	$(0.18948 \pm 0.00344)\times 10^{-3}$	1.8
$gg \to W^{+}\bar{u}dgg$	$(0.62704 \pm 0.01458)\times 10^{-3}$	2.3
$gg \to ZZu\bar{u}gg$	$(0.16217 \pm 0.00420)\times 10^{-6}$	2.6
$gg \to W^{+}W^{-}u\bar{u}gg$	$(0.27526 \pm 0.00752)\times 10^{-5}$	2.7
$d\bar{d} \to Zu\bar{u}gg$	$(0.38811 \pm 0.00569)\times 10^{-5}$	1.5
$d\bar{d} \to W^{+}\bar{c}sgg$	$(0.18765 \pm 0.00453)\times 10^{-5}$	2.4
$d\bar{d} \to ZZgggg$	$(0.99763 \pm 0.02976)\times 10^{-7}$	2.9
$d\bar{d} \to W^{+}W^{-}gggg$	$(0.52355 \pm 0.01509)\times 10^{-6}$	2.9

- **SPHEL approximation based on MHV amplitudes**

$$\sum_{a,\varepsilon} |\mathcal{M}(\{p_i\}_1^n, \{\varepsilon_i\}_1^n, \{a_i\}_1^n)|^2 = 2g^{2n-4} N_c^{n-2}(N_c^2 - 1) \quad \times$$

$$\frac{2^n - 2(n+1)}{n(n-1)} \sum_{1\leq i\leq j\leq n} (p_i \cdot p_j)^4 \sum_{P(2,\ldots,n)} \frac{1}{(p_1 \cdot p_2)(p_2 \cdot p_3)\ldots(p_n \cdot p_1)},$$

Process	t_{SPHEL}	t_{MC}	$t_{\text{MC}}/t_{\text{SPHEL}}$
$gg \to 2g$	0.372×10^{-3}	0.519×10^{-1}	139.52
$gg \to 3g$	0.776×10^{-3}	0.135×10^{0}	173.97
$gg \to 4g$	0.252×10^{-2}	0.364×10^{0}	144.44
$gg \to 5g$	0.122×10^{-1}	0.143×10^{1}	117.21
$gg \to 6g$	0.806×10^{-1}	0.497×10^{1}	61.66
$gg \to 7g$	0.639×10^{0}	0.133×10^{2}	20.81
$gg \to 8g$	0.569×10^{1}	0.334×10^{2}	5.87
$gg \to 9g$	0.567×10^{2}	0.923×10^{2}	1.63
$gg \to 10g$	0.620×10^{3}	0.267×10^{3}	0.43

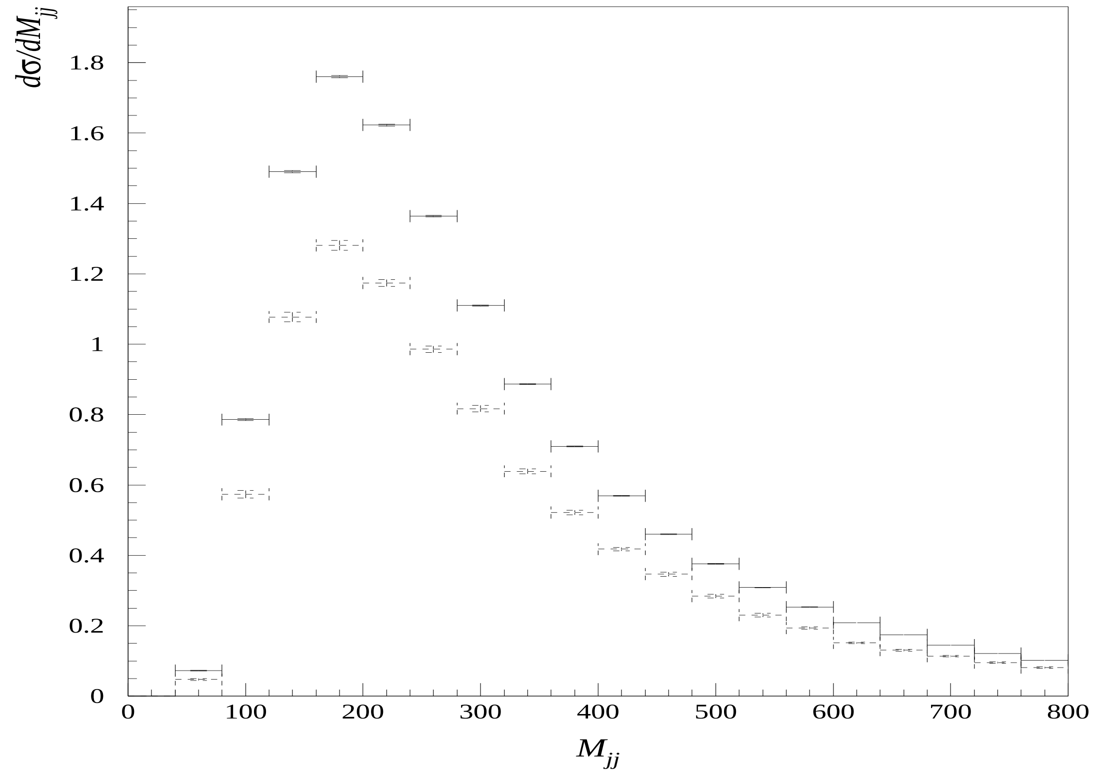
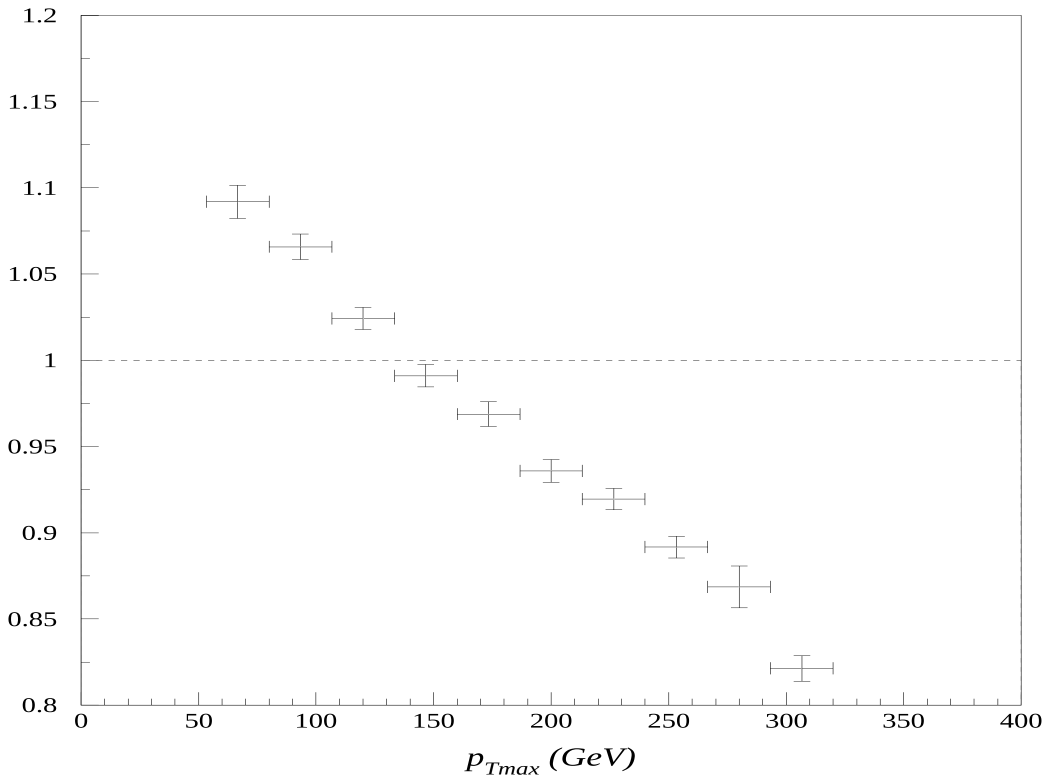
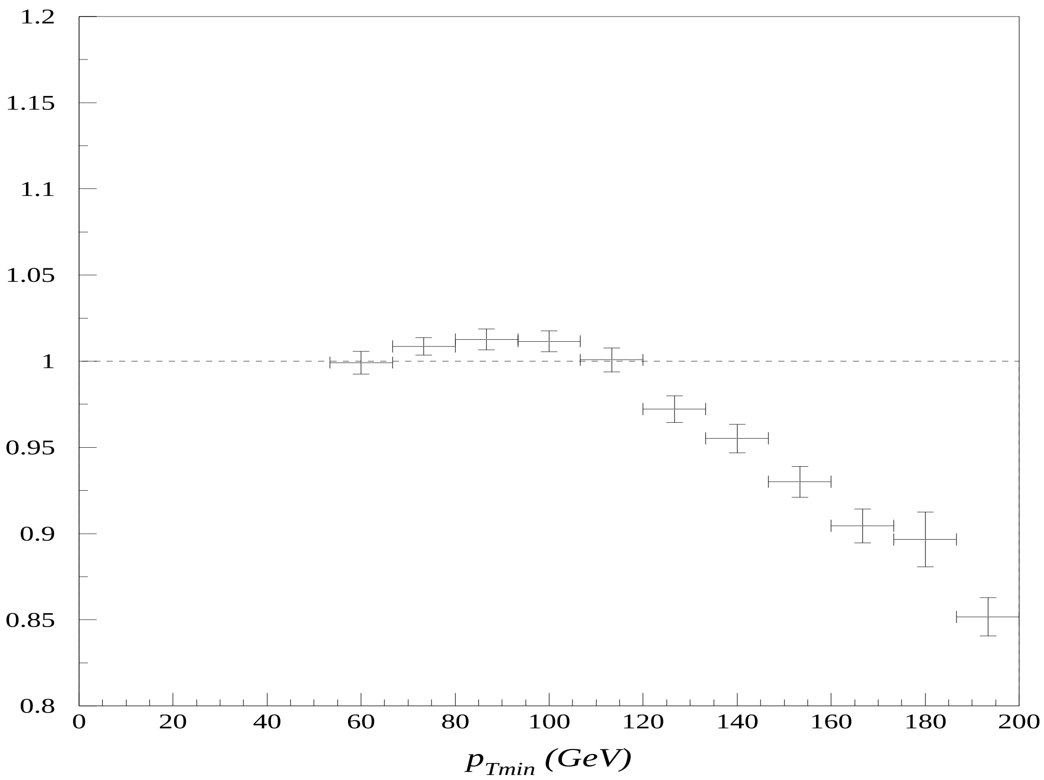


Figure 1: *Invariant mass distribution of 2 gluons in the* $\boldsymbol{gg \rightarrow 5g}$ *process. Solid line crosses denote SPHEL case whereas dashed, the Monte Carlo one.*



1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0
50
100
150
200
250
300
350
400
p_{Tmax} (GeV)


1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0
20
40
60
80
100
120
140
160
180
200
p_{Tmin} (GeV)

Solid line 2jets or 3 jets (HELAC+PYTHIA from $2g \rightarrow 2g$,$2 \rightarrow 3g$,$2 \rightarrow 4g$)

Dashed line 2g or 3g (HELAC $2g \rightarrow 2g$ or $2g \rightarrow 3g$)

Dotted line 2jets or 3 jets (PYTHIA $2g \rightarrow 2g$)

PDF's : CTEQ 5M1 (MSb, NNL)

Cuts: $p_t > 60$ GeV $|\eta| < 2.5$, $\Delta_R > 1.0$

For jets PYCELL subroutine on partons after showering

ISR on

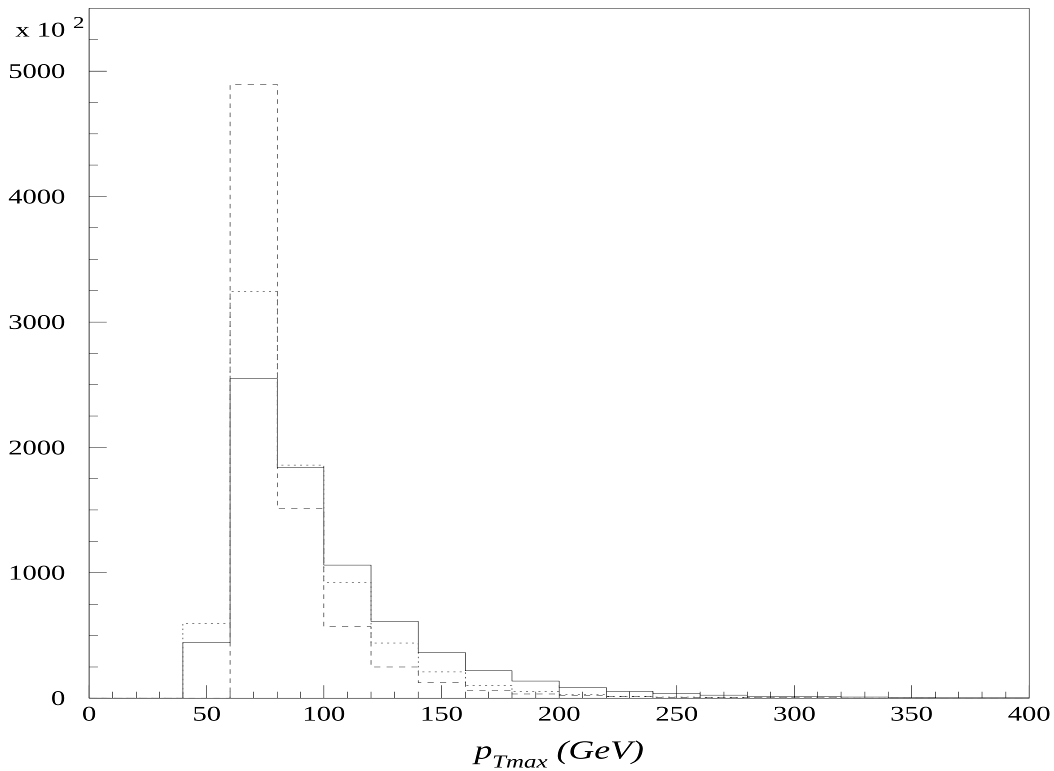
FSR on

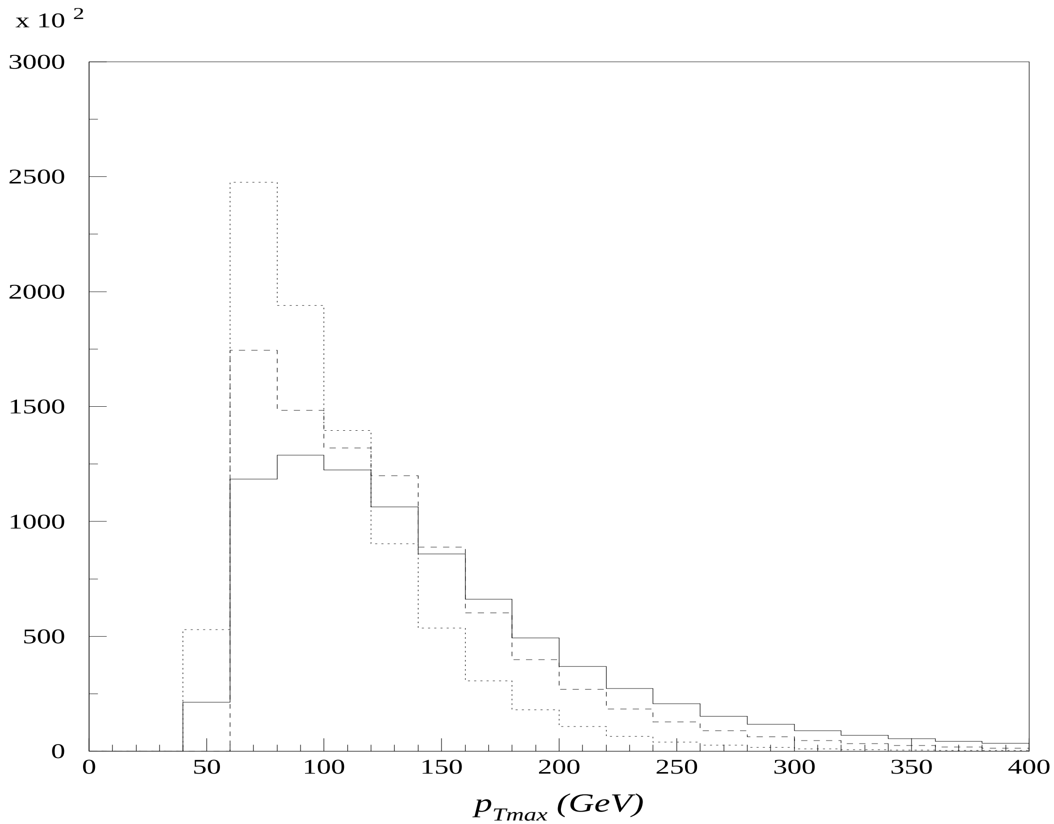
multiple interactions on

fragmentation on

α_s fixed 0.13

PYTHIA version 6.4


x 10 2
5000
4000
3000
2000
1000
0
0
50
100
150
200
250
300
350
400
p_Tmax (GeV)


x 10 2
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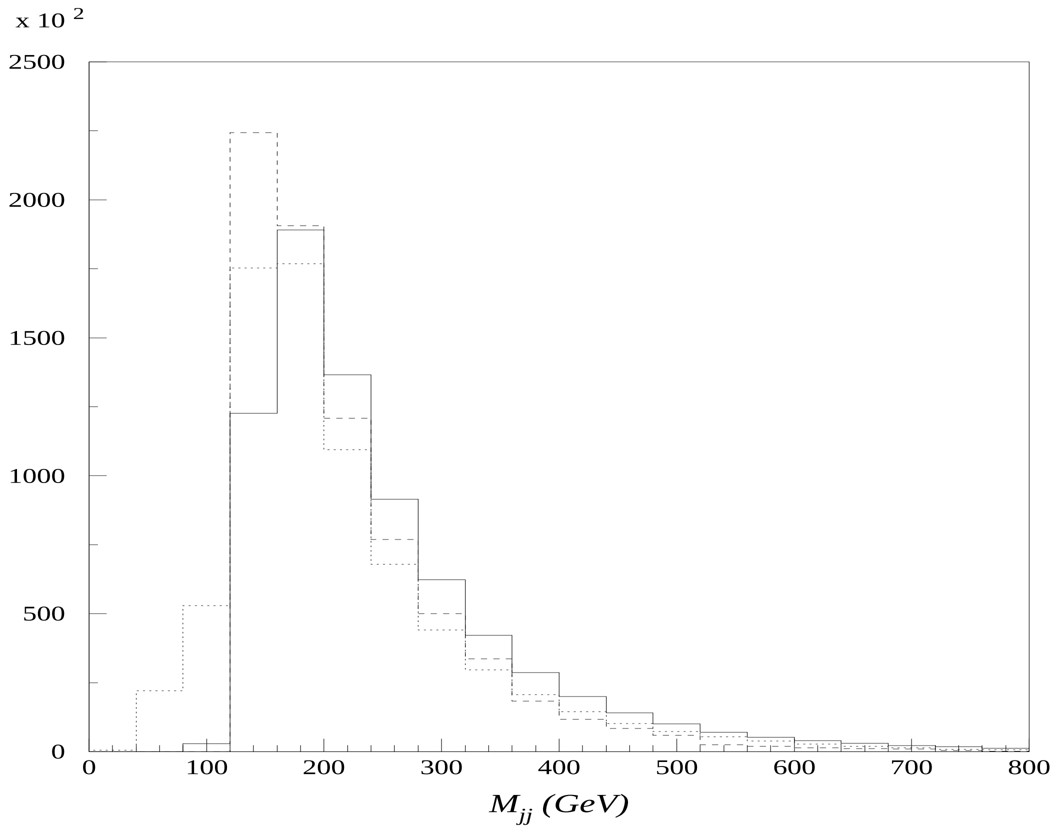
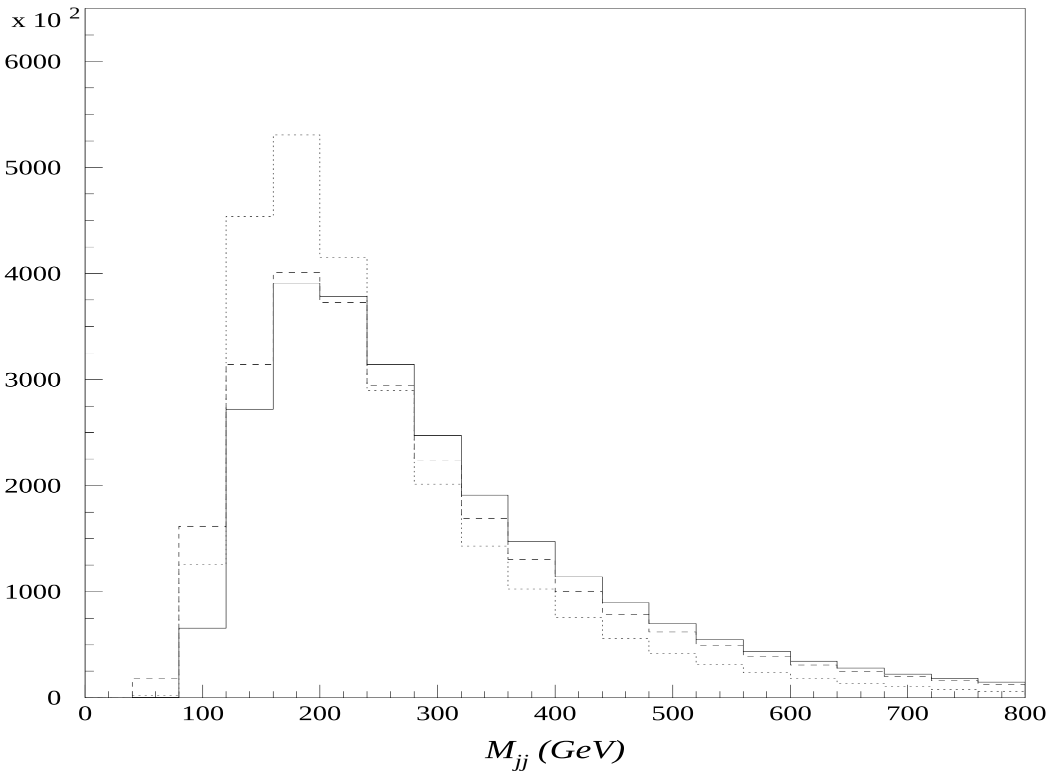
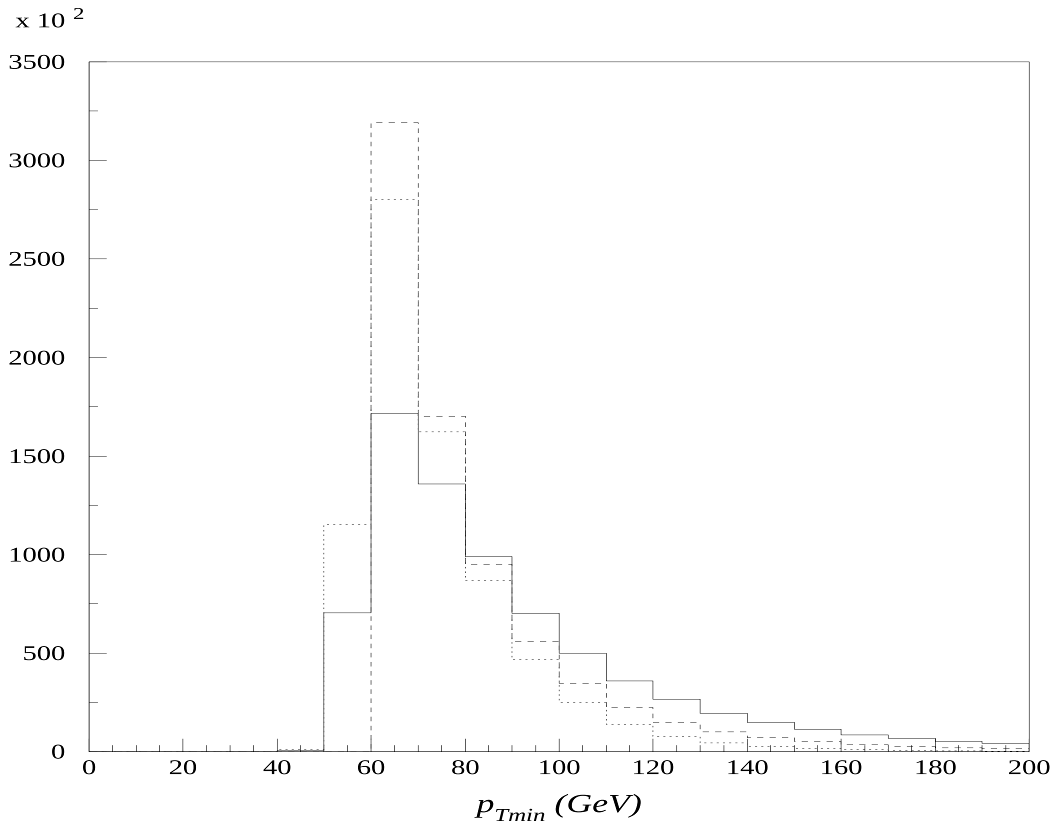
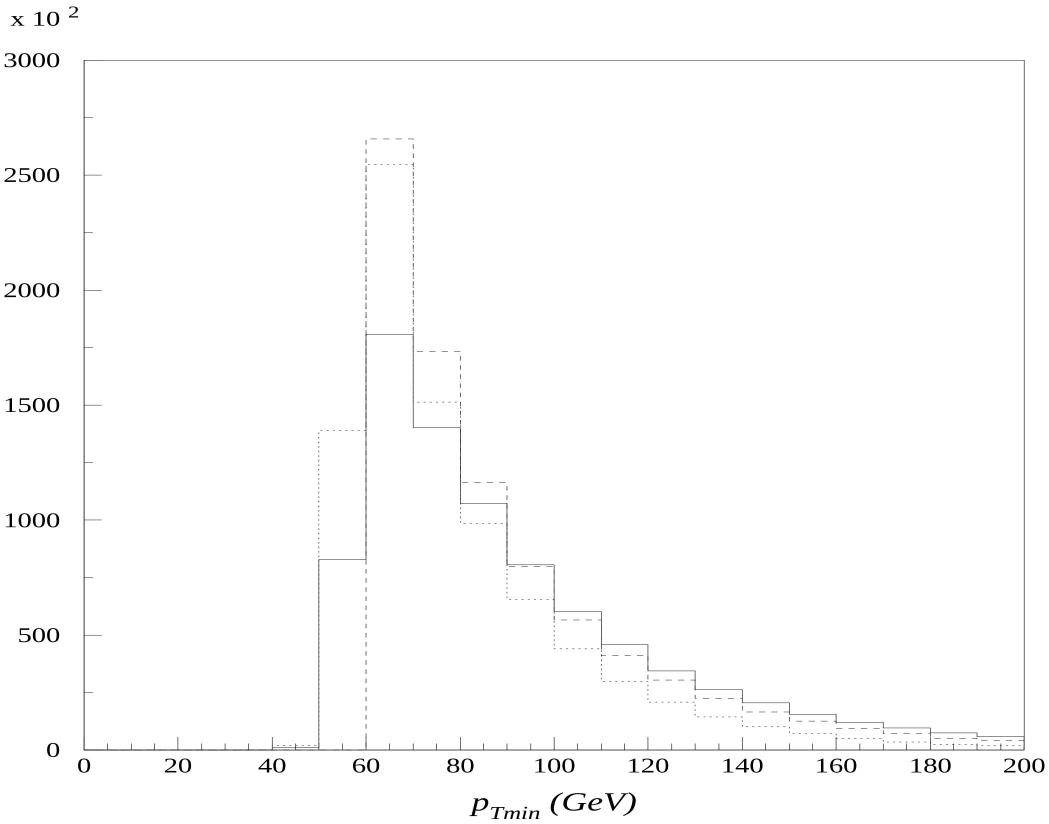
400
p_{Tmax} (GeV)


x 10 2
2500
2000
1500
1000
500
0
0
100
200
300
400
500
600
700
800
M_jj (GeV)



x 10 2
6000
5000
4000
3000
2000
1000
0
0
100
200
300
400
500
600
700
800
M_{jj} (GeV)


x 10 2
3500
3000
2500
2000
1500
1000
500
0
0
20
40
60
80
100
120
140
160
180
200
p_{Tmin} (GeV)



x 10 2
3000
2500
2000
1500
1000
500
0
0
20
40
60
80
100
120
140
160
180
200
p_{Tmin} (GeV)

Multi-jet processes

Beyond any colour treatment a summation over different flavours is also needed.

Up to now the most straightforward way was to count the distinct processes and then multiply with a multiplicity factor, i.e.

process	Flavour
$gg \to ggg$	1
$q\bar{q} \to ggg$	8
$qg \to qgg$	8
$qg \to qgg$	8
$gg \to q\bar{q}g$	5
$q\bar{q} \to q\bar{q}g$	8
$q\bar{q} \to r\bar{r}g$	32
$qq \to qqg$	8
$q\bar{r} \to q\bar{r}g$	24
$qr \to qrg$	24
$qg \to qq\bar{q}$	8
$qg \to qr\bar{r}$	32
$gq \to qq\bar{q}$	8
$gq \to qr\bar{r}$	32

initial-state type		distinct processes	multiplicity factor
A	(gg)	$C_1(n)$	$\chi(n_0, n_1, \ldots, n_f; f)$
B	$(q\bar{q})$	$C_2(n)$	$\chi(n_0, n_2, \ldots, n_f; f-1)$
C	$(gq$ and $qg)$	$C_2(n-1)$	$\chi(n_0, n_2, \ldots, n_f; f-1)$
D	(qq)	$C_2(n-2)$	$\chi(n_0, n_2, \ldots, n_f; f-1)$
E	$(qq'$ and $q\bar{q}')$	$C_3(n-2)$	$\chi(n_0, n_3, \ldots, n_f; f-2)$

In order to clarify what we mean we consider the example of the type A initial state. Each distinct process is defined by an array $(n_0, n_1, \ldots, n_f)$. For instance, in the case of four-jet production we have

$$\begin{array}{ll} (4,0,0,0,0,0) & gg \to gggg \\ (2,1,0,0,0,0) & gg \to ggq\bar{q} \\ (0,2,0,0,0,0) & gg \to q\bar{q}q\bar{q} \\ (0,1,1,0,0,0) & gg \to q\bar{q}r\bar{r} \end{array}$$

$$C_1(n) = \sum_{n_0+2n_1+\ldots+2n_f=n} \Theta(n_1 \geq n_2 \geq \ldots \geq n_f)$$

$$C_2(n) = \sum_{n_0+2n_1+\ldots+2n_f=n} \Theta(n_2 \geq n_3 \geq \ldots \geq n_f)$$

and

$$C_3(n) = \sum_{n_0+2n_1+\ldots+2n_f=n} \Theta(n_3 \geq n_4 \geq \ldots \geq n_f)$$

A distinct process, given by the array $(n_0, n_1, \ldots, n_f)$ has a multiplicity factor :

$$\chi(n_0, n_1, \ldots, n_f; f) = n_f(n_f - 1)\ldots(n_f - j + 1)/j!$$

$$\begin{array}{ccc} j = f & \text{if} & \prod_{i=1}^{f} n_i \neq 0 \\ j = f-1 & \text{if} & \prod_{i=1}^{f-1} n_i \neq 0 \\ \ldots & & \\ j = 1 & \text{if} & n_1 \neq 0 \\ j = 0 & \text{otherwise} & \end{array}$$

Now we can think of a flavour-MC, so the wave function is multiplied by an N_f-dimensional array representing flavour , $\vec{f} = \sqrt{N_f}(f_1, f_2, \ldots)$ such that $\langle f_i f_j \rangle = \delta_{ij}$ with a weight proportional to the relevant pdf for initial state flavours

In that case a process like

$$gg \to ggq\bar{q}q\bar{q}$$

will actually represent a plethora of processes.

The number of distinct processes is now given by

$$9k + 3 \;\text{ if }\; n = 2k \quad \text{and} \quad 9k + 7 \;\text{ if }\; n = 2k + 1$$

# of jets	2	3	4	5	6	7	8	9	10
# of D-processes	12	16	21	24	30	34	39	43	48
# of dist.processes	10	14	28	36	64	78	130	154	241
total # of processes	126	206	621	861	1862	2326	4342	5142	8641

Multi-jet rates

$$p_{T\ i} > 60\ GeV, \quad \theta_{ij} > 30^o \quad |\eta_i| < 3$$

# jets	3	4	5	6	7	8
$\sigma(nb)$	91.41	6.54	0.458	$2.97\ \times 10^{-2}$	$2.21\ \times 10^{-3}$	$2.12\ \times 10^{-4}$
% Gluon	45.7	39.2	35.7	35.1	33.8	26.6

- quark processes relevant
- $gg \longrightarrow ng$ approximation ?
- A new code $\Rightarrow$ `JetI`

PHEGAS

- Phase space

$$d\Phi_n = (2\pi)^{4-3n} \prod_{i=1}^{n} \frac{d^3 p_i}{2E_i} \delta\left(\sum E_i - w\right) \delta^3\left(\sum \vec{p}_i\right)$$

- RAMBO, VEGAS-based nice but completely inefficient!

$$d\sigma_n = \text{FLUX} \times |\mathcal{M}_{2\to n}|^2 d\Phi_n$$

need appropriate mappings of peaking structures, plus optimization!

- Efficiency $\Rightarrow$ to a large number of generators, each one for a specific class of processes.

Multichannel approach

$$\mathcal{I} = \int f(\vec{x})d\mu(\vec{x}) = \int \frac{f(\vec{x})}{p(\vec{x})} p(\vec{x})d\mu(\vec{x})$$

$$p(\vec{x}) = \sum_{i=1}^{M_{ch}} \alpha_i \, p_i(\vec{x}) \qquad \sum_{i=1}^{M_{ch}} \alpha_i = 1$$

$$\mathcal{I} \to \left\langle \frac{f(\vec{x})}{p(\vec{x})} \right\rangle \qquad \mathcal{E}^2 N \to \left\langle \left(\frac{f(\vec{x})}{p(\vec{x})} \right)^2 - \mathcal{I}^2 \right\rangle$$

⋆ **Optimize $\alpha_i \Rightarrow$ Minimize $\mathcal{E}$** ⋆

R.Kleiss and R.Pittau, Comput. Phys. Commun. 83, 141 (1994).

New **Dyson-Schwinger equations: subamplitude is a combination of several peaking structures!**

problem unsolved?

QCD antennas

P.D.Draggiotis, A.van Hameren and R.Kleiss, hep-ph/0004047.

New **Dyson-Schwinger equations: subamplitude is a combination of several peaking structures!**

problem unsolved?

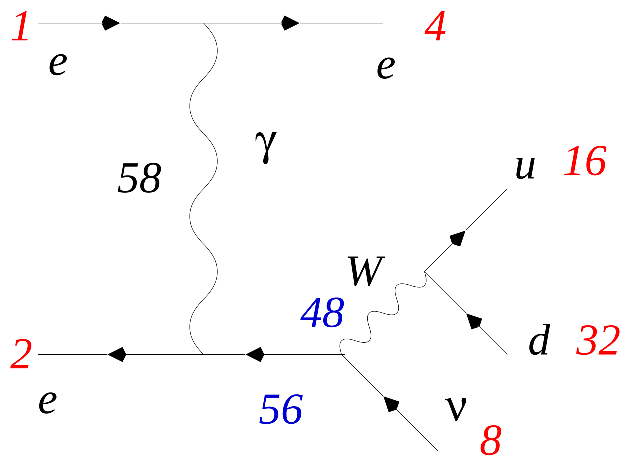
QCD antennas

P.D.Draggiotis, A.van Hameren and R.Kleiss, hep-ph/0004047.

Old **Feynman graphs: exhibit single peaking structure!**

problem solved

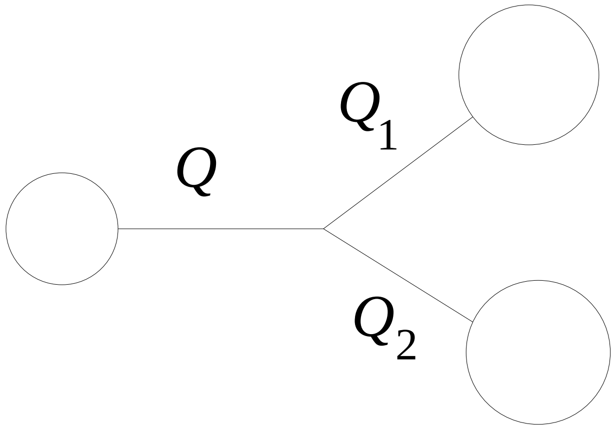
Back to Feynman graphs:



The corresponding intrinsic representation looks like

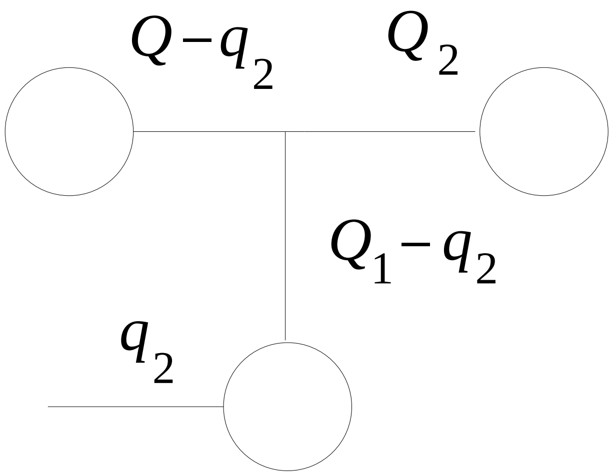
62	**-2**	**4**	**-2**	**58**	**31**
58	**31**	**2**	**-2**	**56**	**2**
56	**2**	**48**	**33**	**8**	**1**
48	**33**	**16**	**-3**	**32**	**4**

Time-like momenta $q^2 \geq 0$



$$
\begin{aligned}
d\Phi_n &= \ldots \frac{dQ_1^2}{2\pi}\,\frac{dQ_2^2}{2\pi}\, d\Phi_2(Q \to Q_1, Q_2) \ldots \\
&= \ldots \frac{dQ_1^2}{2\pi}\,\frac{dQ_2^2}{2\pi}\, d\cos\theta\; d\phi\; \frac{\lambda^{1/2}(Q^2, Q_1^2, Q_2^2)}{32\pi^2\; Q^2} \ldots
\end{aligned}
$$

Space-like momenta



$$d\Phi_n = \ldots \frac{dQ_1^2}{2\pi}\,\frac{dQ_2^2}{2\pi}\,d\Phi_2(Q \to Q_1, Q_2)\ldots$$
$$= \ldots \frac{dQ_1^2}{2\pi}\,\frac{dQ_2^2}{2\pi}\,dt\,d\phi\,\frac{1}{32\pi^2\,Q\,|\vec{q}_2|}\ldots$$

$$t = (Q_1 - q_2)^2 = m_2^2 + Q_1^2 - \frac{E_2}{Q}(Q^2 + Q_1^2 - Q_2^2) + \frac{\lambda^{1/2}}{Q}|\vec{q}_2|\cos\theta$$

- **Find limits of $t(Q_1^2, \cos\theta)$:**

$$t_{\pm} = m_2^2 + Q_1^2 - \frac{E_2}{Q}(Q^2 + Q_1^2 - Q_2^2) \pm \frac{\lambda^{1/2}}{Q}|\vec{q}_2|$$

In order to find the maximum of t_+ we study the function $\partial t_+/\partial Q_1^2$ in the region $Q_{1,min}^2 < Q_1^2 < (Q - Q_2)^2$. Since

$$\frac{\partial^2 t_+}{\partial (Q_1^2)^2} = -4Q^2 Q_2^2 \lambda^{-3/2} \frac{|\vec{q}_2|}{Q} \leq 0$$

and

$$\partial t_+/\partial Q_1^2|_{Q_1^2=(Q-Q_2)^2} \to -\infty$$

we just consider two cases ($|\vec{q}_2| \neq 0$):

1. $\partial t_+/\partial Q_1^2|_{Q_1^2=Q_{1,min}^2} < 0$ in which case $t_{max} = t_{+,max} = t_+(Q_1^2 = Q_{1,min}^2)$, and
2. $\partial t_+/\partial Q_1^2|_{Q_1^2=Q_{1,min}^2} > 0$ in which case one can easily derive

$$t_{max} = t_+(Q_1^2 = x_-) \text{ with}$$

$$x_- = Q^2 + Q_2^2 - 2\, Q\, Q_2 \frac{1 - E_2/Q}{\sqrt{\alpha}}\ , \quad \alpha = \left(1 - \frac{E_2}{Q}\right)^2 - \left(\frac{|\vec{q}_2|}{Q}\right)^2 > 0$$

♠ **Q_1^2-limits:**

The limits for the Q_1^2-integration for given t can now be fixed by the condition $|\cos\theta| \leq 1$ or equivalently

$$\Pi(Q_1^2) \leq 0$$

with

$$\Pi(Q_1^2) = \left(t - Q_1^2 - m_2^2 + \frac{E_2}{Q}(Q^2 + Q_1^2 - Q_2^2)\right)^2 - \left(\frac{|\vec{q}_2|}{Q}\right)^2 \lambda$$

If $y_1 \leq y_2$ are the two roots of the polynomial $\Pi(Q_1^2)$ then we have

1. For $a > 0$, $y_- < Q_1^2 < y_+$, with $y_- = \max(y_1, Q^2_{1,min})$ and $y_+ = \min(y_2, Q^2_{1,max})$
2. For $a < 0$ we have to satisfy two conditions $Q_1^2 < y_1$ or $y_2 < Q_1^2$ and $Q^2_{1,min} < Q_1^2 < Q^2_{1,max}$

- **At the end we get:**

$$d\Phi_n \to \prod ds_i\ p_i(s_i) \prod dt_j\ p_j(t_j) \prod d\phi_k \prod d\cos\theta_l$$

- $p(x)$ **are chosen so that 'singularities' are smoothed out!**

♦ $(s-m^2)^2+m^2\Gamma^2$	**for massive unstable particles,** $W^\pm$, Z.
♦ s^ν	**for time-like massless propagators, e.g.** γ, **gluons, fermions.**
♦ $\lvert t\rvert^\nu$	**for space-like massless propagators.**

$$g\,g \to b\,\bar{b}\,b\,\bar{b}\,W^-\,W^+$$

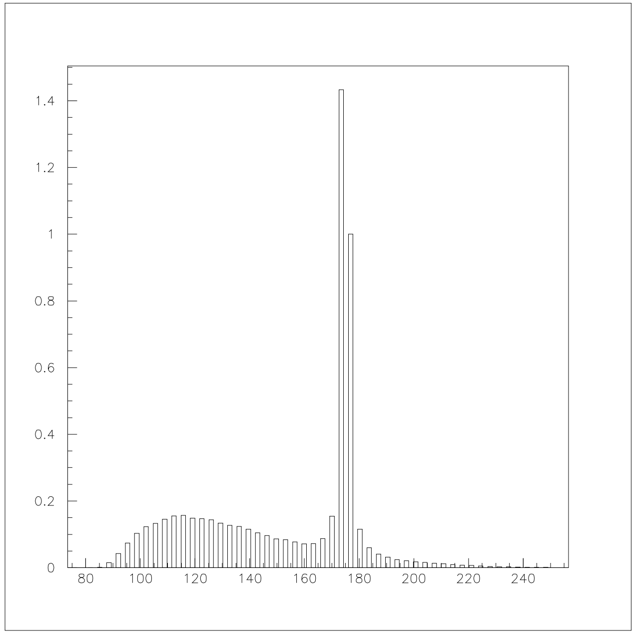
- challenging process, from a computational point of view
- a nice example to demonstrate the ability of PHEGAS/HELAC to deal with QCD processes.
- background of $t\bar{t}H$ production

MC points $w > 0$	result (fb)	error (fb)	efficiency (%)	efficiency $w > 0$ (%)
99442	4.716	0.024	3.3	33

- energy $\sqrt{s} = 500$ GeV and to 1×10^6 MC points.
- Feynman graphs for this process is 960, with 4! colour configurations, without taking into account electroweak contributions from Z and γ intermediate states.

Parameters used are $g_{QCD} = 1$, $m_{top} = 175$ GeV and $\Gamma_{top} = 1.5$ GeV. Moreover the following set of cuts has been applied:

$$M_{q,q'} > 20\text{GeV}, \quad E_q > 20\text{GeV}, \quad |\cos\theta(q, \text{beam})| < 0.9,$$



$p\,p \longrightarrow t\,\bar{t}\,b\,\bar{b}\,b\,\bar{b}$

- **Another challenging process, from a computational point of view**
- **A nice example to demonstrate the ability of** PHEGAS/HELAC **to deal with QCD processes in a realistic setup.**
- **A background of $t\bar{t}HH$ production, which seems interesting in a high-luminosity LHC version for HHH coupling.**
- **Feynman graphs for this process is 1454 (gg), with 5! colour configurations.**
- **A two-phase implementation has been set up.**
- **Srtucture functions and α_s from** PDFLIB**, CTEQ-4L (LO).**
- **Kinematical decays of $t \longrightarrow bW^+$ has been implemented**
- **Cuts: $p_T^b > 20GeV$, $|\eta_b| < 2.5$, $\Delta R > 0.5$**

The result is 1.053 ± 0.073 (fb) @ LHC

Higher-order corrections

- **Fermion-loop corrections have been implemented and studied for up to 3-point vertices.**
- **We have started the computation of 4-point contributions.**
  - `FORM` **has been used to reduce the expressions to Passarino-Veltman coefficient functions**
  - `FF` **has been updated to include level-4 tensor coefficient functions for 4-point integrals.**
  - **Implementation and checking in** `HELAC` **is in progress.**
- **This implementation will allow to study 4 fermion+γ and 6 fermion production including the running of electroweak couplings.**

- **Fermion-loop corrections to six-fermion production process**

$$e^- e^+ \rightarrow \mu^- \bar{\nu}_\mu u \bar{d} \tau^- \tau^+$$

- **Number of Feynman Graphs: 208**
- **Number of DS vertices: 140**
- **Cuts:** $E_l, E_q > 5\text{GeV}$ **and** $m_{ll}, m_{qq} > 10\text{GeV}$
- **Results:** $E = 500\text{GeV}$
- $\sigma_0/ab = 54,96(26)$ $\sigma_1/ab = 57,31(28)$ $K/100 = 4.28(2)$
- **MC data: generated: 1M(961792) used: 404842 time:6 1/2 h**

Outlook

- **PHEGAS / HELAC: a framework for high-energy phenomenology**
- **Standard Model fully included**

⋆ **High color charge processes: multijet production**

P.Draggiotis, R.H.Kleiss and C.G.Papadopoulos, Phys. Lett. B439 (1998) 157;
Eur. Phys. J. C 24 (2002) 447 hep-ph/0202201
C. G. Papadopoulos and M. Worek, arXiv:hep-ph/0512150

⋆ **Higher order corrections**

  - **Direct approach. Ongoing work to better understand Dyson-Schwinger equations and loop calculations: stepping equations, recursive actions, etc.**
  - **Running couplings and masses: 4-point FL contributions and BBC non-local approch to go beyond 4-fermion final states.**

- **SUSY and new particles**

MC4LHC 2006

- **Interface/matching/combination with parton showers**

MC4LHC 2006

- **Interface/matching/combination with parton showers**

- **Beyond LO issues**

MC4LHC 2006

- **Interface/matching/combination with parton showers**
- **Beyond LO issues**
- **Validation of developed tools**