Holographic methods for condensed matter physics

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Holographic superconductors

1. What is superconductivity?
2. BCS theory
3. A holographic superconductor
4. Energy gap and Meissner effect
5. Landscape of superconducting membranes

Superconductivity, symmetry breaking and charge density

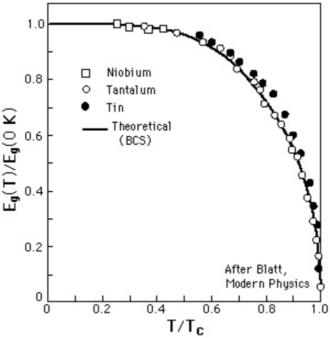
- Superconductivity $\sim$ spontaneous symmetry breaking.
- Breaking a $U(1)$ spontaneously: orthogonal a charge density.
- Should be 'obvious':
  - Charge density breaks Lorentz invariance, a condensate does not.
  - The charge operator $\rho = J^t$ is neutral, so does not break symmetry.
- Suppose we have a Lagrangian with $U(1)$ symmetry. Change variables so that the symmetry only acts by shifting a phase: $\theta \to \theta + \delta\theta$.
- By definition: kinetic term for the phase is: $J^t \partial_t \theta$.
- The charge density $\rho = J^t$ and the phase θ are conjugate:

$$[\theta, \rho] = i\hbar \,.$$

- Therefore a state with definite phase, breaking the symmetry, is maximally different from a state with a definite charge density.

BCS theory (1957)

- Superconductors are charged superfluids, condensation of Cooper pairs of electrons: $\mathcal{O} \sim \psi^\dagger \psi^\dagger$.
- Repulsive Coulomb force between electrons is screened. Attractive force due to the lattice phonons. Instability of Fermi surface.
- Many predictions, for instance $E_g(0) \approx 3.5 T_c$.
- Data:



Minimal ingredients for a holographic superconductor

- Minimal ingredients
  - Continuum theory $\Rightarrow$ have $T^{\mu\nu}$ $\Rightarrow$ need bulk g_{ab}.
  - Conserved charge $\Rightarrow$ have J^{μ} $\Rightarrow$ need bulk A_a.
  - 'Cooper pair' operator $\Rightarrow$ have $\mathcal{O}$ $\Rightarrow$ need bulk ϕ.

- Write a minimal 'phenomenological' bulk Lagrangian

$$\mathcal{L} = \frac{M^2}{2} R + \frac{3M^2}{L^2} - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - \left|\nabla\phi - iqA\phi\right|^2 - m^2 \left|\phi\right|^2 \,.$$

There are four dimensionless quantities in this action.

- The central charge of the CFT is $c = 192(ML)^2$.
- DC conductivity $\sigma_{xx} = \frac{1}{g^2}$.
- $\Delta(\Delta - 3) = (mL)^2$. Either root admissible if $\Delta \geq \frac{1}{2}$.
- The charge q is the charge of the dual operator $\mathcal{O}$.

Two instabilities of a charged AdS black hole

- To get a critical temperature, need a scale. Will work at constant charge density ρ. By dimensional analysis $T_c \propto \sqrt{\rho}$.
- The dual geometry is therefore Reissner-Nordstrom-AdS.
- RN-AdS can be unstable against a (charged) scalar for two reasons.
- Reason 1 (Gubser): Background charge shifts mass:

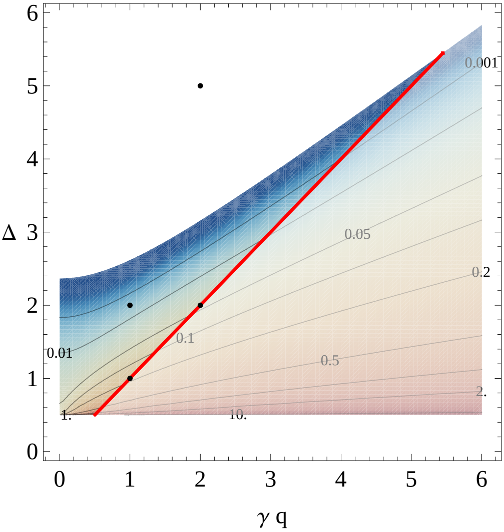
$$m^2_{\text{eff.}} \sim m^2 - q^2 A_t^2 \,.$$

- Reason 2 (SAH-Herzog-Horowitz): Near extremality AdS_2 throat with

$$m^2_{\text{BF-2}} = -\frac{1}{4L_2^2} = -\frac{3}{2L^2} > -\frac{9}{4L^2} = m^2_{BF-4} \,.$$

- Precise criterion for instability at $T = 0$ (Denef-SAH)

$$q^2\gamma^2 \geq 3 + 2\Delta(\Delta - 3) \,.$$


0.001
0.05
0.2
0.1
0.01
0.5
2.
1.
10.
Δ
γ q
0 1 2 3 4 5 6

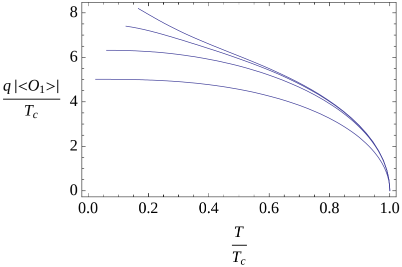
Endpoint – hairy black holes

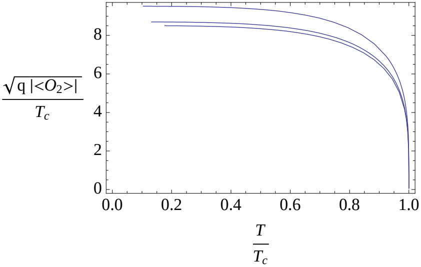
- Endpoint of instability is a hairy black hole:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + \frac{L^2}{r^2}\left(dx^2 + dy^2\right) ,$$

$$A = A_t(r)dt , \qquad \phi = \phi(r) .$$

- Solve numerically (take $m^2 = -2/L^2$). Can obtain $\langle \mathcal{O} \rangle$:





- Compare 8 to ~ 3.5 for BCS and $\sim 5 - 8$ for High-T_C.

Electrical conductivity - some experimental expectations

- BCS theory prediction and some data

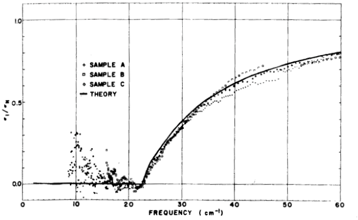
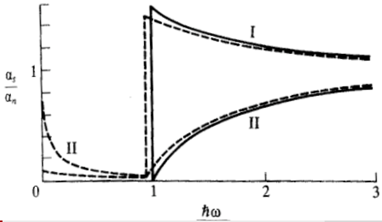


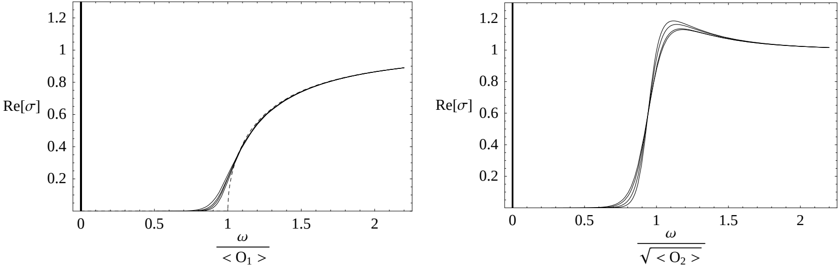
Fig. 2. Measured and calculated values of the conductivity ratio σ_1/σ_n of three superconducting lead films as a function of the photon frequency. [After Palmer (39).]

- A figure from a texbook (Tinkham)



Electrical conductivity - AdS/CFT

- Let's focus on the probe limit ($q \to \infty$) for simplicity.
- We computed the conductivity (2 cases). At $T \sim 0$:



- If the gap is 2Δ then we found that

$$\mathrm{Re}\,\sigma(\omega \to 0) \sim e^{-\alpha\Delta/T}.$$

- $\alpha = 1$ as $q \to \infty$, as in BCS theory, weakly coupled?
- Puzzle: why is there an exact gap?

Magnetically induced currents – Meissner effect

- Essence of Meissner effect is the London equation.
- In the bulk near the boundary

$$A_i = A_i^{(0)} + \frac{r}{L} A_i^{(1)} + \cdots .$$

- In AdS/CFT: $A_i^{(0)}$ will give the boundary background gauge potential (hence magnetic field) and $A_i^{(1)}$ gives the current $\langle J^i \rangle$.
- At low temperature showed that

$$\langle J_i \rangle = -q \langle \mathcal{O}_1 \rangle A_i .$$

This is the London equation and determines the magnetic penetration depth when the theory is coupled to an external photon.

Landscape of superconducting membranes

- There appear to be many vacua of string theory.
- Potentially implies philosophical problems for stringy cosmology or stringy particle physics (only do experiments in one universe).
- Resonates well with atomic physics, which also has a landscape

	Atomic Landscape	String Landscape
Microscopic theory	Standard Model	M theory
Fundamental excitations	Leptons, quarks, photons, etc.	??
Typical vacuum	Atomic lattice	Compactification
Low energy excitations	Dressed electrons, phonons, spinons, triplons, etc.	Gravitons, gauge bosons, moduli, intersectons, etc.
Low energy theory	Various QFTs	Various supergravities

- Logic: Let's look at statistical properties of string AdS_4 vacua from the dual perspective as quantum critical points.

- Studied the stability of $AdS_4 \times X_7$ vacua for Sasaki-Einstein X_7.
- If X_7 has moduli: can build a 3-form mode that linearly decouples from all other modes, even in a background electric field.
- Calabi-Yau cone over X_7 has an anti-self dual closed (3,1)-form. Contracting with $r\partial_r$ get 3-form on Sasaki-Einstein. Mode is

$$\delta C = \phi Y_3 + \text{c.c.}\,.$$

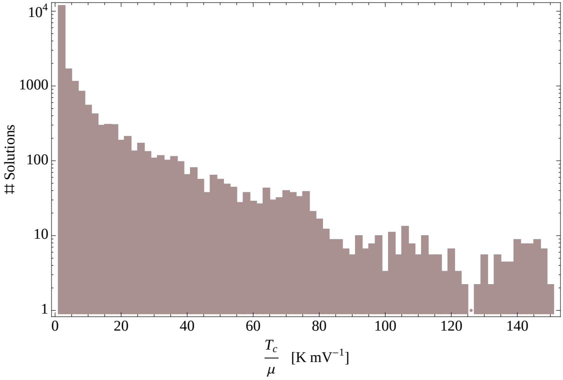
- This mode is always unstable and leads to superconductivity.
- Examples supplied by Brieskorn-Pham cones:

$$z_1^{m_1} + \cdots + z_5^{m_5} = 0\,.$$

- Computed a distribution of superconducting temperatures by scanning.

Distribution of critical temperatures

- Simple scan found 11,821 backgrounds.



- $X_7 = S^7$ has a similar instability
  $\rightarrow$ ABJM theory has a superconducting phase.

Summary

- Superconductivity is due to spontaneous symmetry breaking.
- Established theories of superconductivity depend on having ‘glue’ and ‘electrons’ which are described by quasiparticles.
- Spontaneous symmetry breaking can occur naturally in AdS/CFT.
- Explored basic phenomenology of holographic superconductors.
- Noted a certain ‘resonance’ between string and condensed matter landscapes.
- Much to do: d-wave superconductors needed!