Lecture 2

This lecture: Setting up the tools for making black hole microstates

Starting construction of microstates

Next lecture: Finishing microstate construction

Dynamical issues

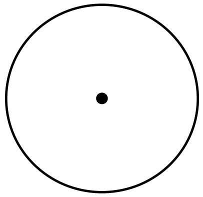
Resolving the information paradox

Last lecture: Application to Cosmology etc.

1. Puzzles with black holes

Puzzles with black holes:

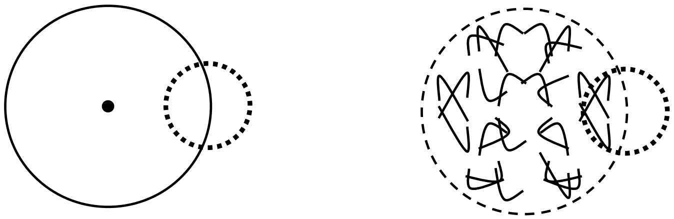
(a) The entropy puzzle: *Does the `Area entropy' correspond to a 'count of states' for the black hole ?*



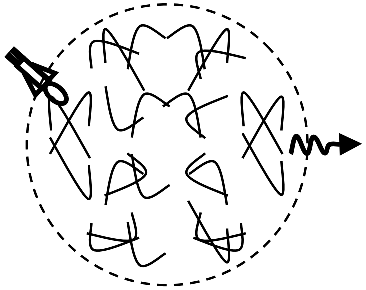
A/4G

(b) The information paradox: *How can the Hawking radiation quanta carry the information in the hole ?*

i.e. Can general relativity and quantum mechanics co-exist ?

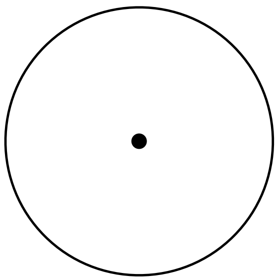


(c) The infall problem: *What does an infalling observer feel ?*



The entropy problem

Black holes behave as if they have an entropy given by their surface area



$$S_{bek} = \frac{A}{4G}$$

(Bekenstein, 72)

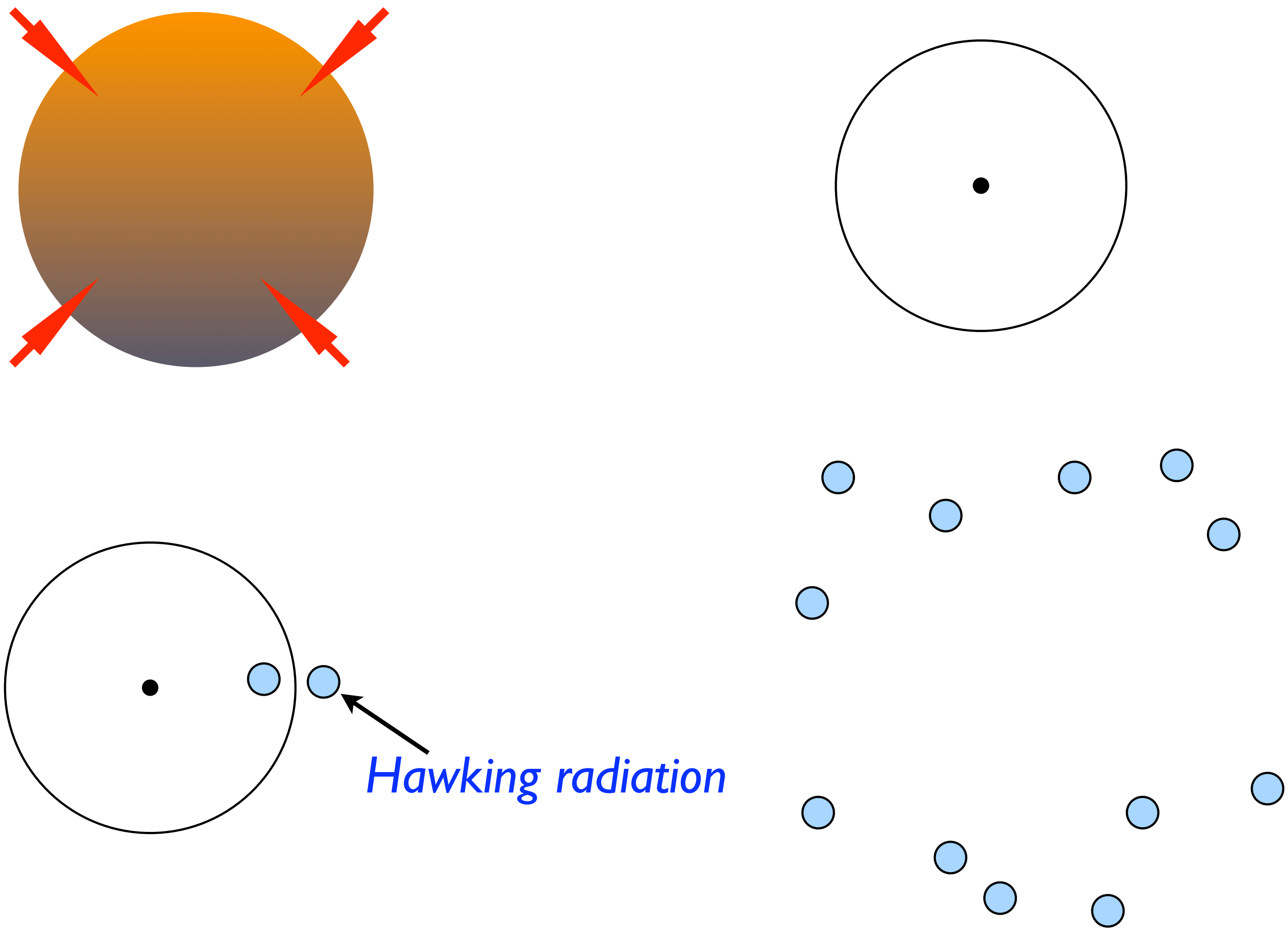
But statistical mechanics then says that there should be

$e^{S_{bek}}$ states of the hole for the same mass and charge

Can we show that there are $e^{S_{bek}}$ states of the hole ?

(Classical relativity finds that black holes have no hair, so there is only *one* state)

The information problem



II. Making black holes in string theory

How do you make black holes in string theory ?

To make a black hole, we need to put a large mass in a small region

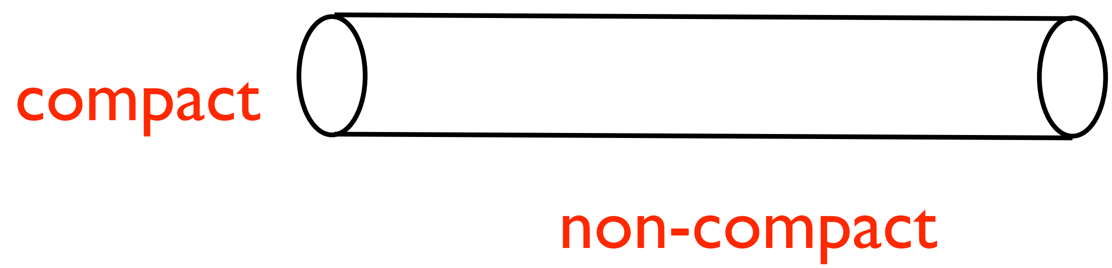
How should we do this ?

String theory is ‘complete’, so we should use only the objects present in the theory ....

Some facts from string theory :

(a) Strings live in 9+1 dimensions

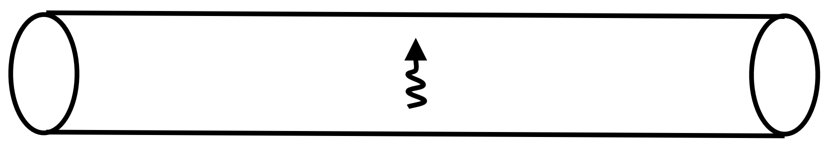
We see only 3+1 dimensions, so the others must be small compact directions



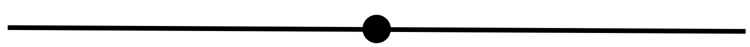
(b) There are many kinds of elementary excitations in the theory, for example gravitons, strings, branes ...

We must make our black holes using these objects ...

Consider a graviton running along the compact direction



To a person who cannot resolve the compact circle,
this looks like a point mass in the noncompact directions



This point mass also carries a 'winding charge',
from the usual idea of Kaluza-Klein reduction

$$g_{y\mu} = A_\mu$$

compact (→ y) non-compact (→ μ)

(c) Such objects are 'BPS objects', i.e., they have 'mass = charge'

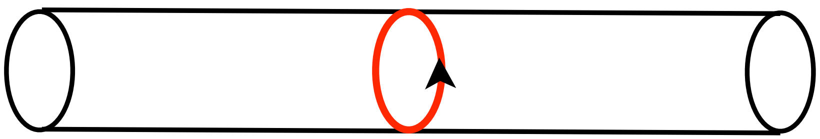
Newtonian mechanics

Gravitational attraction: $-\frac{GM^2}{r^2}$

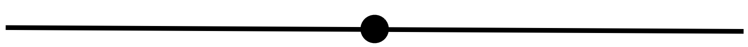
Electromagnetic repulsion: $\frac{Q^2}{r^2}$

These forces exactly cancel for BPS objects

We can also wrap a string around the compact directions



Again, to a person who cannot resolve the compact circle, this looks like a point mass in the noncompact directions



$$B_{y\mu} = \tilde{A}_\mu$$

The string radiates a 2-form gauge field, which looks like a usual gauge field in the non-compact directions

So this string is also a charged object ... it has ‘mass=charge’ (BPS)

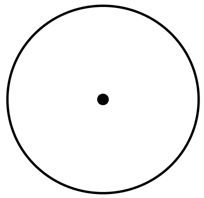
Let us try to make black holes by using such objects.

If we use only one kind of object, we will call it 'one charge'

If we use two kinds of objects, we will call it 'two charge', etc ...

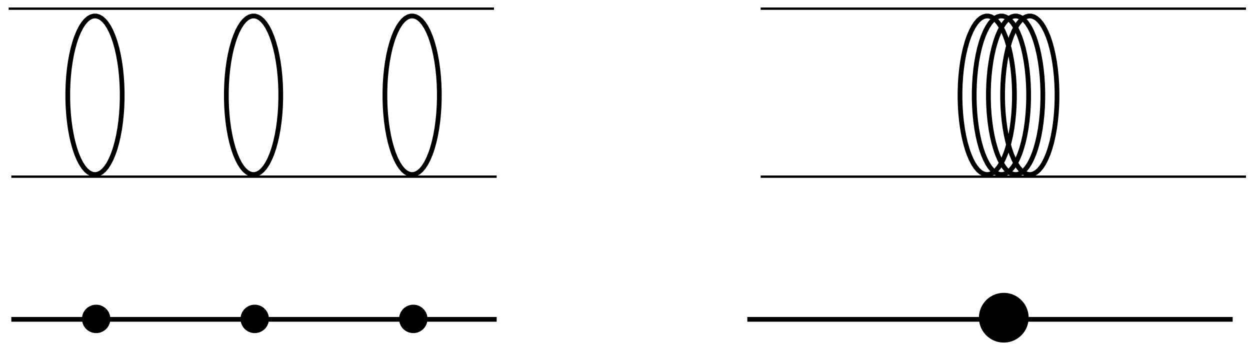
Goal: (a) We can count the number of states the 'brane complex' will have

(b) The branes carry some charges. We can ask how much entropy a classical black hole with those charges will have



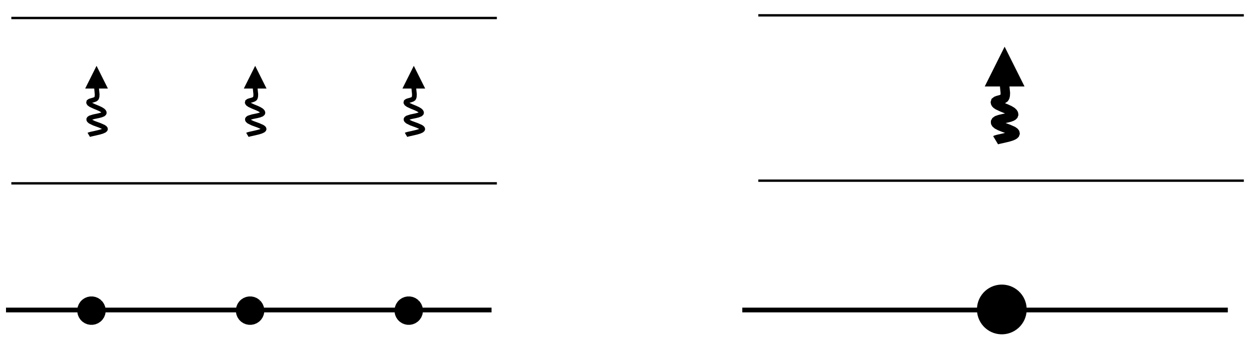
If these quantities agree, then we have obtained a microscopic understanding of black hole entropy

1-charge



Winding charge n_1

Mass = Charge



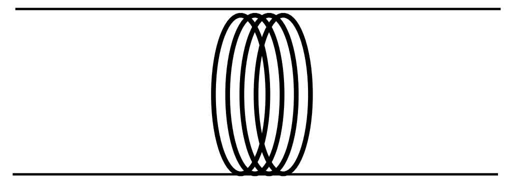
Momentum charge n_p

Mass = Charge

A black hole with winding charge only

$$S_{micro} = \ln[256] \sim 0$$

(Does not grow with n_1)

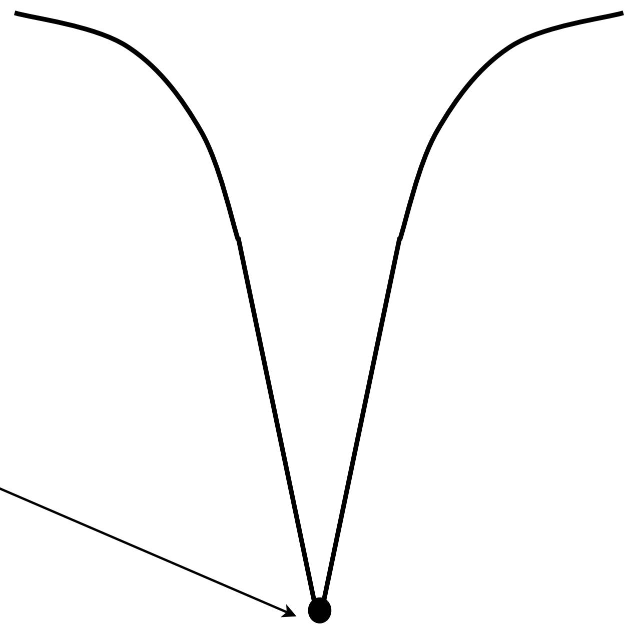


n_1

Horizon is singular

$$A \;=\; 0$$

Bekenstein entropy vanishes

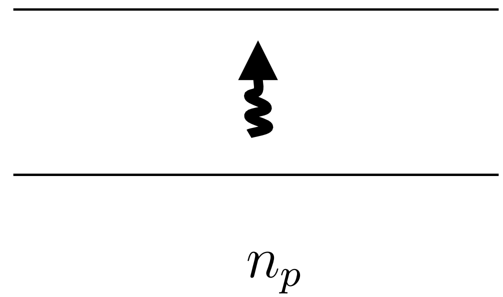


$$S_{micro} = S_{bek} = 0$$

A black hole with momentum charge only

$$S_{micro} = \ln[256] \sim 0$$

(Does not grow with n_p)



Horizon is singular

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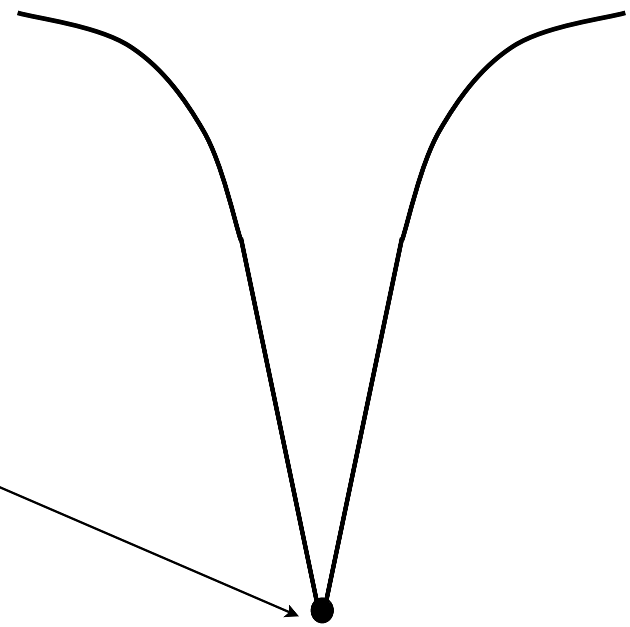
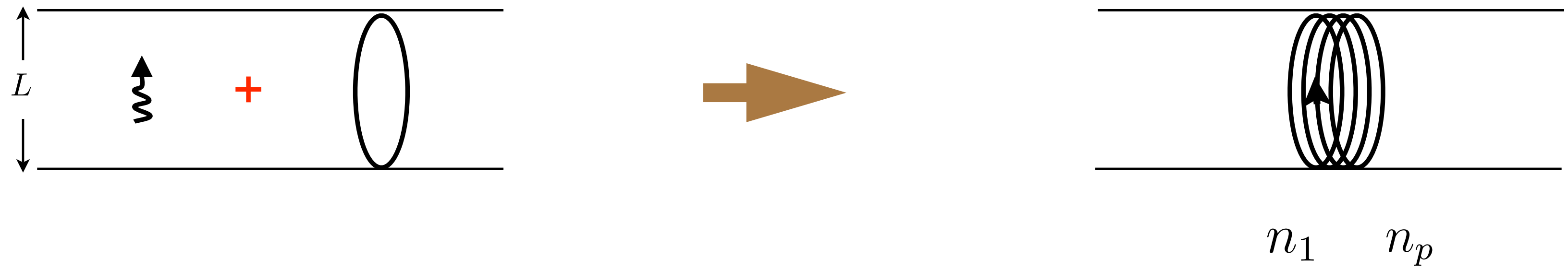


$$S_{micro} = S_{bek} = 0$$

To get something more interesting, let us take TWO kinds of charges

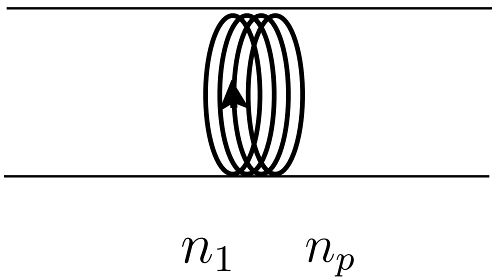


(a) All the strings bind together to make one 'multiwound string'

(b) The momentum binds to this string by becoming traveling waves on the string

We will call this 2-charge system NS1-P
(the elementary string is NS1, the momentum is P)

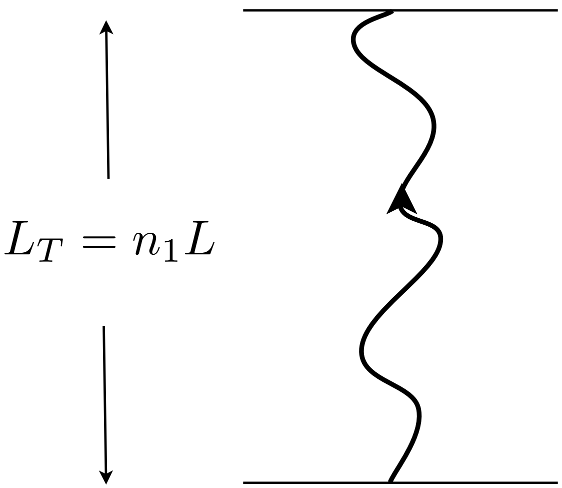
Imagine the string opened up to its full length



We can put all the momentum in the lowest harmonic,

or some in the first harmonic and some in the second ... and so on

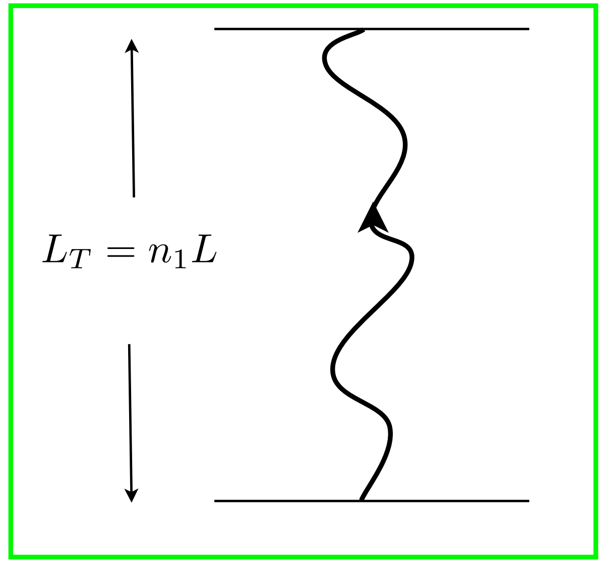
So there are many different states for the same total winding and momentum charges



The count of different states will give the entropy ....

Computing the entropy

The total length of the string is $L_T = n_1 L$



So each quantum of harmonic k carries momentum $\frac{2\pi k}{L_T}$

The total momentum on the string is $P = \frac{2\pi n_p}{L} = \frac{2\pi (n_1 n_p)}{L_T}$

So we have to count all ways of getting a total excitation level $n_1 n_p$

If we have m_k units of the harmonic k, then we need to count all possibilities

$$\sum_k k \; m_k \;=\; n_1 n_p$$

The count of possibilities is $\mathcal{N} \approx e^{2\pi\sqrt{\frac{n_1 n_p}{6}}}$

If the momentum was partitioned among c degrees of freedom, then

$$\mathcal{N} \approx e^{2\pi\sqrt{\frac{c n_1 n_p}{6}}}$$

Thus the count of states gives an entropy $S_{micro} = 2\pi\sqrt{\frac{c n_1 n_p}{6}}$

The string has 8 transverse vibration modes, and susy gives 8 fermions, so

$$c = 8 + \frac{8}{2} = 12$$

Thus we get

$$S_{micro} = 2\sqrt{2}\pi\sqrt{n_1 n_p}$$

(Susskind '93, Sen '94)

Let us put this computation in the context of the black holes that we will consider ....

We will use type IIB string theory

We will compactify spacetime as

$$M_{9,1} \;\rightarrow\; M_{4,1} \times T^4 \times S^1$$

The NS1 is wrapped on S^1 and the P is also along S^1

We can now perform S,T dualities and change to other descriptions of the 2-charge system ...

In particular we can map NS1-P to D1-D5

$$M_{9,1} \rightarrow M_{4,1} \times T^4 \times S^1$$

P

NS1

$$M_{9,1} \rightarrow M_{4,1} \times T^4 \times S^1$$

D1

D5

We perform the following dualities

$$\begin{array}{rcl} & & NS1 - P\ (IIB) \\ S & \rightarrow & D1 - P\ (IIB) \\ T_{6789} & \rightarrow & D5 - P\ (IIB) \\ S & \rightarrow & NS5 - P\ (IIB) \\ T_5 & \rightarrow & NS5 - NS1\ (IIA) \\ T_6 & \rightarrow & NS5 - NS1\ (IIB) \\ S & \rightarrow & D5 - D1\ (IIB) \end{array}$$

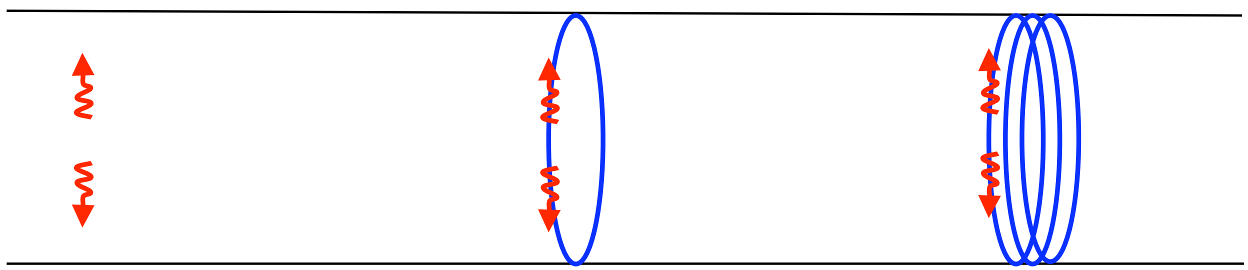
Note that all dualities are in the compact directions ....

What is the nature of the D1D5 bound state ?

Recall the nature of P bound to NS1 :

The total length of the string is $L_T = n_1 L$

So each quantum of harmonic k carries momentum $\dfrac{2\pi k}{L_T}$



$$\Delta E = \frac{4\pi}{L} \qquad \Delta E = \frac{4\pi}{L} \qquad \Delta E = \frac{4\pi}{n_1 L}$$

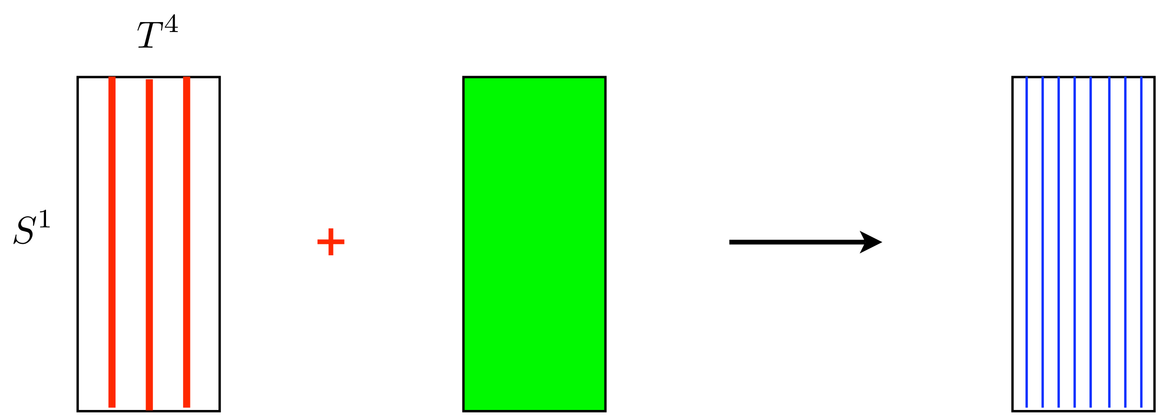
When a P mode is bound to n_1 NS1 branes, then the P modes come in 'fractional units' equal to $\dfrac{1}{n_1}$ of a full P mode

Dualities:

NS1 → D5

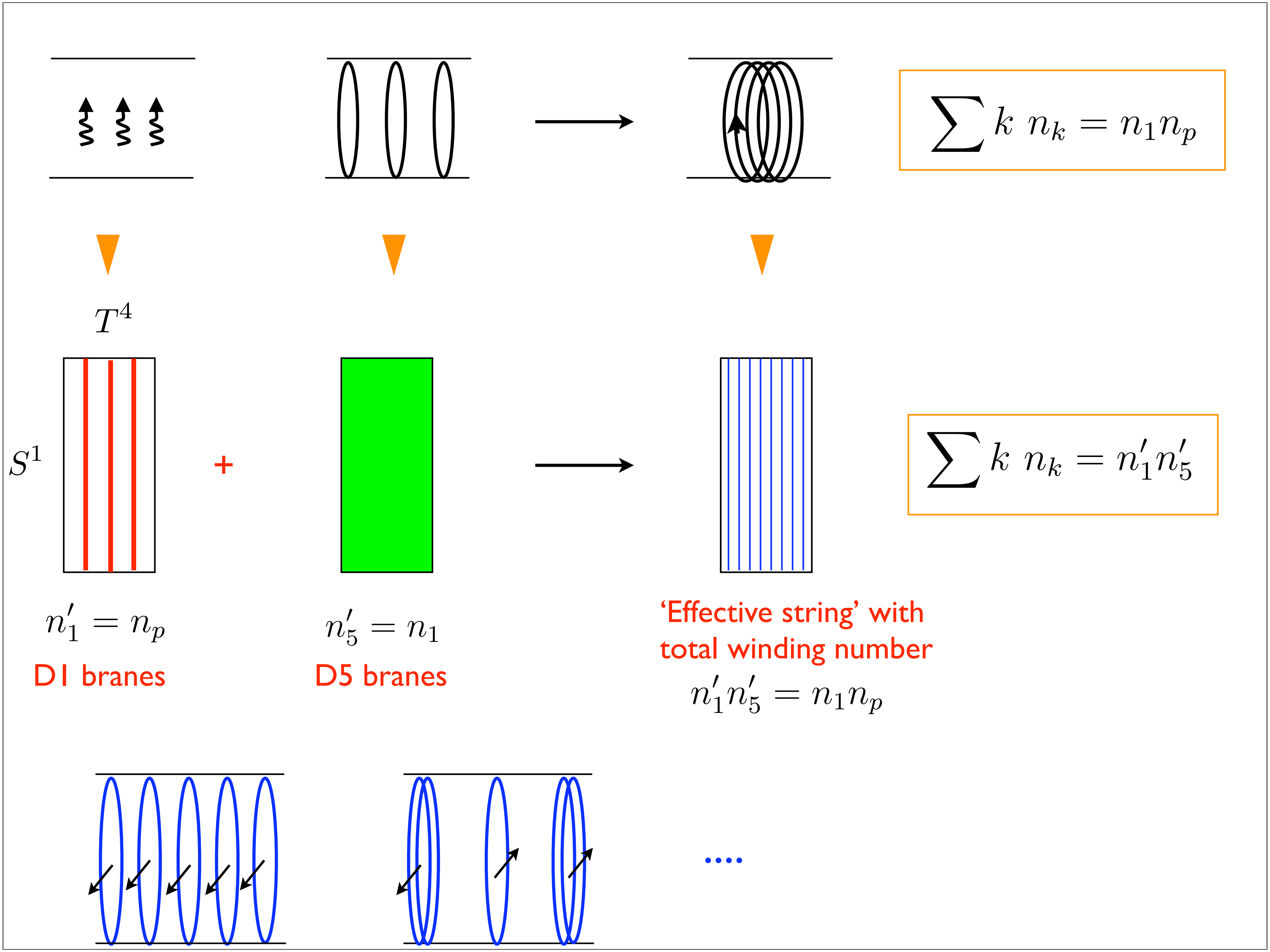
P → D1

When a D1 brane is bound to n_5 D5 branes, then the D1 brane come in 'fractional units' equal to $\frac{1}{n_5}$ of a full D1 brane



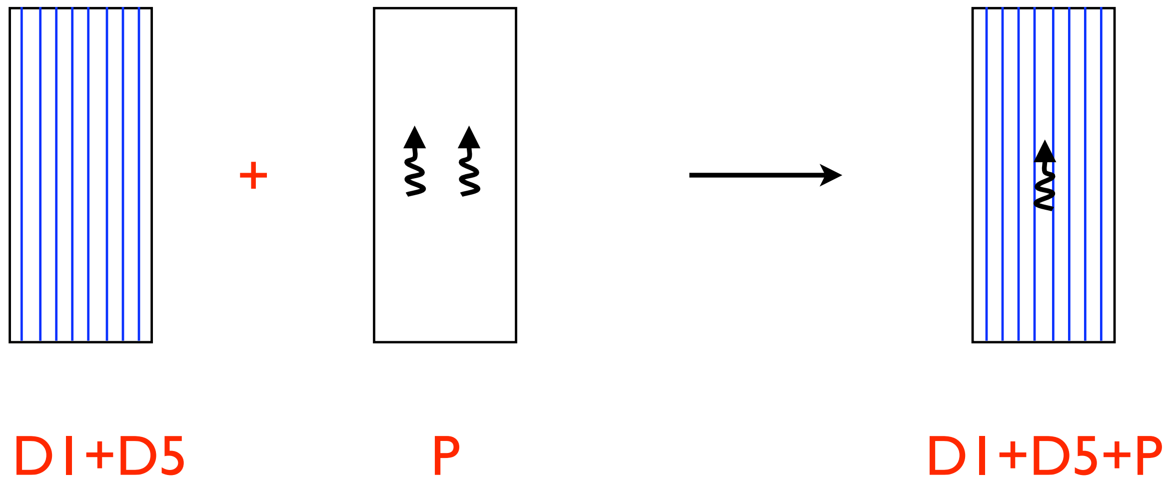
n_1 n_5

'Effective string' with total winding number $n_1 n_5$



$\sum k \; n_k = n_1 n_p$
T^4
S^1
+
$\sum k \; n_k = n_1' n_5'$
$n_1' = n_p$
D1 branes
$n_5' = n_1$
D5 branes
'Effective string' with
total winding number
$n_1' n_5' = n_1 n_p$
....

The 3-charge system



Recall that for a string carrying momentum

$$S_{micro} = 2\pi\sqrt{\frac{c n_1 n_p}{6}}$$

The D1 branes vibrate 'inside' the D5 branes, so we have 4 bosonic modes and 4 fermionic modes

$$c = 4 + \frac{4}{2} = 6$$

This gives

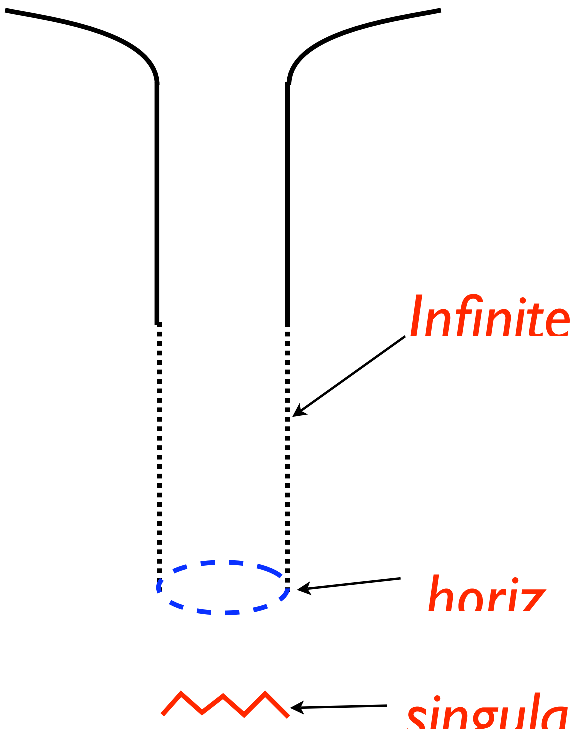
$$\boxed{S_{micro} = 2\pi\sqrt{n_1 n_5 n_p}}$$

(Strominger Vafa 96)

Recall that we had planned to check if this microscopic count of states agrees with the 'horizon entropy'

Go to IIB supergravity.

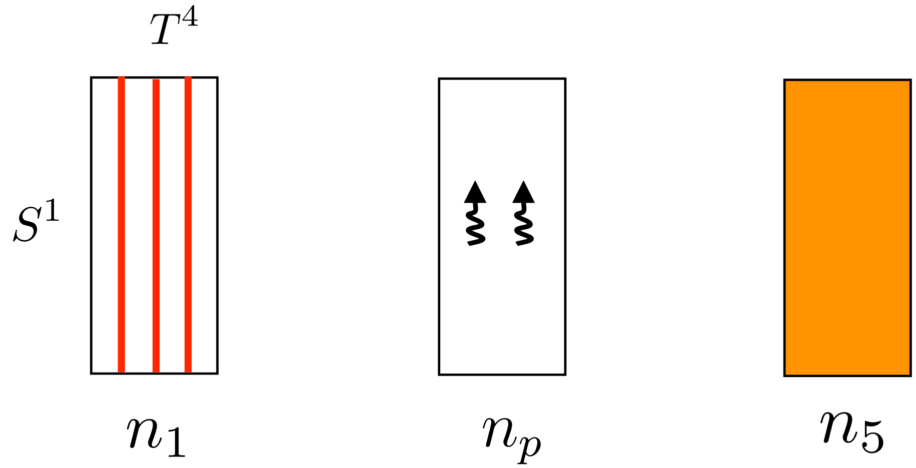
Make an extremal black hole with these charges D1-D5-P, and mass=charge



$$S_{bek} = \frac{A}{4G} = 2\pi\sqrt{n_1 n_5 n_p} = S_{micro}$$

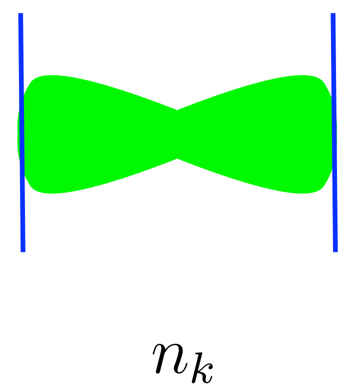
4-charge holes

$$M_{9,1} \to M_{4,1} \times T^4 \times S^1$$



D1, D5, P charges

$$M_{9,1} \to M_{3,1} \times T^4 \times S^1 \times \tilde{S}^1$$



D1,D5,P and KK monopoles with new circle as nontrivial fiber

Microscopic entropy formulae

2-charges $S = 2\sqrt{2}\pi\sqrt{n_1 n_2}$

3-charges $S = 2\pi\sqrt{n_1 n_2 n_3}$

4-charges $S = 2\pi\sqrt{n_1 n_2 n_3 n_4}$

2 charges
+ nonextremality $S = 2\pi\sqrt{n_1 n_2}(\sqrt{n_3} + \sqrt{\bar{n}_3})$

3-charges
+ nonextremality $S = 2\pi\sqrt{n_1 n_2 n_3}(\sqrt{n_4} + \sqrt{\bar{n}_4})$

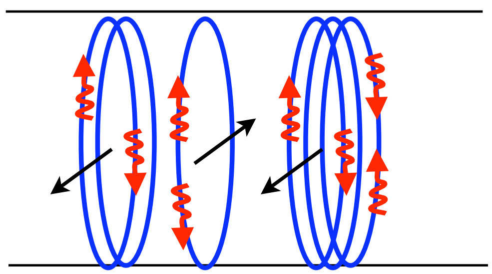
Charges can be permuted by dualities

Charges can be taken as D1, D5, P, KK

Which charge pairs are created can be found by maximizing entropy

Near - extremal holes

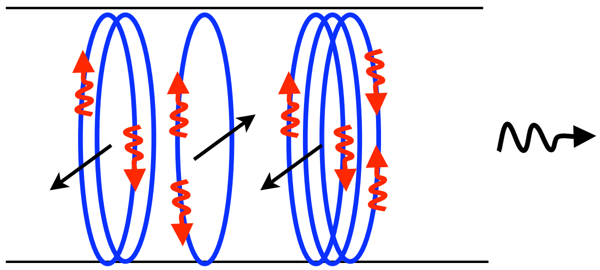
Put both left and right moving excitations

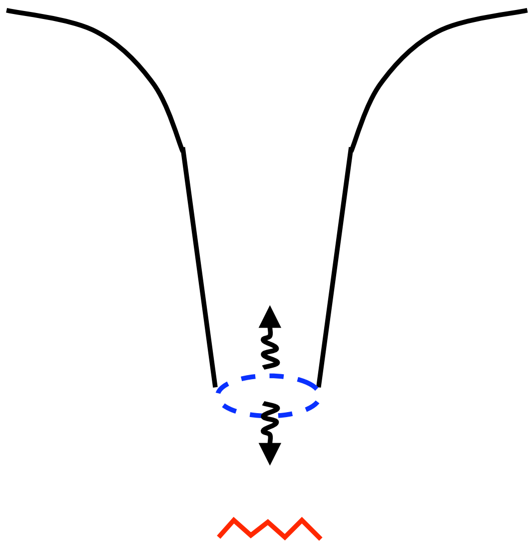


$$S_{micro} = 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p}) = S_{bek}$$

Left and right moving excitations can collide and exit the brane state ....

Hawking radiation





Radiation rates agree (Spins, greybody factors ...)

(Das-Mathur 96, Maldacena-Strominger 96)

But

Unitary radiation
process in CFT

Non-Unitary
radiation from gravity

Can we get UNITARY radiation (information carrying) in the GRAVITY description ??

Some general lessons about entropy

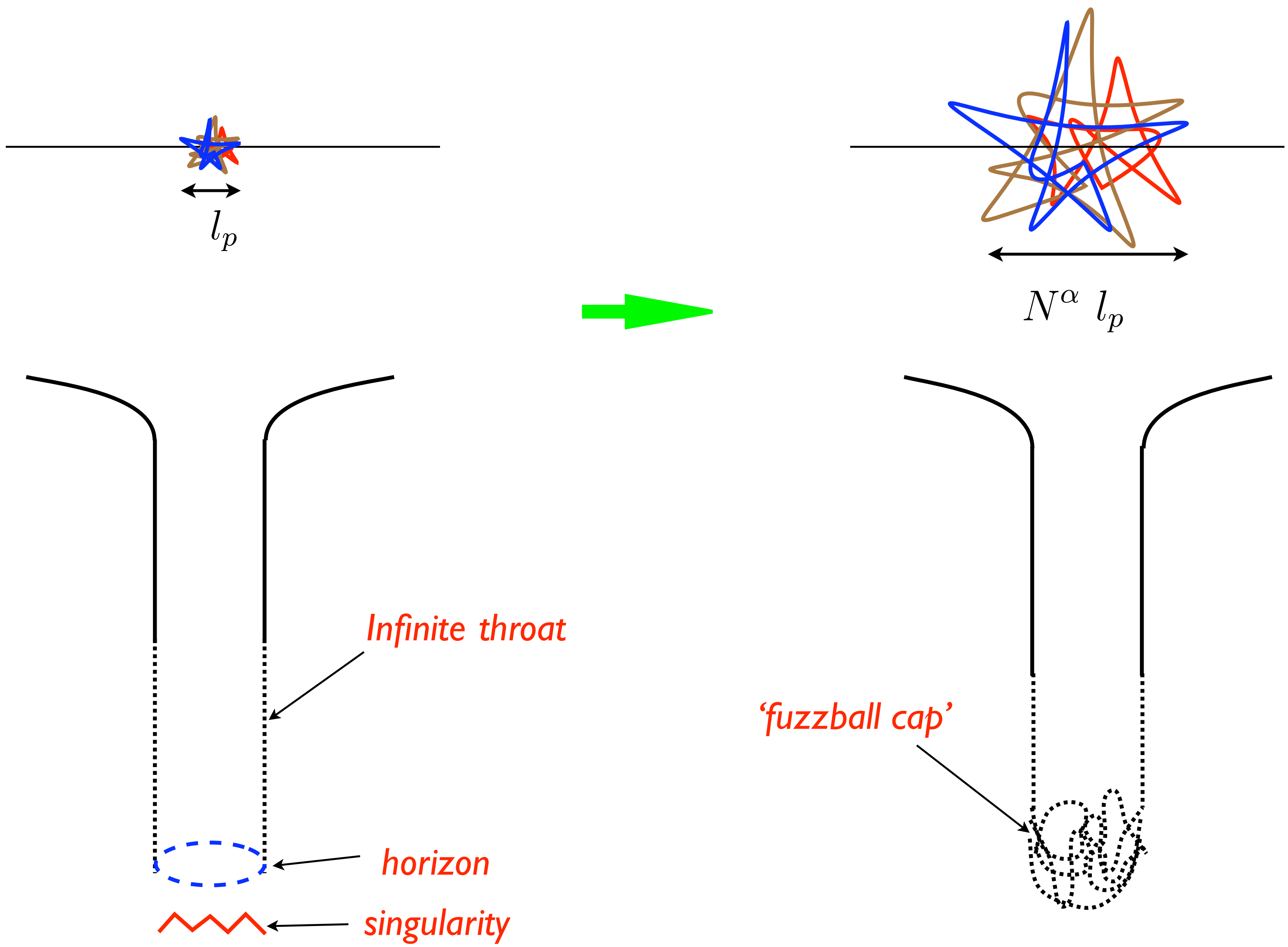
(a) To understand the entropy of black holes in 4+1 noncompact dimensions, we have to see how branes wrap around the OTHER 5 (compact) dimensions

(b) People used to think that string theory is very wasteful ... apart from the graviton, we get all these other excited vibration modes of the string ... but now we see that exactly these vibrations account for the entropy of black holes ... this is a big validation of string theory

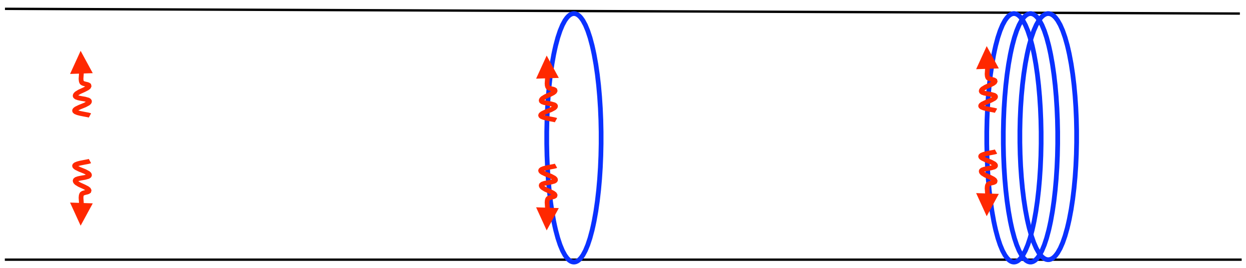
III. How big is a bound state of branes ?
A motivational argument

What we want to show ....

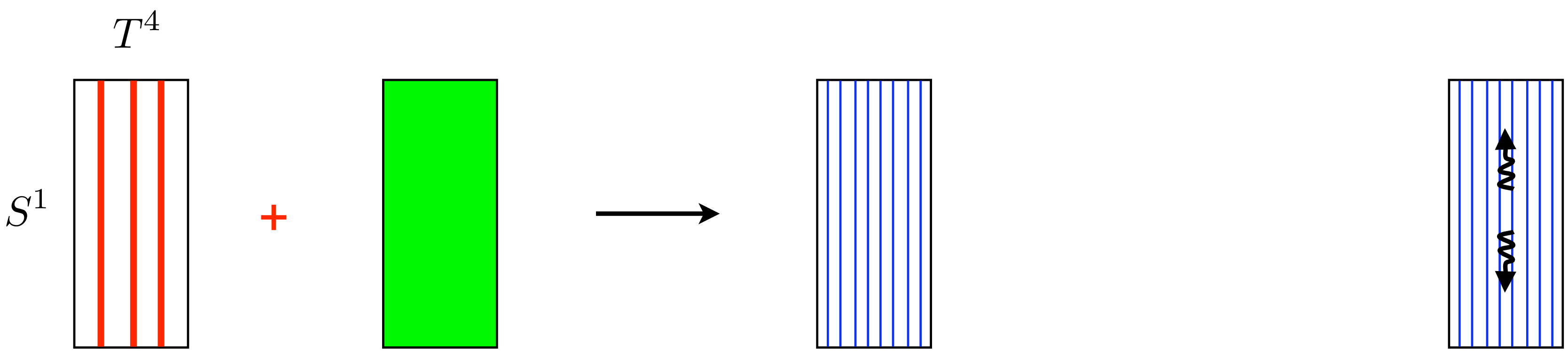
A supersymmetric brane state in string theory: mass=charge



Energy gap (with no net charge)



$$\Delta E = \frac{4\pi}{L} \qquad \Delta E = \frac{4\pi}{L} \qquad \Delta E = \frac{4\pi}{n_1 L}$$



n_1
D1 branes

n_5
D5 branes

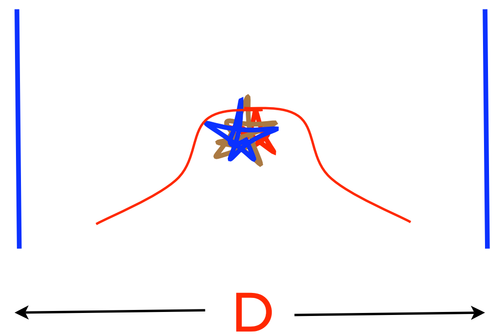
'Effective string' with total winding number $n_1 n_5$

$$\Delta E = \frac{4\pi}{n_1 n_5 L}$$

A motivational argument

Put a D1-D5-P extremal state in a box

$$M_{9,1} \to M_{4,1} \times T^4 \times S^1$$

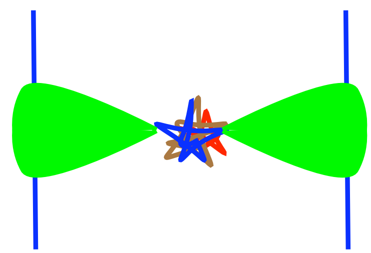


$$S = 2\pi\sqrt{n_1 n_5 n_p}$$

How small should the box be before the state 'feels' the walls of the box?

$$\Delta E \gtrsim \frac{1}{D}$$

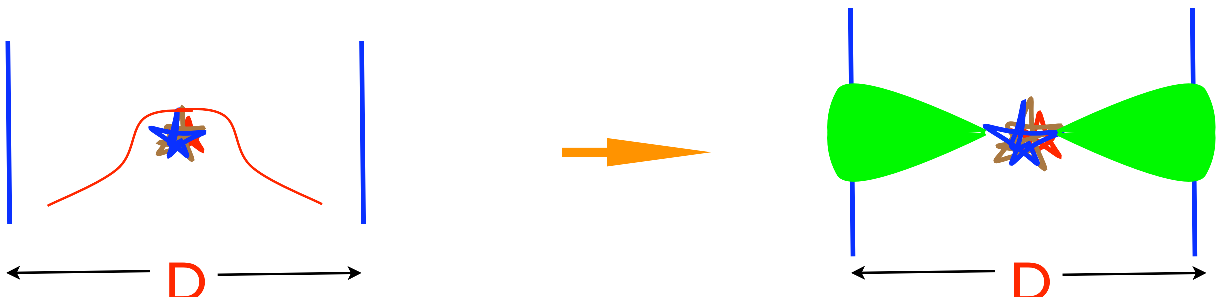
$$M_{9,1} \to M_{3,1} \times T^4 \times S^1 \times \tilde{S}^1$$



The extra energy can go to creating pairs of the fourth charge: KK monopole pairs

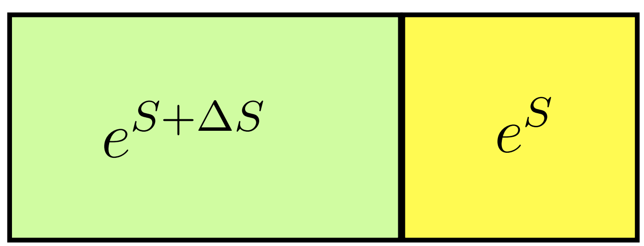
$$S = 2\pi\sqrt{n_1 n_5 n_p}(\sqrt{n_k} + \sqrt{\bar{n}_k})$$

Creating excitations must be *probable*, not just *possible* ....



How small should D be so that $\Delta S \gtrsim 1$?

(i.e. there is 2.7 times more phase space if we use the extra energy to create the pairs as compared to the situation where we do not)



$$S = 2\pi\sqrt{n_1 n_5 n_p(1-f)} + 2\pi\sqrt{n_1 n_5 n_p f}(\sqrt{n_k} + \sqrt{\bar{n}_k})$$

$$n_k = \bar{n}_k = \frac{1}{2}\frac{\Delta E}{m_k} = \frac{1}{2Dm_k}$$

$$m_k \sim \frac{G_5}{G_4^2} \sim \frac{D^2}{G_5}$$

Extremize over f, set

$$\Delta S = S - 2\pi\sqrt{n_1 n_5 n_p} \;=\; 1$$

$$D \sim [\frac{\sqrt{n_1 n_5 n_p} g^2 \alpha'^4}{VR}]^{\frac{1}{3}} \sim R_S$$

V : volume of T^4

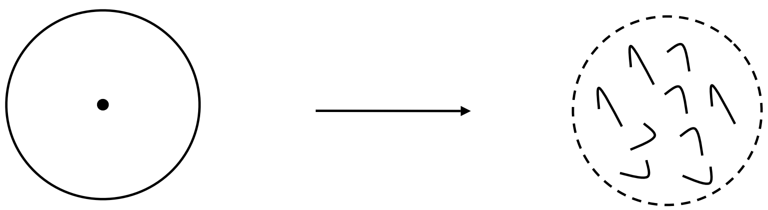
R : radius of S^1

g : string coupling

R_s : Schwarzschild radius of 3-charge extremal black hole

(SDM 97)

Suggests a picture ....



Why does this work?

When we bind together charges n_1, n_2, n_3 then
the excitations of the charge n_4 comes in fractional units $1/n_1 n_2 n_3$

Creates low tension 'effective' objects, which can stretch far ....

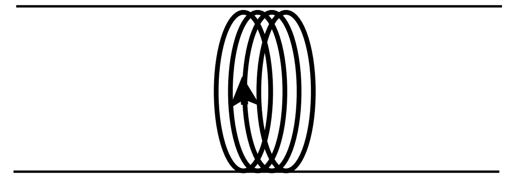
(Das+SDM 96, Maldacena+Susskind 96, SDM 97, Chowdhury+Giusto+SDM 06)

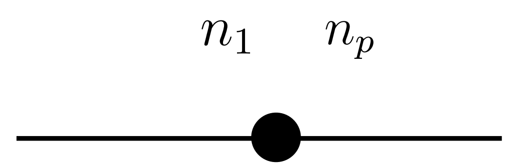
This was only a crude motivational argument ...

But now we should go and construct black hole microstates at strong coupling
(where they are really expected to be black holes)

The simplest hole is the 2-charge hole ...

But we need to include R^2 terms
in the gravitation action

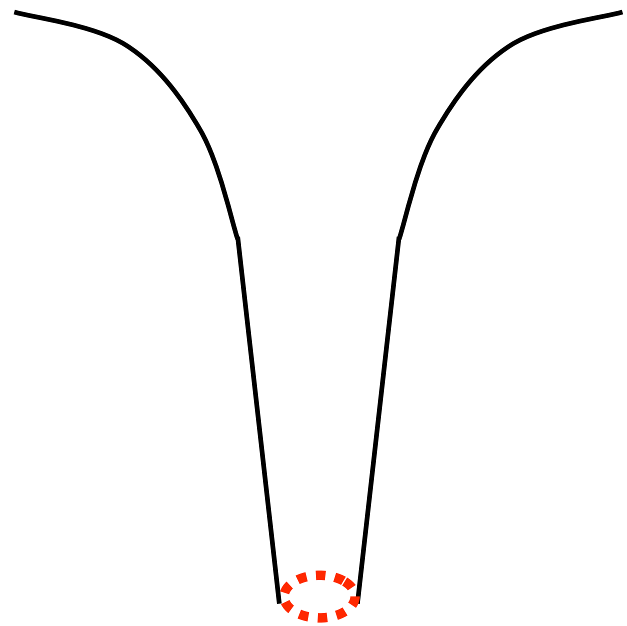




For $K3 \times S^1$ compactification,
geometry gives a Bekenstein - Wald
entropy

$$S_{bek} = \frac{A}{2G} = 4\pi\sqrt{n_1 n_p} = S_{micro}$$

(Cardoso, DeWit, Mohaupt '02, Dabholkar '04)



So the 2-charge hole ('Sen-Vafa hole')
gives a simple story for
black hole entropy in string theory ......