SPLIT SCHOOL OF HIGH ENERGY PHYSICS 2015

LECTURE NOTES
ON
STANDARD MODEL (SM)

- WHAT ARE THE MAIN INGREDIENTS OF THE SM?

FIELDS

$\phi(x) \equiv \phi$ SCALAR

$\psi(x) \equiv \psi$ (DIRAC) FERMION ?

$A^{\mu}(x) \equiv A^{\mu}$ VECTOR-BOSON

SYMMETRIES

INTERNAL

GAUGE SYMMETRIES:
SU(3), SU(2), U(1)
[SU(3) × SU(2) × U(1)]

EXTERNAL

SPACE-TIME SYMMETRY:
LORENTZ SYMMETRY

THESE FIELDS, AT THE LEVEL OF THE SM, SHOULD BE FUNDAMENTAL (ELEMENTARY) OBJECTS (PARTICLES).

* WE HAVE FINALLY OBSERVED FUNDAMENTAL SPIN ZERO PARTICLE (ANNOUNCED ON JULY 4, 2012)

BY THE END OF THESE LECTURES I SHOULD HAVE EXPLAINED TO YOU THE ROLE OF THESE FIELDS AND THESE SYMMETRIES IN THE SM.

$\mu=0,1,2,3$

WE WILL WORK IN UNITS WHERE

$$c=1 \quad \hbar=1 \qquad \Delta x \Delta p \geqslant \hbar = 1$$

$$\Rightarrow [\text{mass}]=[\text{energy}]=[\text{time}]^{-1}=[\text{length}]^{-1}$$

IN THESE UNITS WE HAVE THAT THE DIMENSIONS OF MASS AND ENERGY ARE 1. DIMENSIONS OF TIME AND LENGTH ARE THUS −1.

DIRAC EQUATION (1928) WAS MEANT TO DESCRIBE e.
"The Quantum Theory of the Electron"

ELECTRONS HAVE MASS AND CHARGE. DIRAC FOUND THAT HIS EQUATION PREDICTS EXISTANCE OF THE ANTIPARTICLE! ANTIPARTICLE OF ELECTRON IS POSITRON.

LET US START WITH THINGS YOU PROBABLY KNOW.

DIRAC EQUATION :

MASS

LORENTZ INDEX

$$\mathcal{L} = \bar{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m)\,\psi \quad (*) \qquad \mu = 0,1,2,3$$

→ γ - MATRIX

COVARIANT DERIVATIVE $\partial_{\mu} = \frac{\partial}{\partial x^{\mu}}$

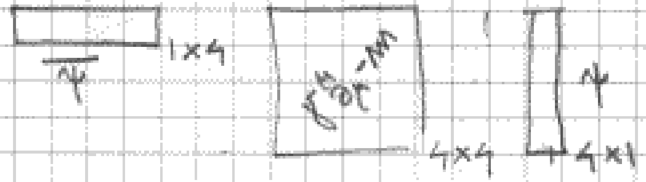
→ LAGRANGE DENSITY

⟹ GAMMA MATRICES SATISFY CLIFFORD ALGEBRA $\{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu}$ TO REPRODUCE $E^2 = m^2 + |\vec{p}\,|^2$.

WHAT IS $\bar{\psi}$?

$$\boxed{\bar{\psi} = \psi^{+}\gamma^{0}}$$

HOW CAN ONE DEPICT THIS LAGRANGIAN DENSITY?



THIS LAGRANGIAN DENSITY DESCRIBES FREE FERMION (SPIN ½) OF MASS m. THE DIRAC EQUATION READS

$$\boxed{(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0}$$

→ THERE IS AN INVISIBLE IDENTITY MATRIX $I_{4\times4}$

NOTE THAT THE TERM (*) IS INVARIANT WITH RESPECT TO THE FOLLOWING GLOBAL TRANSFORMATION

$$\psi \rightarrow e^{i\alpha}\psi \qquad \alpha = \text{const.}$$

THIS IMPLIES EXISTANCE OF A CONSERVED CURRENT

$$\partial_{\mu} J^{\mu} = 0,$$

WHERE $J^{\mu} = \bar{\psi}\gamma^{\mu}\psi$ IS THAT CURRENT.

THIS GLOBAL SYMMETRY IS $U(1)$ SYMMETRY THAT IS COMPACT AND CONTINUOUS. U STANDS FOR UNITARY. FOR AN ELEMENT U OF SUCH A SYMMETRY WE HAVE

$$U^{+}U = UU^{+} = 1$$

WHAT WE WANT IS TO ADD INTERACTION INTO THIS SETUP. IF WE WANT TO REPRODUCE ELECTROMAGNETIC INTERACTIONS WE NEED TO GET THE FOLLOWING TERM:

$$\underbrace{e}_{\text{CHARGE (ELECTRIC)}} J^{\mu} A_{\mu} \longrightarrow \text{CARRIER OF INTERACTION}$$

LET US TRY TO PROMOTE $U(1)$ SYMMETRY INTO A LOCAL ONE AND SEE WHAT HAPPENS.

$$\psi \to e^{i\alpha(x)}\psi$$

$$\overline{\psi} \to \overline{\psi} e^{-i\alpha(x)}$$

$$\Rightarrow \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi \to \overline{\psi}\overbrace{e^{-i\alpha(x)}e^{i\alpha(x)}}^{1}(i\gamma^{\mu}\partial_{\mu} - m)\psi +$$

$$+ \underline{\overline{\psi}\left[- \gamma^{\mu}\partial_{\mu}\alpha(x)\right]\psi} \quad (\Delta)$$

CLEARLY, OUR LAGRANGIAN IS NOT INVARIANT. WE CAN CANCEL ADDITIONAL TERM BY INTRODUCING A FIELD!

$$\partial_{\mu} \to D_{\mu} = \partial_{\mu} - ieA_{\mu}$$

$$\overline{\psi} i\gamma^{\mu} D_{\mu}\psi = \overline{\psi} i\gamma^{\mu}\partial_{\mu}\psi + \overbrace{e\overline{\psi}\gamma^{\mu}A_{\mu}\psi}^{\text{OUR INTERACTION}}$$

IF THE TERM $e\overline{\psi}\gamma^{\mu}A_{\mu}\psi$ IS TO CANCEL SECOND TERM IN (Δ) WE NEED TO MAKE THE FOLLOWING

REQUIREMENT

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

$$\Rightarrow e\bar{\Psi}\gamma^\mu \psi A_\mu \to e\bar{\Psi}\gamma^\mu A_\mu \psi + \underline{\bar{\Psi}[\gamma^\mu \partial_\mu \alpha(x)]\psi}$$

WHAT DID WE LEARN? TO INTRODUCE LOCAL SYMMETRY WE NEED TO INTRODUCE INTERACTION!

$\bar{\Psi} i\gamma^\mu \partial_\mu \Psi$ IS A KINETIC TERM FOR FERMIONS.

WHAT IS A KINETIC TERM FOR GAUGE FIELD A_μ?

$$\mathcal{L} \propto -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

PROBLEM: Find out what this contraction is in terms of E ¡and B ¡.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

NOTE THAT $F_{0i} = E_i$ AND $F_{ij} = \varepsilon_{ijk} B_k$

A LITTLE DETOUR ABOUT DIMENSIONS:

$[\text{energy}] = [\text{mass}] = 1$

$[\Psi] = ? = \frac{3}{2}$ $[x] = [y] = [z] = [t] = -1$

$[A_\mu] = ? = 1$ $\int |\psi|^2 \, dx\, dy\, dz = \#$ (PROBABILITY)

CAN WE ADD MASS TERM FOR THE GAUGE FIELD?

$a\, A_\mu A^\mu$ ✓ $[a] = 2 \Leftarrow [S] = 0$ $[\mathcal{L}] = 4$ $[d^4x] = -4$

→ MASS TERM IS ALLOWED BY LORENTZ SYMMETRY.

$m^2 \cancel{A_\mu A^\mu}$ × → MASS TERM, HOWEVER, IS NOT ALLOWED BY GAUGE SYMMETRY!

THERE EXISTS A MATRIX γ_5 WITH THE FOLLOWING PROPERTIES:

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 \qquad \gamma_5^2 = 1 \qquad \{\gamma_5, \gamma_\mu\} = 0$$

WE CAN WORK IN BASIS WHERE

$$\gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix} \qquad \gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \gamma_5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

LET US NOW DEFINE OPERATORS L AND R:

$$L = \frac{1+\gamma_5}{2} \qquad R = \frac{1-\gamma_5}{2} \qquad L+R=1$$

$$L^2 = LL = \frac{(1+\gamma_5)^2}{4} = \frac{1+2\gamma_5+\gamma_5^2}{4} = \frac{2+2\gamma_5}{4} = L$$

$\Rightarrow$ L AND R ARE THUS PROJECTION OPERATORS!
(RECALL, PROJECTION OPERATORS ARE SINGULAR OPERATORS.)

$$\Rightarrow \psi = L\psi + R\psi = \psi_L + \psi_R$$

$$\psi_L = L\psi = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varphi_L \\ \varphi_R \end{bmatrix} = \begin{bmatrix} \varphi_L \\ 0 \end{bmatrix}$$

$$\psi_R = R\psi = \begin{bmatrix} 0 \\ \varphi_R \end{bmatrix}$$

HOW IS (*) WRITTEN IN TERMS OF ψ_L ADD ψ_R?

$$m\bar{\psi}\psi = m\bar{\psi}[L+R]\psi = m\bar{\psi}L\psi + m\bar{\psi}R\psi$$

$$= m\bar{\psi}L^2\psi + m\bar{\psi}R^2\psi = m\psi^\dagger\gamma^0 L\psi_L + m\psi^\dagger\gamma^0 R\psi_R =$$

$$= m\,\psi^\dagger R \gamma^0 \psi_L + m\,\psi^\dagger L \gamma^0 \psi_R =$$

$$= m\,(R\psi)^\dagger \gamma^0 \psi_L + m\,(L\psi)^\dagger \gamma^0 \psi_R = \underline{m\,\overline{\psi_R}\,\psi_L + m\,\overline{\psi_L}\,\psi_R}$$

$$\boxed{\overline{\psi_L} = \overline{(\psi_L)}}$$

$$\begin{aligned}\overline{\psi}\, i\gamma^\mu \partial_\mu \psi &= \overline{\psi}\, i\gamma^\mu L^2 \partial_\mu \psi + \overline{\psi}\, i\gamma^\mu R^2 \partial_\mu \psi \\ &= \overline{\psi} R\, i\gamma^\mu \partial_\mu \psi_L + \overline{\psi} L\, i\gamma^\mu \partial_\mu \psi_R \\ &= \underline{\overline{\psi_L}\, i\gamma^\mu \partial_\mu \psi_L + \overline{\psi_R}\, i\gamma^\mu \partial_\mu \psi_R}\end{aligned}$$

IT IS THE MASS TERM THAT REPRESENTS THE MARRIAGE OF LEFT AND RIGHT COMPONENTS. KINETIC TERM DOES NOT MIX THEM!

LET US JUST GO BACK TO LORENTZ TRANSFORMATIONS TO UNDERSTAND THE MEANING OF LEFT AND RIGHT.

$$\psi \to \Lambda \psi \qquad \Lambda = e^{i\theta_{\mu\nu}\Sigma^{\mu\nu}}$$

→ ROTATIONS

$$\Sigma^{\mu\nu} = \frac{1}{4i}\left[\gamma^\mu, \gamma^\nu\right] \implies \begin{cases} \Sigma^{ij} = \frac{1}{2}\varepsilon^{ijk}\begin{bmatrix}\sigma^k & 0 \\ 0 & \sigma^k\end{bmatrix} \\ \\ \Sigma^{0i} = \frac{1}{2i}\begin{bmatrix}-\sigma^i & 0 \\ 0 & \sigma^i\end{bmatrix}\end{cases}$$

→ BOOSTS

$$\psi = \begin{pmatrix}\varphi_L \\ \varphi_R\end{pmatrix} \qquad \varphi_{L,R} \to e^{i\frac{\vec{\sigma}}{2}(\vec{\theta} \pm i\vec{\chi})}\,\varphi_{L,R}$$

$$\boxed{2\theta_i = \varepsilon_{ijk}\theta^{jk}} \qquad \boxed{\chi^i = \theta^{0i}}$$

SINCE γ_5 ANTICOMMUTES WITH γ_μ IT COMMUTES WITH $\Sigma_{\mu\nu}$, I.E.,

$$[\gamma_5, \Sigma_{\mu\nu}] = 0 \Rightarrow [L, \Sigma_{\mu\nu}] = 0$$

WHAT ABOUT $e\bar{\psi}\gamma^\mu\psi A_\mu$?

$$\boxed{e\bar{\psi}\gamma^\mu\psi A_\mu = e\overline{\psi_L}\gamma^\mu\psi_L A_\mu + e\overline{\psi_R}\gamma^\mu\psi_R A_\mu}$$

LOOK AT THE MATERIAL ON PAGE 8' BEFORE YOU PROCEED

UNDER PARITY $\psi_L \overset{P}{\longleftrightarrow} \psi_R$ [LEFT-CHIRAL FIELD GOES INTO RIGHT-CHIRAL FIELD!]

$$\begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} \xrightarrow{P} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix}$$

HELICITY:

LET US LOOK AT THE MASSLESS SPINOR:

$$i\gamma^\mu\partial_\mu\psi - m\psi = 0 \Rightarrow i\gamma^\mu\partial_\mu\psi = m\psi = 0$$

$$\Rightarrow \gamma^\mu p_\mu\psi = 0 \qquad p^\mu = (E, \vec{p})$$

$$\begin{bmatrix} 0 & E - \vec{p}\cdot\vec{\sigma} \\ E + \vec{p}\cdot\vec{\sigma} & 0 \end{bmatrix} \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = 0$$

$$\left.\begin{aligned} \Rightarrow \vec{p}\cdot\vec{\sigma}\,\psi_R &= E\psi_R \\ \vec{p}\cdot\vec{\sigma}\,\psi_L &= -E\psi_L \end{aligned}\right\}$$

RECALL: $E^2 = |\vec{p}|^2 + m^2 \Rightarrow E = |\vec{p}|$

PROBLEM: Show that γ_5 does not commute with Hamiltonian. Hamiltonian is given as

$$H = \gamma^0(\gamma^i p^i + m).$$

What does that mean?

GAMMA MATRICES ALSO SATISFY THE FOLLOWING IDENTITY

$$\gamma_0 \gamma_\mu \gamma_0 = \gamma_\mu^+$$

IT IS NEEDED FOR DIRAC EQUATION TO BE HERMITIAN!

$$\Rightarrow \gamma_0^+ = \gamma_0 \qquad \gamma_i^+ = -\gamma_i$$

LET US HAVE A LOOK AT THE CURRENT TRANSFORMATION PROPERTIES UNDER PARITY

$$(\bar{\psi} \gamma^\mu \psi) A_\mu = \underbrace{(\bar{\psi} \gamma^i \psi)}_{\text{ODD}} A_i + \underbrace{(\bar{\psi} \gamma^0 \psi)}_{\text{EVEN}} A_0$$

$$\to (\overline{\psi'} \gamma^i \psi') A'_i + (\overline{\psi'} \gamma^0 \psi') A'_0$$

$$\left[\begin{aligned} \psi' = \gamma^0 \psi \Rightarrow \overline{\psi'} &= (\gamma^0 \psi)^+ \underline{\gamma^0} = \psi^+ \gamma^{0+} \gamma^0 = \\ &= \psi^+ \gamma^0 \overbrace{\gamma^0 \gamma^0}^{1} \gamma^0 = \bar{\psi} \gamma^0 \end{aligned} \right]$$

$$= (\bar{\psi} \overbrace{\gamma^0 \gamma^i \gamma^0}^{\gamma^{i+} = -\gamma^i} \psi) A'_i + (\bar{\psi} \overbrace{\gamma^0 \gamma^0 \gamma^0}^{\gamma^0} \psi) A'_0$$

$$= -(\bar{\psi} \gamma^i \psi) A'_i + (\bar{\psi} \gamma^0 \psi) A'_0$$

SO, UNDER PARITY WE HAVE THAT

$$\psi \xrightarrow{P} P\psi, \text{ WHERE } P = \gamma^0 \text{ !}$$

PROBLEM: Show that helicity operator h commutes with Hamiltonian H. What does that mean?

$$\hat{\vec{p}}\cdot\frac{\vec{\sigma}}{2}\psi_{L,R} = \mp\frac{1}{2}\psi_{L,R}$$

$$\vec{S} \equiv \frac{\vec{\sigma}}{2}$$

$\vec{S}$ → SPIN OPERATOR

$$h = 2\hat{\vec{p}}\cdot\vec{S}$$

h → HELICITY OPERATOR

LOOK AT PAGE 8' BEFORE YOU PROCEED

⇒ FOR MASSLESS PARTICLE HELICITY ≡ CHIRALITY !

PARITY VIOLATION IS MAXIMAL IN THE SM !

LET US NOW LOOK AT THE CASE OF SU(2) SYMMETRY !

$$UU^{\dagger} = U^{\dagger}U = 1$$

$$U = e^{i\theta_a T_a} \qquad U^{\dagger} = e^{-i\theta_a T_a^{\dagger}} \Rightarrow T_a^{\dagger} = T_a$$

T_a → GENERATORS

θ_a → PARAMETERS

⇒ $(\theta_a T_a)$ IS A NORMAL MATRIX !

NORMAL MATRIX A IS SUCH THAT $A^{\dagger}A = AA^{\dagger}$. IT CAN ALWAYS BE DIAGONALIZED BY $RAR^{\dagger} = A^{diag}$, WHERE $RR^{\dagger} = 1$!

PROBLEM:

SHOW THAT

$$\det\left(e^{i\theta_a T_a}\right) = 1$$

GIVES $\mathrm{Tr}\, T_a = 0$!

NOTE:

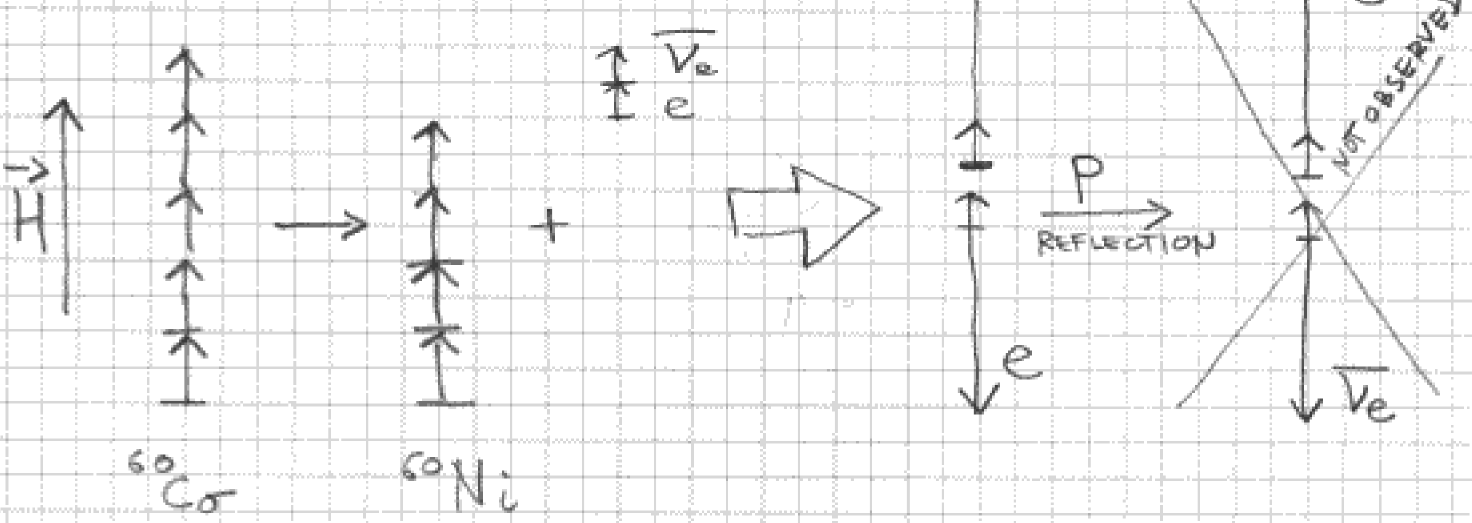
HELICITY IS CONSERVED FOR A FREE PARTICLE BUT IT IS NOT LORENTZ INVARIANT! CHIRALITY IS LORENTZ INVARIANT BUT IT IS NOT CONSERVED. SEE arXiv: 1006.1718.

T.D. LEE AND C.N. YANG: PARITY VIOLATED IN WEAK INTERACTIONS!

C.S. WU CHECKS THIS HYPOTHESIS IN 1957.

$$^{60}Co \longrightarrow {}^{60}Ni + e + \overline{\nu}_e$$

$(J^P = 5^+)$ $(J^P = 4^+)$



EXPERIMENT SHOWS THAT ELECTRONS ARE EMITTED IN THE DIRECTION THAT IS OPPOSIT TO THE SPIN OF THE NUCLEUS ^{60}Co.

$\vec{B}$ AND $\vec{S}$ ARE AXIAL VECTORS!

LET US TRY TO UNIFY FERMIONS USING SU(2).

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix} \to U \begin{pmatrix} u \\ d \end{pmatrix} \qquad U = e^{i\theta_a(x) T_a}, \ a = 1,2,3$$

$$\partial_\mu \to D_\mu = \partial_\mu - i g T_a A^a_\mu$$

RECALL, FOR U(1) WE HAD

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

WHAT WE HAVE IN GENERAL, WHEN THE ELEMENT OF THE GROUP IS U, IS

$$\boxed{A^a_\mu T_a \to U A^a_\mu U^+ - \frac{i}{g}(\partial_\mu U) U^+} \quad (\Delta\Delta)$$

HOMEWORK: CHECK RELATION (ΔΔ) FOR U(1) GROUP.

$$\mathcal{L}_{SU(2)} \propto \bar\psi i \gamma^\mu D_\mu \psi - m \bar\psi \psi$$

$$\partial_\mu \to D_\mu \equiv \partial_\mu - i g T_a A^a_\mu$$

$$\Rightarrow i\bar\psi\gamma^\mu D_\mu\psi = \cdots + g\bar\psi\gamma^\mu T_a A^a_\mu \psi =$$

$$= \cdots + \frac{g}{2}\bar\psi\gamma^\mu \overset{\text{Pauli matrices}}{\sigma_a} A^a_\mu\psi =$$

$$= \cdots + \frac{g}{2}\begin{bmatrix} \bar u & \bar d \end{bmatrix}\gamma^\mu \begin{bmatrix} A^3_\mu & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} =$$

This term is written in SU(2) group space.

$$-\frac{g}{2}(\bar{u}\gamma^\mu u-\bar{d}\gamma^\mu d)A^3_\mu+\frac{g}{\sqrt{2}}\underbrace{\left(\frac{A^1_\mu-iA^2_\mu}{\sqrt{2}}\right)}_{W^+_\mu}\bar{u}\gamma^\mu d+$$

$$+\frac{g}{\sqrt{2}}\underbrace{\left(\frac{A^1_\mu+iA^2_\mu}{\sqrt{2}}\right)}_{W^-_\mu}\bar{d}\gamma^\mu u$$

$\to$ CAN A^3_μ BE A PHOTON?

THE ANSWER IS, OF COURSE, NO! BUT, WE SEE THAT RATIO OF CHARGES IS FIXED. ALSO, IF WE START WITH LEFT-HANDED FIELDS WE WOULD HAVE THAT ONLY THEY TALK TO A^3_μ, W^+_μ AND W^-_μ.

HOW TO FIX CHARGE ASSIGNMENT AND INCORPORATE PARITY VIOLATION?

GLASHOW '61 ⟶ SU(2) x U(1)

$$\partial_\mu \to D_\mu=\partial_\mu-igT_aA^a_\mu-ig'\frac{Y}{2}B_\mu$$

There are two identity matrices of the same dimensions as T_a .

CHARGED CURRENT:

$$[\bar{u}_L\ \bar{d}_L]\gamma^\mu\frac{g}{\sqrt{2}}\begin{bmatrix}0 & \frac{A^1_\mu-iA^2_\mu}{\sqrt{2}}\\ \frac{A^1_\mu+iA^2_\mu}{\sqrt{2}} & 0\end{bmatrix}\begin{bmatrix}u_L\\ d_L\end{bmatrix}=$$

$= \frac{g}{\sqrt{2}} \overline{u_L} \gamma^\nu d_L W_\nu^+ +$ $\qquad W_\nu^+ = \frac{(A_\nu^1 - i A_\nu^2)}{\sqrt{2}}$

$+ \frac{g}{\sqrt{2}} \overline{d_L} \gamma^\nu u_L W_\nu^-$ $\qquad W_\nu^- = \frac{(A_\nu^1 + i A_\nu^2)}{\sqrt{2}}$

NEUTRAL CURRENT:

$$[\overline{u_L} \;\; \overline{d_L}]\left[g T_3 A_\mu^3 + g' \frac{Y}{2} B_\mu\right] \gamma^\mu \begin{bmatrix} u_L \\ d_L \end{bmatrix}$$

$$A_\mu^3 = \sin\theta \, A_\mu + \cos\theta \, Z_\mu \qquad \sin\theta = s_\theta$$

$$B_\mu = \cos\theta \, A_\mu - \sin\theta \, Z_\mu \qquad \cos\theta = c_\theta$$

$$\Rightarrow [\overline{u_L} \;\; \overline{d_L}]\gamma^\mu\left[\left(g T_3 s_\theta + g' \frac{Y}{2} c_\theta\right) A_\mu + \left(g T_3 c_\theta - g' \frac{Y}{2} s_\theta\right) Z_\mu\right]\begin{bmatrix} u_L \\ d_L \end{bmatrix}$$

$$[\overline{u_L} \;\; \overline{d_L}]\gamma^\mu \begin{bmatrix} \frac{g s_\theta}{2} + \frac{g' Y c_\theta}{2} & 0 \\ 0 & -\frac{g s_\theta}{2} + \frac{g' Y c_\theta}{2} \end{bmatrix} \begin{bmatrix} u_L \\ d_L \end{bmatrix} A_\mu$$

$$\overline{u_L}\gamma^\mu\left(\frac{g s_\theta}{2} + \frac{g' Y c_\theta}{2}\right) u_L A_\mu + \overline{d_L}\gamma^\mu\left(-\frac{g s_\theta}{2} + \frac{g' Y c_\theta}{2}\right) d_L A_\mu$$

$$\frac{g s_\theta}{2} + \frac{g' Y c_\theta}{2} = \frac{2}{3} e \qquad -\frac{g s_\theta}{2} + \frac{g' Y c_\theta}{2} = -\frac{1}{3} e$$

$$\Rightarrow g' Y c_\theta = \frac{1}{3} e \qquad \boxed{g s_\theta = e}$$

$$\Downarrow$$

$$\boxed{g' c_\theta = e} \quad \Rightarrow \quad \mathrm{tg}\,\theta = \frac{g'}{g} \qquad \boxed{Y = \frac{1}{3}}$$

$$\begin{bmatrix} u_L \\ d_L \end{bmatrix} \quad \begin{array}{l} Q = \frac{2}{3} = T_3 + \frac{Y}{2} = \frac{1}{2} + \frac{1}{6} = \frac{2}{3} \\ Q = -\frac{1}{3} = T_3 + \frac{Y}{2} = -\frac{1}{2} + \frac{1}{6} = -\frac{1}{3} \end{array}$$

WHEN WE TALKED ABOUT PARITY VIOLATION WE ACTUALLY TALKED ABOUT e AND ν_e.

WHICH WAY IS IT ?

$$\begin{bmatrix} \nu_L \\ e_L \end{bmatrix} \quad \text{OR} \quad \begin{bmatrix} e_L \\ \nu_L \end{bmatrix}$$

$$\begin{bmatrix} e_L \\ \nu_L \end{bmatrix} \quad \begin{array}{l} Q = -1 = \frac{1}{2} + \frac{Y}{2} \Rightarrow Y = -3 \\ Q = 0 = -\frac{1}{2} + \frac{-3}{2} \cancel{= -2} \end{array}$$

$$\Rightarrow \begin{bmatrix} \nu_L \\ \\ e_L \end{bmatrix} \quad \begin{array}{l} Q = 0 = \frac{1}{2} + \frac{Y}{2} \Rightarrow Y = -1 \\ \\ Q = -1 = -\frac{1}{2} - \frac{1}{2} = -1 \end{array}$$

WHAT HAVE WE LEARNED SO FAR ?

	T_3	Y	THE SM #
u_L	$1/2$	$1/3$	$(2, 1/3)$
d_L	$-1/2$	$1/3$	
ν_L	$1/2$	-1	$(2, -1)$
e_L	$-1/2$	-1	

WHAT ARE THE SM QUANTUM NUMBERS OF u_R, d_R, e_R AND ν_R ?

$$u_R \quad Q = \frac{2}{3} = T_3 + \frac{Y}{2} = 0 + \frac{Y}{2} \Rightarrow Y = \frac{4}{3}$$

$$d_R \quad Q = -\frac{1}{3} = T_3 + \frac{Y}{2} = 0 + \frac{Y}{2} \Rightarrow Y = -\frac{2}{3}$$

$$e_R \quad Q = -1 = T_3 + \frac{Y}{2} = 0 + \frac{Y}{2} \Rightarrow Y = -2$$

$$\nu_R \quad Q = 0 = T_3 + \frac{Y}{2} = 0 + \frac{Y}{2} \Rightarrow Y = 0$$

	T_3	Y	THE SM#
u_R	0	$4/3$	$(1, 4/3)$
d_R	0	$-2/3$	$(1, -2/3)$
e_R	0	-2	$(1, -2)$
ν_R	0	0	$(1, 0)$

NOW WE HAVE A PROBLEM!

$$m\overline{\psi}\psi = m\overline{\psi_R}\psi_L + m\overline{\psi_L}\psi_R$$

THE MASS TERM FOR UP QUARK MUST READ

$$m_u \underset{\substack{\downarrow \\ (1,-\frac{4}{3})}}{\overline{u_R}}\, \underset{\substack{\downarrow \\ (2,\frac{1}{3})}}{u_L} + \underbrace{m_u \overline{u_L}\, u_R}_{\hookrightarrow \text{THIS CONTRACTION GIVES } (2,1).}$$

$\Rightarrow$ THIS CONTRACTION IN $SU(2)\times U(1)$ SPACE GIVES $(2,-1)$.

WHAT ABOUT MASS FOR e AND d ?

$$m_d \overline{d_R}\, d_L + \underbrace{m_d \overline{d_L}\, d_R}_{\Longrightarrow (2,-1)}$$

$$\left.\begin{array}{l} \overline{d_R} \downarrow (1, \tfrac{2}{3}) \\ d_L \downarrow (2, \tfrac{1}{3}) \end{array}\right\} \Longrightarrow (2,1)$$

$$m_e \overline{e_R}\, e_L + \underbrace{m_e \overline{e_L}\, e_R}_{\Longrightarrow (2,-1)}$$

$$\left.\begin{array}{l} \overline{e_R} \downarrow (1,2) \\ e_L \downarrow (2,-1) \end{array}\right\} \Longrightarrow (2,1)$$

Let us now talk about the Higgs of the SM.

$$\mathcal{L} \propto \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 \quad \text{(REAL SCALAR)}$$

$$\mathcal{L} \propto \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi \quad \text{(COMPLEX SCALAR)}$$

$$\partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi \longrightarrow$$ IF THERE ARE INTERNAL (GAUGE) SYMMETRIES.

$$D_\mu \phi^\dagger D^\mu \phi - m^2 \phi^\dagger \phi$$

$$V(\phi) = a^2\phi^2 + b\phi^4$$

$$\frac{\partial V}{\partial \phi} = 2a^2\phi + 4b\phi^3 = 0$$

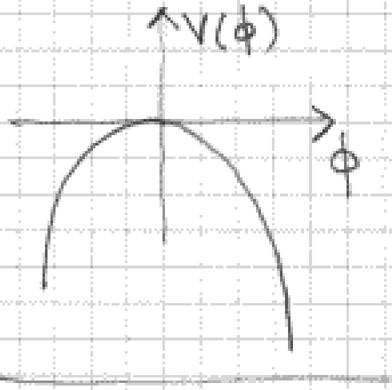
$$\Rightarrow 2a^2\phi_{min} + 4b\phi_{min}^3 = 0 \quad \boxed{\phi_{min} = 0}$$

$$\Rightarrow a^2 + 2b\phi_{min}^2 = 0 \Rightarrow \phi_{min}^2 = -\frac{a^2}{2b}$$

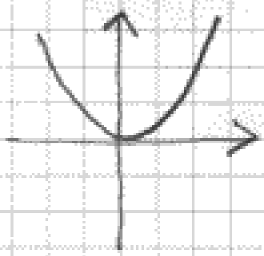
$$\Rightarrow \phi_{min} = \pm\sqrt{-\frac{a^2}{2b}}$$

THERE ARE FOUR OPTIONS WE SHOULD CONSIDER

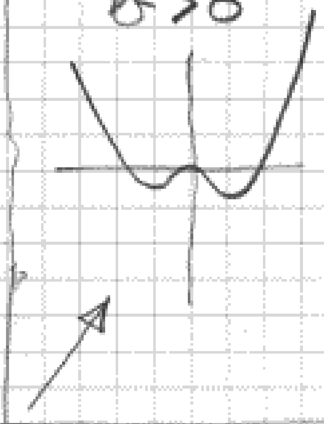
$a^2 = 0$
$b < 0$



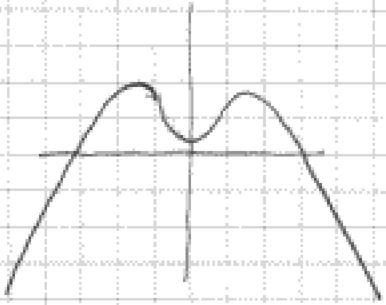
$a^2 = 0$
$b > 0$



$a^2 < 0$
$b > 0$



$a^2 > 0$
$b < 0$



LET US EXPAND IN THE VICINITY OF THE MINIMUM:

$$\phi = \phi_{min} + h = \overset{\substack{\text{CONSTANT (VACUUM EXPECTATION VALUE)} \\ \uparrow}}{v} + \underset{\substack{\uparrow \\ \text{REAL SCALAR}}}{h}$$

$$V(\phi) = a^2 (v+h)^2 + b (v+h)^4 =$$

$$= a^2 (v^2 + \underline{2vh} + h^2) + b (v^2 + \underline{2vh} + h^2)(v^2 + \underline{2vh} + h^2)$$

$$= \underbrace{a^2 \cdot 2vh + 2(b v^2)(2vh)}_{\text{LINEAR IN } h} + \cdots +$$

$$+ \underbrace{a^2 h^2 + 2b v^2 h^2 + 2b(2vh)(2vh)}_{\text{QUADRATIC IN } h} + \cdots$$

$$= \underbrace{\left(a^2 \cdot 2v + 4 \underline{b v^3}\right) h} + \cdots$$

$$\underbrace{2v (a^2 + \underline{2bv^2}) h}$$

$$2v \underbrace{\left(a^2 - \cancel{b} \cancel{2} \frac{a^2}{\cancel{2} \cancel{b}}\right)}_{= 0\,!} h$$

LINEAR TERM VANISHES !

PROBLEM: FIND THE CONSTANT NEXT TO h^2 IN THE EXPANSION OF $V(\phi)$.

THE HIGGS FIELD ϕ HAS TO TRANSFORM AS $(2,1)$

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \quad \begin{array}{l} Q = T_3 + \frac{Y}{2} = \frac{1}{2} + \frac{1}{2} = 1 \\ Q = T_3 + \frac{Y}{2} = -\frac{1}{2} + \frac{1}{2} = 0 \end{array}$$

↑ THIS COMPONENT IS ELECTRICALLY NEUTRAL.

$$\phi_0 = \begin{bmatrix} 0 \\ v \end{bmatrix}$$

→ VACUUM EXPECTATION VALUE (VEV)

→ YUKAWA CONSTANT

$$\mathcal{L}_{YUKAWA} \propto y_e \left[\overline{\nu_L}\ \overline{e_L}\right]_{1\alpha} \phi_{\alpha 1} e_R = \cdots + y_e v \overline{e_L} e_R$$

→ SU(2) INDICES

$$\Rightarrow \boxed{m_e = y_e v}$$

→ YUKAWA CONSTANT

$$\mathcal{L}_{YUKAWA} \propto y_d \left[\overline{u_L}\ \overline{d_L}\right] \phi\, d_R = \cdots + y_d v \overline{d_L} d_R$$

$$\Rightarrow \boxed{m_d = y_d v}$$

→ YUKAWA CONSTANT

$$\mathcal{L}_{YUKAWA} \propto y_u \left[\overline{u_L}\ \overline{d_L}\right]_{1\alpha} \varepsilon_{\alpha\beta} \left[\phi^*\right]_{\beta 1} u_R =$$

$$= \cdots + y_u v \overline{u_L} u_R$$

$$\Rightarrow \boxed{m_u = y_u v}$$

LET US THEN WRITE DOWN THE YUKAWA SECTOR OF THE SM:

$$\mathcal{L}_{Yukawa} \propto \underline{Y_e \left[\overline{\nu_L}\ \overline{e_L}\right] \Phi\, e_R} \ ; \quad \Phi_0 = \begin{bmatrix} 0 \\ v \end{bmatrix}$$

$$\Rightarrow Y_e \overline{e}_L v e_R = Y_e v \overline{e_L} e_R$$

Recall, $Q = T_3 + \frac{Y}{2}$

$$= m_e \overline{e}_L e_R$$

$$\left.\begin{matrix} \begin{bmatrix} \nu_L \\ e_L \end{bmatrix} & \begin{matrix} T_3 = \frac{1}{2} \ ; \ Q = 0 \\ T_3 = -\frac{1}{2} \ ; \ Q = -1 \end{matrix} \end{matrix}\right\} \Rightarrow Y = -1$$

$$\left. e_R \ T_3 = 0 \ ; \ Q = -1 \right\} \Rightarrow Y = -2$$

Y is a $U(1)$ charge. It is thus an additive number. $Y_{tot} = +1 + (X) + (-2) = 0 \Rightarrow X = +1$

$$\Rightarrow \quad Y_e \ (\overline{\nu}\ \overline{e})_L \phi\, e_R$$

$$\Phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \begin{matrix} Q = \frac{1}{2} + \frac{1}{2} = 1 \\ Q = -\frac{1}{2} + \frac{1}{2} = 0 \end{matrix}$$

$$\mathcal{L}_{Yukawa} \propto Y_d \left[\overline{u_L}\ \overline{d_L}\right] \Phi\, d_R =$$

$$= Y_d v \overline{d_L} d_R = m_d \overline{d_L} d_R$$

Recall, $M_W = g v$ → THIS IS THEN FIXED

v → THIS IS MEASURED

M_W → THIS IS MEASURED

What about the mass of the up quarks?

Recall, **under charge conjugation** we have

$$\psi^C = \begin{bmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{bmatrix}\psi^*$$

$$i\sigma_2 = i\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \varepsilon_{\alpha\beta} = \begin{cases} 0 & \alpha=\beta \\ 1, & \alpha=1, \beta=2 \end{cases}$$

$$\mathcal{L}_{\text{Yukawa}} \propto y_u \left[\overline{u_L\, d_L}\right]_{1\alpha} \varepsilon_{\alpha\beta}\, \Phi^*_{\beta 1}\, u_R + \text{h.c.}$$

$$= y_u\, \overline{u}_L\, \varepsilon_{12}\, \Phi^*_2\, u_R$$

$$= y_u v\, \overline{u}_L u_R$$

Now, we have to repeat all of this three times **for three generations**

$$\begin{array}{cccccc} \begin{bmatrix} \nu_{e_L} \\ e_L \end{bmatrix} & \begin{bmatrix} \nu_{\mu_L} \\ \mu_L \end{bmatrix} & \begin{bmatrix} \nu_{\tau_L} \\ \tau_L \end{bmatrix} & \begin{bmatrix} u_L \\ d_L \end{bmatrix} & \begin{bmatrix} c_L \\ s_L \end{bmatrix} & \begin{bmatrix} t_L \\ b_L \end{bmatrix} \\ e_R & \mu_R & \tau_R & u_R & c_R & t_R \\ & & & d_R & s_R & b_R \end{array}$$

$m_e \simeq 0.5$ MeV

$m_t \simeq 170$ GeV

You can look up the masses at the PDG web page.

$$\boxed{\frac{m_e}{m_t} \sim 10^{-6}} \; ?$$

We will leave this question aside!

Can I write

$$\mathcal{L}_{Yukawa} = Y(\bar{\nu}_e\ \bar{e})_L\, \Phi\, u_R \quad ?$$

Recall, again,

$$\boxed{E_0 = mc^2}$$

I should find better notation

$$Y_{ij}[\bar{u}_L\ \bar{d}_L]^i\, \Phi\, d_R^j\ ,\quad i,j=1,2,3$$

$$\left(\begin{bmatrix}u_L\\ d_L\end{bmatrix}, \begin{bmatrix}c_L\\ s_L\end{bmatrix}, \begin{bmatrix}t_L\\ b_L\end{bmatrix}\right) \equiv Q_L^i \qquad d_R^j \equiv \begin{bmatrix}d_R\\ s_R\\ b_R\end{bmatrix}$$

$$\Rightarrow \quad Y^d_{ij}\,\overline{Q_L}^i\,\Phi\, d_R{}^j \qquad Y^u_{ij}\,\overline{Q}^i_{La}\,\varepsilon^{ab}\,\phi^+_b\, u_R^j$$

What defines the particle is its mass and spin! We have to find mass eigenstates:

$$Y^d Y^{d\dagger} = D_L Y^{d^2}_{diag} D_L^\dagger \qquad Y^{d\dagger} Y^d = D_R Y^{d^2}_{diag} D_R^\dagger$$

$$\Rightarrow \boxed{Y^d = D_L Y^d_{diag} D_R^\dagger}$$

$$\left(\overline{Q}_L Y^d d_R\right)_{ij}\phi = \Big(\underbrace{\overline{d}_L D_L}_{\overline{d}_L'}\, Y^d_{diag}\, \underbrace{D_R^\dagger d_R}_{d_R'}\Big)_{ij} v$$

$$\overline{d}_L' = \overline{d}_L D_L \,/\, d_R' = D_R^\dagger d_R$$

$$d_L = D_L d_L' \qquad d_L' = D_L^\dagger d_L \qquad d_R = D_R d_R'$$

The same should be done in all sectors

$$u_R = U_R u'_R$$

$$u_L = U_L u'_L$$

$$\vdots$$

Let us look at the quark sector only:

Kinetic term

$$\frac{i}{2}\overline{u_L}\gamma^\mu\partial_\mu u_L + \frac{i}{2}\overline{u_R}\gamma^\mu\partial_\mu u_R + \dots$$

Charged current term

$$\underbrace{\frac{g}{\sqrt{2}}(\overline{u_L})_i\gamma^\nu(d_L)_i W_\nu^+}$$

$$\frac{g}{\sqrt{2}}\overline{u_L}\underbrace{U_L^+ D_L}_{V_{CKM}}\gamma^\nu d_L W_\nu^+$$

(Cabbibo-Kobayashi-
- Maskawa matrix)

CKM is a mismatch between left handed rotations in the up-type and the down-type quark sectors.

How many parameters there are in CKM? V_{CKM} is a unitary matrix (3x3):

$\Rightarrow N=3 \quad 3^2$ parameters

3 angles
6 phases

$$R_{12} = \begin{bmatrix} c_{\theta_{12}} & s_{\theta_{12}} & 0 \\ -s_{\theta_{12}} & c_{\theta_{12}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{\theta_{23}} & s_{\theta_{23}} \\ 0 & -s_{\theta_{23}} & c_{\theta_{23}} \end{bmatrix}$$

$$R_{13} = \begin{bmatrix} c_{\theta_{13}} & 0 & s_{\theta_{13}} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{\theta_{13}} \end{bmatrix}$$

$$V_{CKM} = \begin{bmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{bmatrix} R_{23} R_{13} R_{12} \begin{bmatrix} e^{i\alpha_4} & 0 & 0 \\ 0 & e^{i\alpha_5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now we know why $t \to Wb$!