Draft version 0.3

LHCEWWG

Di-Boson Group Report

Fiducial cross section and triple gauge coupling definitions for di-boson measurements

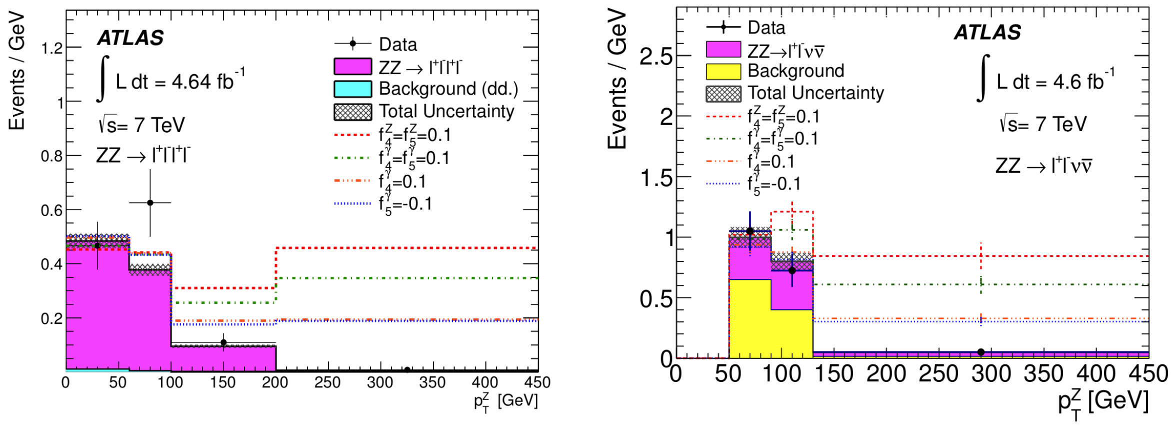
ATLAS, CMS

Abstract

This note presents a summary of preliminary fiducial cross section definitions and anomalous triple gauge coupling parametrisations planned to be used in di-boson measurement with ATLAS and CMS on the full 2012 data set.

Contents

1 Introduction



(a) $ZZ \to \ell^+\ell^-\ell'^+\ell'^-$ (b) $ZZ \to \ell^+\ell^-\nu\bar{\nu}$

Figure 1: The transverse momentum distribution of the Z boson with the highest transverse momentum. The predicted distributions for four different aTGC values at $\Lambda = 3$ TeV are shown as dashed lines [1].

2 ZZ analyses

2.1 ATLAS 7 TeV ZZ Analysis

The ATLAS analysis of ZZ production uses two decay channels, $ZZ \to \ell^+\ell^-\nu\bar{\nu}$ and $ZZ \to \ell^+\ell^-\ell'^+\ell'^-$. As in the $W^\pm Z$ analysis, ℓ refers to an electron or a muon. The $ZZ \to \ell^+\ell^-\ell'^+\ell'^-$ selection is broken into two categories, ZZ and ZZ^* where the Z^* boson is off shell. This section is summarizing the selection details, background estimation, and uncertainties from Reference [1] used to extract the anomalous triple gauge coupling intervals. Plots of the reconstructed Z p_{T} distributions used to extract the aTGC intervals are shown in Figure 1.

2.1.1 Event Selection: $ZZ \to \ell^+\ell^-\ell'^+\ell'^-$

The event is required to have exactly four leptons and fired at least a single electron or single muon event trigger. All pairs of leptons in the event must be separated by $\Delta R > 0.2$. Out of the four leptons, at least one muon must have $p_{\mathrm{T}} > 20$ GeV or at least one electron must have $p_{\mathrm{T}} > 25$ GeV and be matched to the online electron or muon that triggered the event to ensure a high trigger efficiency. The remaining leptons have a $p_{\mathrm{T}} > 7$ GeV requirement. All the leptons must pass an isolation requirement where the calorimeter and track isolation measurements within a ΔR cone of 0.2 must be less than 30% and 15% of the lepton p_{T} respectively. Generally, electrons must have a pseudorapidity of $|\eta| < 2.47$ and muons must have $0.1 < |\eta| < 2.5$.

The overall selection is extended by looking for electrons and/or muons from an extended $|\eta|$ range. These muons come in two categories, calorimeter tagged muons from a region with poor muon spectrometer coverage, $|\eta| < 0.1$, and forward spectrometer muons, $2.5 < |\eta| < 2.7$. These muons must have $p_{\mathrm{T}} > 10$ GeV and a calorimeter isolation within a $\Delta R < 0.2$ cone that is less than 15% of the muon p_{T}. Calorimeter tagged muons are built with calorimeter clusters are are matched to inner detector tracks with $p_{\mathrm{T}} > 20$ GeV. Only one muon from each of these categories is allowed and it must be paired with a non-extended muon in forming a Z boson.

The extended electrons come from the range $2.5 < |\eta| < 3.16$ and are required to have $p_{\mathrm{T}} > 20$ GeV. As this is outside the inner detector fiducial coverage region, no track or charge information is available for these electrons. Electron identification comes entirely from the longitudinal and transverse shower profile in the calorimeter. No isolation requirement is imposed. Only one is allowed in an event and it is assigned the opposite charge as the same flavor lepton it is paired with.

Same flavor, oppositly charged lepton pairs are used to reconstruct Z boson candidates. Two Z boson candidates are required for each event. In the channels with four same flavor leptons, the pairs are selected to minimize the difference between the pair's invariant mass and the global average mass of the Z boson. For the aTGC measurement, both Z boson candidates must have an invariant mass between 66 and 116 GeV.

With these selections, 66 $ZZ \to \ell^+\ell^-\ell'^+\ell'^-$ candidate events are observed in data. Seven of these candidates contain extended $|\eta|$ range leptons.

2.1.2 Event Selection: $ZZ \to \ell^+\ell^-\nu\bar{\nu}$

The leptons in these events are required to have $p_{\mathrm{T}} > 20$ GeV and have the same trigger matching requirements as the $ZZ \to \ell^+\ell^-\ell'^+\ell'^-$ selection. The isolation requirements are changed as the isolation cone size is increased to $\Delta R < 0.3$. Both track and calorimeter isolation measurements are required to be less than 15% of the lepton p_{T}. Each event must have exactly two same flavor, oppositely charged leptons with an invariant mass between 76 and 106 GeV. The lepton pair must also be separated by $\Delta R > 0.3$ reflecting the larger isolation cone used.

Jets are used as a veto for background events in the $ZZ \to \ell^+\ell^-\nu\bar{\nu}$ channel. The jets are reconstructed with the anti-k_t algorithm [2] with a radius of $R = 0.4$. The veto jets are required to have $p_{\mathrm{T}} > 25$ GeV and $|\eta| < 4.5$. A requirement that the scalar sum of the track p_{T} associated with the jet and originating from the primary vertex be at least 75% of the total scalar sum of the p_{T} of all the tracks associated with the jet is used to reduce the impact of pile up jets. Any jets that lie within $\Delta R < 0.3$ of a lepton is discarded. If any veto jets are found, the event is rejected. The event is also rejected if there are any jets with $p_{\mathrm{T}} > 20$ GeV that did not result from the proton-proton collision.

The last requirement for these events is to have a large missing transverse energy, indicative of the second Z boson decaying to neutrinos. The ZZ boson pair are expected to be produced back to back, the $E_{\mathrm{T}}^{\mathrm{miss}}$ is modified into something called axial-$E_{\mathrm{T}}^{\mathrm{miss}}$ to further eliminate backgrounds. Axial-$E_{\mathrm{T}}^{\mathrm{miss}}$ is defined as $-\vec{E}_{\mathrm{T}}^{\mathrm{miss}} \cdot \vec{p}^{Z}/p_T^Z$. The axial-$E_{\mathrm{T}}^{\mathrm{miss}}$ must be greater than 75 GeV. A final distinguishing variable is the fractional p_{T} difference, $|E_{\mathrm{T}}^{\mathrm{miss}} - p_{\mathrm{T}}^{Z}|/p_{\mathrm{T}}^{Z}$. The fractional p_{T} difference must be less than 0.4. $W^{\pm}Z$ events making it through the selection are reduced by rejecting any events with a third lepton with $p_{\mathrm{T}} > 10$ GeV. After all these selections, 87 $ZZ \to \ell^+\ell^-\nu\bar{\nu}$ candidates are observed in data.

2.1.3 Background Estimation

The background estimate for $ZZ \to \ell^+\ell^-\ell'^+\ell'^-$ was performed using data driven techniques similar to those described in Section **??** in the $W^{\pm}Z$ analysis. A fake factor, f, is calculated using cut regions with one Z boson plus real and fake leptons. f used to estimate the number of background events that make it into the signal selection region by extrapolating from two regions dominated by background, those with three selected leptons and a fourth failing select cuts and those with two selected leptons with two additional leptons failing select cuts. See Reference [1] for exact details. Table 1 contains the number of events observed, the expected signal and the expected background in each Z p_{T} bin.

The $ZZ \to \ell^+\ell^-\nu\bar{\nu}$ has far more processes contributing to its background and a much wider variety of techniques were used to estimate it. The background contribution from $t\bar{t}$, Wt, W^+W^- and $Z \to \tau^+\tau^-$ was calculated with a data driven method that looked for events with one electron and one muon passing the lepton selection. These background events were then extrapolated into the signal region using the ratio of the efficiencies of ee and $\mu\mu$ to the efficiency of $e\mu$. Backgrounds from $W^{\pm}Z$ events where one lepton is lost were estimated with simulated $W^{\pm}Z$ events. Backgrounds from the Z+jets process used a data driven template method (see [1]). Finally, the background from events with a misidentified lepton were

Source	$0 < p_T^Z < 60$ GeV	$60 < p_T^Z < 100$ GeV	$100 < p_T^Z < 200$ GeV	$p_T^Z > 200$ GeV
Data	28	25	11	2
ZZ SM Signal	27.9	14.6	9.3	1.6
Background	0.6	0.2	0.1	0.1

Table 1: Observed and predicted events entering into aTGC extraction in the $ZZ \to \ell^+\ell^-\ell'^+\ell'^-$ channel.

Source	$50 < p_T^Z < 90$ GeV	$90 < p_T^Z < 130$ GeV	$p_T^Z > 130$ GeV
Data	42	29	16
ZZ SM Signal	13.6	15.7	10.1
MC Backgrounds	8.5	8.4	4.1
Data Driven Backgrounds	17.5	7.6	0.8

Table 2: Observed and predicted events entering into aTGC extraction in the $ZZ \to \ell^+\ell^-\bar{\nu}\nu$ channel.

extracted from data using the matrix method [1], [3]. Table 2 contains the number of events observed, the expected signal and the expected background in each Z p_T bin.

2.1.4 Uncertainties

All the uncertainties used in the aTGC interval extraction are listed in Tabels 3 and 4. The luminosity uncertainty was 3.9% in all bins and all channels. The "Systematics" uncertainty listed in the tables comes from the combination of lepton efficiency, lepton energy/momentum, lepton isolation and impact parameter, jet and E_T^{miss} modeling, the jet veto, and trigger efficiencies. These uncertainties affect the signal estimation and any background estimations made with MC. The ZZ theory uncertainty comes from the combination of PDF, renormalization and factorization scale uncertainties. The data driven systematic uncertainty for the $ZZ \to \ell^+\ell^-\ell'^+\ell'^-$ background comes from the combined uncertainty on the calculated fake factor, f. The systematics on the data driven background for the $ZZ \to \ell^+\ell^-\bar{\nu}\nu$ channel come from systematics on MC backgrounds used in the calculations and from uncertainties on extrapolation factors. With the exception of the statistical uncertainties, all the uncertainties were considered correlated across the Z p_T bins and the 4ℓ and $2\ell 2\nu$ decay channels.

2.2 CMS 7 TeV ZZ Analysis

The CMS analysis of the ZZ aTGCs at $\sqrt{s} = 7$ TeV used 5.0 fb^{-1}of recorded data. The data was searched for events with two same flavor opposite charge pairs of leptons (muons or electrons) that could be the decay products of a ZZ boson pair. This section will summarize the event selection, backgrounds, and systematics used in the CMS 7 TeV ZZ analysis found in Reference [4].

2.2.1 Event Selection

Events were selected by searching for pairs of oppositely charged electrons or muons. Electron candidates had to fall within $|\eta| < 2.5$ with $p_T > 7$ GeV and muons had to fall within $|\eta| < 2.4$ with $p_T > 5$ GeV. One of the leptons was required to have $p_T > 20$ GeV and the second had to have $p_T > 10$ GeV. The invariant mass of the lepton pair was required to fall within $60 < m_{\ell\ell} < 120$ GeV. Both of these leptons had an isolation requirement where the pile corrected total energy in tracks, the EM calorimeter, and the hadronic calorimeter within a $\Delta R < 0.3$ cone be less than 27.5% of the magnitude of the transverse momentum of the lepton. The lepton pair meeting these requirements with invariant mass closest Z boson mass was selected as Z_1 while second Z boson was labeled as Z_2.

Uncertainty	$0 < p_{\mathrm{T}}^{Z} < 60$ GeV	$60 < p_{\mathrm{T}}^{Z} < 100$ GeV	$100 < p_{\mathrm{T}}^{Z} < 200$ GeV	$p_{\mathrm{T}}^{Z} > 200$ GeV
ZZ statistics	0.9%	1.2%	1.5%	3.9%
Luminosity	3.9%	3.9%	3.9%	3.9%
Systematics	3.5%	3.8%	4.0%	4.5%
ZZ Theory	6.4%	7.1%	8.5%	16.1%
Data Driven Background Systematics	76.2%	75.0%	77.8%	72.7%
Data Driven Background Statistics	119.0%	120.0%	111.1%	90.9%

Table 3: ATLAS uncertainties on signal and background in the $ZZ \rightarrow \ell^{+}\ell^{-}\ell'^{+}\ell'^{-}$ channel used in the aTGC interval extraction. The first three uncertainties apply to the ZZ prediction, the final two apply to the data driven backgrounds.

Uncertainty	$50 < p_{\mathrm{T}}^{Z} < 90$ GeV	$90 < p_{\mathrm{T}}^{Z} < 130$ GeV	$p_{\mathrm{T}}^{Z} > 130$ GeV
ZZ statistics	1.6%	1.8%	1.4%
Luminosity	3.9%	3.9%	3.9%
Systematics	2.0%	3.1%	3.9%
ZZ Theory	9.5%	10.5%	13.3%
Data Driven Background Systematics	6.4%	9.0%	24.4%
Data Driven Background Statistics	25.1%	35.5%	217.9%
MC Background Systematics	2.2%	2.3%	2.2%
MC Background Statistics	9.1%	8.8%	15.9%

Table 4: ATLAS uncertainties on signal and background in the $ZZ \rightarrow \ell^+\ell^-\bar{\nu}\nu$ channel used in the aTGC interval extraction. The first three uncertainties apply to the ZZ prediction, the final two apply to the data driven backgrounds.

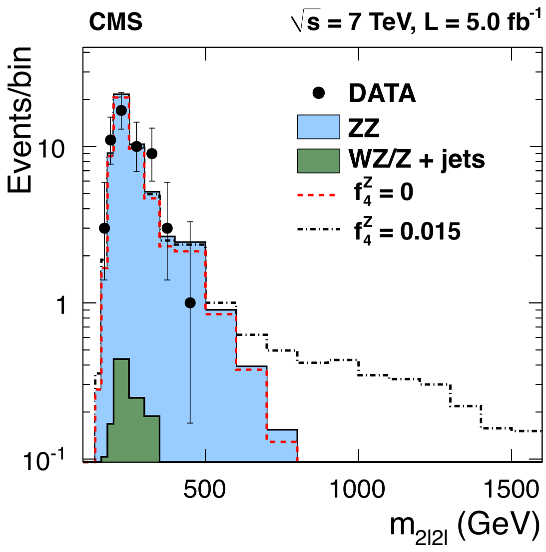


Figure 2: The distribution of the four lepton invariant mass for the sum of the $eeee$, $ee\mu\mu$, and the $\mu\mu\mu\mu$ channels [4].

2.2.2 Background Estimation

As with the ATLAS analysis, the CMS analysis estimated its background using a data driven technique. The rate for non-isolated leptons that are misidentified as isolated leptons was measured using a control region with no ZZ signal contribution. Events that contained the selected Z_1 boson and only a single probe electron or muon that has no isolation requirement were used. The misidentification rate was then calculated as the ratio of the number of probes that pass the isolation to the total number of probe candidates. This rate was measure as a function of p_{T} and η for muons and electrons respectively. The background estimation in the signal region was then estimated by measuring the number of events in a third control region that had all the Z_1 and Z_2 selection requirements except the isolation requirements were reversed. Table 5 contains the observed events, the estimated SM signal, and the estimated background events in bins of m_{ZZ} used in the aTGC extraction.

2.2.3 Uncertaities

The uncertainties used in the CMS aTGC extraction are outlined in Table 6. The same relative uncertainty value was used in each m_{ZZ} bin. The data driven background uncertainty was treated as correlated across the m_{ZZ} bins but uncorrelated across the three decay channels: $4e$, 4μ, and $2e2\mu$. The data driven background uncertainty was measured on the values of the misidentification rates and the limited quantity of data in the control regions. The uncertainty on ZZ includes the PDF uncertainty, renormalization and factorization scales uncertainty, reconstruction uncertainty, uncertainty on aTGC signal reweighting and statistical uncertainty.

m_{ZZ} bin	(100, 200) GeV	(200, 300) GeV	(300, 400) GeV	(400, 600) GeV	(600, 800) GeV
		$ZZ \to \mu^+\mu^-\mu^+\mu^-$			
Data	4	8	1	1	0
ZZ SM Signal	2.8	8.7	2.1	0.8	0.2
Background	0.2	0.4	0.1	0.0	0.0
		$ZZ \to e^+e^-\mu^+\mu^-$			
Data	8	15	8	0	0
ZZ SM Signal	5.0	14.1	3.2	1.4	0.3
Background	0.2	0.3	0.2	0.0	0.0
		$ZZ \to e^+e^-e^+e^-$			
Data	2	4	3	0	0
ZZ SM Signal	2.0	5.5	1.2	0.5	0.1
Background	0.1	0.1	0.1	0.0	0.0

Table 5: CMS observed and predicted events entering into aTGC extraction.

Source	(%)
ZZ Signal modeling (reweighintg, statistics, reconstruction)	13.4
ZZ Theory (PDF, scale)	4.0
Data Driven Background	30.0
Luminosity	2.2

Table 6: CMS uncertainties used in the aTGC extraction.

3 ZZ combination

The combination of anomalous coupling results by ATLAS and CMS collaboration was performed using published results for ZZ production channels with full 7 TeV dataset.

A likelihood is used to extract the 95% confidence interval with using Baysian integration and Neyman construction techniques.

Systematic uncertainties are included as nuisance parameters, correlations and bin-to-bin migrations are taken into account.

There are many differences between ATLAS and CMS anomalous coupling measurements. First, different observables were used. Anomalous couplings results with increase of a cross section at high energies, therefore diboson system mass and boson transverse momentum are particularly sensitive. ATLAS is using leading Z transverse momentum while CMS uses diboson system mass. Theoretical uncertainties on the signal are p_T dependent but flat in diboson mass, this results with signal shape dependent uncertainty used by ATLAS and flat uncertainty used by CMS.

Anomalous coupling signal model continuous in anomalous coupling parameters was built differently in two experiments as described in section 3.2.

Several differences were also found in the statistical methods for limit setting. These are described in detail in section 3.3.

3.1 Theoretical framework for anomalous couplings

Neutral trilinear gauge couplings are forbidden at the tree level, but allowed in some extensions of the SM . The ZZ production enables to probe the existence of anomalous couplings in the ZZZ and γZZ vertices.

Neutral couplings $V^{(*)}ZZ$ $(V = Z, \gamma)$ can be described using the effective Lagrangian [5]:

$$\mathcal{L}_{\mathrm{VZZ}} = -\frac{e}{M_{\mathrm{Z}}^2}\left\{\left[f_4^{\gamma}\left(\partial_\mu F^{\mu\alpha}\right) + f_4^{Z}\left(\partial_\mu Z^{\mu\alpha}\right)\right] Z_\beta\left(\partial^\beta Z_\alpha\right) - \left[f_5^{\gamma}\left(\partial^\mu F_{\mu\alpha}\right) + f_5^{Z}\left(\partial^\mu Z_{\mu\alpha}\right)\right]\tilde{Z}^{\alpha\beta} Z_\beta\right\}, \quad (1)$$

where Z represents the Z boson and $F_{\mu\alpha}$ represents the electromagnetic field tensor. The coefficients f_i^{γ} and f_i^{Z} correspond to couplings $\gamma^{(*)}ZZ$ and $Z^{(*)}ZZ$ where the terms corresponding to f_4^{V} parameters violate the CP symmetry, and the terms corresponding to f_5^{V} parameters conserve CP.

ATLAS analysis includes anomalous coupling measurement with two form factor scales, $\Lambda_{FF} = 2$ TeV and $\Lambda_{FF} = \infty$, while CMS analysis uses the approach without form factor equivalent to $\Lambda_{FF} = \infty$. Therefore combined limit was derived for approach without form factor.

3.2 Anomalous coupling signal modeling (Re-weighting Procedure)

Simulated events generated with *SHERPA* [6] generator were used by both ATLAS and CMS to model anomalous coupling signal.

In CMS analysis events were generated and simulated with several non Standard Model values of anomalous neutral $ZZ\gamma$ coupling. Two anomalous coupling parameters were varied at the same time, (f_4^γ, f_4^Z) and (f_5^γ, f_5^Z), while other parameters were set to Standard Model value. Signal model continous in anomalous coupling parameters was achieved by performing two dimensional second order polynomial fit on simulated expected yield in every observable bin.

In ATLAS analysis reweighting of Sherpa events was performed using the BaurRainwater [7, 8] and BHO [9] MC generators. The re-weighting procedure allows to re-weight a sample of event simulated with a given set of coupling parameters to another arbitrary set of couplings parameters. It is possible to generate ZZ events with any anomalous TGC $(f_4^\gamma, f_4^Z, f_5^\gamma)$ with . Each event has a vector of 15 weights $\{w_0 \ldots w_{14}\}$ which can be reweighted to another anomalous TGC phase space point. The weight at a new point is given by

$$\begin{aligned} w(f_4^\gamma, f_4^Z, f_5^\gamma, f_5^Z) &= w_0 + (f_4^\gamma)^2 w_1 + (f_4^Z)^2 w_2 + (f_5^\gamma)^2 w_3 + (f_5^Z)^2 w_4 \\ &+ 2f_4^\gamma w_5 + 2f_4^Z w_6 + 2f_5^\gamma w_7 + 2f_5^Z w_8 \\ &+ 2f_4^\gamma f_4^Z w_9 + 2f_4^\gamma f_5^\gamma w_{10} + 2f_4^\gamma f_5^Z w_{11} + 2f_4^Z f_5^\gamma w_{12} + 2f_4^Z f_5^Z w_{13} + 2f_5^\gamma f_5^Z w_{14} \end{aligned} \tag{2}$$

To re-weight to another value of the form factor Λ_{FF}, the anomalous TGC parameters $\boldsymbol{\alpha} = (f_4^\gamma, f_4^Z, f_5^\gamma, f_5^Z)$ are multiplied by $\zeta = (1 + \frac{\hat{s}}{\Lambda_{FF}^2})^3 (1 + \frac{\hat{s}}{\Lambda_{FF}'^2})^{-3}$, where Λ_{FF} is the cutoff used in generating the original sample, and Λ'_{FF} is the target value. This is equivalent to adjusting the event weights $\{w_0 \ldots w_{14}\}$ as

$$w_i \to \begin{cases} w_i & \text{for } i = 0 \\ w_i\zeta & \text{for } i = 5, 6, 7, 8 \\ w_i\zeta^2 & \text{for } i = 1, 2, 3, 4, 9, 10, 11, 12, 13, 14 \end{cases} \tag{3}$$

After applying these factors, the event weights are accumulated for the MC signal events that pass the selection and correct with additional scale factors related to reconstruction, trigger and pile-up description. The end result is the expected number of signal events N_s^i in our data sample in the form of

$$\begin{aligned} N_s^i(\boldsymbol{\alpha}) &= W_0^i + (f_4^\gamma)^2 W_1 + (f_4^Z)^2 W_2 + (f_5^\gamma)^2 W_3 + (f_5^Z)^2 W_4 \\ &+ 2f_4^\gamma W_5 + 2f_4^Z W_6 + 2f_5^\gamma W_7 + 2f_5^Z W_8 \\ &+ 2f_4^\gamma f_4^Z W_9 + 2f_4^\gamma f_5^\gamma W_{10} + 2f_4^\gamma f_5^Z W_{11} + 2f_4^Z f_5^\gamma W_{12} + 2f_4^Z f_5^Z W_{13} + 2f_5^\gamma f_5^Z W_{14} \end{aligned} \tag{4}$$

for each bin in a histogram.

Both approaches of signal description in anomalous coupling parameter space are consistent, making the translation between them trivial. Sets of two parameters were varied simultanaously providing a two dimensional model in parameter space by both experiments. Coefficients $\{W_j^i\}$ are used in the anomalous TGC limit setting procedure described in section 3.3.

3.3 Statistical method used for the combination

To set limits on the anomalous TGC paramters, two different limit approaches were used, frequentist and delta log-likelihood method. The reweighting procedure described in the previous section allows us to express expected number of signal events $N_{\rm sig}^i$ in observable bin as a function of anomalous TGC parameters.

The likelihood is built from a series of components. First is the model prediction of the number of events in each bin and these are defined as

$$N^i_{\text{sig}}(\boldsymbol{\alpha},\boldsymbol{\theta}) = N^i_{\text{sig}}(\boldsymbol{\alpha})\prod_j^J (1+\delta^{ij})^{\theta^j} \quad (5)$$

$$N^i_{\text{bkg}}(\boldsymbol{\theta}) = N^i_{\text{bkg}}\prod_j^J (1+\delta^{ij})^{\theta^j} \quad (6)$$

resulting with the nuisance effect following log−normal distribution. Here, N^i_{bkg} is the background prediction in bin i, δ^{ij} is the value of the j^{th} uncertainty in bin i, and θ^j is the nuisance parameter associated with the j^{th} uncertainty. Another approach would be to use Gaussian distribution as done in the ATLAS measurement:

$$N^i_{\text{sig}}(\boldsymbol{\alpha},\boldsymbol{\theta}) = N^i_{\text{sig}}(\boldsymbol{\alpha})\prod_j^J (1+\theta^j\delta^{ij}) \quad (7)$$

$$N^i_{\text{bkg}}(\boldsymbol{\theta}) = N^i_{\text{bkg}}\prod_j^J (1+\theta^j\delta^{ij}) \quad (8)$$

where also a different approach for total effect from all uncertainties was used:

$$N^i_{\text{sig}}(\boldsymbol{\alpha},\boldsymbol{\theta}) = N^i_{\text{sig}}(\boldsymbol{\alpha})(1+\sum_j^J \theta^j\delta^{ij}) \quad (9)$$

$$N^i_{\text{bkg}}(\boldsymbol{\theta}) = N^i_{\text{bkg}}(1+\sum_j^J \theta^j\delta^{ij}) \quad (10)$$

For a short hand, define $\psi^i(\boldsymbol{\alpha},\boldsymbol{\theta}) = N^i_{\text{sig}}(\boldsymbol{\alpha},\boldsymbol{\theta}) + N^i_{\text{bkg}}(\boldsymbol{\theta})$. The likelihood of observing N^i_{data} events given $\psi^i(\boldsymbol{\alpha},\boldsymbol{\theta})$ is then described by a Poisson distribution. The likelihood is completed multiplying the Poisson distribution by the constraint on the nuisance parameters.

$$L(\boldsymbol{\alpha},\boldsymbol{\theta}) = \prod_{i=1}^I \frac{[\psi^i(\boldsymbol{\alpha},\boldsymbol{\theta})]^{N_{\text{data}}} e^{-\psi^i(\boldsymbol{\alpha},\boldsymbol{\theta})}}{N^i_{\text{data}}!} \times \frac{1}{(2\pi)^J} e^{-\frac{1}{2}\boldsymbol{\theta}^2}, \quad (11)$$

The most likely estimators (MLE) for the aTGCs and nuisance parameters are then found by finding the minimum of the negative log of Equation 11. Finding the 95% confidence interval in the Frequentist sense now means a comparison must be made to many other "experiments". Since the experiment cannot be repeated many times, pseudo-experiments are used. These pseudo experiments are just a count of events that are generated for each bin i. The count of events are generated by randomly sampling a Poisson distribution with a mean of $\psi^i(\boldsymbol{\alpha}_{\text{test}},\hat{\hat{\boldsymbol{\theta}}})$. Parametric bootstrap scheme was used where $\hat{\hat{\boldsymbol{\theta}}}$ are the nuisance parameter values that maximize the likelihood acting on N_{data} when $\boldsymbol{\alpha}$ is held fixed at $\boldsymbol{\alpha}_{\text{test}}$. The likelihood for the pseudo-experiment is modified to the following form

$$L(\boldsymbol{\alpha},\boldsymbol{\theta}) = \prod_{i=1}^I \frac{[\psi^i(\boldsymbol{\alpha},\boldsymbol{\theta})]^{N_{\text{pseudo}}} e^{-\psi^i(\boldsymbol{\alpha},\boldsymbol{\theta})}}{N^i_{\text{pseudo}}!} \times \frac{1}{(2\pi)^J} e^{-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\theta_0})^2} \quad (12)$$

The $\boldsymbol{\theta_0}$ represents a small random Gaussian shift in the central values of the nuisance parameters that is added for each pseudo-experiment to represent how a different experiment would have different estimates for its uncertainties. Ten thousand pseudo-experiments are generated to determine the p-value at a test point, $\boldsymbol{\alpha}_{\text{test}}$. The p-value at a test point is defined as

$$\text{p-value}(\boldsymbol{\alpha}_{\text{test}}) = \frac{\text{Number of pseudo experiments with less likely results than observed}}{\text{Total number of pseudo experiments}} \tag{13}$$

Evaluating the numerator in Equation 13 requires a way to compare how likely a pseudo-experiment is to N_{data}. This is accomplished through the use of the profile likelihood ratio, $\lambda(\boldsymbol{\alpha})$.

$$\lambda(\boldsymbol{\alpha}_{\text{test}}) = \frac{L(\boldsymbol{\alpha}_{\text{test}}, \hat{\hat{\boldsymbol{\theta}}})}{L(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\theta}})} \tag{14}$$

$\boldsymbol{\alpha}_{\text{test}}$ is the aTGC point being tested, $\hat{\hat{\boldsymbol{\theta}}}$ are the nuisance parameter values that maximize the likelihood at $\boldsymbol{\alpha}_{\text{test}}$, and $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\theta}}$ are the values of $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$ that maximize the likelihood together. As a result, $0 < \lambda(\boldsymbol{\alpha}) < 1$. Pseudo-experiments with $\lambda(\boldsymbol{\alpha})$ that is less than the $\lambda(\boldsymbol{\alpha})$ found on data are considered less likely. Points are tested moving out from $\hat{\boldsymbol{\alpha}}$ in the positive and negative directions until the upper and lower points are found with p-value equal to 5%. These points bound the 95% confidence interval.

The limit setting criteria described above is called the Feldman-Cousins method [10].

A much faster method for extracting the intervals involves defining a test statistic that uses the profile likelihood ratio.

$$t_\alpha = -2 \ln \lambda(\boldsymbol{\alpha}) \tag{15}$$

In this case, t_α is assumed to follow a χ^2 distribution so the probability is read directly as a result of this value. The test points where $t_\alpha = 3.84$ bound the 95% confidence interval when only a single aTGC value is allowed to float. This method for extracting the 95% confidence interval is commonly referred to as the delta log-likelihood method as another way to write the test statistic is as $t_\alpha/2 = \ln[L(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\theta}})] - \ln[L(\boldsymbol{\alpha}_{\text{test}}, \hat{\hat{\boldsymbol{\theta}}})]$

Expected intervals are also calculated for the combined ATLAS and CMS inputs. All intervals presented were calculated using an pre−fit Asimov dataset. An Asimov dataset is a special kind of pseudo-experiment used for quickly extracting expected limits. Nominally and also used in ATLAS measurement paper, extracting expected limits involes running many pseudo-experiments in the same manner as used to build the p-value distribution discussed earlier in this section but only at the standard model expectation (background only). Each of these pseudo-experiments then has its interval calculated with one or both of the methods described earlier. The average of the upper and lower intervals for all the pseudo-experiments is then taken as the expected interval. An Asimov dataset takes the most average version of the pseudo-experiments, $\psi^i(\mathbf{0}, \tilde{\boldsymbol{\theta}})$, where $\tilde{\boldsymbol{\theta}}$ is the nominal value of nuisances, and extracts a single interval with it. As this pseudo-experiment is the average, the 95% confidence interval should converge to the values found on the average interval as the number of pseudo-experiments tested approaches infinity.

In ATLAS measurement the Feldman-Cousins method was used to set the limits on anomalous couplings, while in CMS measurement the modified frequentist construction CL_S method [11, 12, 13] was used.

3.4 Treatment of systematic uncertainties

For the combination of ATLAS and CMS data only luminosity, PDF and QCD scale uncertainty on signal are treated as 100% correlated. Other uncertainties are statistical or detector related and these are treated as uncorrelated.

Source	Affecteed processes	Uncertainty value (ATLAS/CMS)
Luminosity	ZZ signal, MC driven background	3.9% / 2.2%
PDFs+α_S	ZZ signal	6.4%-16.1% (shape) / 4%

Table 7: Table of 100% correlated uncertainties

Source	Affecteed processes	Uncertainty (ATLAS/CMS)
MC statistics	ZZ signal, MC driven background	shape, uncorrelated bins
data-driven method statistics	data driven background	shape / flat
data-driven method systematics	data driven background	shape / flat
MC systematics	MC driven background	shape / -
Reconstruction	ZZ signal, MC driven background	shape / flat

Table 8: Table of 100% uncorrelated uncertainties

Luminosity uncertainty is correlated since it is driven by machine-dependent uncertainties.

Theoretical uncertainties on signal, due to QCD scales and PDFs, are calculated seperately in ATLAS and CMS. Since the source of uncertainties is the same the uncertainties are 100% correlated between CMS and ATLAS.

Expected background contribution in both analysis is mainly (or completely) derived from the data. The methods are not identical and reconstructions in detectors are different therefore the uncertainties on the estimated backgrounds are used as uncorrelated.

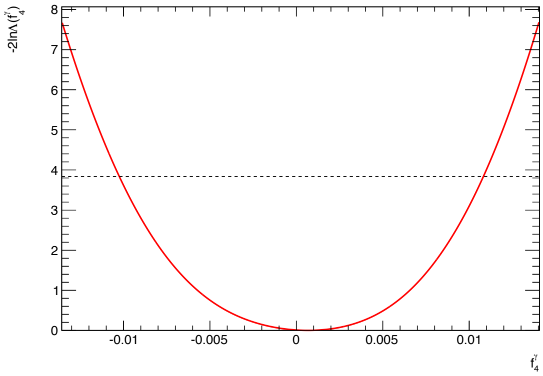
Full list of uncertainties can be found in Tables 7 and 8.

3.5 Combination Results

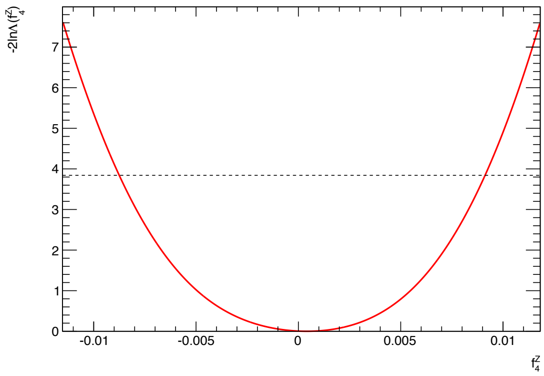
Expected deltaNLL limit	f_4^γ		f_4^Z	
	ATLAS	CMS	ATLAS	CMS
Gaussian	[-0.0121, 0.0125]	[0.0120, 0.0125]	[-0.0103, 0.0105]	[-0.0103, 0.0105]
log-Normal	[-0.0120, 0.0123]	[0.0119, 0.0123]	[-0.0102, 0.0104]	[-0.0102, 0.0104]
	f_5^γ		f_5^Z	
Gaussian	[-0.0127, 0.0121]	[0.0126, 0.0121]	[-0.0106, 0.0104]	[0.0106, 0.0104]
log-Normal	[-0.0126, 0.0119]	[0.0125, 0.0120]	[-0.0105, 0.0103]	[0.0105, 0.0103]
Observed deltaNLL limit	f_4^γ		f_4^Z	
	ATLAS	CMS	ATLAS	CMS
Gaussian	[-0.0103, 0.0108]	[0.0103, 0.0109]	[-0.00875, 0.00912]	[0.00874, 0.00913]
log-Normal	[-0.0102, 0.0108]	[0.0102, 0.0108]	[-0.00870, 0.00907]	[0.00871, 0.00909]
	f_5^γ		f_5^Z	
Gaussian	[-0.0108, 0.0103]	[0.0108, 0.0104]	[-0.00907, 0.00891]	[0.00909, 0.00891]
log-Normal	[-0.0108, 0.0103]	[0.0108, 0.0103]	[-0.00902, 0.00885]	[0.00906, 0.00886]

Table 9: Table of expected and observed intervals for the combined ATLAS and CMS inputs. Intervals were all extracted with the delta log-likelihood method.

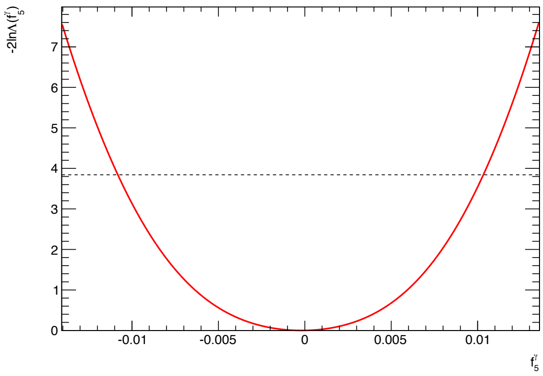
3.6 Comment on unitarization issues



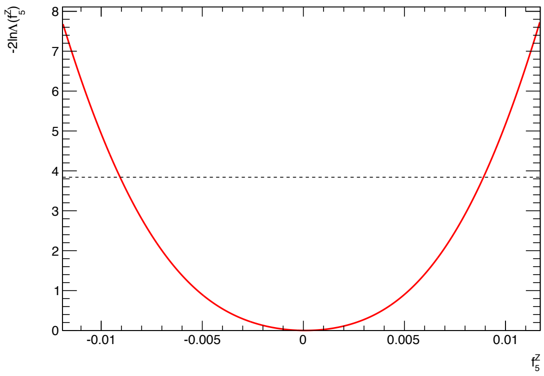
(a) f_4^γ



(b) f_4^Z

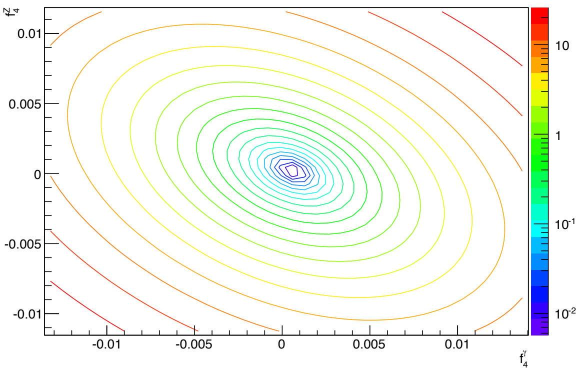


(c) f_5^γ

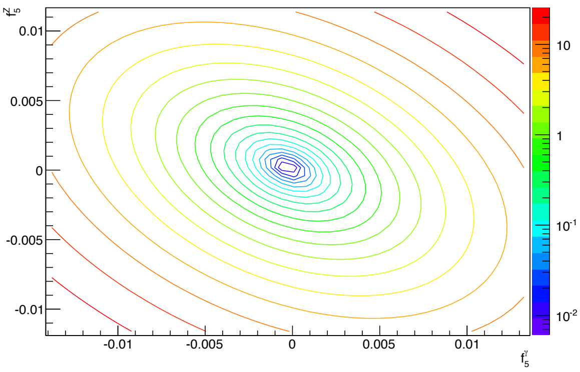


(d) f_5^Z

Figure 3: Plots of the test statistic, t_α, as a function of aTGC value made with truncated Gaussian constraints. Dashed line marks the 95% C.I. cutoff values.

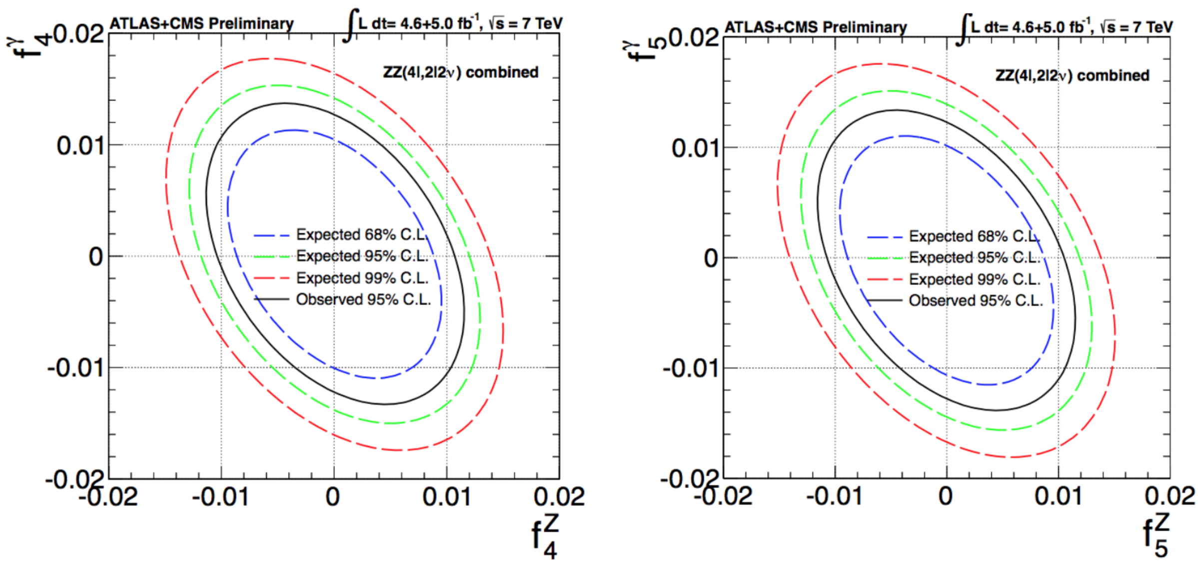


(a) f_4^Z *vs* f_4^γ



(b) f_5^Z *vs* f_5^γ

Figure 4: Plots of the test statistic, t_α, as a function of aTGC value made with truncated Gaussian constraints.



(a) f_4^Z vs f_4^γ (b) f_5^Z vs f_5^γ

Figure 5: 2D expected and observed deltaNLL combined limits with log-normal constraints.

4 Conclusions

References

[1] ATLAS Collaboration Collaboration, G. Aad et al., *Measurement of ZZ production in pp collisions at $\sqrt{s} = 7$ TeV and limits on anomalous ZZZ and ZZγ couplings with the ATLAS detector*, JHEP **1303** (2013) 128, arXiv:1211.6096 [hep-ex].

[2] M. Cacciari, G. P. Salam, and G. Soyez, *The Anti-k(t) jet clustering algorithm*, JHEP **0804** (2008) 063, arXiv:0802.1189 [hep-ph].

[3] ATLAS Collaboration Collaboration, G. Aad et al., *Measurement of the top quark pair cross section with ATLAS in pp collisions at sqrt(s) = 7 TeV using final states with an electron or a muon and a hadronically decaying τ lepton*, Phys.Lett. **B717** (2012) 89–108, arXiv:1205.2067 [hep-ex].

[4] CMS Collaboration Collaboration, S. Chatrchyan et al., *Measurement of the ZZ production cross section and search for anomalous couplings in 2 l2l ' final states in pp collisions at $\sqrt{s} = 7$ TeV*, JHEP **1301** (2013) 063, arXiv:1211.4890 [hep-ex].

[5] K. Hagiwara et al., *Probing the weak boson sector in $e^+e^- \to W^+W^-$*, Nucl. Phys. B **282** (1987) 253.

[6] T. G. et al., *Event generation with Sherpa 1.1*, JHEP **0902** (2009) 007, arXiv:0811.4622 [hep-ph].

[7] U. Baur and D. L. Rainwater, *Probing neutral gauge boson self interactions in ZZ production at hadron colliders*, Phys. Rev. D **62** (2000) 113011, arXiv:0011016 [hep-ph].

[8] J. O. U. Baur, T. Han, *WZ production at hadron colliders: Effects of non-standard WWZ couplings and QCD corrections*, Phys. Rev. D **51** (1995) 3381, arXiv:9410266 [hep-ph].

[9] J. O. U. Baur, T. Han, *QCD corrections and anomalous couplings in Z production at hadron colliders*, Phys. Rev. D **57** (1998) 2823, arXiv:9710416 [hep-ph].

[10] R. D. C. Gary J. Feldman, *A Unified Approach to the Classical Statistical Analysis of Small Signals*, JHEP Phys.Rev **D57** (1998) 3873, arXiv:9711021 [hep-ex].

[11] T. Junk, *Confidence level computation for combining searches with small statistics*, Nucl. Instrum. Meth. **A434** (1999) 435, arXiv:9902006 [hep-ex].

[12] A. Read, *Presentation of search results: the CLs technique*, J. Phys. **G28** (2002) 2693.

[13] A. collaboration, *Procedure for the LHC Higgs boson search combination in summer 2011*, CERN, Geneva Switzerland (2011).