Resolving the information paradox

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CERN 2009

Lecture 1: What is the information paradox ?

Lecture 2: Making black holes in string theory : fuzzballs

Lecture 3: Dynamical questions about black holes

Lecture 4: Applying lessons to Cosmology, open questions

Two basic points:

The Hawking 'theorem' : If

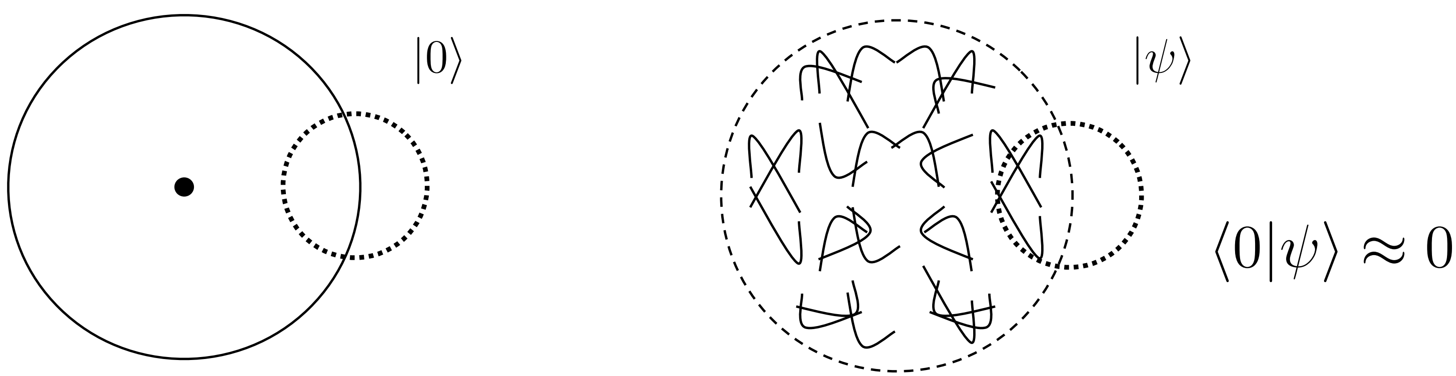
(a) All quantum gravity effects are confined to within a given distance like planck length or string length

(b) The vacuum is unique

Then there WILL be information loss

String theory: Bound states of quanta in string theory have a 'large' size

This size grows with the number of branes in the bound state, making the state a 'horizon sized quantum fuzzball'

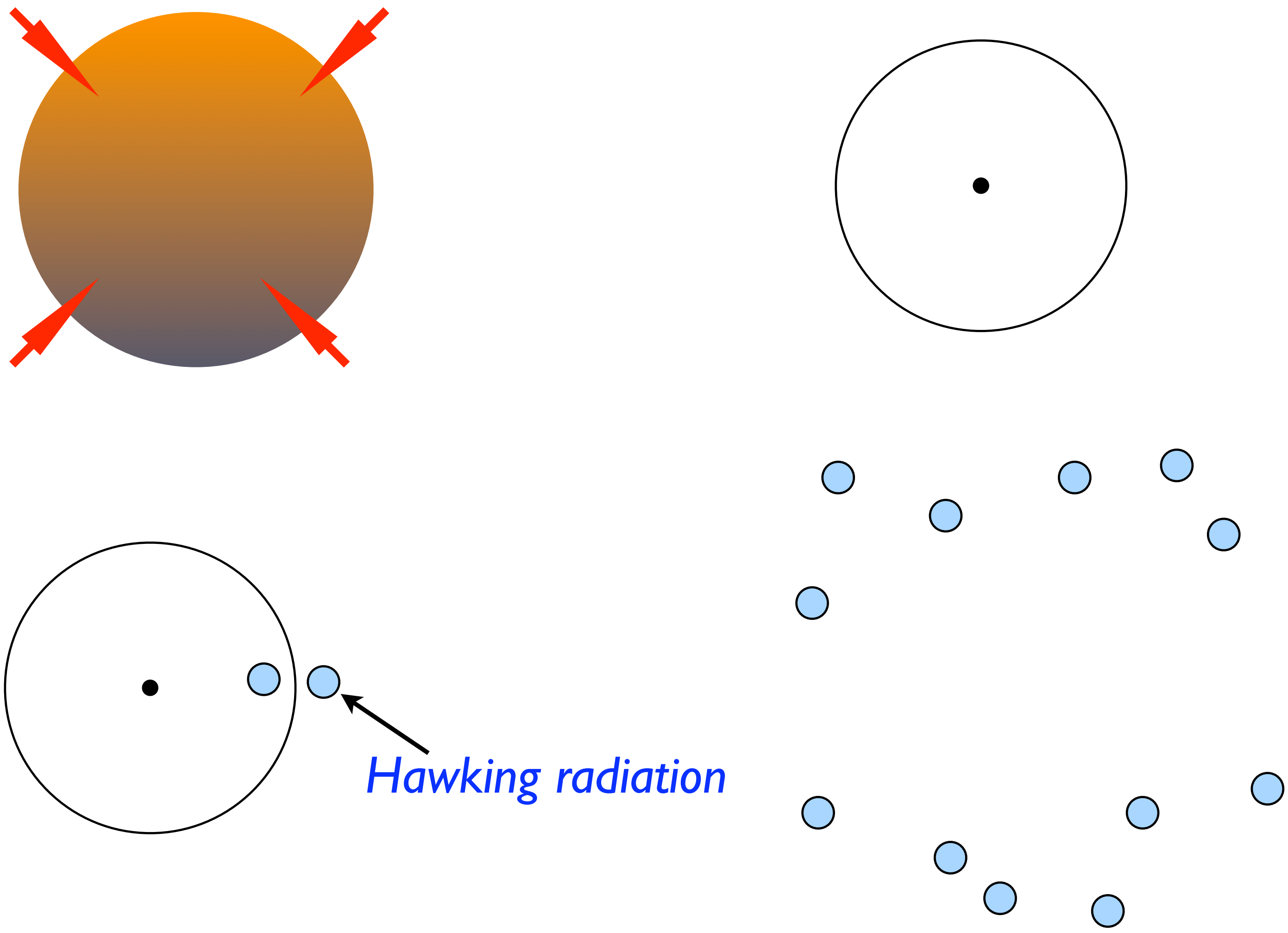


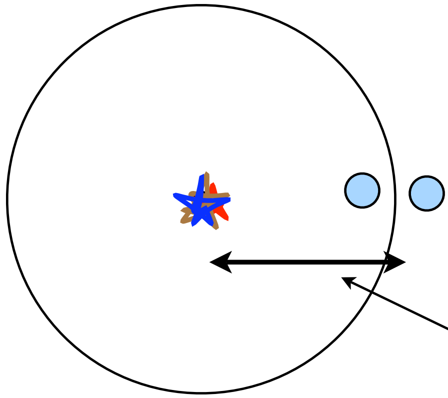
Lecture 1

What exactly is the black hole information paradox?

(arXiv 0803.2030)

The information problem: a first pass





Large distance
(much bigger than planck length)

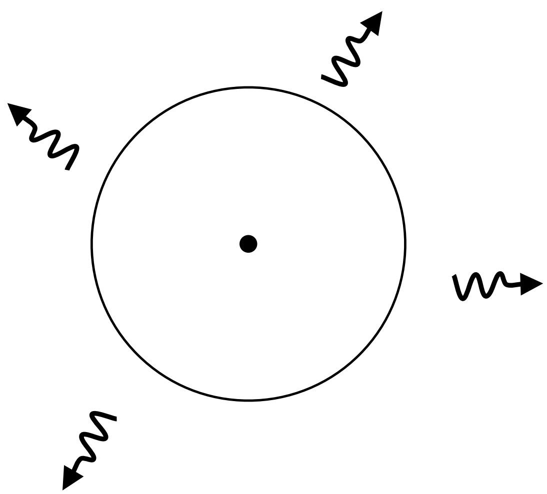
How can the Hawking radiation carry the information of the initial matter ?

If the radiation does not carry the information, then the final state cannot be determined from the initial state, and there is no Schrodinger type evolution equation for the whole system.

So we lose quantum theory ...

We would like the information to come out of the hole in the details of the Hawking radiation ...

This would save us from having to contemplate more complicated solutions of the puzzle (remnants, wormholes, baby Universes, modifications of quantum mechanics ...)

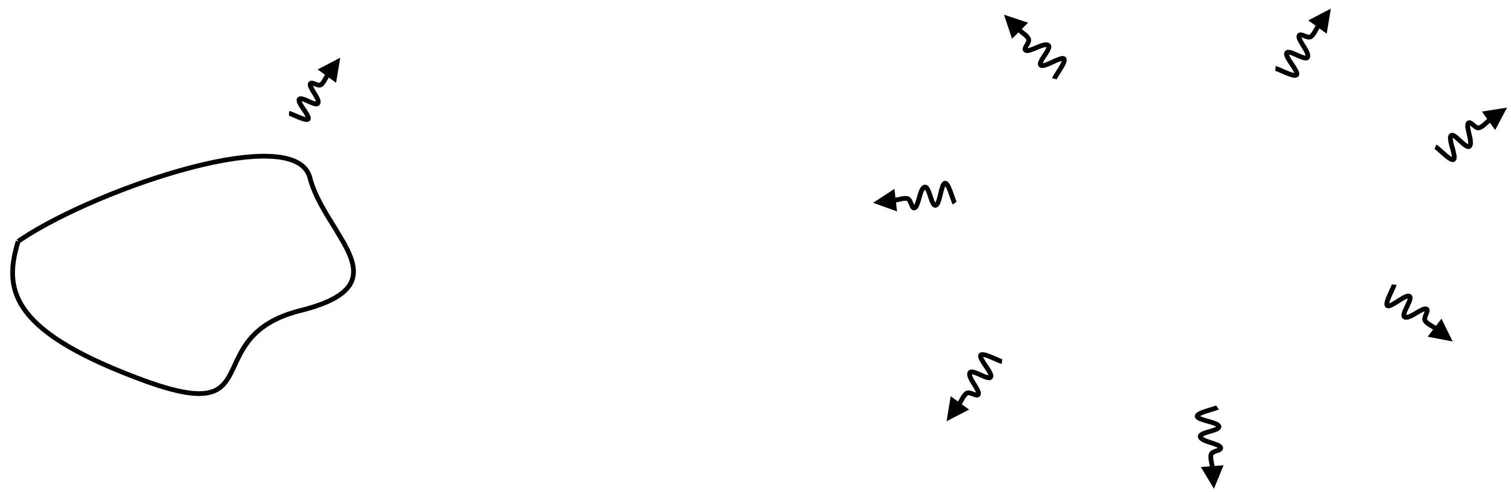


Is it possible for this to happen ?

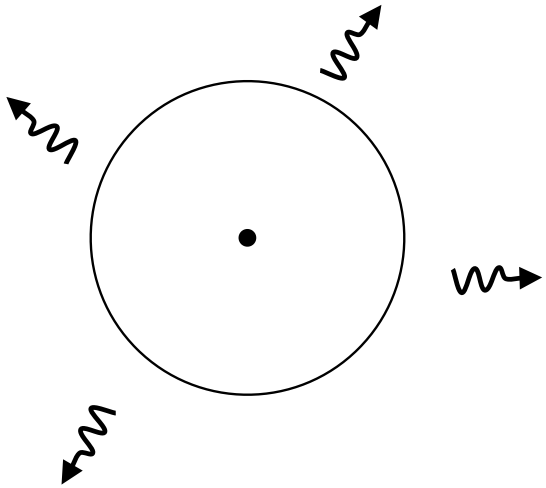
The central issue

When a piece of coal burns away, a large number of photons are produced.

The information of the coal is hidden in ‘delicate correlations’ between these photons’



Can this happen in a black hole ?



Hawking's computation: semiclassical
(gravity classical, matter quantum) ⟹

Get no correlations among radiation quanta

What if we use quantum gravity ?

Quantum gravity effects will be small

But we need 'subtle correlations', so maybe the delicate effects of quantum gravity will generate just the needed correlations ?

This is incorrect !!

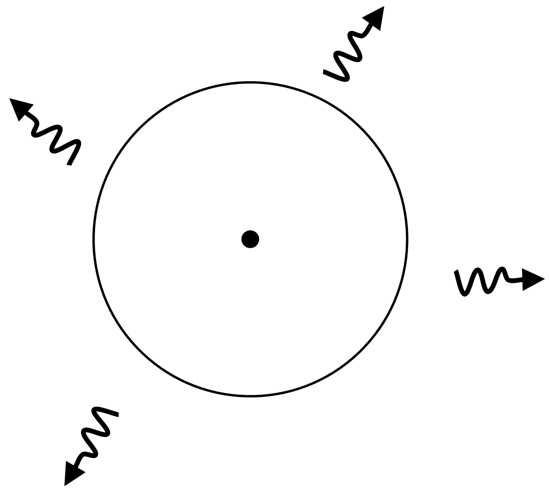
The effects of quantum gravity have to change the state of the radiation by order unity ....

Basic point:

SUBTLE correlations can carry information, as in the case of the burning coal

But this CANNOT be achieved by SMALL corrections to the Hawking emission process

We need an ORDER UNITY change in the evolution of field modes at the horizon



Plan of the talk

(A) Particle creation in curved space, black holes

(B) Entangled nature of the produced pairs

(C) The essential problem created by this entanglement

(D) Formulating Hawking's argument as a 'theorem'

If

(a) All quantum gravity effects are confined to within a given distance like planck length or string length

(b) The vacuum is unique

Then there WILL be information loss

(A) Particle creation in curved space and black holes

Particle creation in curved space

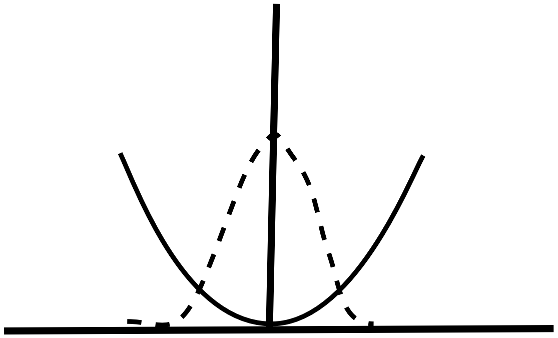
Take a scalar field with Lagrangian

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi$$

Expand the field in fourier modes, and look at a particular mode.
Let the amplitude of this mode be a

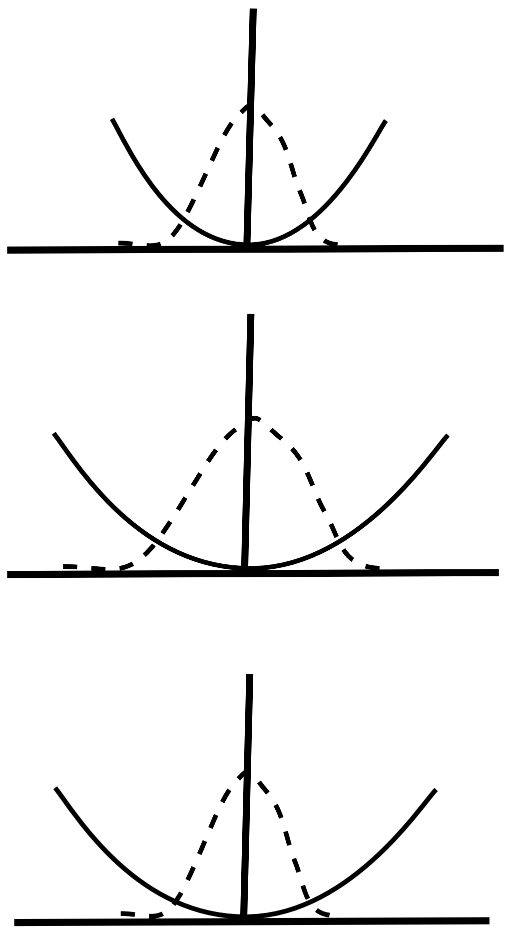
The Lagrangian for a is

$$L = \frac{1}{2}\dot{a}^2 - \frac{1}{2}\omega^2 a^2$$



So we have a harmonic oscillator. Wavefunction in ground state gives vacuum, excited states mean particles in that mode

Suppose the space expands. Then the frequency of the mode changes



Original mode, vacuum state

If the frequency changes slowly, the vacuum state goes over to the new vacuum state

If the frequency changes very quickly, the wavefunction remains the old one, so the state is NOT the vacuum for the new potential

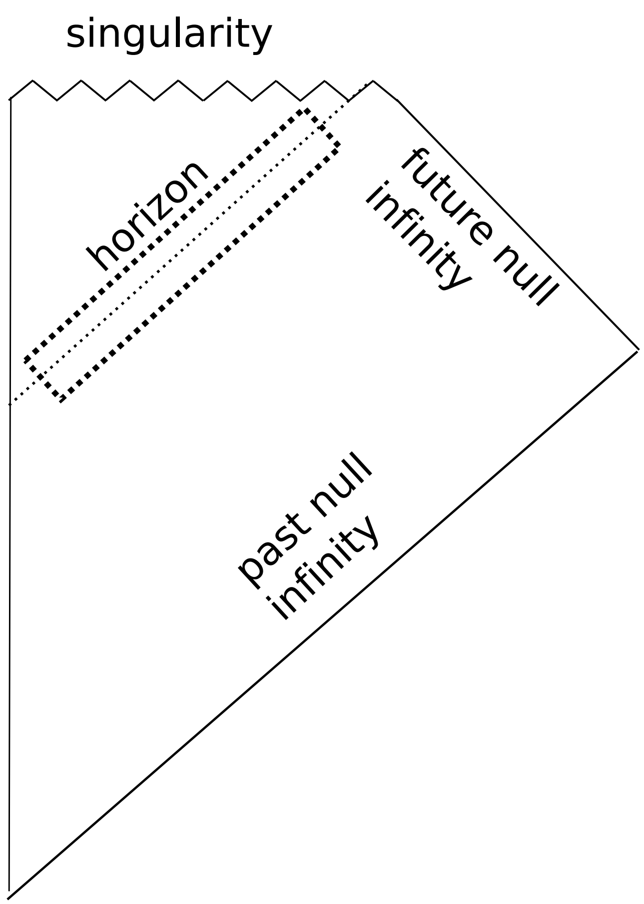
$$\boxed{|0\rangle_\omega = c_0|0\rangle_{\omega'} + c_2|2\rangle_{\omega'} + c_4|4\rangle_{\omega'} + \ldots}$$

How do frequencies change in the black hole geometry ?

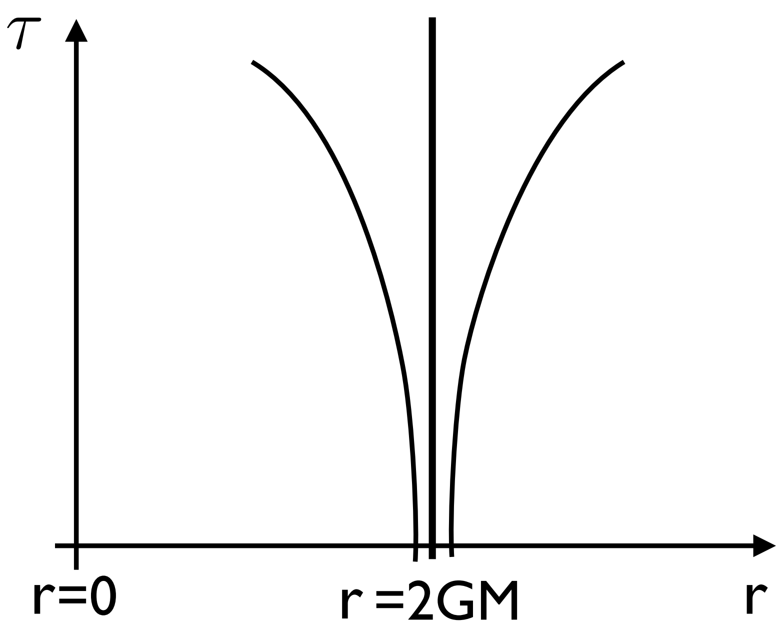
$$ds^2 = -(1 - \frac{2GM}{r})dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

The metric looks time independent,
but this covers only the outside of the hole

The full geometry is given by the Penrose diagram, and we see that the horizon is a null surface



Why do frequencies change in the black hole geometry?

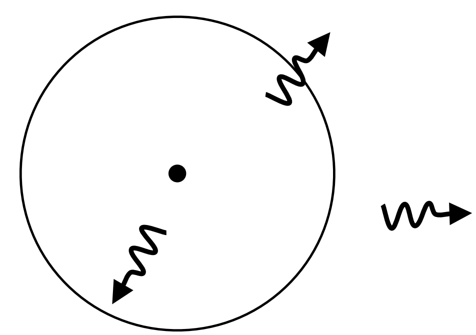


The horizon is a null geodesic

Geodesics starting just outside eventually escape outwards

Geodesics staring just inside fall into the hole

Thus there is a 'stretching' going on at the horizon

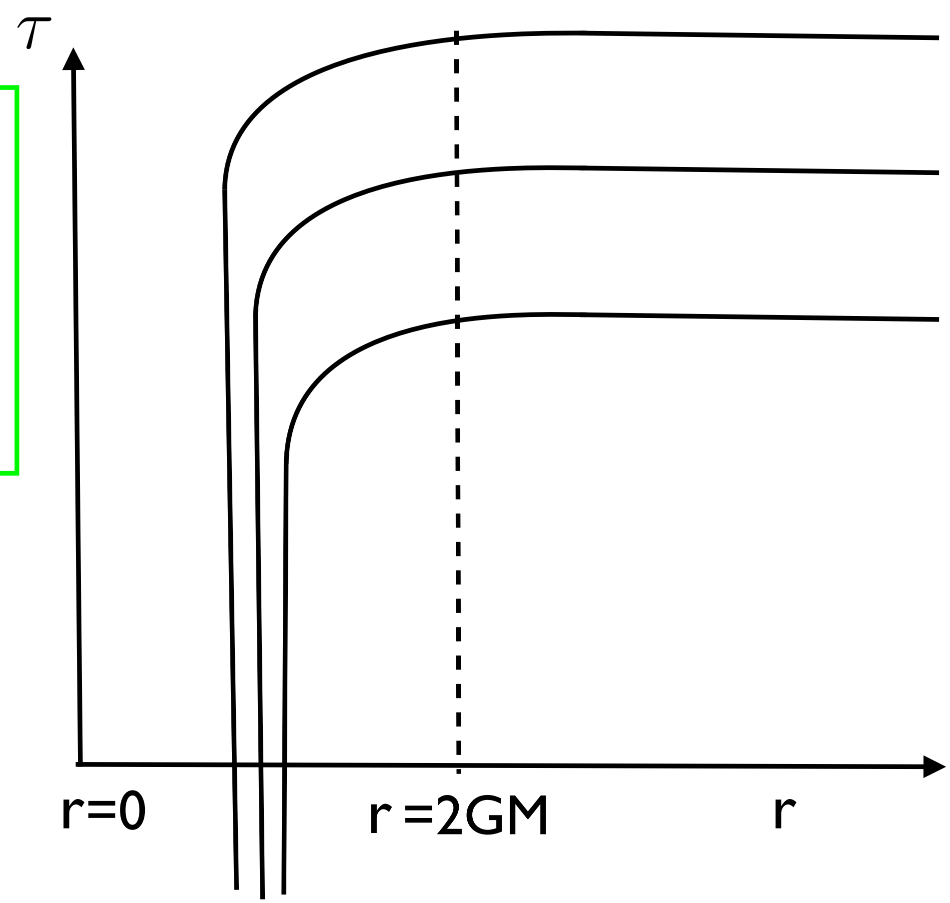


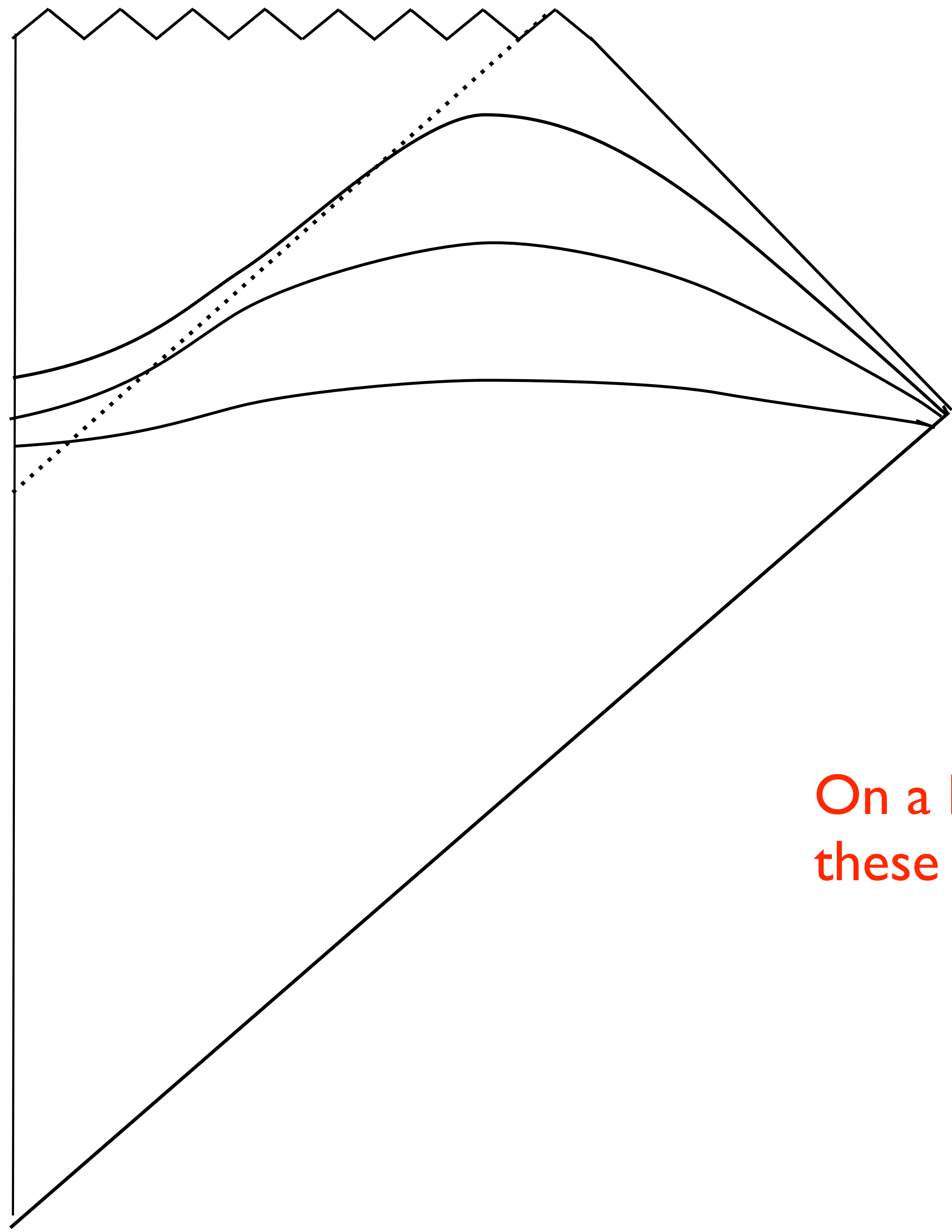
Spacelike slices for the black hole

$$ds^2 = -(1 - \frac{2GM}{r})dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Outside the horizon, constant time is spacelike

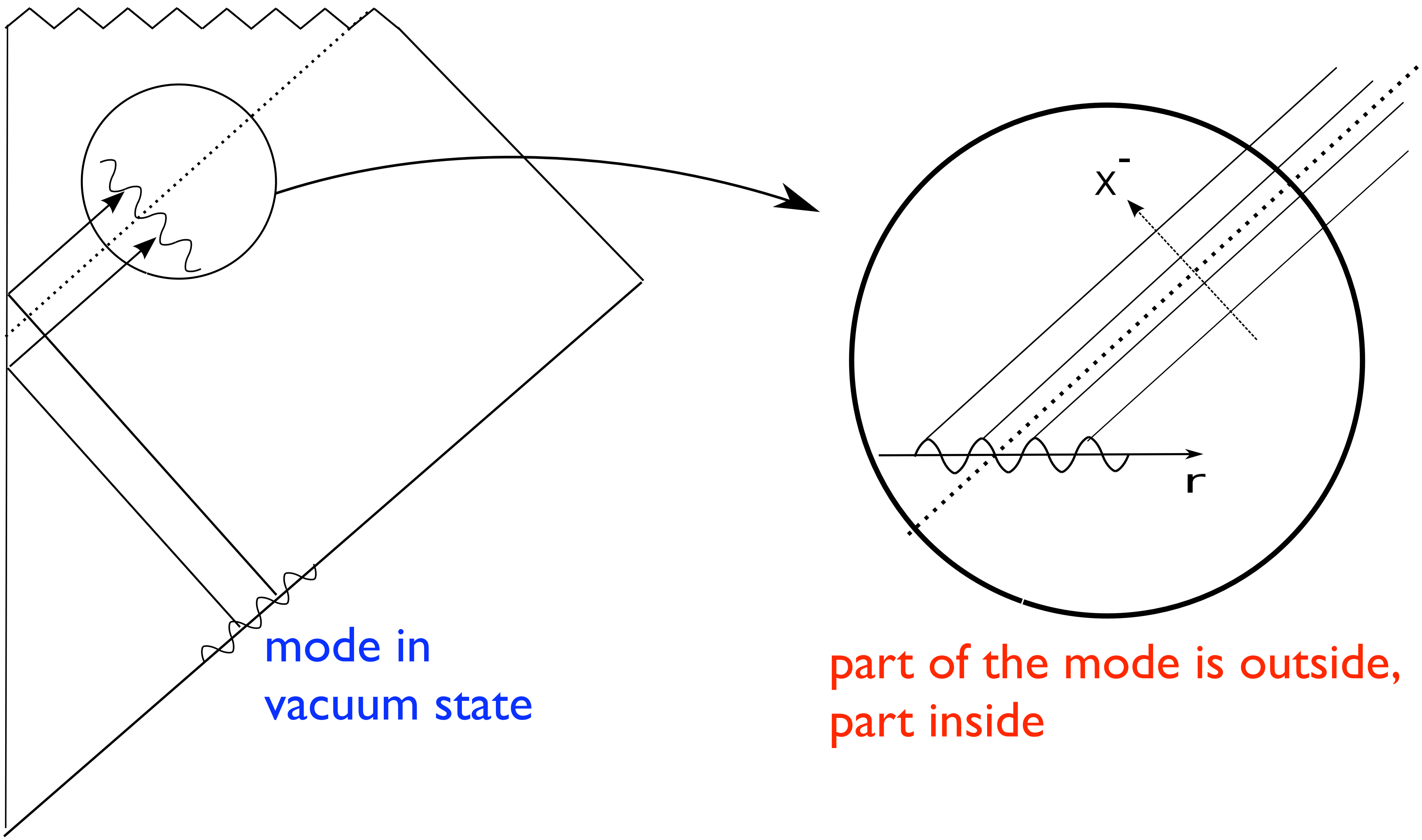
Inside the horizon, constant r is spacelike



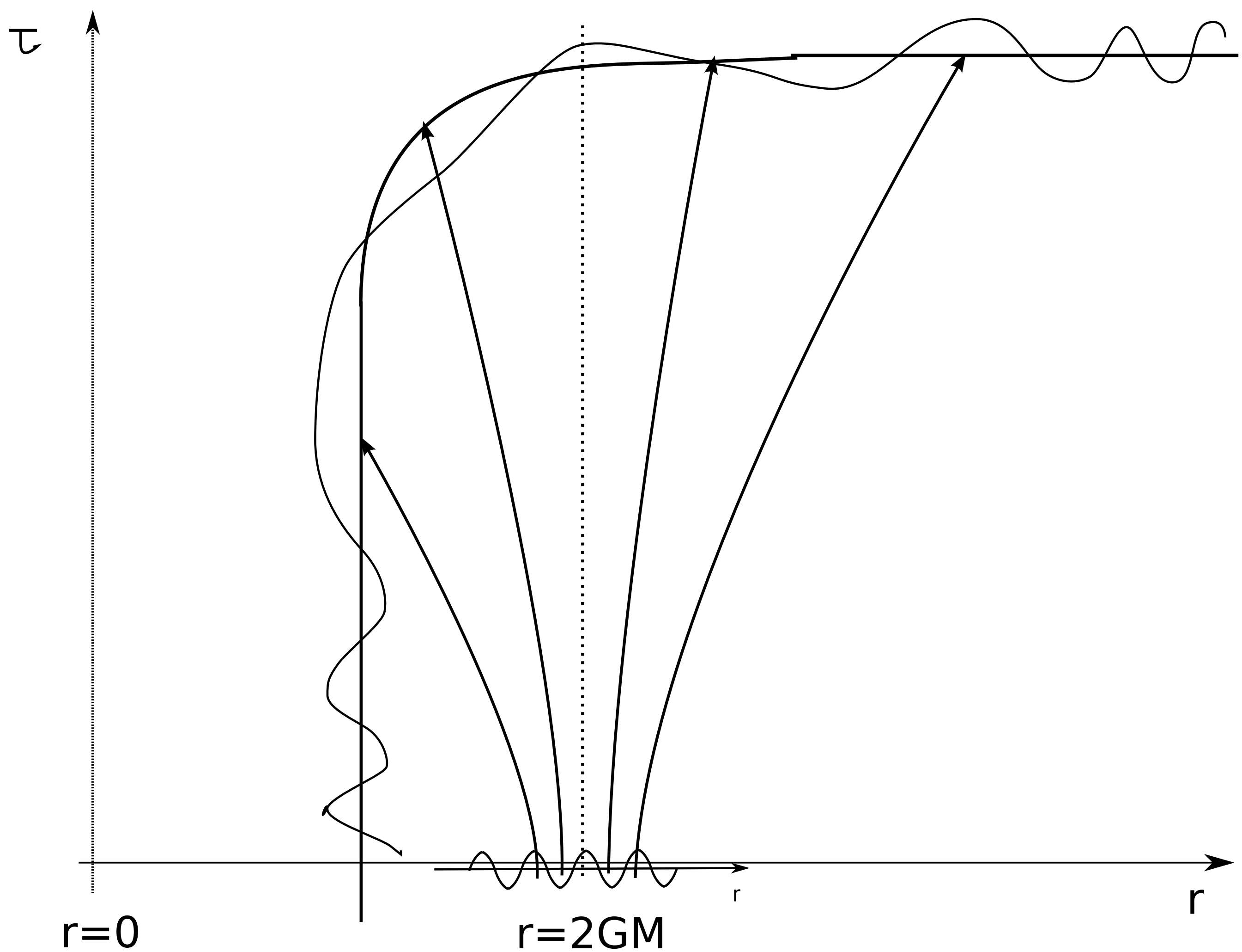


On a Penrose diagram,
these slices look like this ...

Evolution of wavemodes: To leading order, we can evolve wavemodes by letting the lines of constant phase be null geodescics



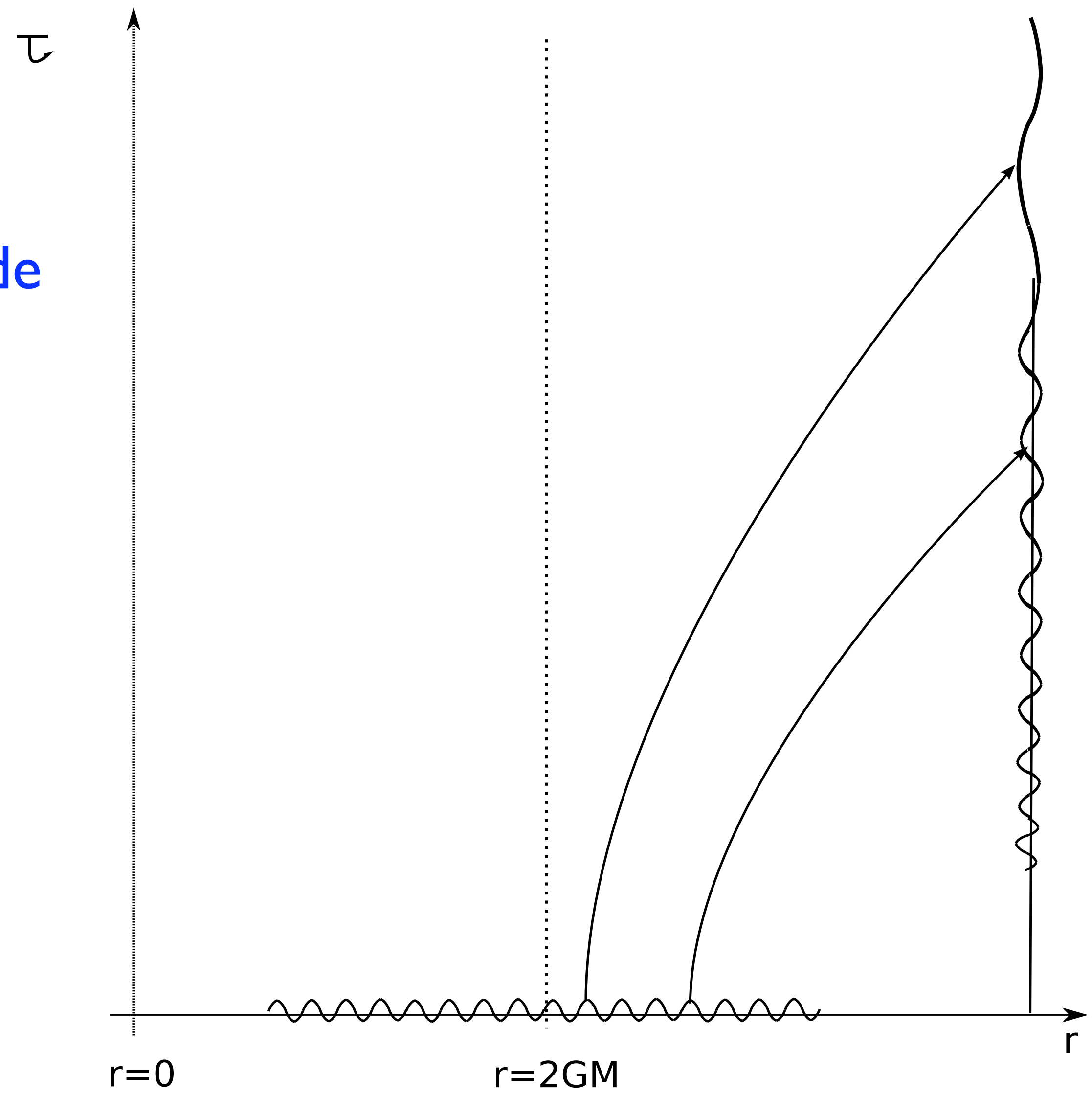
On the schematic diagram we can see that the wavemode stretches, and this will create particles ...



The stretching is logarithmic, as we come near the horizon, so it is *nonlinear*

The part of the mode outside escapes to infinity,

The part inside falls to the singularity



In the quantum field, annihilation operators multiply positive frequency modes and creation operators multiply negative frequency modes

$$\hat{\phi} = \sum_{\vec{k}} \frac{1}{\sqrt{V}} \frac{1}{\sqrt{2\omega}} \left(\hat{a}_{\vec{k}} e^{i\vec{k}\cdot\vec{x} - i\omega t} + \hat{a}^{\dagger}_{\vec{k}} e^{-i\vec{k}\cdot\vec{x} + i\omega t} \right)$$

flat space

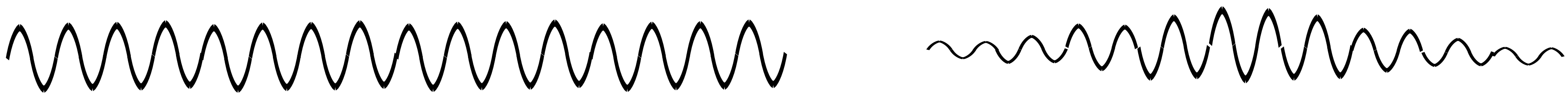
$$\hat{\phi}(x) = \sum_{n} \left(\hat{a}_n f_n(x) + \hat{a}^{\dagger}_n f^{*}_n(x) \right)$$

curved space

Particle creation happens when the stretching mixes the positive frequency modes with the negative frequency modes ...

Wavepackets

We can break up the wavemode into wavepackets without losing the essential physics

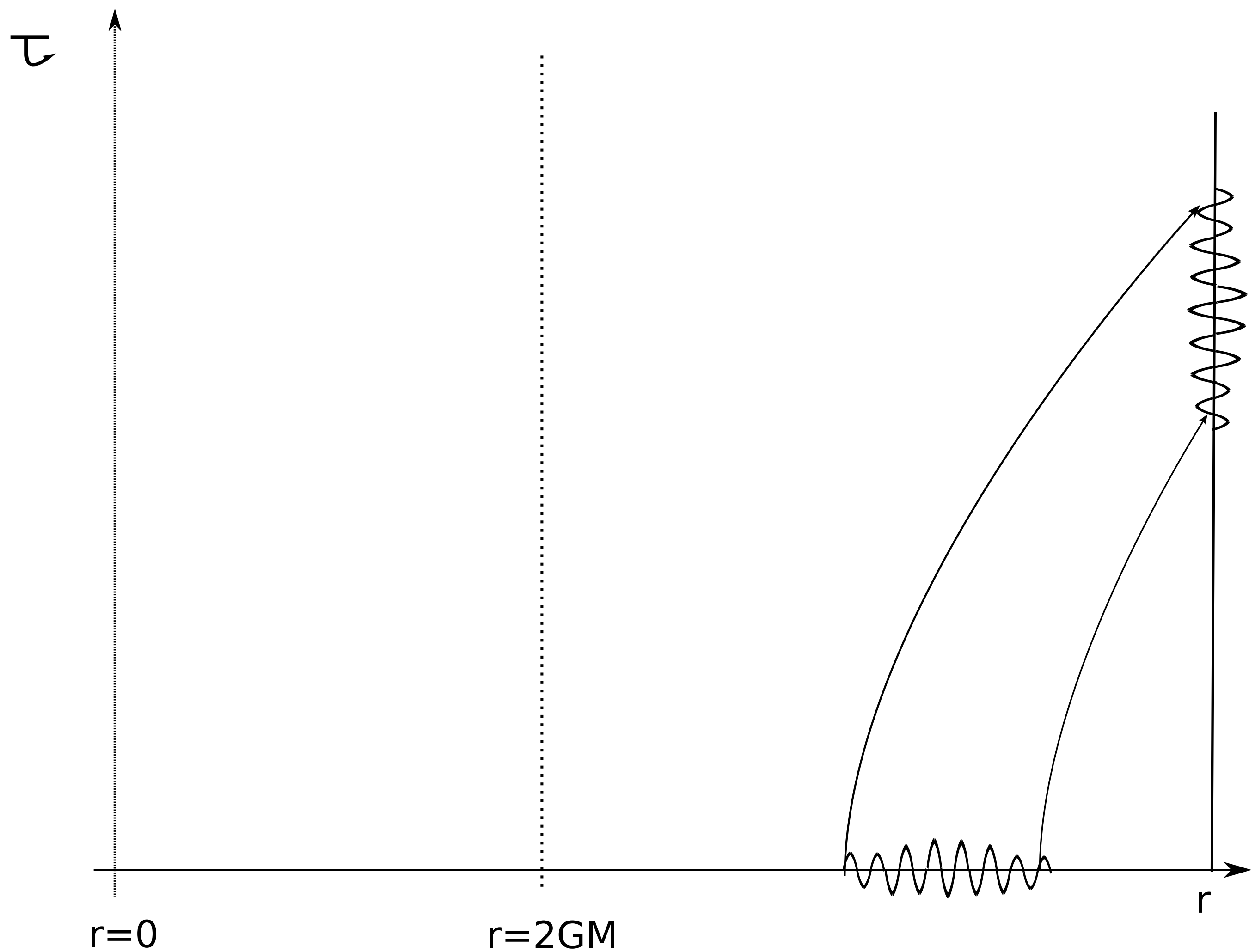


But to make a wavepacket we need at least a few complete oscillations of the wave

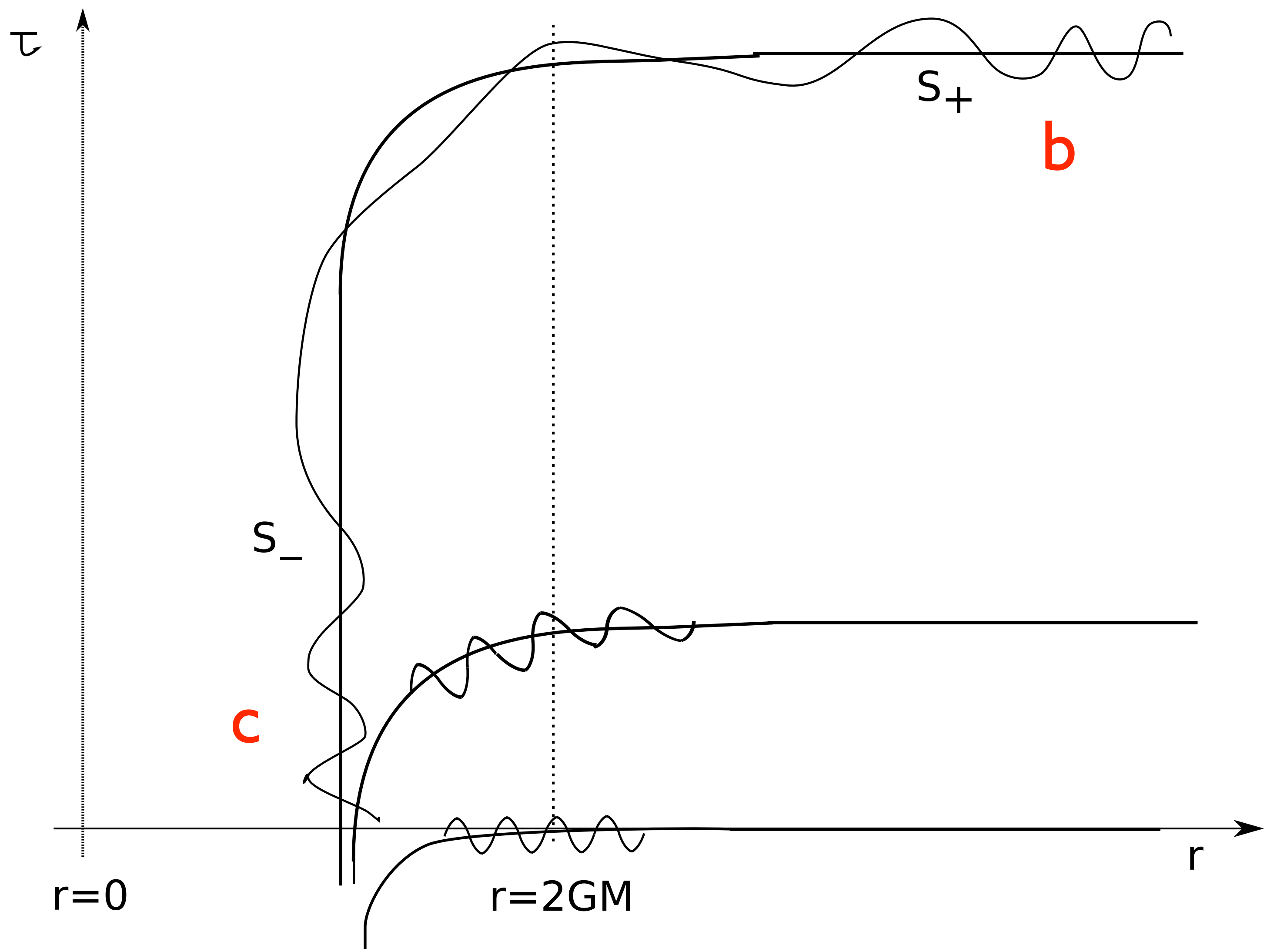
We will break up our initial wavemode into wavepackets, so that we can study exactly which part of the wavemode leads to particle creation

A wavepacket that is localized outside the horizon stretches, but the stretching is approximately constant over the wavepacket

Thus positive frequencies do not mix with negative frequencies, and there is no particle creation



If a wavepacket sits across the horizon, then we will get particle creation. The mode gets cut in two parts ...



(B) Entangled nature of the produced pairs

The nature of the created pairs

The temperature scale is set by the natural length scale in the geometry

$$T = \frac{1}{8\pi GM}$$

But the important fact about the state is that the b, c quanta are in an entangled state

$$|\psi\rangle_1 = Ce^{\gamma \hat{b}_1^\dagger \hat{c}_1^\dagger}|0\rangle_{b_1}|0\rangle_{c_1}$$

The entangled nature of the state $|\psi\rangle_1 = Ce^{\gamma \hat{b}_1^\dagger \hat{c}_1^\dagger}|0\rangle_{b_1}|0\rangle_{c_1}$

$$\begin{aligned} |\psi\rangle_1 &= C\left(|0\rangle_{b_1}\otimes|0\rangle_{c_1} + \gamma\hat{b}_1^\dagger|0\rangle_{b_1}\otimes\hat{c}_1^\dagger|0\rangle_{c_1} + \frac{\gamma^2}{2}\hat{b}_1^\dagger\hat{b}_1^\dagger|0\rangle_{b_1}\otimes\hat{c}_1^\dagger\hat{c}_1^\dagger|0\rangle_{c_1} + \dots\right) \\ &= C\left(|0\rangle_{b_1}\otimes|0\rangle_{c_1} + \gamma|1\rangle_{b_1}\otimes|1\rangle_{c_1} + \gamma^2|2\rangle_{b_1}\otimes|2\rangle_{c_1} + \dots\right) \end{aligned}$$

This state is entangled between the b and c spaces

Consider a system of two spins

Factored states

$$\begin{aligned} |\psi\rangle &= |\uparrow\rangle_1\otimes|\downarrow\rangle_2 \\ |\psi\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 + |\downarrow\rangle_1)\otimes\frac{1}{\sqrt{2}}(\uparrow\rangle_2 + |\downarrow\rangle_2) \end{aligned}$$

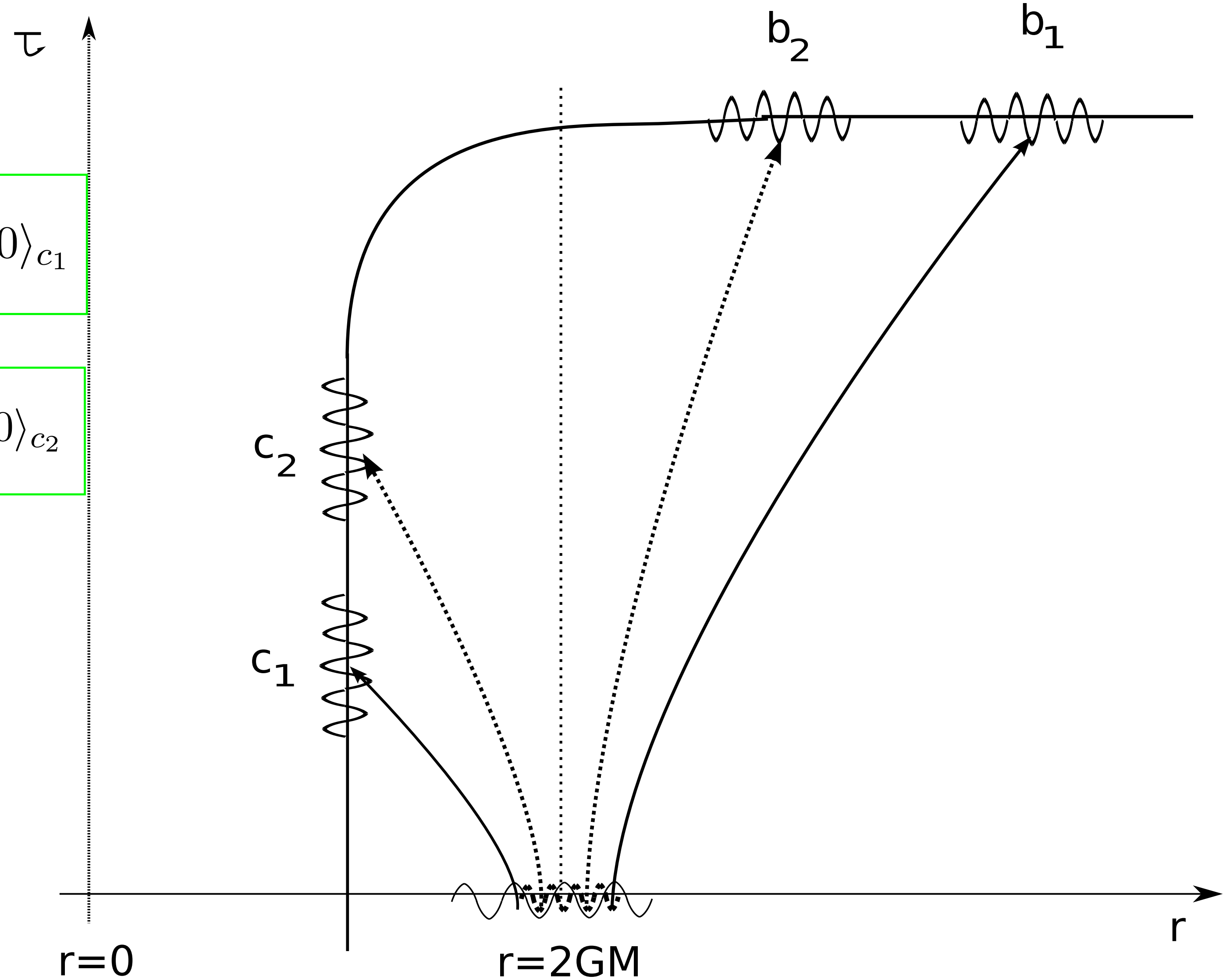
Entangled states

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1\otimes|\downarrow\rangle_2 + |\downarrow\rangle_1\otimes|\uparrow\rangle_2\right)$$

The total wavefunction is a tensor product of the wavefuntions of these pairs .....

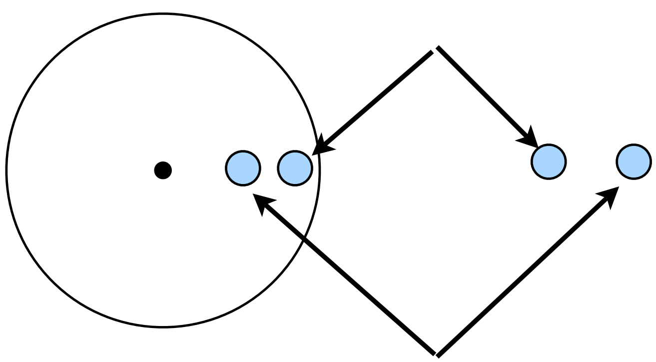
$$|\psi\rangle_1 = Ce^{\gamma \hat{b}_1^\dagger \hat{c}_1^\dagger}|0\rangle_{b_1}|0\rangle_{c_1}$$

$$|\psi\rangle_2 = Ce^{\gamma \hat{b}_2^\dagger \hat{c}_2^\dagger}|0\rangle_{b_2}|0\rangle_{c_2}$$

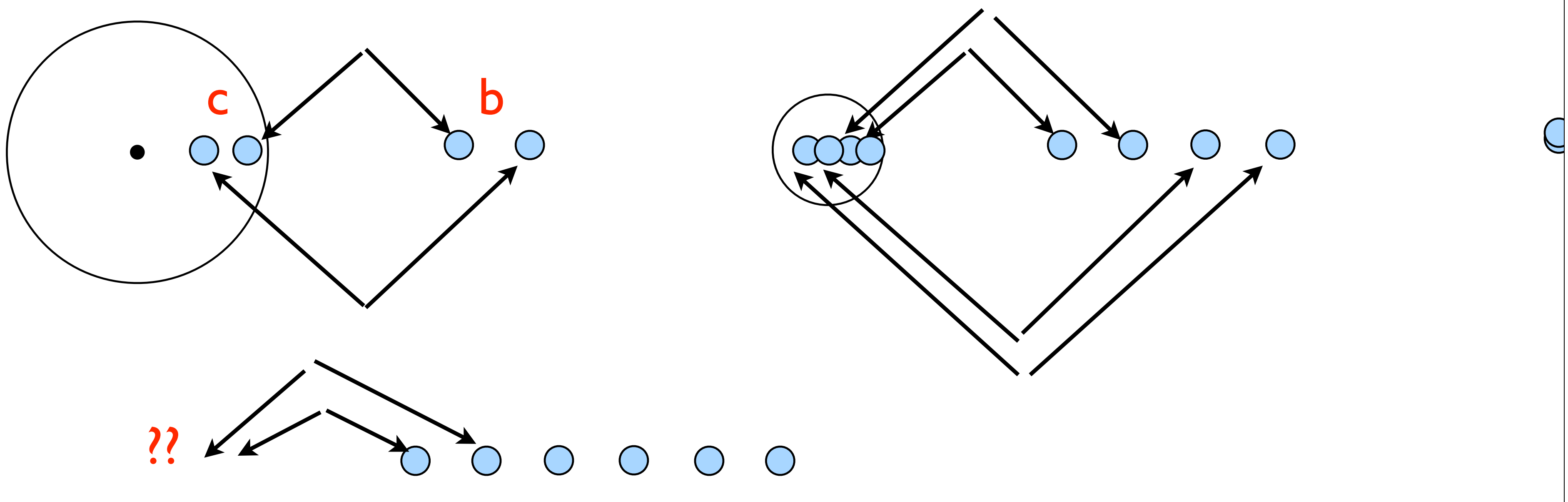


The problem of information loss (loss of unitarity)

$$
\begin{aligned}
|\Psi\rangle &= [|0\rangle_{b_1}|0\rangle_{c_1} + |1\rangle_{b_1}|1\rangle_{c_1}] \\
&\otimes [|0\rangle_{b_2}|0\rangle_{c_2} + |1\rangle_{b_2}|1\rangle_{c_2}] \\
& \quad \ldots \\
&\otimes [|0\rangle_{b_1}|0\rangle_{c_1} + |1\rangle_{b_1}|1\rangle_{c_1}]
\end{aligned}
$$



Entangled pairs



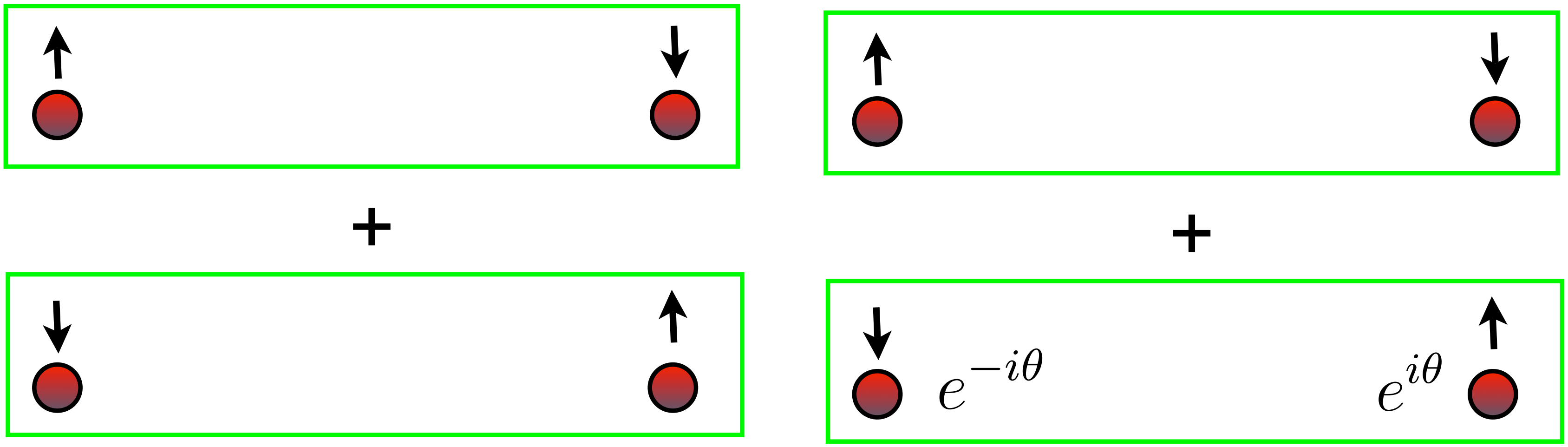
(a) The b quanta are entangled with the c quanta

(b) Thus there is no state as such for the b quanta alone,
but there is a state for the b and c quanta together

(c) If the black hole vanishes, then the b quanta are
left 'entangled with nothing'

(d) There is not supposed to be any such state in quantum mechanics !!

Suppose you have an entangled state



Suppose the left atom vanishes

What is the state of the right atom ?

In fact, there is no state, quantum theory is violated

Essential problem # 1: The state of the Hawking radiation is entangled with what is left in the black hole, and when the black hole vanishes, the radiation is not in any ‘pure’ quantum state

Can small changes to this state make it ‘non-entangled’ ?

NO, that is not possible ......

That is the essential strength of the information paradox

Our state is of this essential form

$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}\left(|0\rangle_{b_1} \otimes |0\rangle_{c_1} + |1\rangle_{b_1} \otimes |1\rangle_{c_1}\right)$$

A factored state would be of the form

$$|\psi\rangle_1 = (C_0|0\rangle_{b_1} + C_1|1\rangle_{b_1} + \ldots) \otimes (D_0|0\rangle_{c_1} + D_1|1\rangle_{c_1} + \ldots)$$

The essential point is that a small change in our state will not make it a factored state :

$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}\left(1.1|0\rangle_{b_1} \otimes |0\rangle_{c_1} + 0.9|1\rangle_{b_1} \otimes |1\rangle_{c_1}\right)$$

is almost as entangled as the initial state we had

Thus a small change in the evolution of the wavemode will NOT solve the information problem

We need a change of ORDER UNITY

How does the information come out when we burn a piece of coal ?

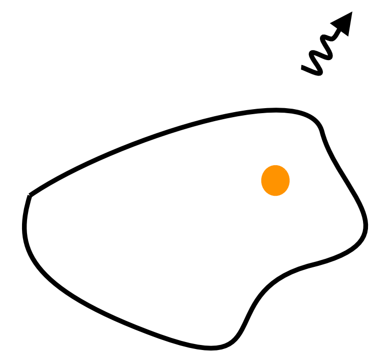
We will find TWO important differences between the coal and the black hole

First difference:

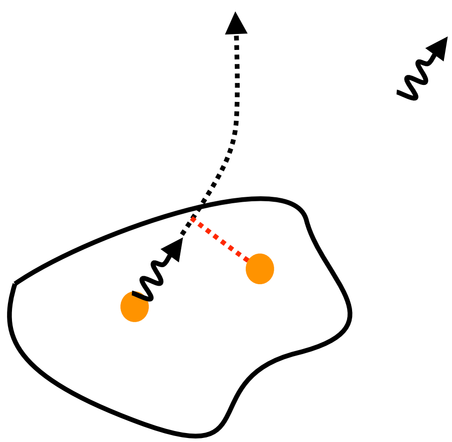
Suppose an atom ‘a’ emits a photon ‘p’

The spins of these can be entangled, so this looks like the situation in the black hole

$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_a \otimes |\downarrow\rangle_p + |\downarrow\rangle_a \otimes |\uparrow\rangle_p\right)$$



Now consider the next emitted photon



This photon can interact with the first atom and its state will be affected by the spin of the first atom

This gives subtle correlations between emitted photons ...

By contrast, In the black hole each particle pair is produced in a place that does not interfere with the earlier pairs

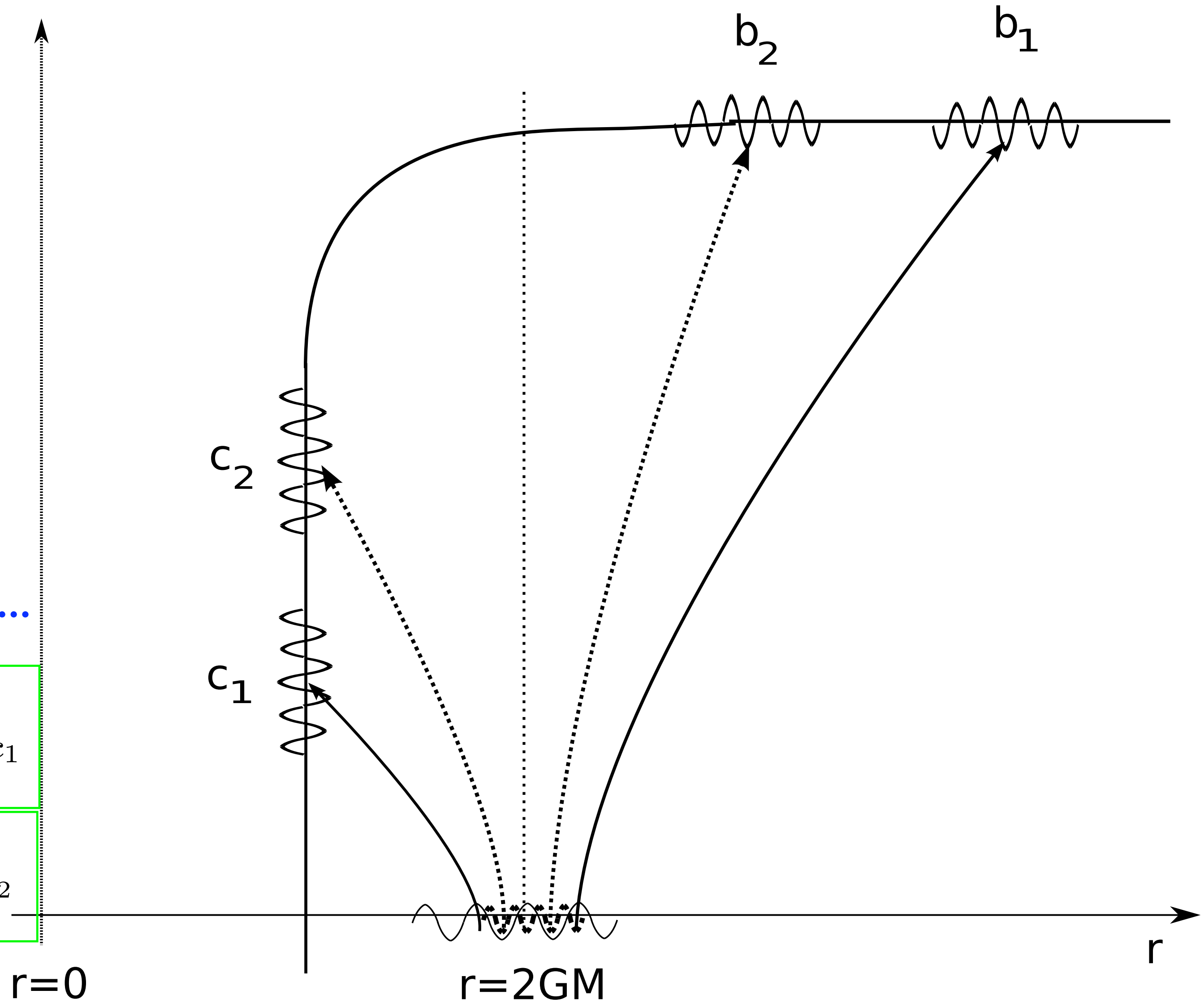
A pair is made at the horizon,

Then the surface stretches, moves pair apart,

Then a new pair is made at the horizon ...

$$|\psi\rangle_1 = Ce^{\gamma \hat{b}_1^\dagger \hat{c}_1^\dagger}|0\rangle_{b_1}|0\rangle_{c_1}$$

$$|\psi\rangle_2 = Ce^{\gamma \hat{b}_2^\dagger \hat{c}_2^\dagger}|0\rangle_{b_2}|0\rangle_{c_2}$$

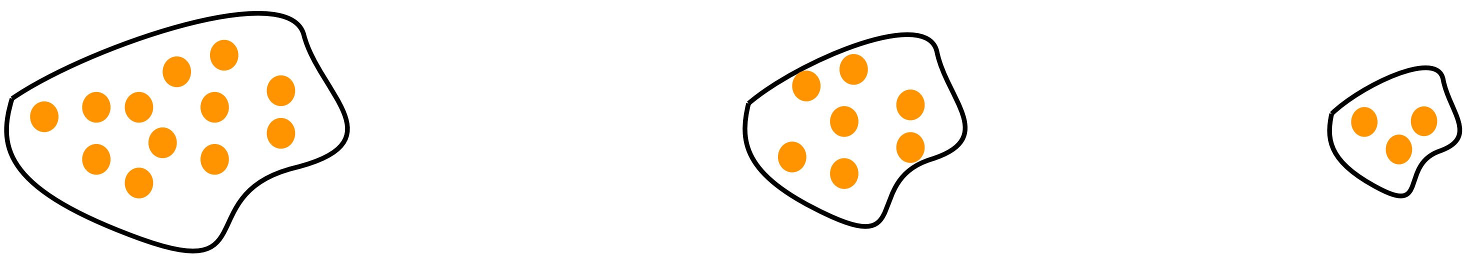


Second difference

As the coal radiates, there is less stuff left in the coal

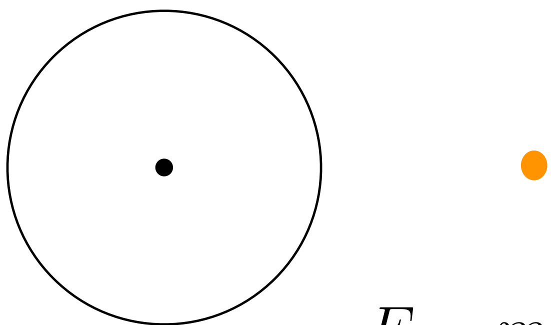
The atom left behind can float out as ash, so all the information goes out finally

Nothing can be left behind, because if there is no mass, there is no matter possible



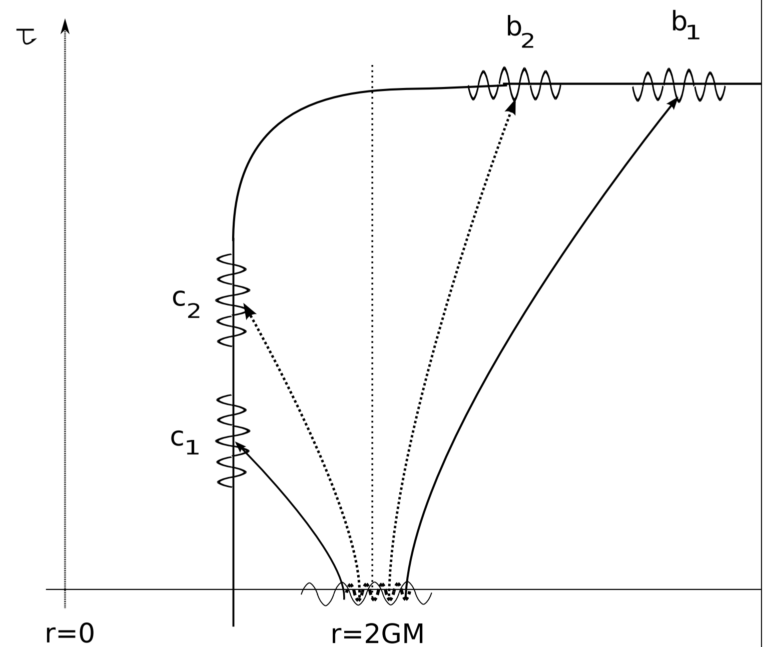
In the black hole, the quanta 'c' falling in have negative energy

$$E = mc^2$$



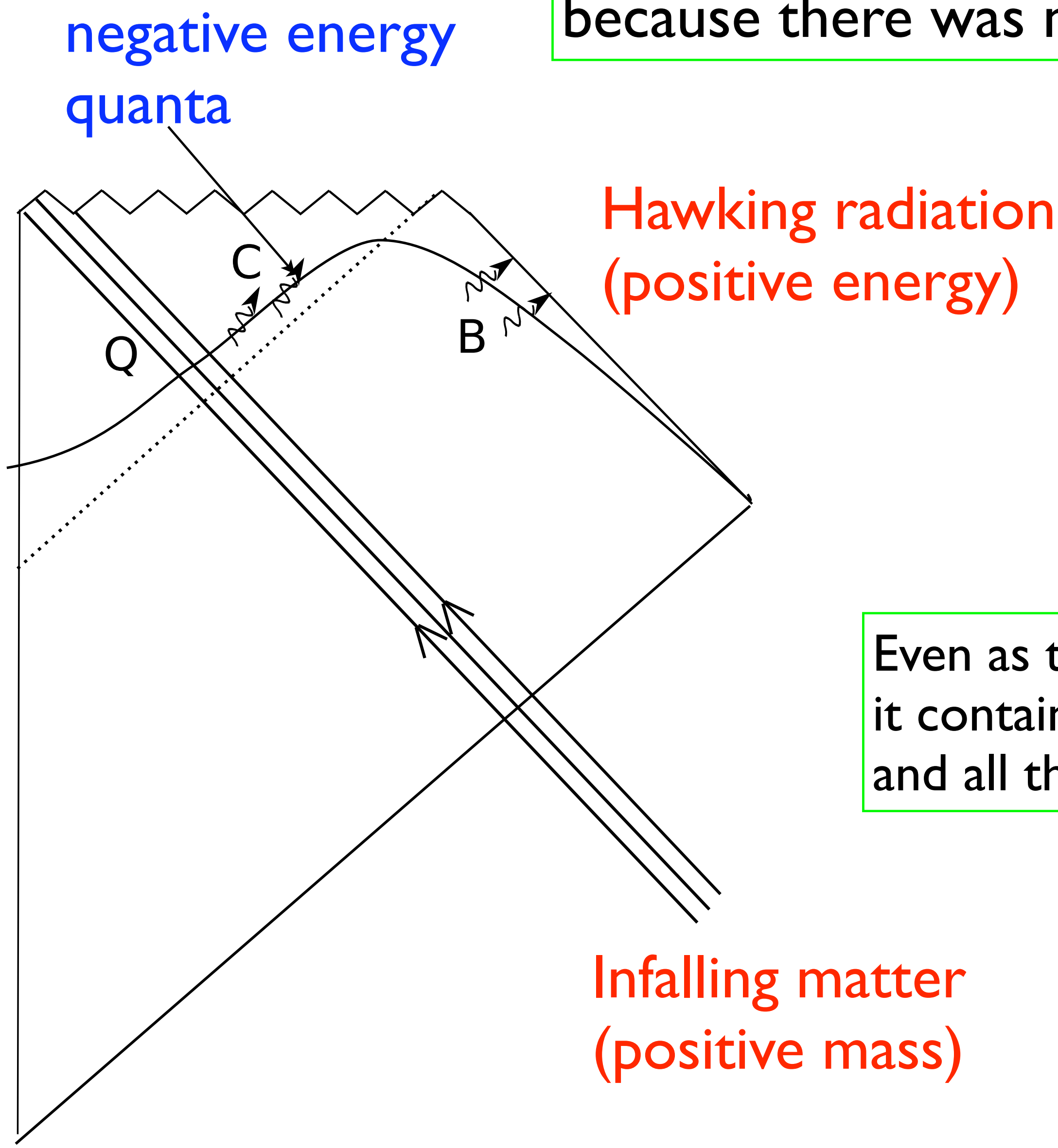
$$E = mc^2 - \frac{GMm}{r}$$

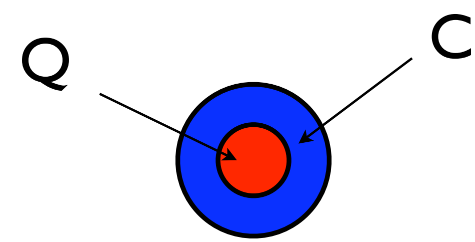
$E \sim 0$ when $r \sim \dfrac{GM}{c^2}$



Where is the mass inside the black hole ?

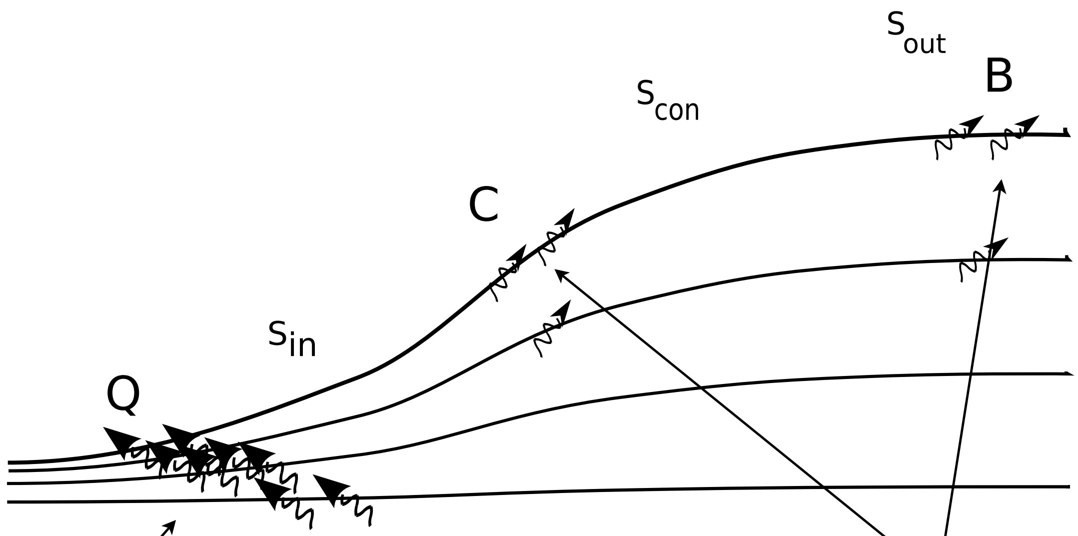
The black hole vanishes because the energy of Q cancels the energy of C, not because there was nothing left in there





Even as the black hole becomes tiny, it contains all the information of Q and all the details of C

The nature of the evolution on spacelike slices looks like this



B and C are entangled

Q does nothing but provide
the mass needed
to make the geometry

What we need is for B to be a 'non-entangled' state
carrying the information of Q

Essential problem # 1: The state of the Hawking radiation is entangled with what is left in the black hole, and when the black hole vanishes, the radiation is not in any ‘pure’ quantum state

Essential problem #2: Even if we manage to dis-entangle b, c, we would still need the state of b to carry the information of Q. But Q is nowhere near the place where b, c are being created, so how can this happen?

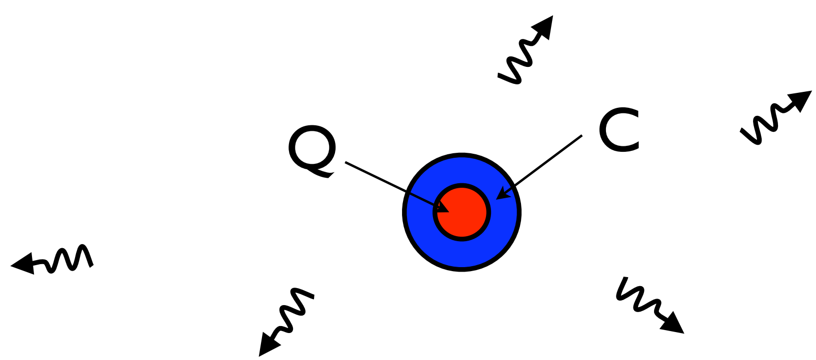
What are the possibilities allowed by this paradox ?

What happens if we accept this situation, and have these b quanta entangled with the c quanta ... ?

(A) We can accept that quantum mechanics is violated ... (Hawking 1974). This is bad because quantum mechanics works so well in all other contexts

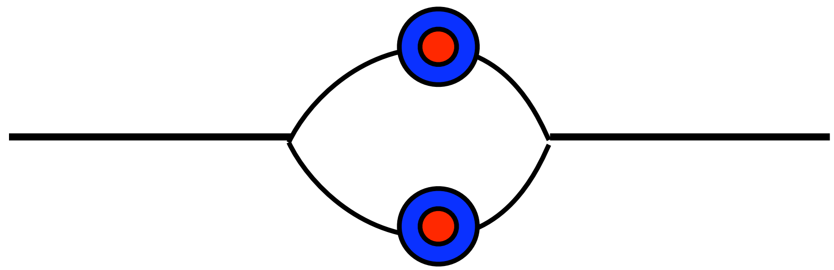
(B) REMNANTS: After the hole becomes very small (planck size), quantum gravity effects can stop the process of pair creation

The black hole does not evaporate away, the information in Q stays in the hole, and the b quanta stay entangled with the c quanta in the hole



Problem with remnants: Since the starting hole can be as big as we want, there will have to by an infinite number of remnants with planck mass

It is unusual for a theory to have an infinite number of states for a bounded energy and volume

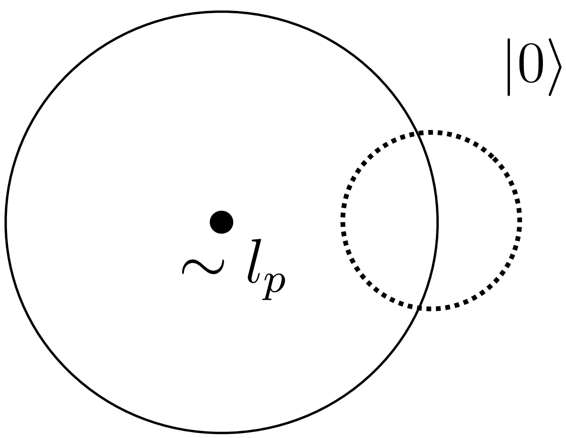


Loop amplitude will diverge because of infinitely many flavors of remnants

(C) We can find some mechanism to change the evolution of field modes by order unity near the horizon

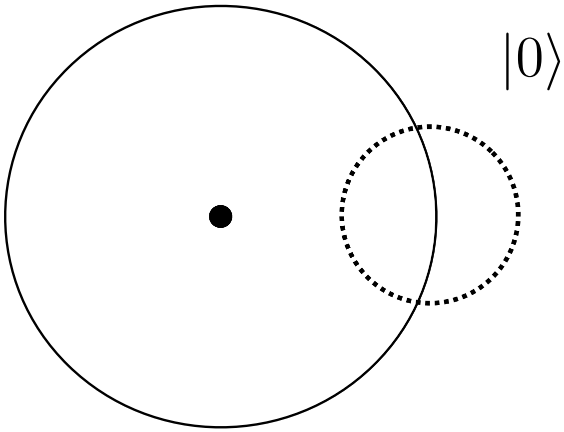
How can this happen?

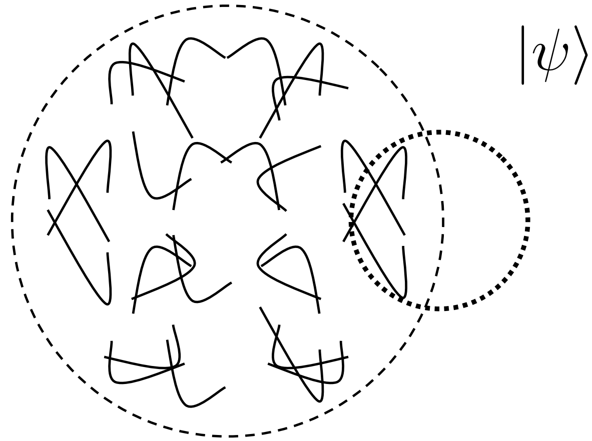
From $c, \ \hbar, \ G$ the only length scale is $\sim l_p$



So it would seem that quantum gravity effects must stay near the singularity

But a black hole is made of a large number of quanta N , so we must ask if the relevant length scales are $\sim l_p$ or $\sim N^{\alpha}\, l_p$





$$\langle 0|\psi\rangle \approx 0$$

In this case the black hole would be replaced by a horizon - sized quantum ‘fuzzball’.

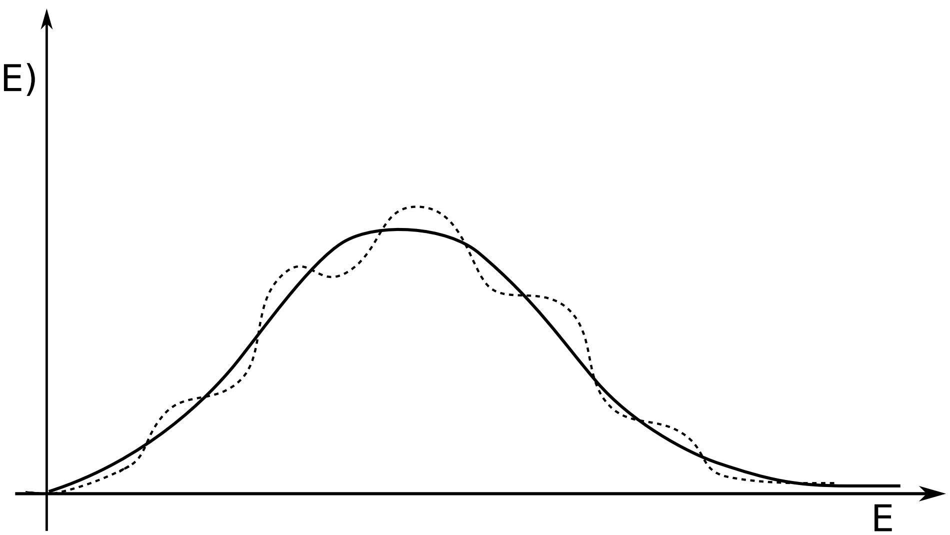
String theory computations suggest that such is the case ...

Some misconceptions, and summary

Some common misconceptions (in my opinion)

(A) 'To see if the information comes out we should see if the spectrum departs from thermality'

This is incorrect. Black hole radiation has 'grey body factors anyway, so it is not planckian



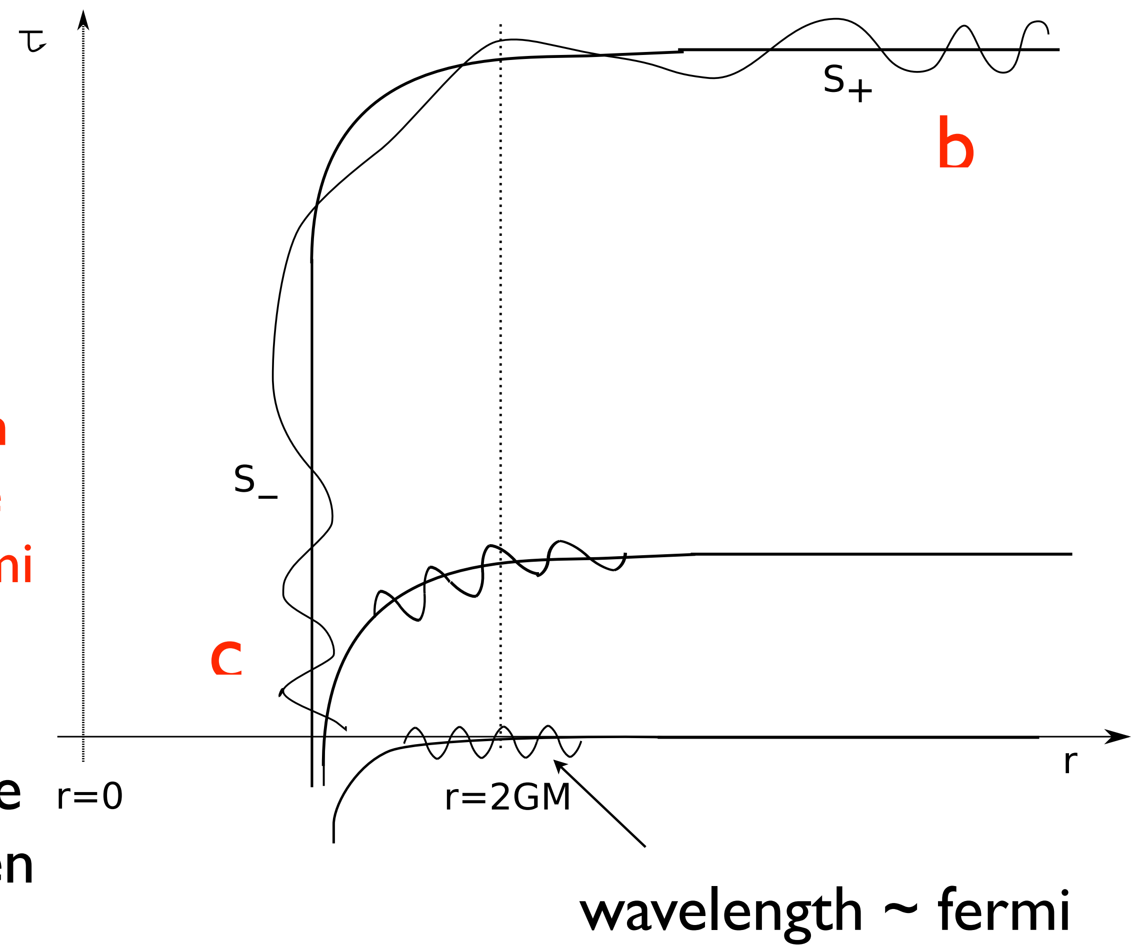
The problem is the entangled nature of the state. We can make it very entangled and still make the 1-point function exactly planckian

(B) The `Transplankian problem': Hawking radiation arises from modes that were very high frequency in the past. Can we trust their evolution? Maybe they carry information after all ...

This is incorrect. We can argue in the following steps:

(A) Look at the mode when it is still << M, but when we know the physics, say ~ fermi

(B) Spacetime is vacuum at the horizon. We assume the vacuum is unique. Then we know the physics of fermi scale modes



(C) There are two possibilities:

(i) The mode is not in the vacuum state. Then there will be particles giving nuclear mass density at the horizon, which contradicts the traditional black hole picture.

(ii) The mode is in the vacuum state. In that case it will evolve as noted before into entangled pairs

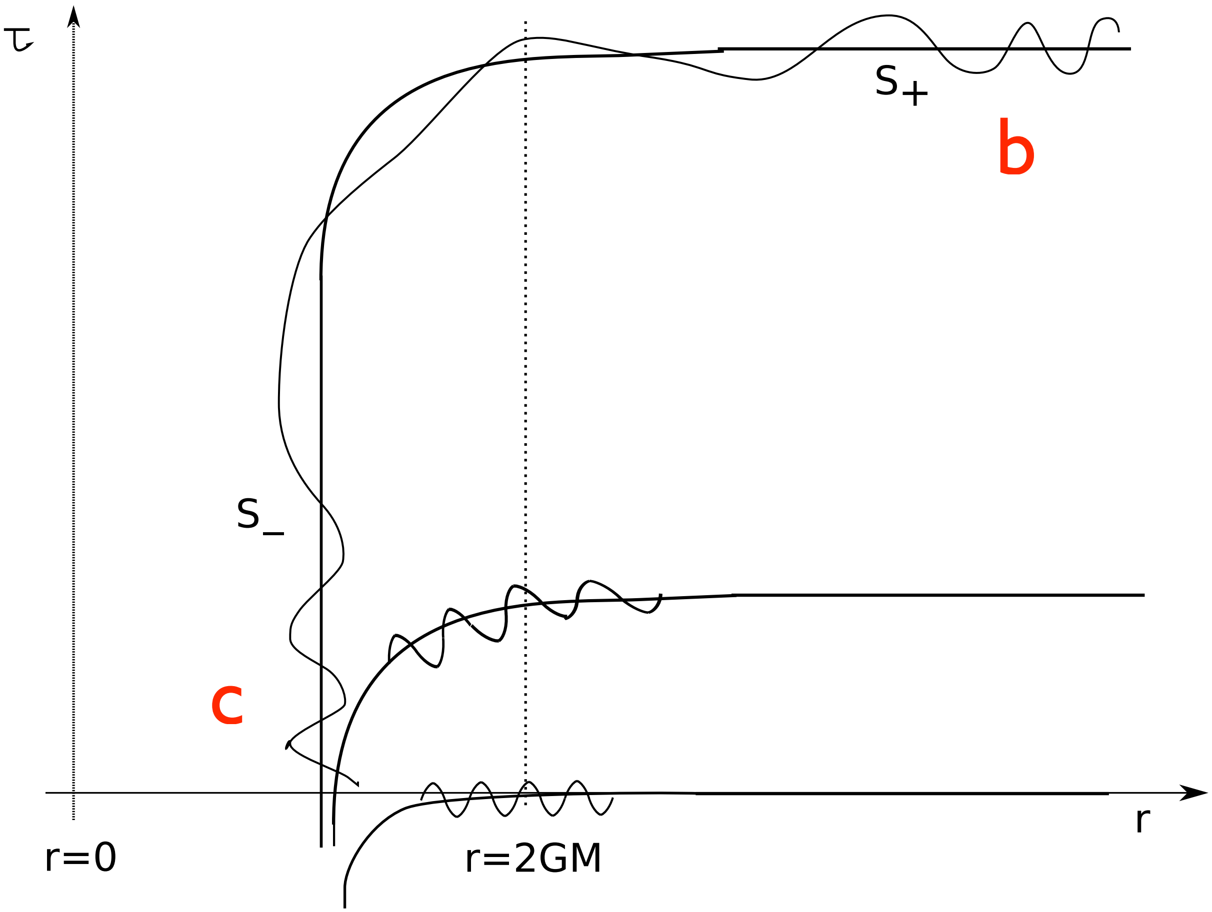
SUMMARY

'Small' quantum gravity effects CANNOT encode the information in subtle correlations in the Hawking radiation quanta.

We need to alter this evolution completely

But this is evolution of wavelength ~M modes on smooth vacuum spacetime

So how can we change it so much ??



Mathematical statement of information paradox:

In the classical black hole solution, the state at the horizon is the vacuum $|0\rangle$

This state evolves to give entangled pairs, we get information loss

We need quantum effects to change this state at the horizon

$$|0\rangle \rightarrow |\psi\rangle$$

such that $$\langle 0|\psi\rangle \approx 0$$

The paradox is that all such attempts failed, quantum corrections always turned out to give

$$\langle 0|\psi\rangle \approx 1$$