F-theory work group: Part I

CERN, February 10, 2009

F-theory: Definition

Setting: IIB SUGRA on a Kähler threefold B_3, i.e. $\dim_{\mathbb{R}}(B_3) = 6$.
B_3 is *not* a global Calabi-Yau, even in perturbative models.
Field content:

$$\tau = C_0 + i\, e^{-\phi}\,, \quad H_3 + \tau\, F_3\,, \quad F_2 \in H^2(D7)\,.$$

BPS equations $\Rightarrow$ τ must be holomorphic in coordinates of B_3.
Since τ is only well-defined up to $SL(2,\mathbb{Z})$ transformation

$$\tau \to \frac{a\,\tau + b}{c\,\tau + d}\,,$$

$\rightsquigarrow$ τ defines an *elliptic fibration* over B_3.

Requiring $\mathcal{N} = 1$ in $d = 4$ $\Rightarrow$ resulting fourfold X_4 must be Calabi-Yau.

Fibre degeneration

Weierstrass model of elliptic fibration:

$$y^2 = x^3 + f(B_3)\,x + g(B_3)\,, \quad x, y, z \in \mathbb{C}^3$$

will degenerate at complex codimension one locus S:

$$\Delta = 4\,f^3 + 27\,g^2 = 0\,.$$

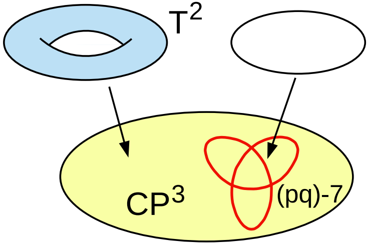
Fibre degeneration $\Leftrightarrow$ presence of a 7-brane.
Simplest degeneration:

$$\tau \to \tau + 1\,,$$

signals presence of a perturbative D7-brane.

More general monodromy $\Rightarrow$ presence of a (p, q)-brane.

Degeneration



The fiber degenerates ons surface S, where the discriminant vanishes:

$$\Delta \equiv 4\, f^3 + 27\, g^2 = 0 \quad \Rightarrow \quad (p,q) - \text{brane lurking}$$

'Mild' singularities: Fibre singular, fourfold still smooth.
'Enhanced' singularities: Fibre singular, fourfold singular.

Local constructions

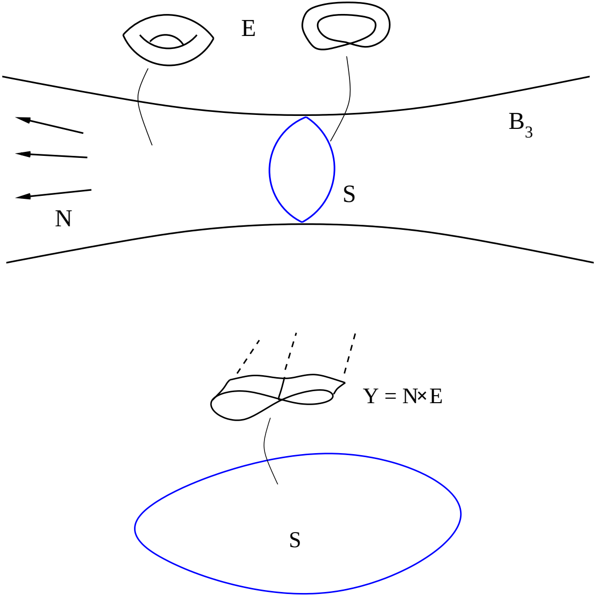
Instead of building full X_4, build the 'zoomed in' version around 7-brane: X_4 is non-compact, but S is compact.

Combine normal bundle of S with elliptic fibration over B_3 to write X_4 as fibration over S.
Can model singularity locally as non-compact K3-surface Y_2 in $\mathbb{C}^3$ fibered over S.

$$\begin{array}{ccc} Y_2 & \longrightarrow & X_4 \\ & & \big\downarrow \pi \\ & & S \end{array},$$

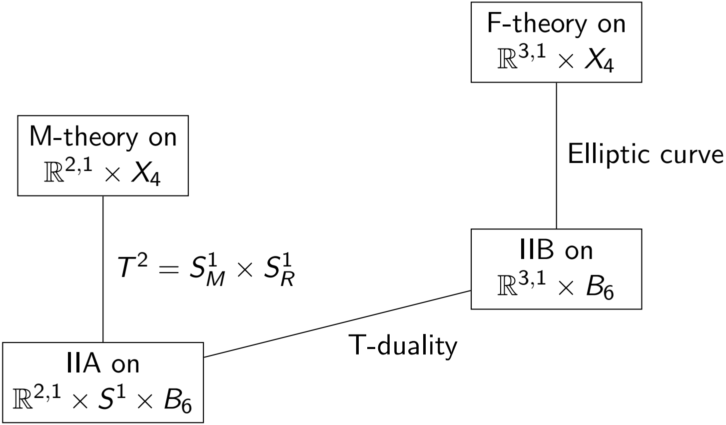
$\rightsquigarrow$ ADE classification easy to use.

Local models



Relation to M-theory

F-theory is dual to M-theory via fiberwise T-duality:



Singularity enhancement

$(x, y, z) \in \mathbb{C}^3$, with z be normal coordinate to S. Define Y as

$$y^2 + x^2 = \prod_i^{n+1} (z + t_i)\,, \quad \text{with} \quad \epsilon \in \mathbb{R}\,.$$

n two-cycles touching at roots of polynomial, in shape of A_n Dynkin diagram.
Send $t_i \to 0 \Rightarrow$ two-cycles vanish $\rightsquigarrow A_n$ singularity.

By M/F duality, vanishing two-cycles $\Leftrightarrow$ massless M2-branes
$\Rightarrow$ enhanced A_n gauge group.

Groups in A and D series are realized in perturbative IIB.
Groups in E series only possible in F-theory.
Non-trivial monodromies along S itself $\rightsquigarrow$ non-simply laced groups:
E. g. $SO(2\,n+1), Sp(2\,n)$.

Twisted gauge theory on 7-brane and 4d EFT

Gauge theory on flat 7-brane: Reduction of $d = 10$ maximally SYM down to maximally $d = 8$ SYM.

In this case, worldvolume is $\mathbb{R}^{3,1} \times S$, where S has non-trivial spin bundle $K_P^{1/2}$. For S a Del Pezzo, it is highly non-trivial.
$\Rightarrow$ theory must be 'twisted' in order to preserve $\mathcal{N} = 1$ in $d = 4$.

'Twisted' Content:

- 'twisted fermions' become even-degree (anti-)holomorphic forms on S.
- Two former scalars parametrizing transverse coordinates $\rightsquigarrow$ $(0, 2)$ and $(2, 0)$-forms.

More specifically, given a gauge bundle P on S, we have

$$\phi \text{ section of } K_S \times \mathrm{ad}(P)\,, \quad \bar{\phi} \text{ section of } \overline{K_S} \times \mathrm{ad}(P)\,.$$

Twisted gauge theory on 7-brane and 4d EFT

ϕ gives deformations of discriminant locus:

$$y^2 + x^2 = \prod_{i=1}^{n+1}(z + t_i) = z^{n+1} + \sum s_k(t_1, \ldots, t_{n+1})\, z^{n+1-k}\,,$$

where $s_k(t) = s_k(t)$(coordinates on S).
$\Rightarrow s_k \leftrightarrow$ Casimir's of ϕ.

$\Rightarrow$ compatible with ϕ section of $K_S \times \mathrm{ad}(P)$.

4d EFT

Given a gauge group G on 8-d theory, can turn on non-trivial flux F to break it

$$G \to \Gamma \times H\,.$$

adjoint representation of G decomposes
$\rightsquigarrow$ 4-d chiral fields classified by bundle-valued cohomologies.

However, vanishing theorems on dP's kill a subsector of these s.t.
$\Rightarrow$ Yukawa couplings all vanish

$$\sim \int_S A \wedge A \wedge \phi = 0\,.$$

Solution: Intersecting 7-branes.

Intersecting branes: Colliding singularities

Consider the deformed A_{m+n} singularity:

$$y^2 + x^2 = z^m (z - t)^{n+1} ,$$

with S at $z = 0$ and $t = t(\mathrm{S})$ s. t. $t = 0$ along a Riemann surface $\Sigma \subset S$.

Structure:

- A_{m-1} singularity along $z = 0$;
- A_n singularity along $z - t = 0$;
- *enhanced* A_{m+n} singularity along $\Sigma : \{z = 0\} \cap \{t = 0\}$.

Intersecting branes: Defect field theory on intersection

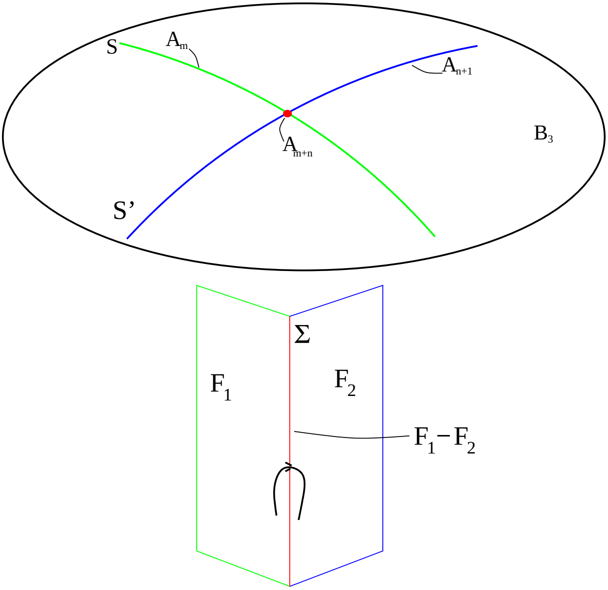
Theory along an intersection/defect with $G_S \times G_{S'} \subset G_\Sigma$ can be treated in two ways:

1. Constructing 6-d partially twisted SYM consistent:
   - $\mathcal{N} = 1$ in $d = 4$.
   - Decomposition of adjoint representation of G_Σ into adjoints and bifundamentals of G_S and $G_{S'}$.
   - SUSY, gauge invariant coupling to 'bulk' theory on S.
2. Treat defect as a cosmic string: i.e. treat defect as non-trivial profile for ϕ with $\phi = 0$ along Σ, and look for zero-modes trapped along defect.

So far, this creates vector-like pairs of bifundamental fermions in 4-d. Switching on non-trivial fluxes $F_S, F_{S'}$ breaks the symmetry. Net chiral number is given by

$$n = \int_\Sigma F_S - F_{S'}.$$

Intersecting branes: Chiral matter



Yukawa couplings

Possible mechanisms:

- Generated on a single 7-brane on S $\rightsquigarrow$ always vanish on Del Pezzo's.
- Intersections on two 7-branes: involves a 'bulk' field from S $\rightsquigarrow$ not a Yukawa cpling.
- Intersections of three 7-branes, i.e. when two Σ's intersect along S: involves a non-SM field $\rightsquigarrow$ not a Yukawa cpling.
- Intersections of four 7-branes, i.e. when three Σ's intersect along S: bona fidae Yukawa cpling $\rightsquigarrow$ can smile.
  A priori non-generic in alg. geom., but Vafa claims they are generic for E-groups.
  Non-perturbative couplings can be generated even for $SU(5)$ models.

D-instantons: counting zero-modes

Witten's result: Take Euclidean M5-brane wrapping six-cycle P in $\mathbb{R}^{3,1} \times X_4$. Then fermionic zero-modes of the instanton are counted by:

$$\mathrm{chiral}: h^{0,0}, h^{0,2}, \quad \mathrm{anti-chiral}: h^{0,1}, h^{0,3}.$$

To generate superpotential:

- Sufficient condition: all $h^{(0,i)}$ vanish.
- Necessary condition: $\chi_{holom}(P) = \sum_i (-)^i \, h^{(0,i)} = 1$.

D-instantons: counting zero-modes

By M/F-duality only 'horizontal' divisors become D3-instantons in IIB.
In IIB, zero-mode counting splits up. For D3 on four-cycle S:

- 3-3 zero-modes: These are counted by

$$H^0(S, K_S \otimes \mathrm{ad}(P))\,, \quad H^1(S, \mathrm{ad}(P))\,, \quad H^2(S, K_S \otimes \mathrm{ad}(P))$$

- 3-7 zero-modes, for 7-brane on S: These are given by intersection product of D3 and 7-brane:

$$H^0(S, K_S \otimes V_7)\,, \quad H^1(S, \otimes V_7)\,, \quad H^2(S, K_S \otimes \otimes V_7)$$

- 3-7 zero-modes, for 7-brane on S' with $S \cap S' = \Sigma$:

$$H^0(\Sigma, K_\Sigma^{1/2} \otimes V_7)$$