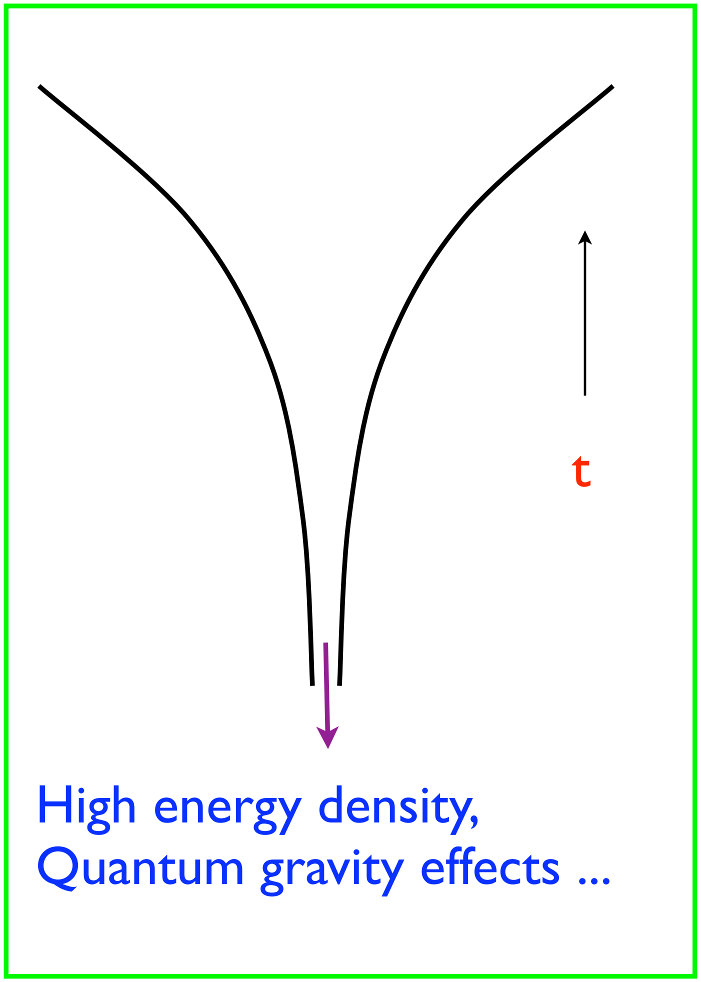
What do black holes suggest about the early Universe ?

Work in collaboration with
Borun D. Chowdhury
(see also work of Kalyanrama)

Consider an expanding Universe



Where else do we encounter high energy densities ?

When matter collapses to form a black hole ....

String theory has had good success in understanding the quantum structure of the black hole interior...

What do the results suggest about the early Universe?

Note:

In the last 3 lectures, we did well defined computations in string theory to resolve the black hole information paradox

The ideas in the present talk will be purely speculative, and are just one way that we can start to think about the very early Universe

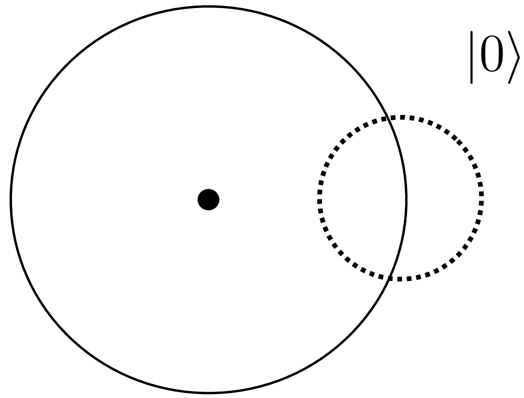
Summary of results from last 3 lectures:

The Hawking 'theorem' : If

(a) All quantum gravity effects are confined to within a given distance like planck length or string length

(b) The vacuum is unique

Then there WILL be information loss

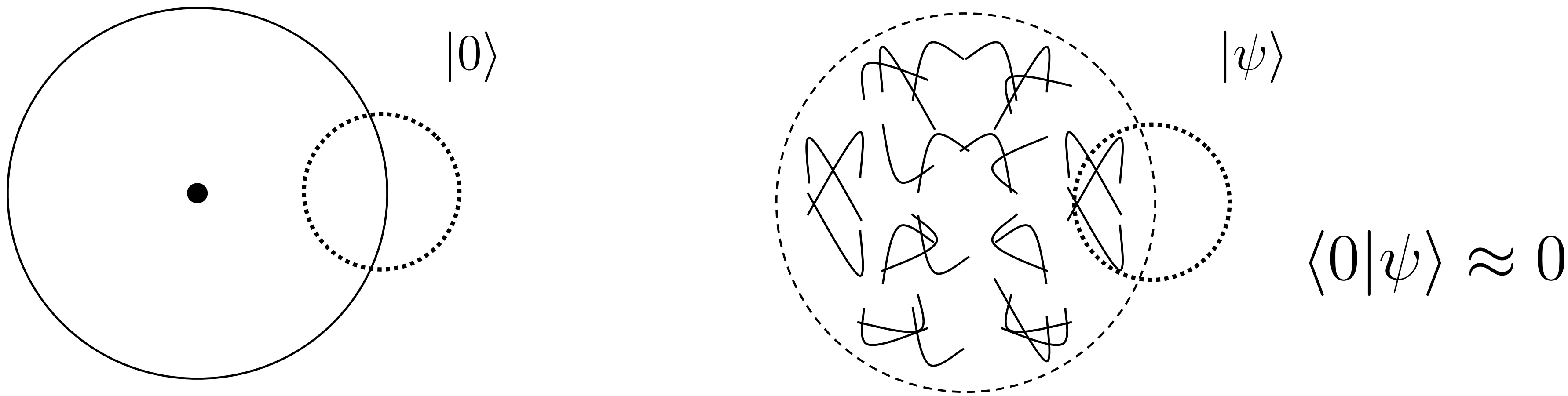


Hawking gives an explicit construction of the evolution of the vacuum state near the horizon, and shows that it gives entangled pairs

$$\boxed{\begin{array}{c} |0\rangle \quad \rightarrow \quad |\psi\rangle \\ \langle 0|\psi\rangle \quad \approx \quad 0 \end{array}}$$

If we can show that the state is not $|0\rangle$, then we resolve the problem

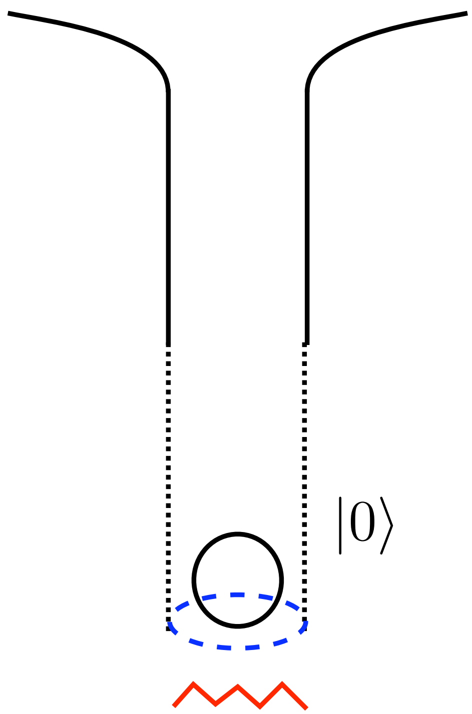
But a black hole is made of a large number of quanta N, so we must ask if the relevant length scales are $\sim l_p$ or $\sim N^{\alpha}\, l_p$



In this case the black hole would be replaced by a horizon - sized quantum 'fuzzball'.

String theory computations suggest that such is the case ...

Earlier attempts to find 'hair' did not succeed ....they looked for perturbative deformations of the traditional black hole metric

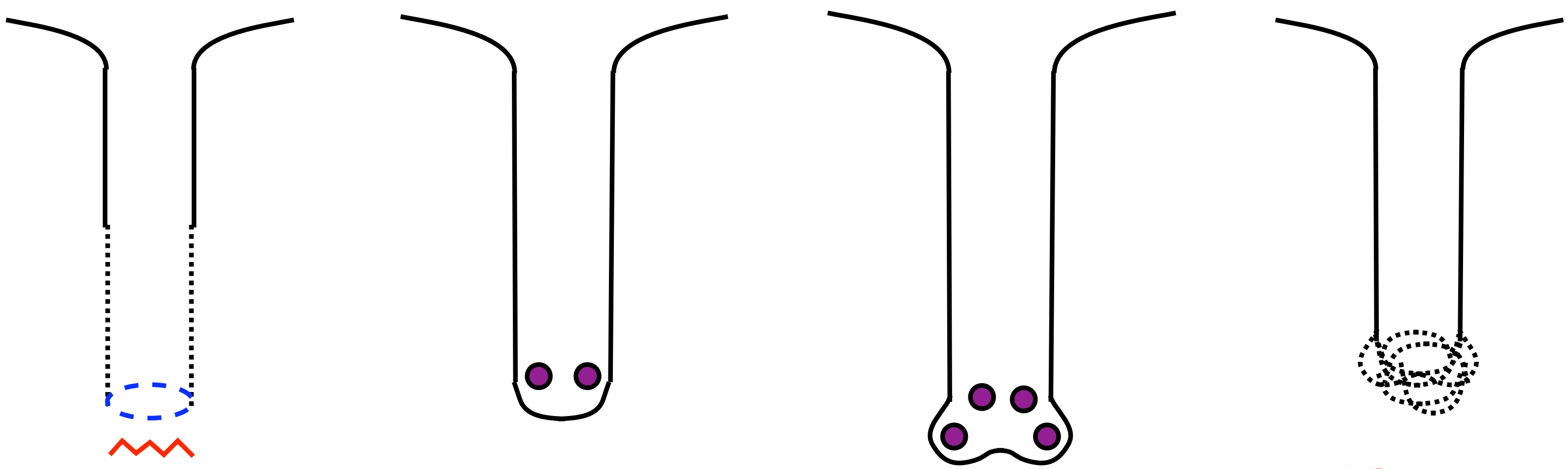


Thus we kept getting

$$\langle 0|\psi\rangle \approx 1$$

We used string theory to understand the whole class of black hole states ... this allowed us to start from simple states and move to more complicated ones

$|\psi_1\rangle$ $\quad$ $|\psi_2\rangle$ $\quad$ ..... $\quad$ $|\psi_n\rangle$



$\langle 0|\psi_1\rangle \approx 0$ $\qquad$ $\langle 0|\psi_2\rangle \approx 0$ $\qquad$ 'Quantum fuzzball'

Thus the generic state is expected to have $\quad \langle 0|\psi\rangle \approx 0$

and not $\quad \langle 0|\psi\rangle \approx 1$

We are resolving a *paradox*. All we have to show is that there is a physical way out of the Hawking construction.

We do not need to make all states in all detail.

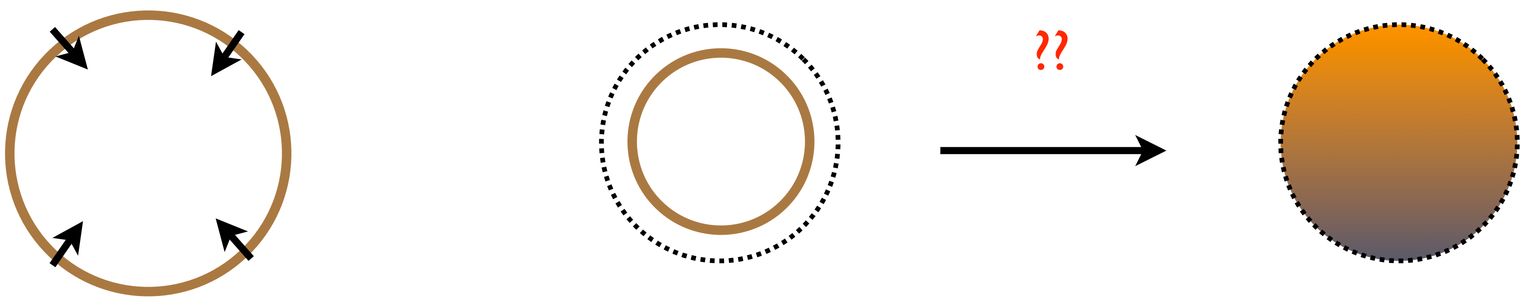
If someone wants to still argue there is a paradox, then he has to show that other states will *not* behave this way

Concrete computations: 2-charge extremal, 3-charge extremal, some non-extremal, Hawking radiation from nonextremal

We do have to show a way out of the simple 'dynamical puzzle':

Once a shell collapses inside its horizon, how can information ever come out ?

We cannot assume that the black hole interior is classical ...



Amplitude to tunnel $e^{-S} \sim e^{-GM^2}$

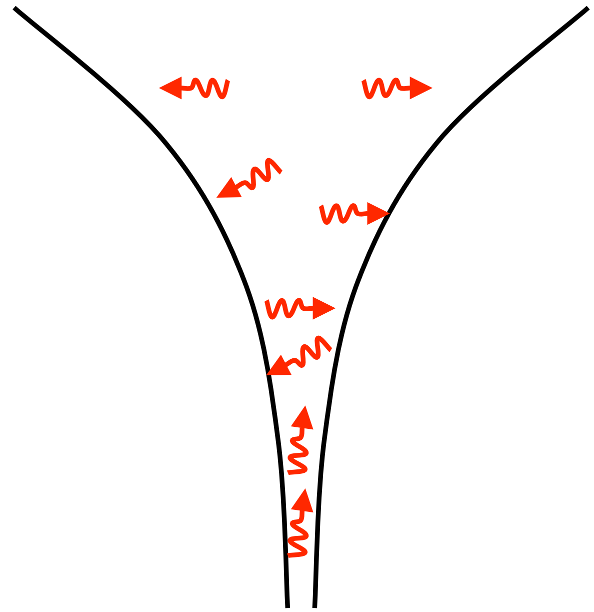
Number of states that we can tunnel to

$$e^{S_{bek}} \sim e^{GM^2}$$

The infalling shell can tunnel into a linear combination of fuzzball states, and the time for this tunneling is shorter than Hawking evaporation time (SDM 08)

(A) An overview of the main point: Entropy in the early Universe

In the early days of Cosmology,
one assumed that the early Universe
was filled with radiation



Why did we do this ?

We held fixed the volume of the
Universe at a given time,
and looked for the most entropic state
with the given energy

(Note that when the Universe expands,
the entropy can increase, because the
box size increases)

What happens if we find configurations with more entropy ?

Start with a box of volume V

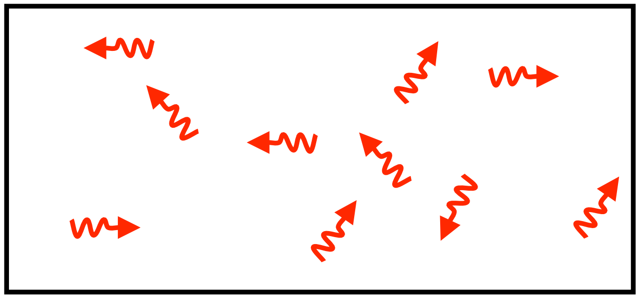
In the box put energy E

> Question: What is the state of maximal entropy S, and how much is S(E) ?

This appears to be a well defined question in string theory, though we should worry about the fact that energy density forces expansion, so we may go out of equilibrium.

For the moment, we will just assume equilibrium at all times, just like for radiation in the early Universe

1-charge objects: radiation



Radiation

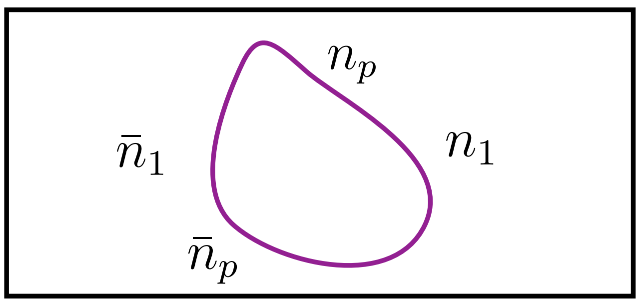
$$\boxed{S \sim E^{\frac{D-1}{D}}}$$

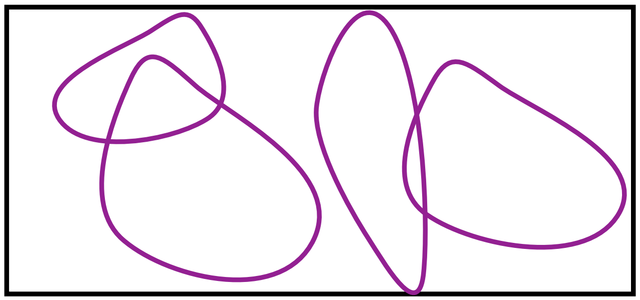
Note that there is no net momentum, so we have both P and anti-P modes (the state is not BPS)

The entropy also depends on the volume V

We hold V fixed, so we do not write it

But we can get more entropy if we use two kinds of charges ...





String gas
'Hagedorn phase'

$$\boxed{S \sim E \sim \sqrt{E}\sqrt{E}}$$

(Brandenberger+Vafa)

Fix V ...

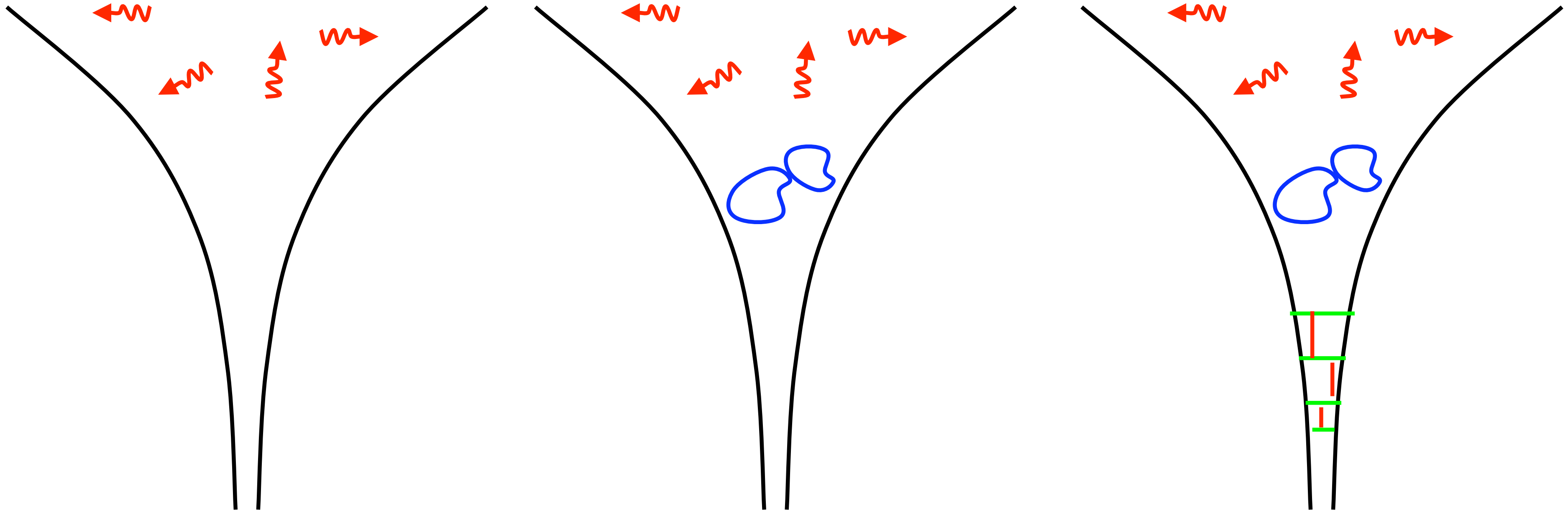
For high enough E, this entropy is more than the entropy of radiation

If we use 3 kinds of charges, the entropy behaves as

$$S = 2\pi\sqrt{n_1 n_2 n_3}$$

We have $\quad n_i \sim E$

Which gives $\quad S \sim E^{\frac{3}{2}}$



Does string theory force us to consider more and more types of branes in the early Universe ?

Plan of the talk :

(a) Review what is known about entropy for *non-extremal* black holes

(b) Get an equation of state for the early Universe based on the physics of black holes

(c) Solve for the evolution with this physics

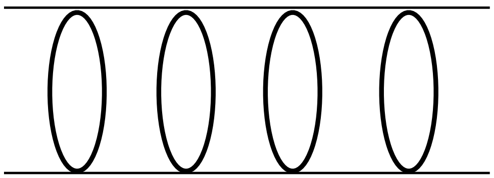
(d) Conjectures about late time evolution

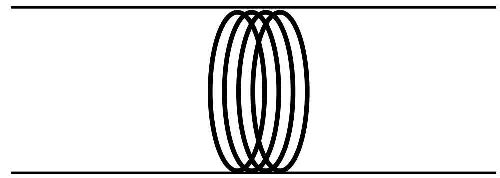
(e) Summary

Some general emerging principles about physics at very high energy densities

Entropy of non-extremal black holes

One charge extremal



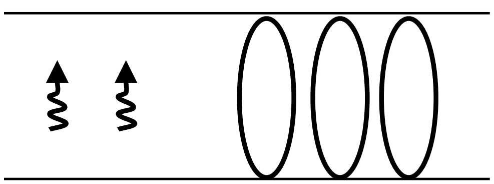


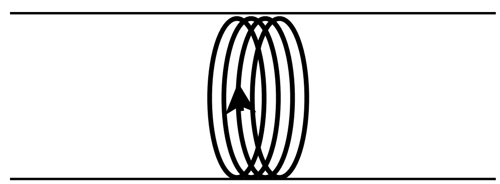
Degeneracy = 256
(independent of n)

$S = \ln 256 \sim 0$

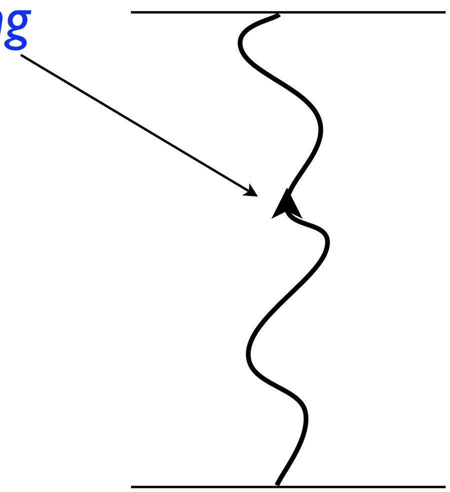
bound state

Two charge extremal





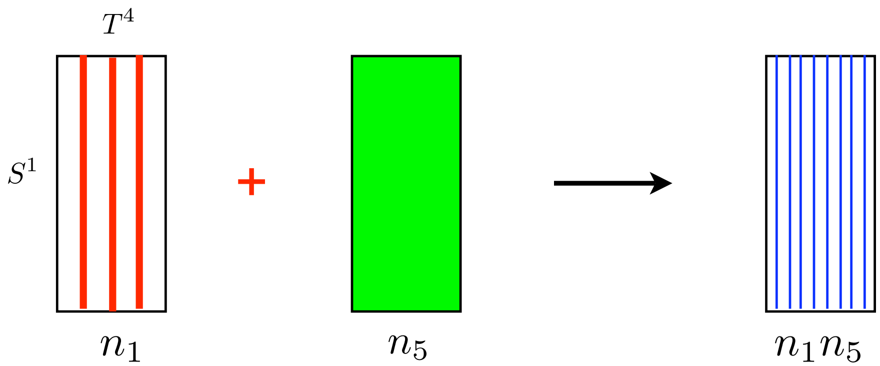
travelling waves on string



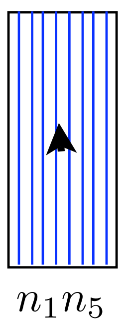
Many ways to partition momentum among different harmonics -- *large entropy*

$$S = 2\pi\sqrt{2}\sqrt{n_1 n_p}$$

Three charge extremal

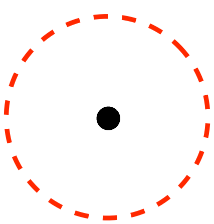


D1-D5



D1-D5-P

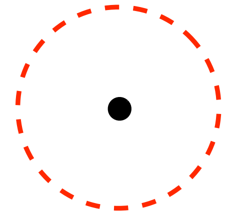
$$S_{micro} = 2\pi\sqrt{n_1 n_5 n_p}$$

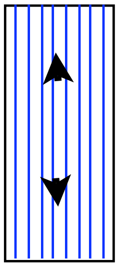


$$\boxed{S_{micro} = S_{bek}}$$

(Strominger + Vafa '96)

Two large charges + nonextremality





$n_1 n_5$

$$\boxed{S_{micro} = 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p}) = S_{bek}}$$

$$n_p - \bar{n}_p = \hat{n}_p$$

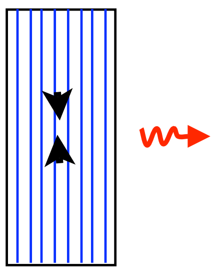
$$E = m_1 n_1 + m_5 n_5 + m_p(n_p + \bar{n}_p)$$

(Callan + Maldacena ' 96)

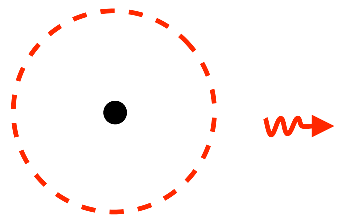
Thus we see that we reproduce the Bekenstein entropy by assuming that the momentum and anti-momentum excitations do not interact -- the energy is the sum of the two energies and the entropy is the sum of the two entropies

Do we trust this picture of excitations ?

Radiation from near-extremal D1-D5 system



$n_1 n_5$



$P\,\bar{P}$ excitations collide and create gravitons

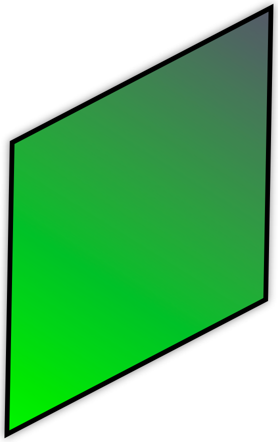
Semiclassical Hawking radiation from black hole

Exact agreement of radiation rate, spin dependence, grey-body factors

$$\boxed{\Gamma_{micro} = \Gamma_{hawking}}$$

(Das+SDM '96, Strominger+Maldacena '96)

One large charge (D5) + nonextremality



$$S_{micro} = 2\pi\sqrt{n_5}(\sqrt{n_1}+\sqrt{\bar{n}_1})(\sqrt{n_p}+\sqrt{\bar{n}_p})$$
$$= S_{bek}$$

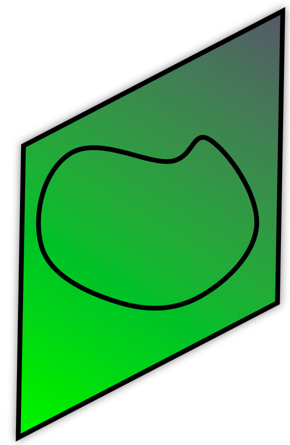
(Maldacena '96)

$$n_1 - \bar{n}_1 = \hat{n}_1$$

$$n_p - \bar{n}_p = \hat{n}_p$$

$$E = m_5 n_5 + m_1(n_1+\bar{n}_1) + m_p(n_p+\bar{n}_p)$$

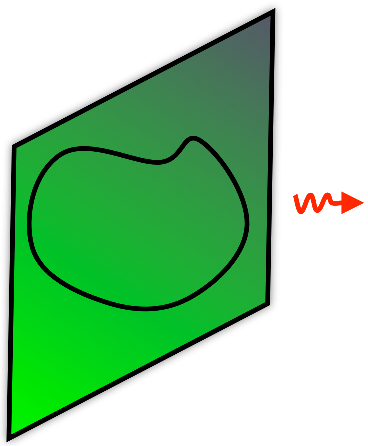
Maximize the formal expression for S_{micro} subject to these constraints



Effective string with fractional tension $\frac{1}{n_5}T_{D1}$

Do we trust this picture of excitations ?

Radiation from near-extremal D5



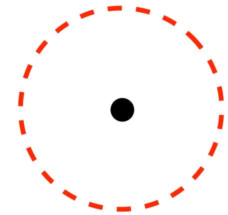
$$\boxed{\Gamma_{micro} = \Gamma_{hawking}}$$

Effective string with fractional tension $\frac{1}{n_5} T_{D1}$

(Klebanov+SDM '97)

No large charges

$$S_{micro} = 2\pi(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p})$$



Maximize S_{micro} *subject to the constraints*

$$\boxed{S_{micro} = S_{bek}}$$

$$n_5 - \bar{n}_5 = \hat{n}_5$$

$$n_1 - \bar{n}_1 = \hat{n}_1$$

$$n_p - \bar{n}_p = \hat{n}_p$$

$$E = m_5(n_5 + \bar{n}_5) + m_1(n_1 + \bar{n}_1) + m_p(n_p + \bar{n}_p)$$

(Horowitz, Maldacena, Strominger '96)

Take a neutral hole and add charges by boosting + dualities. This relates it to a near extremal hole, and we can find the emission from microscopics:

$$\boxed{\Gamma_{micro} = \Gamma_{hawking}}$$

(Das, SDM, Ramadevi '98)

Note that boosting in a compact direction is not an exact symmetry, but is presumably a good approximation for large charges (similar to the idea of Matrix theory)

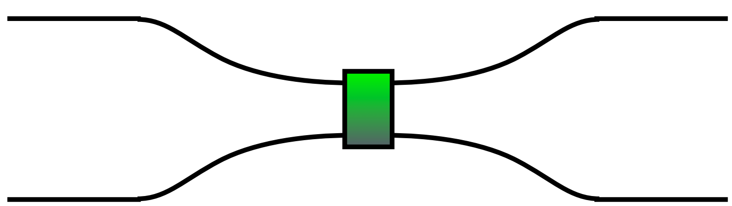
We have seen that the energies of the branes and antibranes just 'add', as if there were no interactions.

We get a similar simplification for 'pressures'

The compact directions have a size that relaxes gradually to its value at infinity.

From this behavior, we can deduce the 'pressure' on the compact directions from the branes.

One finds that this pressure is given by summing the tensions of all the branes and anti-branes (no interactions)



$$P = \sum_i (n_i + \bar{n}_i)\ p_i$$

Black holes in 3+1 dimensions

$$M_{9,1} \to M_{3,1} \times T^4 \times S^1 \times \tilde{S}^1$$

$D5$

$D1$

P

KK

Nontrivial fiber direction

$$\boxed{\begin{aligned} S_{micro} &= 2\pi(\sqrt{n_5}+\sqrt{\bar{n}_5})(\sqrt{n_1}+\sqrt{\bar{n}_1})(\sqrt{n_p}+\sqrt{\bar{n}_p})(\sqrt{n_{kk}}+\sqrt{\bar{n}_{kk}}) \\ &= S_{bek} \end{aligned}}$$

(Horowitz, Lowe, Maldacena '96)

$$\boxed{\Gamma_{micro} = \Gamma_{hawking}}$$

Basic lessons:

(a) Even if we have a neutral black hole, the energy of the hole goes to creating appropriate types of branes and anti-branes.

(b) These branes/anti-branes are of several different types, and the entropy is of the form

$$S_{micro} = 2\pi(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p})$$

(c) The total energy is just the sum of the masses of the branes and anti-branes

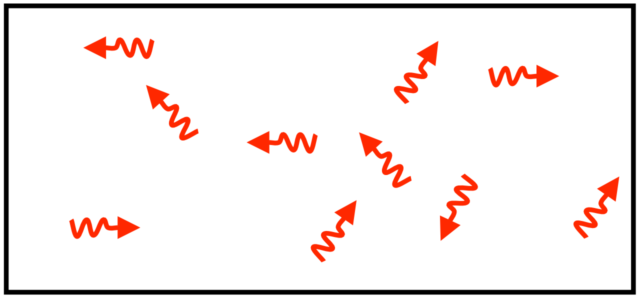
(d) The pressures on the directions wrapped by the branes are just given by adding the tensions produced by the branes and anti-branes

Let us apply these principles to the early Universe.

We let the Universe be a torus with volume V

Branes can wrap around all the cycles of this torus

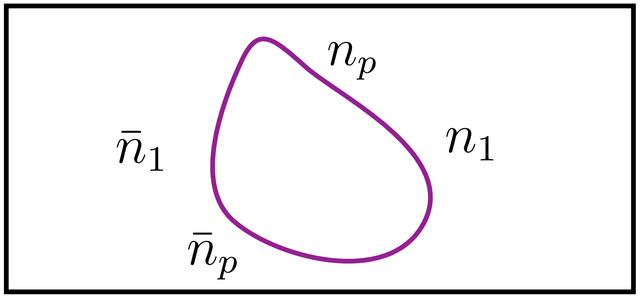
We look for the configuration with maximum entropy



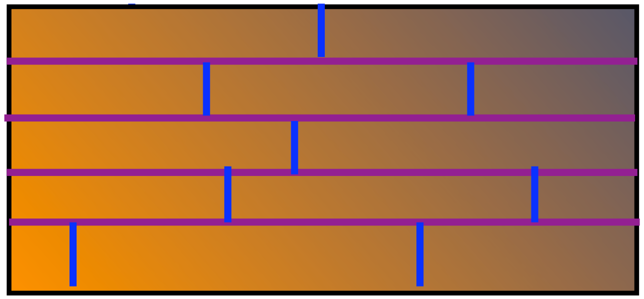
Radiation

$$\boxed{S \sim E^{\frac{D-1}{D}}}$$

Two charges



$$S = 2\pi\sqrt{2}(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p}) \sim \sqrt{E}\sqrt{E} \sim E$$



Three charges (4+1 d black hole)

$$S = 2\pi(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_p} + \sqrt{\bar{n}_p}) \sim E^{\frac{3}{2}}$$

This needs 5 compact directions ...

Four charges (3+1 d black hole) ... *this uses 6 compact directions*

$$S = 2\pi(\sqrt{n_1}+\sqrt{\bar{n}_1})(\sqrt{n_2}+\sqrt{\bar{n}_2})(\sqrt{n_3}+\sqrt{\bar{n}_3})(\sqrt{n_4}+\sqrt{\bar{n}_4}) \sim E^2$$

In M-theory language, we have 10 spatial directions and one time ...
so we have 10 cycles to wrap objects on

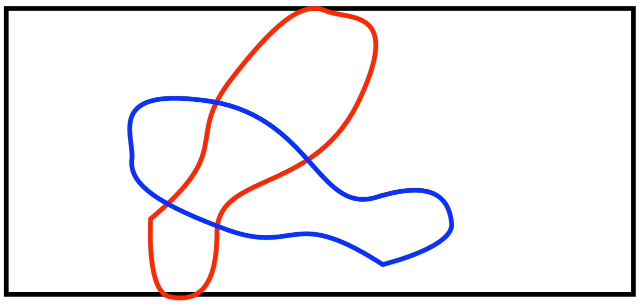
N charges,
postulate .....

$$\boxed{S = A_N \prod_{i=1}^{N}(\sqrt{n_i}+\sqrt{\bar{n}_i}) \sim E^{\frac{N}{2}}}$$

We will call such a state the 'Fractional brane state'

Fractional brane stated vs Brane gas

This state looks like a 'brane gas', but is actually different in many ways ...



In a brane gas, we have a dilute set of branes filling the Universe. The branes carry vibrations on their surface, and can interact when they cross

(a) The entropy of a brane gas is S ~ E, i.e. Hagedorn type, since it comes from vibrations of the brane surface

The entropy of the fractional brane state is

$$S = A_N \prod_{i=1}^{N} (\sqrt{n_i} + \sqrt{\bar{n}_i}) \sim E^{\frac{N}{2}}$$

(b) In a brane gas, we can take any set of branes to exist

In the fractional brane state, the energy goes to specific brane sets.
For example, if we have D1, then we can have D5 but not D3.
The branes we can have are such that they are pairwise BPS

Only in this case does the entropy grow as the product of brane numbers

We can take upto 9 kinds of branes in the fractional brane state.

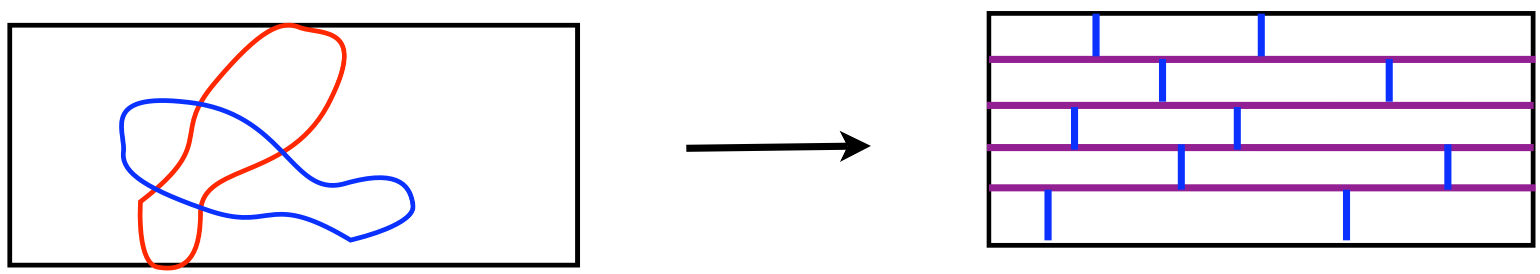
$$\boxed{S = A_N \prod_{i=1}^{N} (\sqrt{n_i} + \sqrt{\bar{n}_i}) \sim E^{\frac{N}{2}}}$$

(c) To understand the fractional brane state, start with a brane gas.
At low densities, the branes float around, with rare interactions

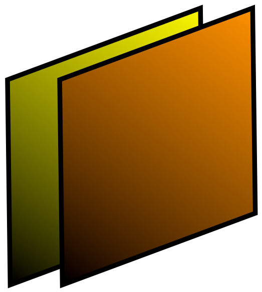
Increase the density of branes so that they are squeezed tightly together, tighter than planck spacing.

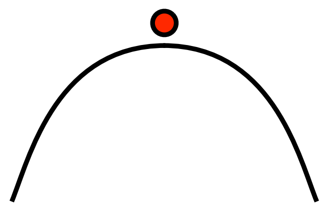
At this point, we might have expected annihilation, but instead the energy goes to creating a set of specific branes and antibranes in a specific kind of bound state. This state is metastable, decaying very very slowly.

This is the fractional brane state that we are after



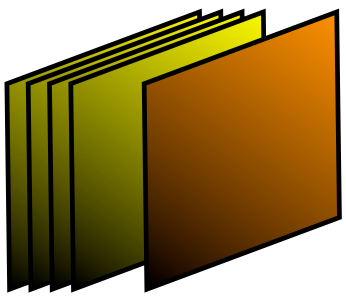
Why don't the branes and antibranes annihilate immediately?

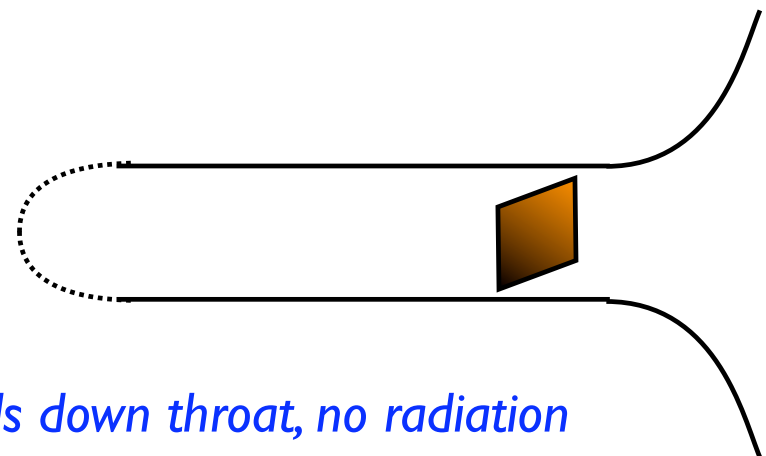




Tachyon at top of potential

(Sen '99)



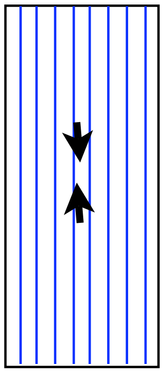


Antibrane falls down throat, no radiation emerges for a long time ...

Dhar, Mandal, Wadia, Yogendran '99
Lunin, SDM, Park, Saxena '03

Excitations of
the effective string
come in units of

$$\frac{2\pi k}{n_1 n_5 L}$$

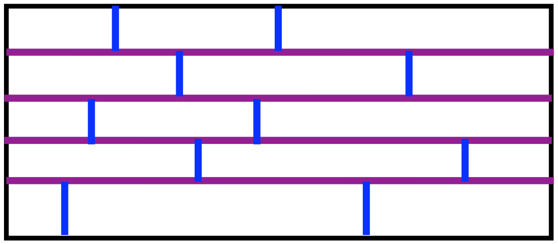


$$\frac{2\pi m}{L}$$

A graviton outside
the branes has to
have an integral
momentum

(a) Fractionation makes the excitations very low energy

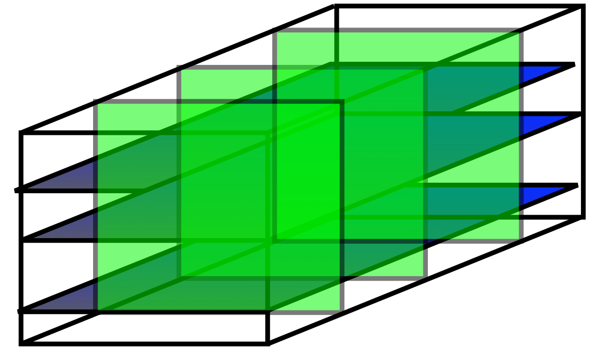
(b) An excitation with k = 7 cannot annihilate against k = -5 ... can annihilate with k = -7



Fractional branes and antibranes have to 'find' each other before they can annihilate ...

Solving for the evolution

Plan of the computation :



Assume Universe is a torus ...

Branes can wrap all directions of space ...

Assume entropy relation like the one for black holes ..

$$\boxed{S = A_N \prod_{i=1}^{N} (\sqrt{n_i} + \sqrt{\bar{n}_i}) \sim E^{\frac{N}{2}}}$$

We will work for general N, though in principle string theory should fix this (N=9?)

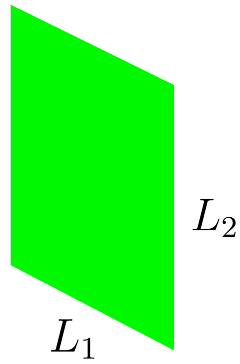
Just as for black holes, the energy and pressure are taken to be a simple sum over the energies and pressures of the branes/antibranes.

Solve for the evolution !

Step A : Find the number of branes and anti-branes by maximizing entropy S for given total mass of branes/anti-branes

Mass of a brane is given by its tension times its area

$$m_i = T_p \prod_j L_j$$



The Universe is neutral, so $\quad n_i = \bar{n}_i$

Maximize S for given total energy E

$$\tilde{S} = S - \lambda(E_{branes} - E) = A \prod_{i=1}^{N} \sqrt{n_i} - \lambda(2 \sum_i m_i n_i - E)$$

We find

$$n_k = \bar{n}_k = \frac{E}{2Nm_k}$$

Energy is equi-partitioned among different types of branes

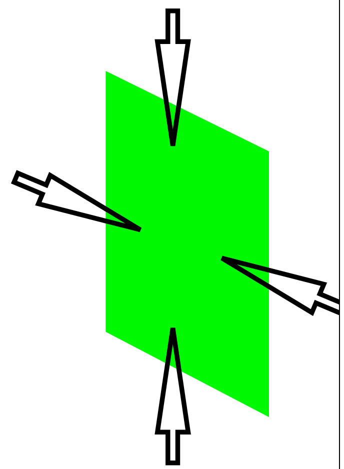
$$E_k = n_k m_k = \frac{E}{2N}$$

Step B : Find the stress tensor due to the branes/anti-branes

A brane has tension (negative pressure) along the directions where it extends, and zero pressure in the remaining directions

$$T^{(p)k}{}_{k} = -T_p \prod_{i=p+1}^{D-1} \hat{\delta}(x_i - \bar{x}_i), \qquad k = 1, \ldots, p$$

$$T^{(p)k}{}_{k} = 0, \qquad k = p+1, \ldots, (D-1)$$

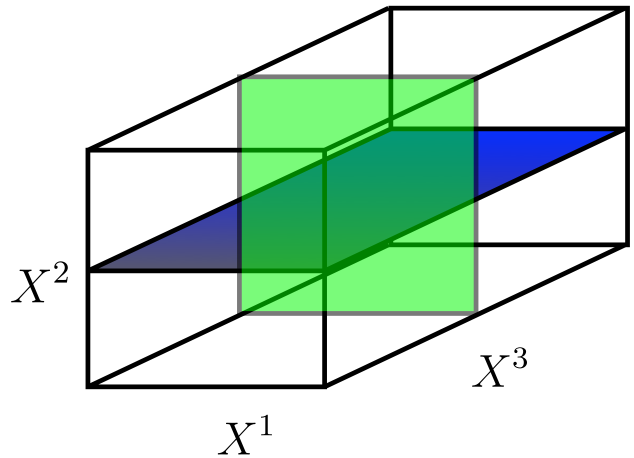


Following what we learnt from black holes, we will simply add the stress tensors from all the branes and anti-branes

Let there be N different types of branes/antibranes

Let N_i of these types extend along the direction X^i

Define $$w_i \equiv -\frac{N_i}{N}$$



$$N \;=\; 2$$

$$w = \{1, .5, .5\}$$

Then we find that when the entropy is maximized, the pressure in the direction X^i is given by

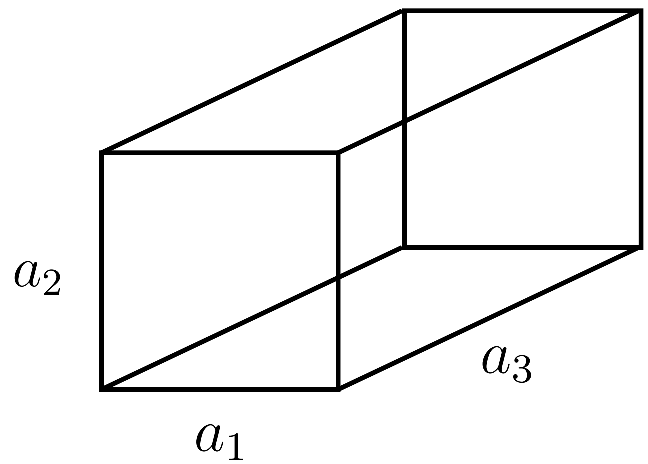
$$\boxed{p_i \;=\; w_i \; \rho}$$

where ρ is the energy density of the Universe

Step C : Solving Einstein's equations

We take a 'Kasner-type' metric ansatz

$$ds^2 = -dt^2 + \sum_{i=1}^{D-1} a_i^2(t) dx_i^2$$



And solve the Einstein equations with $p_i = w_i \, \rho$

Interestingly, the problem can be solved in closed form

(B. Chowdhury + SDM, 2006)

(some earlier work with similar equations had found numerical solutions)

The solution

Define the constants

$$W \equiv \sum_i w_i, \qquad U \equiv \sum_i w_i^2$$

(Recall that $w_i \equiv -\frac{N_i}{N}$)

Compute the constants

$$\begin{aligned} K_1 &= \frac{(D-1-W)}{2(D-2)} \\ K_2 &= -\frac{1}{2}[\frac{1-W}{D-2} W + U] \end{aligned}$$

$$\delta_k = \frac{1}{2}[\frac{1-W}{D-2} + w_k]$$

Then

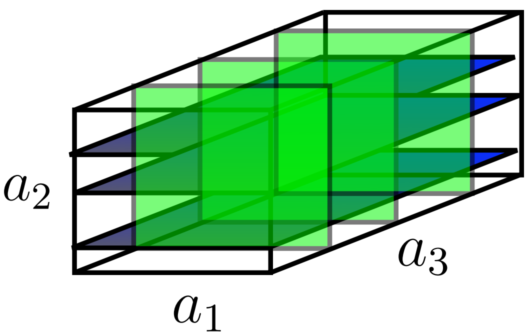
$$a_k = C_k \, (\tau - r_1)^{\frac{2(\delta_k r_1 + f_k)}{(K_1+K_2)(r_1-r_2)}} (\tau - r_2)^{-\frac{2(\delta_k r_2 + f_k)}{(K_1+K_2)(r_1-r_2)}}$$

where τ is an auxiliary time parameter given by

$$(t - t_0) = \frac{1}{A_4} \int_0^{\tau} (\tau' - r_1)^{\frac{2(-r_1 K_2 + A_2)}{(K_1+K_2)(r_1-r_2)}} (\tau' - r_2)^{-\frac{2(-r_2 K_2 + A_2)}{(K_1+K_2)(r_1-r_2)}} d\tau'$$

Recall that this integral is just the incomplete Beta function

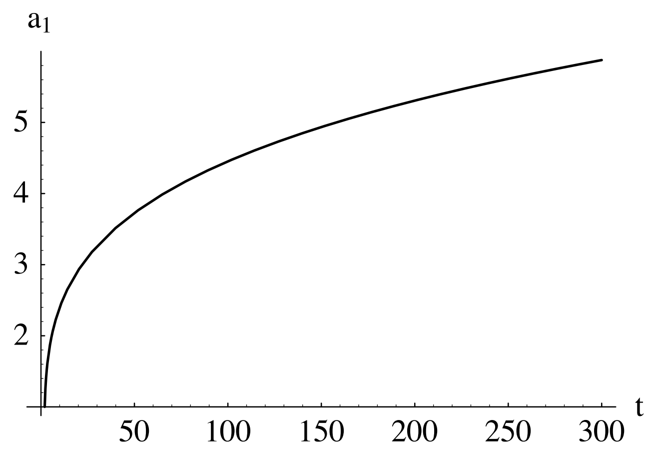
$$B_x(p, q) = \int_0^x s^{p-1}(1-s)^{q-1} ds$$



At late times the evolution becomes power law ...

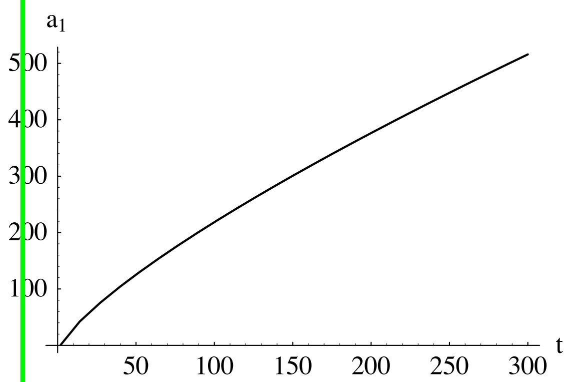
$$w_i = -.2 \text{ for all } i$$

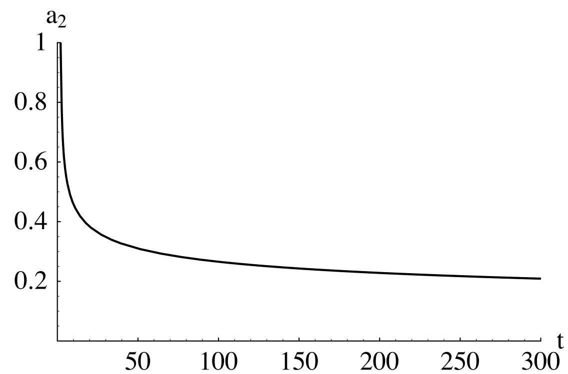
All a_i equal

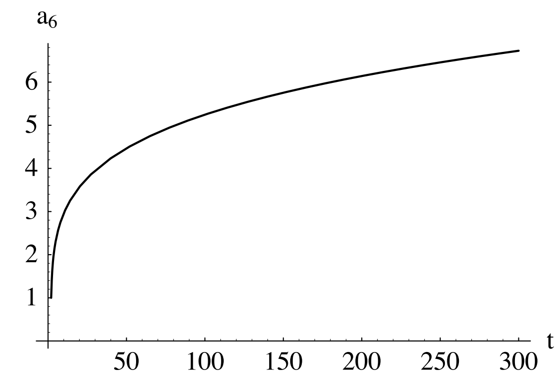


$$w_i = \{.9, -.9, -.9, -.9, -.9, -.1, -.1, -.1, -.1, -.1\}$$

All a_i start off with same value and same time derivative







(Several other cases and related ideas studied by Kalyanrama 2007)

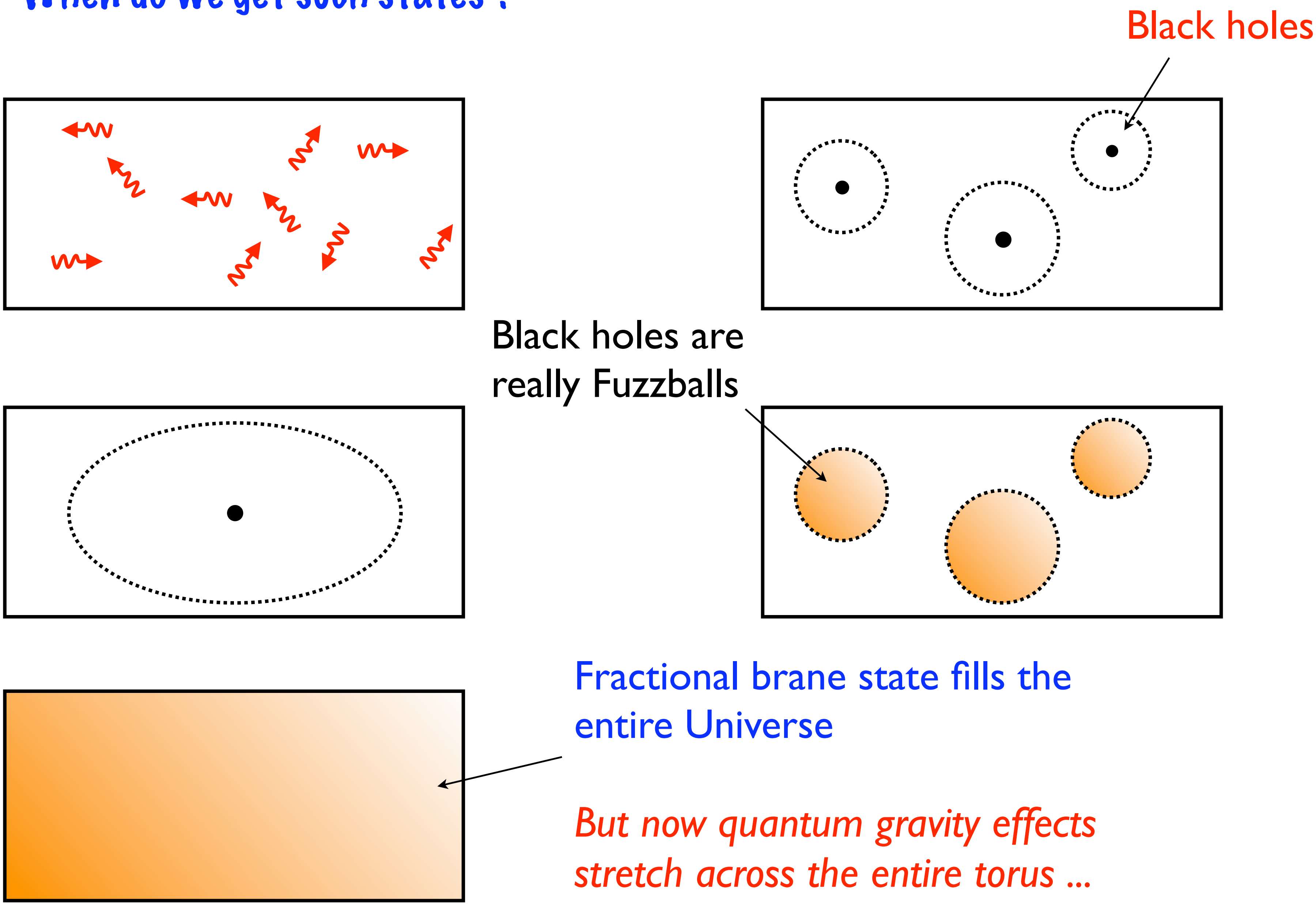
We do not seem to get an inflationary evolution ...

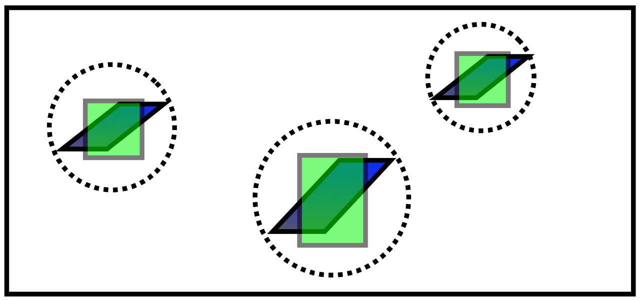
But quantum nonlocal effects can stretch all across
the Universe ....

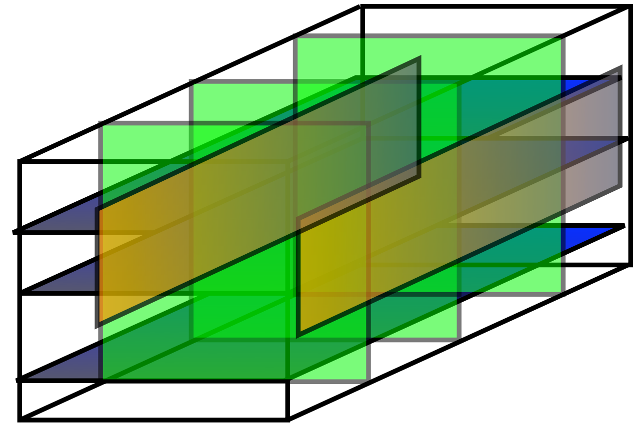
What is the physics of this Universe ? When do we get into a phase
like the one that we are studying ?

What kind of states should we get ?

When do we get such states ?







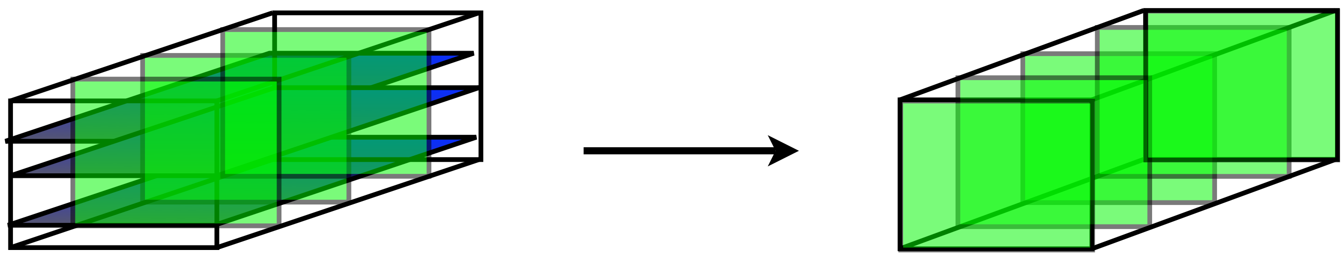
Black holes have a structure all the way to the horizon ...

Packing in more energy creates more of the 'same stuff' ,
we can keep increasing the density of the same stuff, getting
any amount of E in a given V

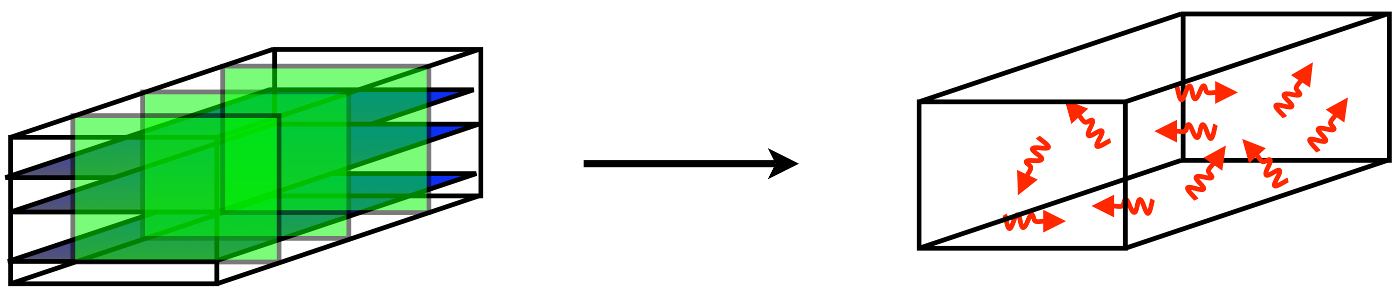
Changes in the fractional brane gas state (work in progress)

As the Universe evolves, the different cycles of the torus expand at different rates.

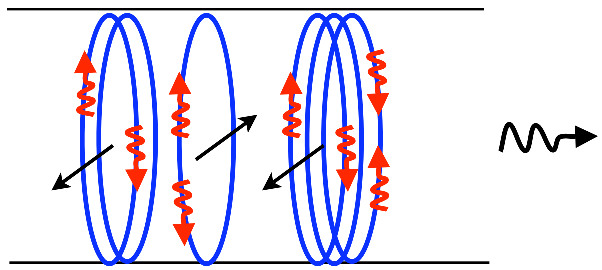
Some branes thus become much heavier than other branes, and it becomes entropically favorable to transfer their energy to the other sets of branes

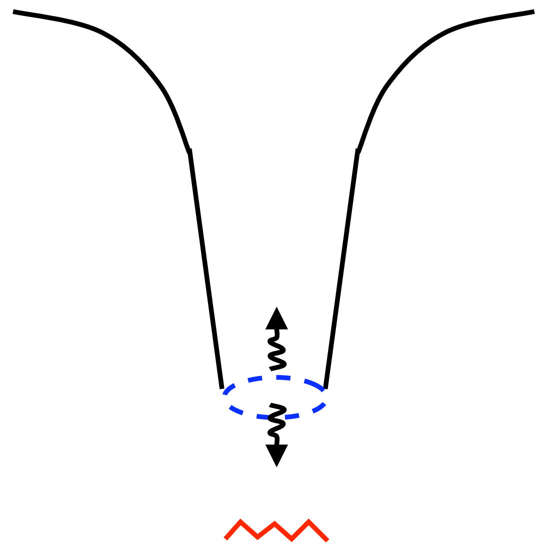


After sufficient expansion, it becomes entropically favorable to transfer all energy to radiation



But these annihilation processes are SLOW, since they are given by the Hawking radiation process for black holes





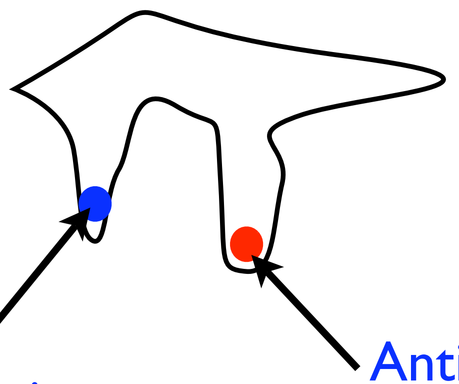
Radiation rates agree (Spins, greybody factors ...)

(Das-Mathur 96, Maldacena-Strominger 96)

Thus the system can be left out of equilibrium for long times ... when it does return to equilibrium, we can get a substantial entropy generation

Some possibilities to explore ....

If some branes do not annihilate, but stay trapped in different 'wells' of the Calabi-Yau manifold, then this could give the starting point for brane inflation models



CY manifold describing compact directions

Brane extending in noncompact directions

Antibrane extending in noncompact directions

If some fractional branes do not annihilate till today, they could be part of missing matter ....

Summary

A simple picture seems to emerge for matter at very high densities

The basic elements are:

(a) Fractionation: *When different kinds of branes are bound together, they 'fractionate' each other*, so that we get get a large number of objects with very low mass.

This large number of fractional objects gives the large black hole entropy, and the low mass gives very long distance effects, that stretch upto horizon radius.

Thus we get quantum gravtity effects over macroscopic distances

(b) Brane-antibrane pairs: If we have energy but no charge, then we get the maximal entropic state by using the energy to make *brane-antibrane pairs*, which then fractionate as above.

(c) Quasi-free constituents: *These fractional objects seem to be essentially free*, so that we get the total energy, pressure, entropy by just adding the contributions from individual fractional branes.

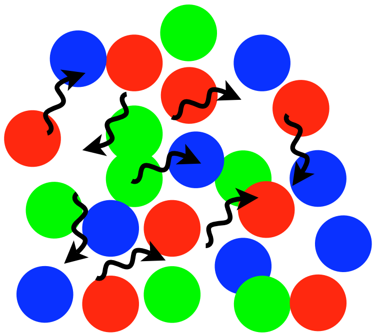
Analogy: Quark-Gluon plasma:

The dynamics of hadrons is very complicated

But if we go to very high densities and very high energies, the physics simplifies

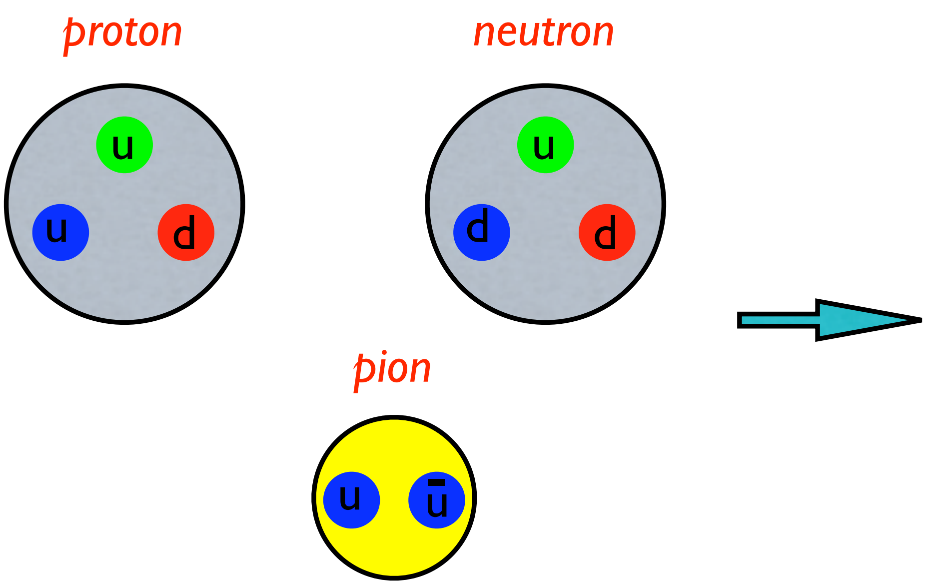
To see this simplified physics, we must use the correct dynamical objects - the quarks and gluons

At high energy density the quarks and gluons are essentially free ...



These simple observations suggest that there is a deeper detailed theory of matter at high energy densities ...

This would be similar to the case of strong interactions ... From hadron classification and scattering *quarks* were deduced, but QCD came later ....



$$\mathcal{L} = \int dx [-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \frac{i}{2} \bar{\psi} \partial \psi + \ldots]$$

Questions :

(a) Is it a correct principle to ask for maximal entropy in the early Universe ?

(b) Does the Hartle-Hawking process lead to such a state ?

(c) How much of the energy should be in a 'brane bound state' ?

String theory should be able to supply all the other answers ...