LECTURE NOTES ON
STANDARD MODEL (SM)

WHAT ARE THE MAIN INGREDIENTS OF THE SM?

FIELDS

$\phi(x) = \phi$ SCALAR
$\gamma(x) = \gamma$ FERMION?
$A^\mu(x) = A^\mu$ VECTOR-BOSON

SYMMETRIES

GUAGE SYMMETRIES:

$SU(3) \times SU(2) \times U(1)$

SPACE-TIME SYMMETRY:

LORENTZ SYMMETRY

THESE FIELDS, AT THE LEVEL OF THE SM, SHOULD BE FUNDAMENTAL (ELEMENTARY) OBJECTS (PARTICLES).

* We have finally observed fundamental spin zero particle (announced on July 4, 2012)

BY THE END OF THESE LECTURES I SHOULD HAVE EXPLAINED TO YOU THE ROLE OF THESE FIELDS AND THESE SYMMETRIES IN THE SM.

$\mu = 0, 1, 2, 3$
WE WILL WORK IN UNITS WHERE

\[ c = 1 \quad \hbar = 1 \quad \Delta x \Delta p \gg \hbar = 1 \]

\[ \Rightarrow [\text{mass}] = [\text{energy}] = [\text{time}]^{-1} = [\text{length}]^{-1} \]

IN THESE UNITS WE HAVE THAT THE DIMENSIONS OF MASS AND ENERGY ARE 1. DIMENSIONS OF TIME AND LENGTH ARE THUS -1.

DIRAC EQUATION (1928) WAS MEANT TO DESCRIBE E. "THE QUANTUM THEORY OF THE ELECTRON"

ELECTRONS HAVE MASS AND CHARGE. DIRAC FOUND THAT HIS EQUATION PREDICTS EXISTENCE OF THE ANTI-PARTICLE. ANTI-PARTICLE OF ELECTRON IS POSITRON.
Let us start with things you probably know.

**Dirac Equation:**

\[ L = \overline{\Psi} \left( i \gamma^\mu \partial_\mu - m \right) \Psi \quad (\ast) \quad \mu = 0, 1, 2, 3 \]

- Mass
- Lorentz Index
- \( \overline{\Psi} \rightarrow \Gamma - \text{matrix} \)
- Covariant Derivative \( \partial_\mu = \frac{\partial}{\partial x^\mu} \)
- Lagrange Density

\[ \rightarrow \text{Gamma matrices satisfy Clifford algebra} \]

\[ \{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu \nu} \]

To reproduce \( E^2 = m^2 + \overrightarrow{p}^2 \).

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**What is \( \overline{\Psi} \)?**

\[ \overline{\Psi} = \Psi^+ \gamma^0 \]

---

**How can one depict this Lagrangian density?**

\[ 1 \times 4 \]

\[ 4 \times 4 \]

\[ 4 \times 1 \]

---

This Lagrangian density describes free fermion (spin 1/2) of mass \( m \). The Dirac equation reads

\[ (i \gamma^\mu \partial_\mu - m) \Psi = 0 \]

---

**Note that the term (\ast) is invariant with respect to the following global transformation**

\[ \Psi \rightarrow e^{i\lambda} \Psi \quad \lambda = \text{const.} \]

This implies existence of a conserved current

\[ \partial_\mu J^\mu = 0, \]

where \( J^\mu = \overline{\Psi} \gamma^\mu \Psi \) is that current.
This global symmetry is $U(1)$ symmetry that is compact and continuous. $U$ stands for unitary. For an element $U$ of such a symmetry we have

$$U^+ U = U U^+ = 1.$$  

What we want is to add interaction into this setup. If we want to reproduce electromagnetic interactions we need to get the following term:

$$e \int \mu \mathbf{A}_\mu \rightarrow \text{carrier of interaction}$$

$$\rightarrow \text{charge (electric)}$$

Let us try to promote $U(1)$ symmetry into a local one and see what happens.

$$\psi \rightarrow e^{i \phi(x)} \psi$$

$$\bar{\psi} \rightarrow \bar{\psi} e^{-i \phi(x)}$$

$$\rightarrow \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi \rightarrow \bar{\psi} e^{-i \phi(x)} (\gamma^\mu \partial_\mu - m - e A^\mu) \psi$$

$$+ \bar{\psi} \gamma^\mu \partial_\mu \phi(x) \psi$$

Clearly, our Lagrangian is not invariant. We can cancel additional term by introducing a field

$$\phi^\mu \rightarrow D_\mu = \partial_\mu - ie A_\mu$$

$$\bar{\psi} i \gamma^\mu D_\mu \psi = \bar{\psi} (i \gamma^\mu \partial_\mu - m - e A^\mu) \psi + \bar{\psi} \gamma^\mu \partial_\mu \phi(x) \psi$$

If the term $e \bar{\psi} \gamma^\mu A_\mu \psi$ is to cancel second term in (4) we need to make the following
Requirement

\[ A_\mu \rightarrow A_\mu + \frac{i}{e} \partial_\mu \phi(x) \]

\[ \Rightarrow e^{\mp \frac{i}{2} A_\mu} A_\mu e^{\pm \frac{i}{2} A_\mu} \phi(x) \]

What did we learn? To introduce local symmetry we need to introduce interaction!

\( \bar{\psi} i \gamma^\mu \nabla_\mu \psi \) is a kinetic term for fermions.

What is a kinetic term for gauge field \( A_\mu \)?

\[ L \propto -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]

Note that \( F_{0i} = E_i \) and \( F_{ij} = \epsilon_{ijk} B_k \).

A little detour about dimensions:

- Energy \([\text{energy}] = [\text{mass}] = 1\)
- \([x] = [\psi^\dagger \psi] = [\phi] = [-1] = -1\)
- \([A_\mu] = [1] \quad \int \psi^\dagger \psi \ dx \ dy \ dz = \# \quad (\text{probability})\)

Can we add mass term for the gauge field?

\[ \alpha A_\mu A^\mu \]

\([\alpha] = 2 \leq [\phi] = 0 \quad [\frac{\phi^2}{2}] = 4 \quad [d^4x] = -4\]

Mass term is allowed by Lorentz symmetry.

\[ m^2 A_\mu A^\mu \]

Mass term, however, is not allowed by gauge symmetry!
There exists a matrix $\chi_5$ with the following properties:

$$\chi_5^1 = i \chi_0 \chi_5 \chi_2 \chi_3 \chi_5^2 = 1 \quad \{\chi_5, \chi_5^2\} = 0$$

We can work in basis where

$$\chi_i = \begin{bmatrix} 0 & 5 \iota \\ -5 \iota & 0 \end{bmatrix} \quad \chi_0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \chi_5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Let us now define operators $L$ and $R$:

$$L = \frac{1 + \chi_5}{2} \quad R = \frac{1 - \chi_5}{2} \quad L + R = 1$$

$$L^2 = LL = \left(\frac{1 + \chi_5}{2}\right)^2 = \frac{1 + 2\chi_5 + \chi_5^2}{4} = \frac{2 + 2\chi_5}{4} = L$$

$\Rightarrow L$ and $R$ are thus projection operators!

(Recall, projection operators are singular operators.)

$\Rightarrow \psi = L\psi + R\psi = \psi_L + \psi_R$\n
$$\psi_L = L\psi = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = \begin{bmatrix} \psi_L \\ 0 \end{bmatrix}$$

$$\psi_R = R\psi = \begin{bmatrix} 0 \\ \psi_R \end{bmatrix}$$

How is $(\star)$ written in terms of $\psi_L$ and $\psi_R$?

$$m\psi_L = m\psi_L [L + R]\psi = m\psi_L L\psi + m\psi_L R\psi = m\psi_L \chi_5 L\psi + m\psi_L \chi_5 R\psi = m\psi_L L\psi + m\psi_L R\psi$$
\[
\Psi = m \Psi_R \gamma^0 \Psi_L + m \Psi_L \gamma^0 \Psi_R = m \Psi_R \gamma^0 \Psi_L + m \Psi_L \gamma^0 \Psi_R
\]

\[\Psi_L = (\Psi_L)\]

\[
\overline{\Psi} i \gamma^\mu \partial_\mu \Psi = \overline{\Psi} i \gamma^\mu L^\mu \Psi + \overline{\Psi} i \gamma^\mu R^\mu \Psi
\]

\[= \overline{\Psi} R i \gamma^\mu L^\mu \Psi_L + \overline{\Psi} L i \gamma^\mu R^\mu \Psi_R
\]

\[= \overline{\Psi}_L i \gamma^\mu L^\mu \Psi_L + \overline{\Psi}_R i \gamma^\mu R^\mu \Psi_R\]

It is the mass term that represents the marriage of left and right components. Kinetic term does not mix them!

Let us just go back to Lorentz transformations to understand the meaning of left and right.

\[
\Psi \rightarrow \Lambda \Psi \quad \Lambda = e^{i \theta_{\mu \nu} N_{\mu \nu}} \Rightarrow \begin{cases}
\Lambda^R = e^{i \frac{1}{2} \Theta^R_{\mu \nu} \left[ \begin{array}{cc}
+i & 0 \\
0 & i
\end{array} \right]} \\
\Lambda^L = e^{i \frac{1}{2} \Theta^L_{\mu \nu} \left[ \begin{array}{cc}
+i & 0 \\
0 & -i
\end{array} \right]}
\end{cases}
\]

\[
\Psi = \left( \begin{array}{c}
\Psi_L \\
\Psi_R
\end{array} \right) \quad \Psi_{L,R} \rightarrow e^{i \frac{\Theta}{2} \left( \Theta^R + i \chi^R \right)} \Psi_{L,R}
\]

\[\Theta^i = \Theta^0 i^i \\
\chi^i = \Theta^0 i^i \]

[2 \Theta^i = \Theta^0 i^i]
SINCE $\phi_5$ ANTONCOMMUTES WITH $\phi^\mu$, IT COMMUTES WITH $\Sigma_{\mu\nu}$, I.E.,

$$[\phi_5, \Sigma_{\mu\nu}] = 0 \Rightarrow [\phi_5, \Sigma_{\mu\nu}] = 0$$

WHAT ABOUT $e^{\psi^{\gamma^\mu}} \phi^\gamma A_\mu$?

$$e^{\psi^{\gamma^\mu}} \phi^\gamma A_\mu = e^{\psi^{\gamma^\mu}} \phi^\gamma A_\mu + e^{\psi^{\gamma^\mu}} \phi^\gamma A_\mu$$

LOOK AT THE MATERIAL ON PAGE 8 Before you proceed.

UNDER PARITY $\psi_L \leftrightarrow \psi_R$ [LEFT-CHIRAL FIELD GOES INTO] RIGHT-CHIRAL FIELD!

$$\begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix}$$

HELICITY:

LET US LOOK AT THE MASSLESS SPINOR:

$$i \gamma^\mu \partial_\mu \psi - m \psi = 0 \Rightarrow i \gamma^\mu \partial_\mu \psi = m \psi = 0$$

$$\Rightarrow \gamma^\mu \partial_\mu \psi = 0 \quad \partial^\mu = (E, p^\perp)$$

$$\begin{bmatrix} 0 & E - p^\perp, \vec{p} \\ E + p^\perp, \vec{p} \end{bmatrix} \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = 0$$

$$\Rightarrow \vec{p} \cdot \vec{p} \psi_R = E \psi_R \quad \{ \text{RECALL:} \} \quad E^2 = p^0 p^0 + m^2 \Rightarrow E = \sqrt{p^0}$$

PROBLEM: Show that $\gamma_5$ does not commute with Hamiltonian. Hamiltonian is given as

$$H = \gamma^0 (\gamma^i p^i + m).$$

What does that mean?
GAMMA MATRICES ALSO SATISFY THE FOLLOWING IDENTITY

\[ \gamma_0 \gamma_\alpha \gamma_0 = \gamma_\alpha \]

IT IS NEEDED FOR DIRAC EQUATION TO BE HERMITIAN.

\[ \Rightarrow \gamma_0^\dagger = \gamma_0 \quad \gamma_i^\dagger = -\gamma_i \]

LET US HAVE A LOOK AT THE CURRENT TRANSFORMATION PROPERTIES UNDER PARITY

\[
\begin{align*}
(\overline{\psi} \gamma^- \nu \psi) A_\mu &= (\overline{\nu} \gamma^- i \gamma_0 \psi) A_i + (\overline{\nu} \gamma^- \gamma_0 \psi) A_0.
\end{align*}
\]

\[
\Rightarrow (\overline{\psi} \gamma^- \nu \psi) A_1 + (\overline{\psi} \gamma^- \gamma_0 \psi) A_0
\]

\[
\begin{align*}
\gamma^1 = \gamma^0 \gamma \Rightarrow \overline{\psi}^1 &= (\psi^0 \gamma^0 + \psi^1 \gamma^0) = \gamma^1 \gamma^0 \gamma^0 \gamma^0 = \gamma^1 \gamma^0 \gamma^0.
\end{align*}
\]

\[
\gamma^1 = \gamma^0 \gamma \Rightarrow \gamma^1 = \gamma^1 \gamma^0 \gamma^0 = \gamma^1 \gamma^0
\]

\[
\begin{align*}
\gamma^i &=-\gamma^i
\end{align*}
\]

\[
\begin{align*}
&= (\overline{\psi} \gamma^- \gamma^0 \gamma^- \gamma^0 \psi) A_i + (\overline{\psi} \gamma^- \gamma^0 \gamma^- \gamma^0 \psi) A_0.
\end{align*}
\]

\[
\begin{align*}
&= -(\overline{\psi} \gamma^- \nu \psi) A_i + (\overline{\psi} \gamma^- \gamma_0 \psi) A_0.
\end{align*}
\]

SO, UNDER PARITY WE HAVE THAT

\[
\psi \rightarrow \gamma^0 \psi \quad \text{WHERE} \quad \gamma^0 = \psi \]

PROBLEM: Show that helicity operator $h$ commutes with Hamiltonian $H$. What does that mean?

Let us now look at the case of SU(2) symmetry:

$$UU^T = U^T U = 1$$

$$U = e^{i\Theta_a T_a}, \quad U^T = e^{-i\Theta_a T_a} \Rightarrow T_a^T = T_a$$

$$(\Theta_a T_a)$$ is a normal matrix.

Normal matrix $A$ is such that $A^T A = A A^T$. It can always be diagonalized by $R A R^T = A \text{diag}$, where $RR^T = 1$.

PROBLEM:

Show that

$$\det (e^{i\Theta_a T_a}) = 1$$

gives $Tr T_a = 0$.
**Note:**

Helicity is conserved for a free particle but it is not Lorentz invariant!

Chirality is Lorentz invariant but it is not conserved. See arXiv: 1006.1718.

T.D. Lee and C.N. Yang: Parity violated in weak interactions!

C.S. Wu checks this hypothesis in 1957.

\[ { }^{60}\text{Co} \rightarrow { }^{60}\text{Ni} + e^- + \bar{\nu}_e \]

\( (J^P = 5^+) \quad (J^P = 4^+) \)

---

**Diagram:**

Experiment shows that electrons are emitted in the direction that is opposite to the spin of the nucleus \( { }^{60}\text{Co} \).

\( \vec{B} \) and \( \vec{S} \) are axial vectors!
Let us try to unify fermions using SU(2).

\[ \psi = \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow U(u) \begin{pmatrix} u \\ d \end{pmatrix}, \quad U = e^{i \Theta_a(x) T_a}, \quad a = 1, 2, 3 \]

\[ \partial_\mu \rightarrow D_\mu = D_\mu - ig T_a A_\mu^a \]

Recall, for U(1) we had

\[ A_\mu \rightarrow A_\mu + \frac{1}{e} \Theta_\mu (x) \]

What we have in general, when the element of the group is U, is

\[ A_\mu^a T_a \rightarrow U A_\mu^a U^+ - \frac{i}{g} (\Theta_\mu U) U^+ \]

Homework: check relation (\sigma \sigma) for U(1) group.

\[ L_{SU(2)} = \bar{\psi} i \gamma^\mu D_\mu \psi - m \bar{\psi} \psi \]

\[ \partial_\mu \rightarrow D_\mu = \partial_\mu - ig T_a A_\mu^a \]

\[ \Rightarrow i \bar{\psi} \gamma^\mu D_\mu \psi = \cdots + g \bar{\psi} \gamma^\mu T_a A_\mu^a \psi = \]

\[ = \cdots + \frac{g}{2} \bar{\psi} \gamma^\mu \sigma^a b_a A_\mu^a \psi = \]

\[ = \cdots + \frac{g}{2} \left[ \begin{pmatrix} A_\mu^3 \\ A_\mu^1 - i A_\mu^2 \\ A_\mu^1 + i A_\mu^2 \end{pmatrix} \right] \left[ \begin{pmatrix} u \\ d \end{pmatrix} \right] = 2 \]

This term is written in SU(2) group space.
\[ \frac{g}{2} \left( u \gamma^\mu u - d \gamma^\mu d \right) A_\mu^3 + \frac{g}{12} \left( \frac{A_\mu^1 - i A_\mu^2}{\sqrt{2}} \right) \bar{u} \gamma^\mu d + \frac{g}{\sqrt{2}} \left( \frac{A_\mu^1 + i A_\mu^2}{\sqrt{2}} \right) \bar{d} \gamma^\mu u \]

\[ \Rightarrow \text{CAN } A_\mu^3 \text{ BE A PHOTON?} \]

---

THE ANSWER IS, OF COURSE, NO! BUT, WE SEE THAT RATIO OF CHARGES IS FIXED. ALSO, IF WE START WITH LEFT-HANDED FIELDS WE WOULD HAVE THAT ONLY THEY TALK TO \( A_\mu^3, W^+_{\mu}, \) AND \( W^-_{\mu} \).

HOW TO FIX CHARGE ASSIGNMENT AND INCORPORATE PARITY VIOLATION?

GLASHOW '61 \( \rightarrow \) SU(2) \( \times \) U(1)

---

\[ \partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu - ig T^a A^a_\mu - ig \frac{\gamma^\mu}{2} B_\mu \]

CHARGED CURRENT:

\[ [u_L, d_L] \gamma^\mu \frac{g}{\sqrt{2}} [0, -i \frac{A_\mu^1 + i A_\mu^2}{\sqrt{2}}] [u_L] = [\frac{A_\mu^1 - i A_\mu^2}{\sqrt{2}}, 0] [d_L] \]

There are two identity matrices of the same dimensions as \( T^a \).
\[
\begin{align*}
\frac{g}{\sqrt{2}} u_L \bar{d}_L W^+ &+ \frac{g}{\sqrt{2}} d_L \bar{u}_L W^- = & & (A^\nu - i A^\nu_Y) \\
& & & \sqrt{2} \\
W^+ & = & & (A^\nu + i A^\nu_Y) \\
& & & \sqrt{2}
\end{align*}
\]

Neutral Current:

\[
\begin{bmatrix} u_L \\ d_L \end{bmatrix} \begin{bmatrix} gT_3 A^3 + q_1 \frac{Y}{2} B & A_\mu \\ A_\mu & \end{bmatrix} \begin{bmatrix} u_L \\ d_L \end{bmatrix}
\]

\[
A_\mu = \sin \theta A_\mu + \cos \theta Z_\mu \\
\sin \theta = \frac{1}{3}
\]

\[
B_\mu = \cos \theta A_\mu - \sin \theta Z_\mu \\
\cos \theta = \frac{1}{2}
\]

\[
\Rightarrow \begin{bmatrix} u_L \\ d_L \end{bmatrix} \begin{bmatrix} gT_3 \frac{Y}{2} c_\theta + g_1 \frac{Y}{2} c_\theta \sin \theta \cos \theta A_\mu + (gT_3 c_\theta - g_1 \frac{Y}{2} c_\theta) Z_\mu \\ A_\mu \end{bmatrix}
\]

\[
\begin{bmatrix} u_L \\ d_L \end{bmatrix} \begin{bmatrix} gT_3 \frac{Y}{2} c_\theta + g_1 \frac{Y}{2} c_\theta \sin \theta \cos \theta & 0 \\ 0 & -gT_3 \frac{Y}{2} c_\theta + g_1 \frac{Y}{2} c_\theta \end{bmatrix} \begin{bmatrix} u_L \\ d_L \end{bmatrix}
\]

\[
\begin{align*}
\frac{gT_3 \frac{Y}{2} c_\theta + g_1 \frac{Y}{2} c_\theta}{2} u_L A_\mu + d_L \begin{bmatrix} gT_3 \frac{Y}{2} c_\theta + g_1 \frac{Y}{2} c_\theta \\ \end{bmatrix} d_L A_\mu \\
gT_3 \frac{Y}{2} c_\theta + g_1 \frac{Y}{2} c_\theta = \frac{2}{3} e \\
-\frac{gT_3 \frac{Y}{2} c_\theta + g_1 \frac{Y}{2} c_\theta}{2} = -\frac{1}{3} e
\end{align*}
\]

\[
\Rightarrow g_1 Y c_\theta = \frac{1}{3} e \\
\Rightarrow +g_\theta = \frac{g_1}{g} \\
Y = \frac{1}{3}
\]
\[
\begin{pmatrix}
u_L \\ d_L
\end{pmatrix}
Q = \frac{2}{3} = T_3 + \frac{Y}{2} = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}
\]
\[
\begin{pmatrix}
u_L \\ d_L
\end{pmatrix}
Q = -\frac{1}{3} = T_3 + \frac{Y}{2} = -\frac{1}{2} + \frac{1}{6} = -\frac{1}{3}
\]

When we talked about parity violation, we actually talked about \(\nu\) and \(\nu_e\).

Which way is it?

\[
\begin{pmatrix}
u_L \\ e_L
\end{pmatrix}
\text{ or }
\begin{pmatrix}
e_L \\ \nu_L
\end{pmatrix}
\]

\[
\begin{pmatrix}
e_L \\ \nu_L
\end{pmatrix}
Q = -1 = \frac{1}{2} + \frac{Y}{2} \Rightarrow Y = -3
\]
\[
\begin{pmatrix}
\nu_L \\ e_L
\end{pmatrix}
Q = 0 = -\frac{1}{2} + \frac{3}{2} = -\frac{1}{2}
\]

\[
\begin{pmatrix}
u_L \\ e_L
\end{pmatrix}
Q = 0 = \frac{1}{2} + \frac{Y}{2} \Rightarrow Y = -1
\]

\[
\begin{pmatrix}
u_L \\ e_L
\end{pmatrix}
Q = -1 = -\frac{1}{2} - \frac{1}{2} = -1
\]

What have we learned so far?

<table>
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<tr>
<th>(T_3)</th>
<th>(Y)</th>
<th>\text{THE SM #}</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_L)</td>
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<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>(d_L)</td>
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<td>(\frac{1}{3})</td>
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<tr>
<td>(\nu_L)</td>
<td>(\frac{1}{2})</td>
<td>(-1)</td>
</tr>
<tr>
<td>(e_L)</td>
<td>(-\frac{1}{2})</td>
<td>(-1)</td>
</tr>
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</table>
WHAT ARE THE SM QUANTUM NUMBERS OF \( u_R, d_R, e_R \) AND \( \nu_R \)?

\[ u_R \quad Q = \frac{2}{3} = T_3 + \frac{Y}{2} = 0 + \frac{2}{2} \Rightarrow Y = \frac{4}{3} \]

\[ d_R \quad Q = -\frac{1}{3} = T_3 + \frac{Y}{2} = 0 + \frac{1}{2} \Rightarrow Y = -\frac{2}{3} \]

\[ e_R \quad Q = -1 = T_3 + \frac{Y}{2} = 0 + \frac{2}{2} \Rightarrow Y = -2 \]

\[ \nu_R \quad Q = 0 = T_3 + \frac{Y}{2} = 0 + \frac{2}{2} \Rightarrow Y = 0 \]

<table>
<thead>
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<th></th>
<th>( T_3 )</th>
<th>( Y )</th>
<th>THE SM#</th>
</tr>
</thead>
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<tr>
<td>( u_R )</td>
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<td>( \frac{2}{3} )</td>
<td>( (1, \frac{4}{3}) )</td>
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<tr>
<td>( d_R )</td>
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<td>-( \frac{2}{3} )</td>
<td>( (1, -\frac{2}{3}) )</td>
</tr>
<tr>
<td>( e_R )</td>
<td>0</td>
<td>-2</td>
<td>( (1, -2) )</td>
</tr>
<tr>
<td>( \nu_R )</td>
<td>0</td>
<td>0</td>
<td>( (1, 0) )</td>
</tr>
</tbody>
</table>

NOW WE HAVE A PROBLEM!

\[ m \bar{\Psi} \Psi = m \bar{\Psi}_R \Psi_L + m \bar{\Psi}_L \Psi_R \]

THE MASS TERM FOR UP QUARK MUST READ

\[ m_{u_R} u_R u_L + m_{u_L} u_L u_R \]

\( \Rightarrow \) THIS CONTRACTION GIVES \( (2, 1) \).

\[ (1, -\frac{4}{3}) \]

\( \Rightarrow \) THIS CONTRACTION IN \( SU(2) \times U(1) \) SPACE GIVES \( (2, -1) \).
What about mass for $e$ and $d$?

\[
md \, \overline{d}_R \, d_L + md \, \overline{d}_L \, d_R \\
\downarrow \quad \downarrow \quad (1, 2/3) \quad (2, 1/2) \\
\quad \downarrow \quad \downarrow \\
(1, 2) \quad (2, -1) \\
\rightarrow (2, 1)
\]

\[
me \, \overline{e}_R \, e_L + me \, \overline{e}_L \, e_R \\
\downarrow \quad \downarrow \\
(1, 2) \quad (2, -1) \\
\rightarrow (2, 1)
\]

Let us now talk about the Higgs of the SM.

\[
L = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 \quad \text{(REAL SCALAR)}
\]

\[
L = \partial^\mu \phi^* \partial_\mu \phi - m^2 \phi^* \phi \quad \text{(COMPLEX SCALAR)}
\]

\[
\partial^\mu \phi^* \partial_\mu \phi - m^2 \phi^* \phi \rightarrow \text{IF THERE ARE INTERNAL (GAUGE) SYMMETRIES}
\]

\[
D_\mu \phi^+ D^\mu \phi - m^2 \phi^+ \phi
\]
\[ V(\phi) = a^2 \phi^2 + b \phi^4 \]
\[ \frac{\partial V}{\partial \phi} = 2 a^2 \phi + 4 b \phi^3 = 0 \]
\[ \Rightarrow 2 a^2 \phi_{\text{min}} + 4 b \phi_{\text{min}}^3 = 0 \]
\[ \Rightarrow a^2 \phi_{\text{min}}^2 + 2 b \phi_{\text{min}}^2 = 0 \Rightarrow \phi_{\text{min}}^2 = -\frac{a^2}{2b} \]
\[ \Rightarrow \phi_{\text{min}} = \pm \sqrt{-\frac{a^2}{2b}} \]

**THERE ARE FOUR OPTIONS WE SHOULD CONSIDER**

- \( a^2 = 0 \)
- \( b < 0 \)
- \( a^2 > 0, b < 0 \)
- \( a^2 < 0, b > 0 \)
LET US EXPAND IN THE VICINITY OF THE MINIMUM:

\[ \phi = \phi_{\text{min}} + h = \nu + h \]

\[ V(\phi) = a^2 (\nu + h)^2 + b (\nu + h)^4 = \]

\[ = a^2 \left( \nu^2 + 2\nu h + h^2 \right) + b \left( \nu^2 + 2\nu h + h^2 \right)^2 \left( \nu^2 + 2\nu h + h^2 \right) \]

\[ = a^2 \cdot 2\nu h + 2(b - \nu^2)(2\nu h) + \ldots + \]

\[ \text{LINEAR IN } h \]

\[ + a^2 h^2 + 2b - \nu^2 h^2 + 2b(2\nu h)(2\nu h) + \ldots \]

\[ \text{QUADRATIC IN } h \]

\[ = (a^2 \cdot 2\nu + 4b - \nu^2) h + \ldots \]

\[ 2\nu \left( a^2 + 2b - \nu^2 \right) h \]

\[ \frac{2\nu \left( a^2 - \nu \frac{a^2}{2b} \right) h}{2b} \]

\[ = 0 ! \]

\[ \text{LINEAR TERM VANISHES !} \]
PROBLEM: FIND THE CONSTANT NEXT TO \( h^2 \) IN THE EXPANSION OF \( V(\phi) \).
The Higgs field $\phi$ has to transform as $(2,1)$

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \quad Q = T_3 + \frac{Y}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

This component is electrically neutral.

$$\phi_0 = \begin{bmatrix} 0 \\ \nu^- \end{bmatrix} \quad \text{vacuum expectation value (VEV)}$$

$$\text{Yukawa} \propto Y_e \begin{bmatrix} \nu_L \\ \bar{e}_L \end{bmatrix} \phi e_R = \ldots + Y_e \nu^- \bar{e}_L e_R$$

$$\implies m_e = Y_e \nu^-$$

$$\text{Yukawa constant}$$

$$\text{Yukawa} \propto Y_d \begin{bmatrix} u_L \\ d_L \end{bmatrix} \phi d_R = \ldots + Y_d \nu^- \bar{d}_L d_R$$

$$\implies m_d = Y_d \nu^-$$

$$\text{Yukawa constant}$$

$$\text{Yukawa} \propto Y_u \begin{bmatrix} u_L \\ d_L \end{bmatrix} \epsilon_{\gamma\delta} \frac{1}{2} \phi^* a_1 u_R = \ldots + Y_u \nu^- \bar{u}_L u_R$$

$$\implies m_u = Y_u \nu^-$$
Let us then write down the Yukawa sector of the SM:

\[ L_{\text{Yukawa}} \propto Y_e \begin{bmatrix} V e_L^c \end{bmatrix} \Phi \begin{bmatrix} e_R \end{bmatrix} ; \begin{bmatrix} q \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \]

Recall, \( q = T_3 + \frac{1}{2} \bar{Q} \Rightarrow Y_e = \begin{bmatrix} V e_L e_R \end{bmatrix} = m_e \begin{bmatrix} e_L e_R \end{bmatrix} \)

\[
\begin{bmatrix} V e_L \end{bmatrix} \begin{bmatrix} T_3 = \frac{1}{2} ; Q = 0 \end{bmatrix} \begin{bmatrix} e_R \end{bmatrix} \begin{bmatrix} T_3 = -\frac{1}{2} ; Q = -1 \end{bmatrix} \Rightarrow Y = -1
\]

\( e_L T_3 = 0 ; Q = -1 \Rightarrow Y = 2 \)

\( Y \) is a U(1) charge, thus an additive number.

\( Y_{\text{dir}} = +1 + (X) + (2) = 0 \Rightarrow X = +1 \)

\( Y e \begin{bmatrix} \Phi e_R \end{bmatrix} \)

\[
\begin{bmatrix} \Phi \end{bmatrix} = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} ; \begin{bmatrix} Q = \frac{1}{2} + \frac{1}{2} = 1 \end{bmatrix} \begin{bmatrix} Q = -\frac{1}{2} + \frac{1}{2} = 0 \end{bmatrix}
\]

\[ Y_d \begin{bmatrix} u_L \bar{d}_R \end{bmatrix} \Phi \begin{bmatrix} d_R \end{bmatrix} = \]

\[ = Y_d \begin{bmatrix} \bar{u}_L d_R \end{bmatrix} = m_d \begin{bmatrix} u_L \bar{d}_R \end{bmatrix} \]

Recall, \( M_W = g W \rightarrow \text{This is then fixed} \)

\[ \rightarrow \text{This is measured} \]

\[ \rightarrow \text{This is measured} \]
What about the mass of the up quarks?

Recall, under charge conjugation we have

\[ N^c = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} N^* \]

\[ i \sigma_2 = i \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \varepsilon \]

\[ \varepsilon_{1 \beta} = \varepsilon_{1 \beta} \]

\[ L = \sum \mu \left[ \varepsilon_{1 \beta} \right] \frac{1}{\sqrt{2}} \varepsilon \beta^1 \left[ U_R \right] + \text{h.c.} \]

\[ = \sum \mu \varepsilon_{1 \beta} \left[ U_R \right] \frac{1}{\sqrt{2}} \varepsilon \beta^1 \]

\[ = \sum \mu \overline{U_L} U_R \]

Now, we have to repeat all of this three times for three generations.

\[
\begin{bmatrix}
\nu_{eL} \\
e_{eL}
\end{bmatrix}
\begin{bmatrix}
\nu_{\mu L} \\
\mu_{eL}
\end{bmatrix}
\begin{bmatrix}
\nu_{\tau L} \\
\tau_{eL}
\end{bmatrix}
\begin{bmatrix}
u_L \\
\nu_R
\end{bmatrix}
\begin{bmatrix}
l_L \\
l_R
\end{bmatrix}
\]

\[ m_e = 0.5 \text{ MeV} \]

\[ m_t = 170 \text{ GeV} \]

You can look up the masses at the PDG website.

\[ \frac{m_e}{m_t} \approx 10^{-6} \]

We will leave this question aside!
Can I write
\[ y_{\nu_e} = y_{\nu_e} (\nu_e e) L \Phi \Delta R \]?

Recall, again,
\[ E_0 = mc^2 \]

I should find better notation
\[ Y_{ij} [u_L d_L]^i \Phi d_R^j, \quad i, j = 1, 2, 3 \]

\[ \begin{pmatrix} [u_L] & [c_L] & [e_L] \\ [d_L] & [s_L] & [b_L] \end{pmatrix} \equiv Q_L \]
\[ \begin{pmatrix} d_R^i \equiv [d_R] \\ D_R \end{pmatrix} \]

\[ \Rightarrow \]
\[ Y_{ij} d_R^j Q_L^i \Phi d_R^j \]

\[ Y_{ij} u \Phi u_{\nu}^c \]
\[ Y_{ij} a_{\nu_e} a_{\nu_e} \]

What defines the particle is its mass and spin! We have to find mass eigenstates:

\[ Y d \; Y d^+ = D_L \; Y d^2 \; D_L^+ \quad Y d^+ \; Y d = D_R \; Y d^2 \; D_R^+ \]

\[ \Rightarrow \]
\[ Y d = D_L Y d^2 D_L^+ \]

\[ \begin{pmatrix} Q_L \; Y d \; d_R^j \end{pmatrix} \phi = \begin{pmatrix} d_L^i \; Y d^2 \; d_R^j \end{pmatrix} \]

\[ \begin{pmatrix} d_L^i \; Y d^2 \; d_R^j \end{pmatrix} \phi = \begin{pmatrix} D_R \; d_R^j \end{pmatrix} \phi \]

\[ d_L = D_L \; d_L \]
\[ d_R = D_R \; d_R \]
The same should be done in all sectors

\[ u_R = U_R u_R' \]
\[ u_L = U_L u_L' \]

Let us look at the quark sector only:

Kinetic term

\[ \frac{i}{2} \bar{u}_L \gamma^\mu \not{v} u_L + \frac{i}{2} \bar{u}_R \gamma^\mu \not{v} u_R + \ldots \]

Changed current term

\[ \frac{g}{\sqrt{2}} \bar{u}_L \gamma^\nu (d_L): W^+ \]

\[ \frac{g}{\sqrt{2}} \bar{u}_L U_L^\dagger D_L \gamma^\nu d_L W^+ \]

\[ V_{\text{CKM}} \] (Cabibbo–Kobayashi–Maskawa matrix)

CKM is a mismatch between left-handed rotations in the up-type and the down-type quark sectors.
How many parameters there are in CKM? $V_{CKM}$ is a unitary matrix (3x3):

$\Rightarrow N = 3 \times 3^2$ parameters

3 angles
6 phases

$R_{12} = \begin{bmatrix} \cos \theta_{12} & -\sin \theta_{12} & 0 \\ \sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{bmatrix}$

$R_{13} = \begin{bmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{i\delta} & 0 & \cos \theta_{13} \end{bmatrix}$

$V_{CKM} = \begin{bmatrix} e^{i\delta_1} & 0 & 0 \\ 0 & e^{i\delta_2} & 0 \\ 0 & 0 & e^{i\delta_3} \end{bmatrix} R_{23} R_{13} R_{12} \begin{bmatrix} e^{i\theta_1} & 0 & 0 \\ 0 & e^{i\theta_2} & 0 \\ 0 & 0 & e^{i\theta_3} \end{bmatrix}$

Now we know why $t \rightarrow Wb$!