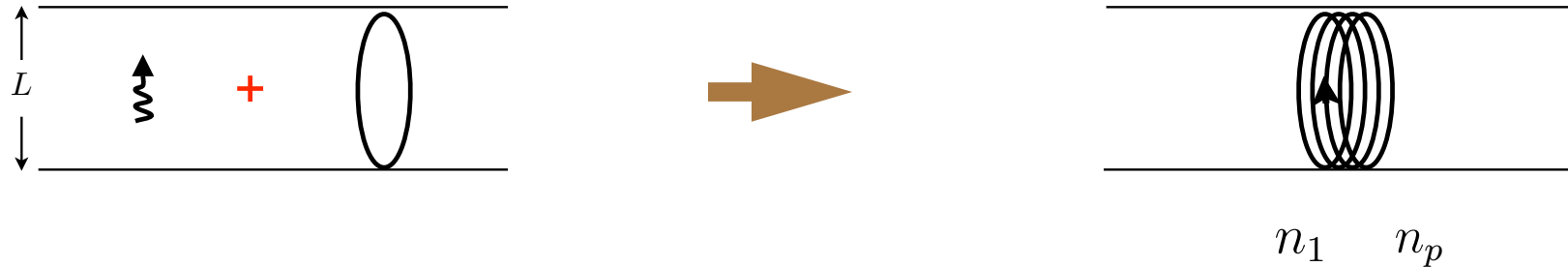


Lecture 3

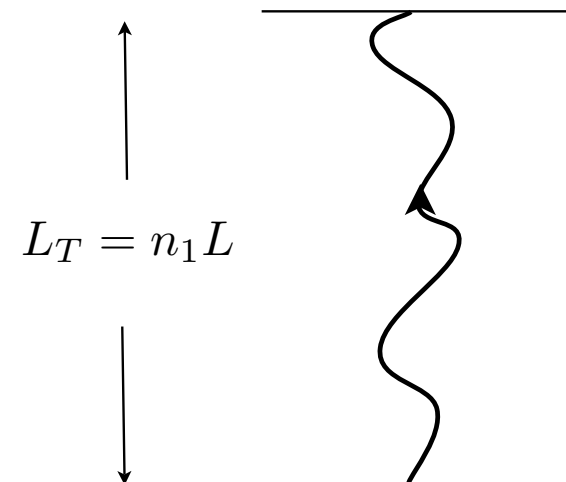
Constructing Fuzzballs

Dynamical behavior: results and conjectures

Recall the way we made the 2-charge black hole ...



This allowed us to count the states of the black hole, so we solve the entropy problem, but what about the information puzzle?



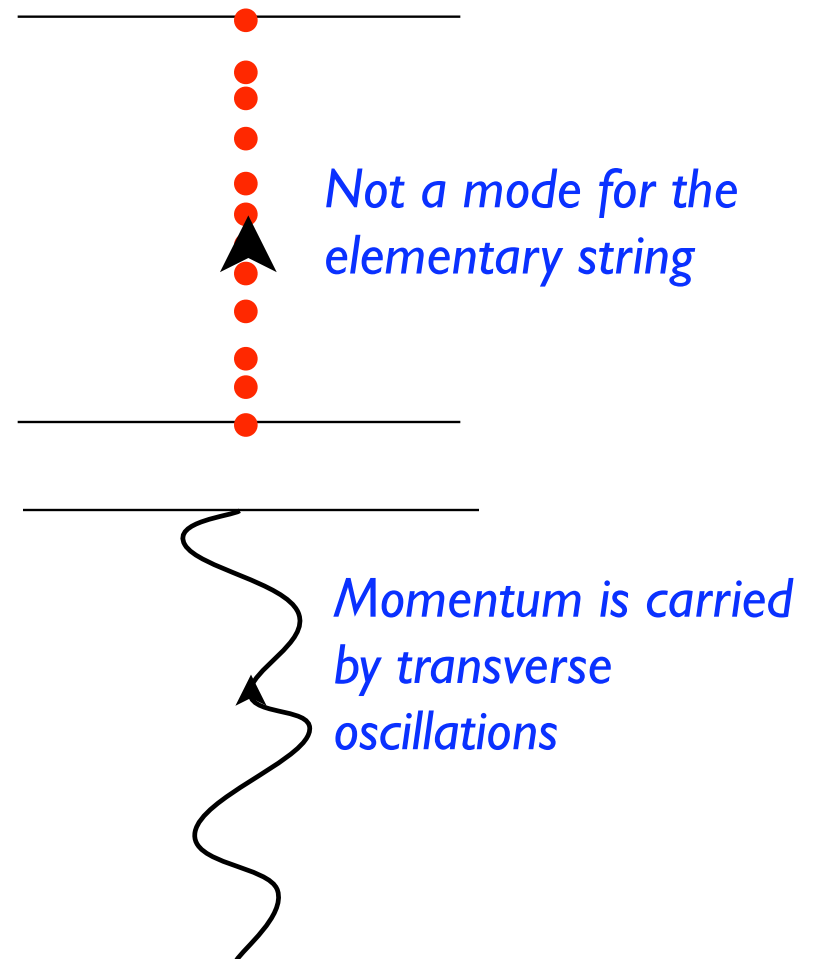
A key point

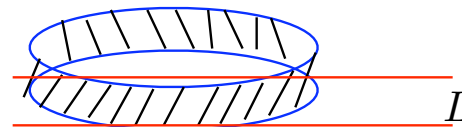
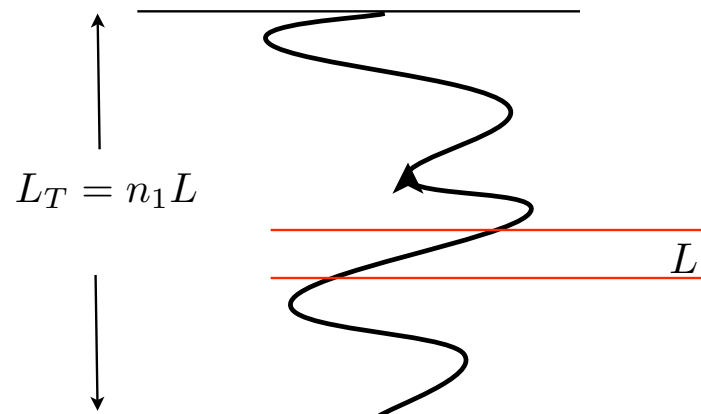
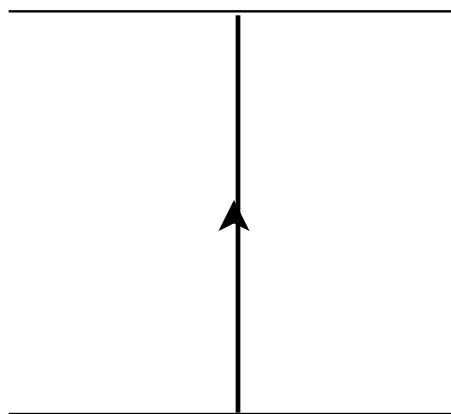
The elementary string (NSI) does not have any LONGITUDINAL vibration modes

This is because it is not made up of 'more elementary particles'

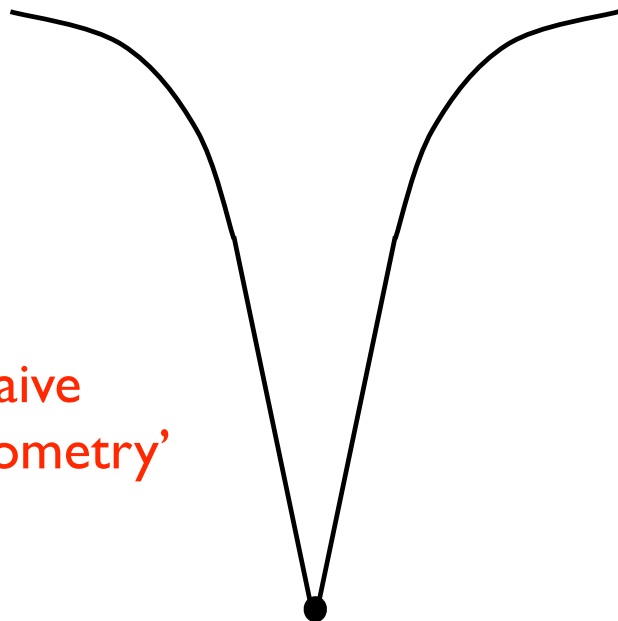
Thus only transverse oscillations are permitted

This causes the string to spread over a nonzero transverse area

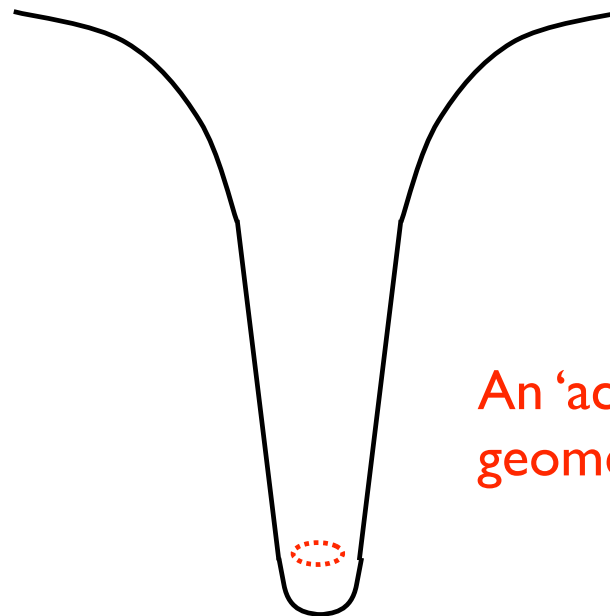




‘Naive
geometry’



An ‘actual
geometry’



Making the geometry

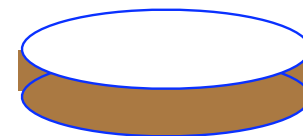
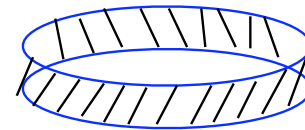
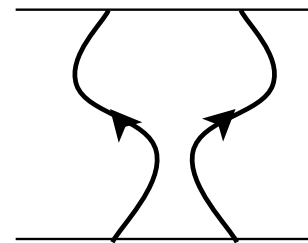
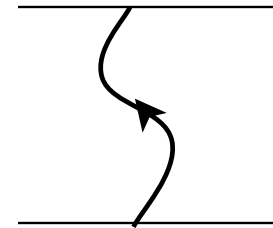
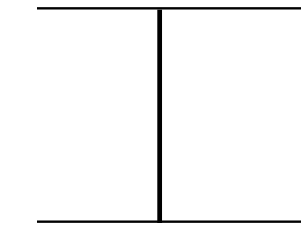
We know the metric of one straight strand of string

We know the metric of a string carrying a wave -- 'Vachaspati transform'

We get the metric for many strands by superposing harmonic functions from each strand

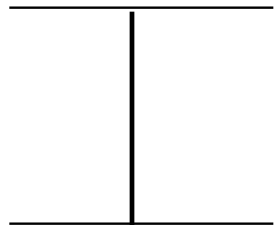
(Dabholkar, Gauntlett, Harvey, Waldram '95, Callan, Maldacena, Peet '95)

In our present case, we have a large number of strands, so we 'smear over them to make a continuous 'strip' (Lunin+SDM '01)



Let us now carry out these steps:

Step I: We write the metric of a single strand of string



$$ds_{string}^2 = H_1^{-1}[-dt^2 + dy^2] + \sum_{i=1}^8 dx_i dx_i$$

$$e^{2\phi} = H_1^{-1}$$

$$H_1 = 1 + \frac{Q_1}{r^6}$$

Step 2: Adding momentum

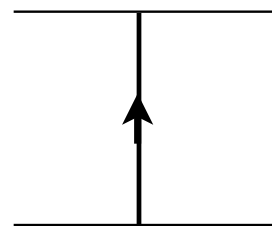
The naive metric is

$$ds_{string}^2 = H[-dudv + Kdv^2] + \sum_{i=1}^4 dx_i dx_i + \sum_{a=1}^4 dz_a dz_a$$

$$B_{uv} = -\frac{1}{2}[H - 1]$$

$$e^{2\phi} = H$$

$$H^{-1} = 1 + \frac{Q_1}{r^2}, \quad K = \frac{Q_p}{r^2}$$



Step 2: Done correctly, actual metric

$$ds_{string}^2 = H[-dudv + Kdv^2 + 2A_idx_idv] + \sum_{i=1}^4 dx_idx_i + \sum_{a=1}^4 dz_adz_a$$

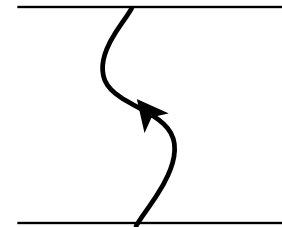
$$B_{uv} = -\frac{1}{2}[H - 1], \quad B_{vi} = HA_i$$

$$e^{2\phi} = H$$

$$H^{-1}(\vec{x}, y, t) = 1 + \frac{Q_1}{|\vec{x} - \vec{F}(t - y)|^2}$$

$$K(\vec{x}, y, t) = \frac{Q_1|\dot{\vec{F}}(t - y)|^2}{|\vec{x} - \vec{F}(t - y)|^2}$$

$$A_i(\vec{x}, y, t) = -\frac{Q_1\dot{F}_i(t - y)}{|\vec{x} - \vec{F}(t - y)|^2}$$



Step 3: Adding over strands

$$ds_{string}^2 = H[-dudv + Kdv^2 + 2A_i dx_i dv] + \sum_{i=1}^4 dx_i dx_i + \sum_{a=1}^4 dz_a dz_a$$

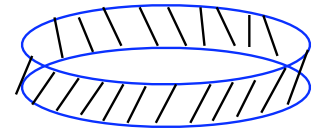
$$B_{uv} = -\frac{1}{2}[H - 1], \quad B_{vi} = HA_i$$

$$e^{2\phi} = H$$

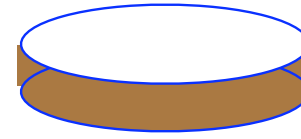
$$H^{-1}(\vec{x}, y, t) = 1 + \sum_s \frac{Q_1^{(s)}}{|\vec{x} - \vec{F}^{(s)}(t - y)|^2}$$

$$K(\vec{x}, y, t) = \sum_s \frac{Q_1^{(s)} |\dot{\vec{F}}^{(s)}(t - y)|^2}{|\vec{x} - \vec{F}^{(s)}(t - y)|^2}$$

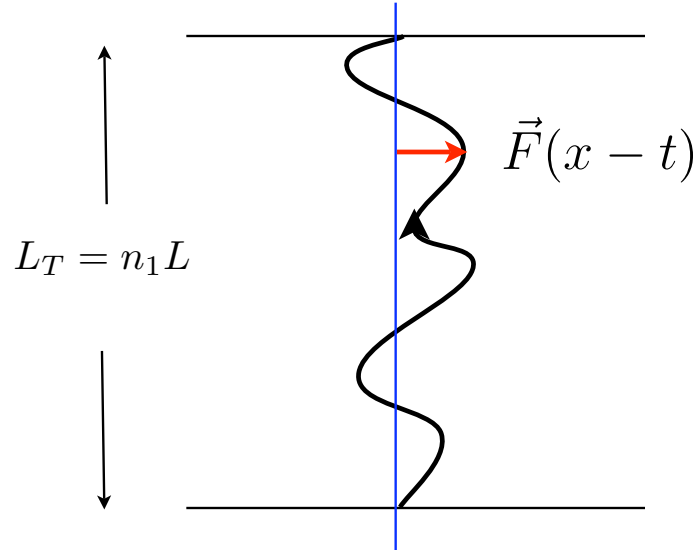
$$A_i(\vec{x}, y, t) = - \sum_s \frac{Q_1^{(s)} \dot{F}_i^{(s)}(t - y)}{|\vec{x} - \vec{F}^{(s)}(t - y)|^2}$$



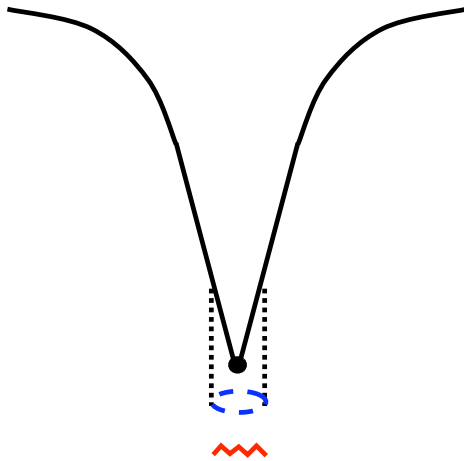
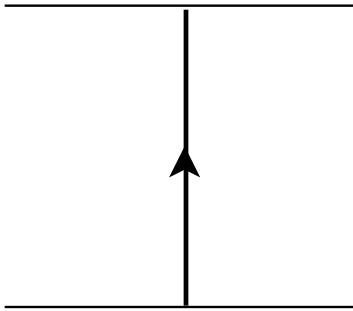
Step 4: Smoothing over strands:



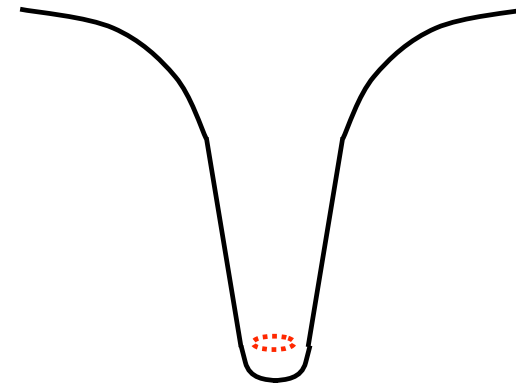
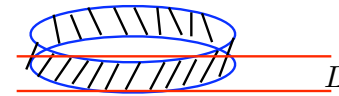
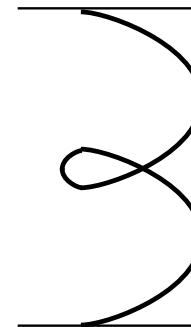
$$\begin{aligned}
 ds_{string}^2 &= H[-dudv + Kdv^2 + 2A_idx_idv] + \sum_{i=1}^4 dx_idx_i + \sum_{a=1}^4 dz_adz_a \\
 B_{uv} &= \frac{1}{2}[H - 1], & B_{vi} &= HA_i \\
 e^{2\phi} &= H
 \end{aligned}$$



$$\begin{aligned}
 H^{-1} &= 1 + \frac{Q_1}{L_T} \int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2} \\
 K &= \frac{Q_1}{L_T} \int_0^{L_T} \frac{dv(\dot{F}(v))^2}{|\vec{x} - \vec{F}(v)|^2} \\
 A_i &= -\frac{Q_1}{L_T} \int_0^{L_T} \frac{dv\dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2}
 \end{aligned}$$

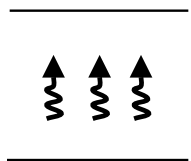


‘Naive NSI-P
geometry’



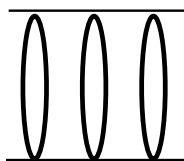
Actual NSI-P
geometry

DI-D5 \longleftrightarrow NSI-P



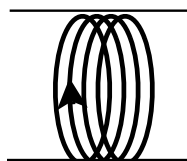
$$n'_p = n_1$$

P

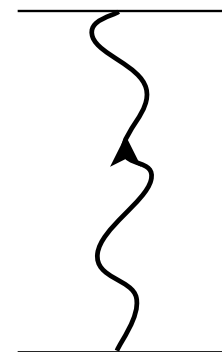


$$n'_1 = n_5$$

NSI



String carrying
 $n'_p n'_1$ units
of lightest excitation



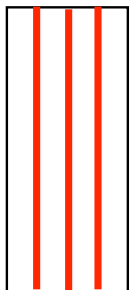
$$\sum k m_k = n'_p n'_1$$



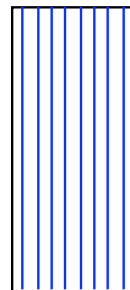
T^4



S^1



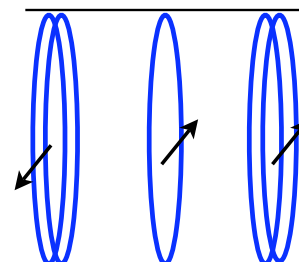
+



n_1
DI branes

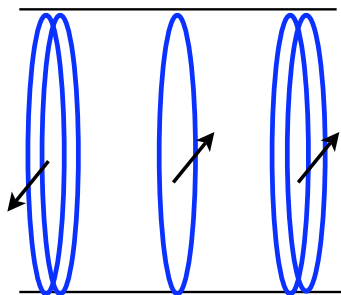
n_5
D5 branes

'Effective string' with
total winding number
 $n_1 n_5$

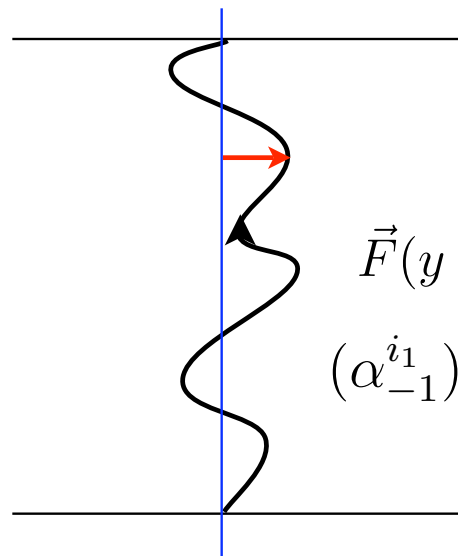


$$\sum k m_k = n_1 n_5$$

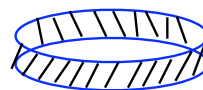
DI-D5
CFT state



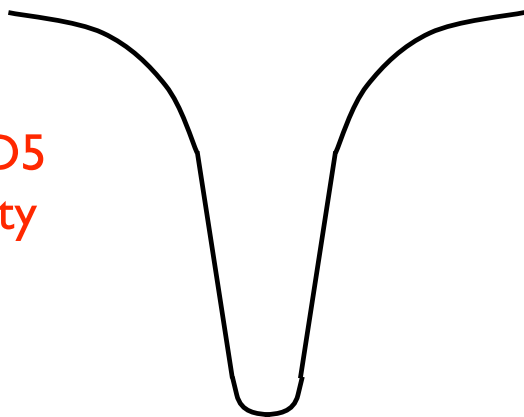
NSI-P
state



$$\vec{F}(y - ct) = (\alpha_{-1}^{i_1})^{n_1} (\alpha_{-2}^{i_2})^{n_2} \dots |0\rangle$$



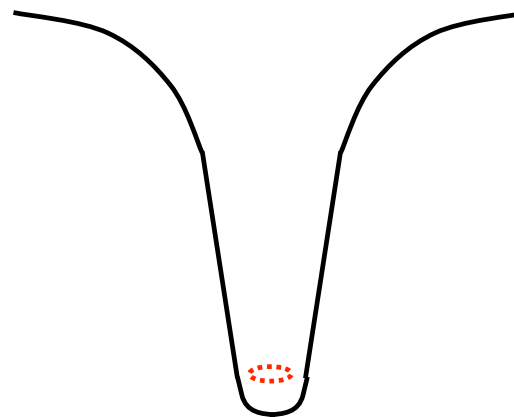
DI-D5
gravity
dual



S,T
dualities



NSI-P
geometry



Geometry for D1-D5

$$ds^2 = \sqrt{\frac{H}{1+K}} [-(dt - A_i dx^i)^2 + (dy + B_i dx^i)^2] \\ + \sqrt{\frac{1+K}{H}} dx_i dx_i + \sqrt{H(1+K)} dz_a dz_a$$

$$H^{-1} = 1 + \frac{Q}{L_T} \int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2} \\ K = \frac{Q}{L_T} \int_0^{L_T} \frac{dv (\dot{F}(v))^2}{|\vec{x} - \vec{F}(v)|^2} \\ A_i = -\frac{Q}{L_T} \int_0^{L_T} \frac{dv \dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2}$$

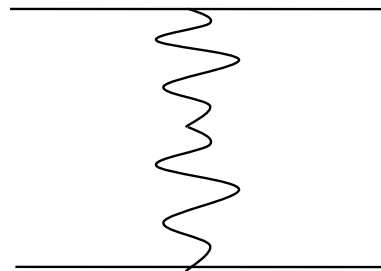
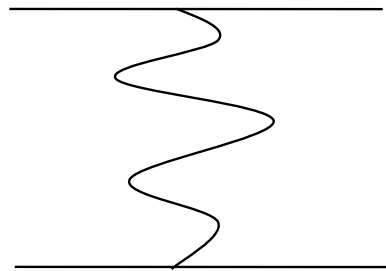
$$dB = - *_4 dA$$

(Lunin+SDM '01,
also
'Supergravity supertubes'
(Empanan+Mateos+Townsend '01))

(a) Size depends on mean harmonic number

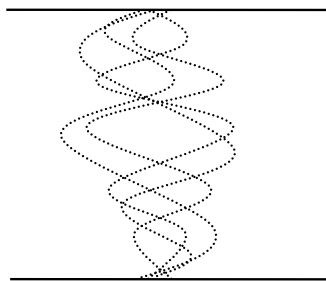
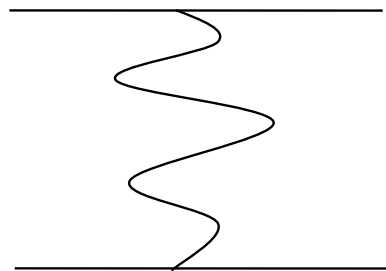
$$\sum k m_k = n_1 n_p$$

(b) Fluctuations depend on occupation number



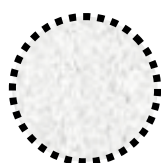
*Put energy in a few
harmonics, large
occupation number
for each harmonic*

Coherent
states



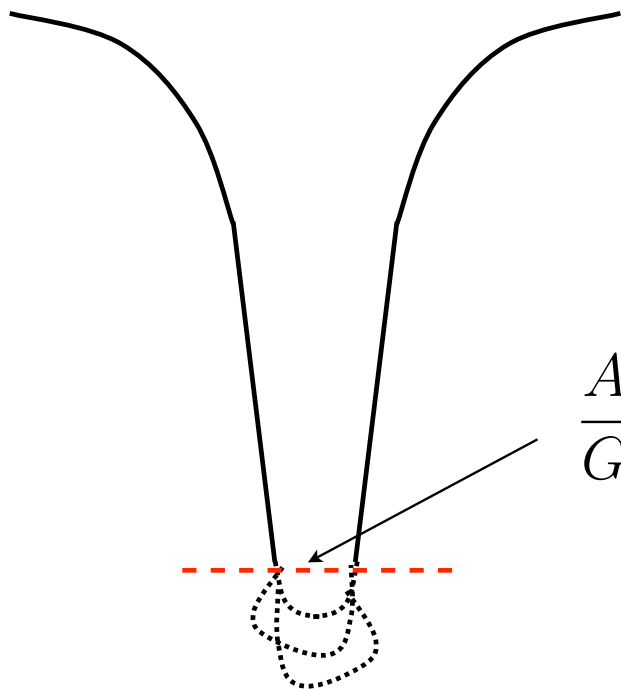
*Energy in
many harmonics,
occupation number
order unity in each*

Generic
quantum
state



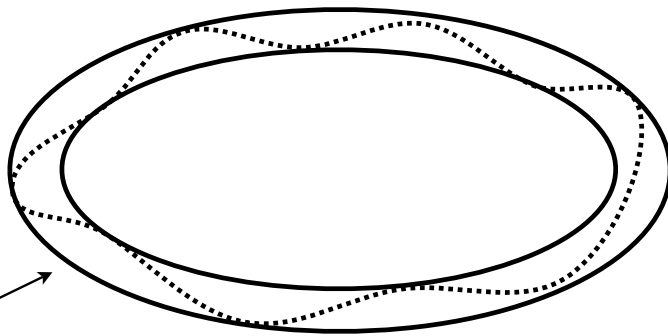
'Fuzzball'

Scale of the 'fuzzball'



$$\frac{A}{G} \sim \sqrt{n_1 n_p} \sim S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_p}$$

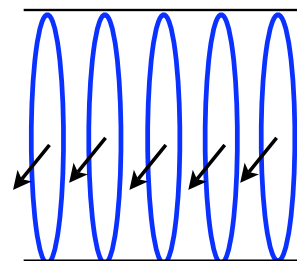
(Lunin+SDM '02)



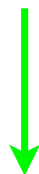
$$\frac{A}{G} \sim \sqrt{n_1 n_p - J} \sim S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_p - J}$$

A simple example

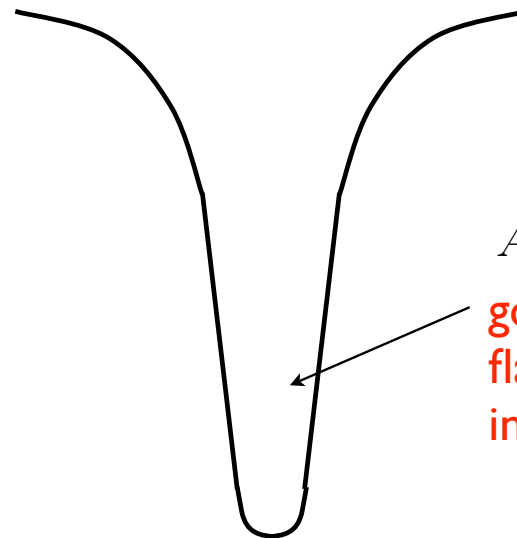
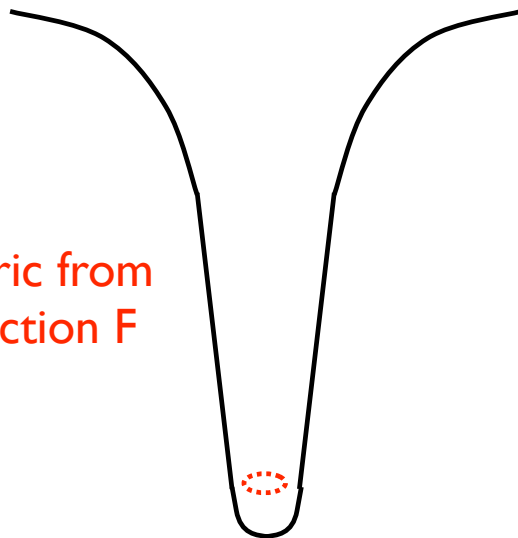
NSI- P : one turn
of a uniform helix



D1-D5: CFT state
has all loops
'singly wound',
and all spins aligned



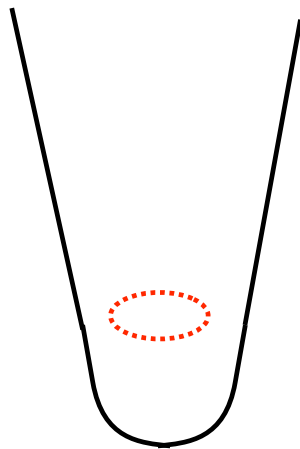
Make metric from
profile function F



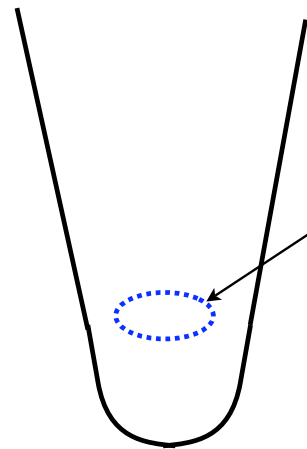
$AdS_3 \times S^3$
going over to
flat space at
infinity

$$\begin{aligned}
ds^2 = & -H_1^{-1}(dt^2 - dy^2) + H_5 f \left(d\theta^2 + \frac{dr^2}{r^2 + a^2} \right) - \frac{2a\sqrt{Q'_1 Q'_5}}{H_1 f} (\cos^2 \theta dy d\psi + \sin^2 \theta dt d\phi) \\
& + H_5 \left[\left(r^2 + \frac{a^2 Q'_1 Q'_5 \cos^2 \theta}{H_1 H_5 f^2} \right) \cos^2 \theta d\psi^2 + \left(r^2 + a^2 - \frac{a^2 Q'_1 Q'_5 \sin^2 \theta}{H_1 H_5 f^2} \right) \sin^2 \theta d\phi^2 \right] \\
& + dz_a dz_a
\end{aligned}$$

$$f = r^2 + a^2 \cos^2 \theta, \quad H_1 = 1 + \frac{Q'_1}{f}, \quad H_5 = 1 + \frac{Q'_5}{f}$$



NSI-P

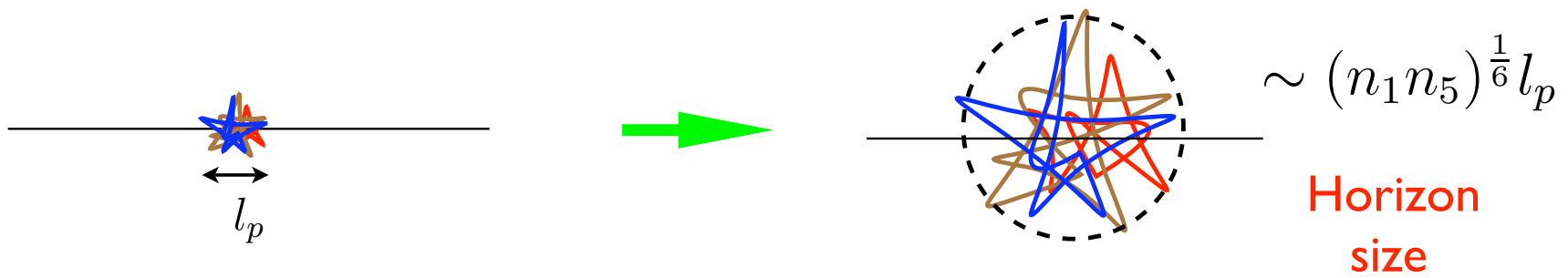


DI-D5

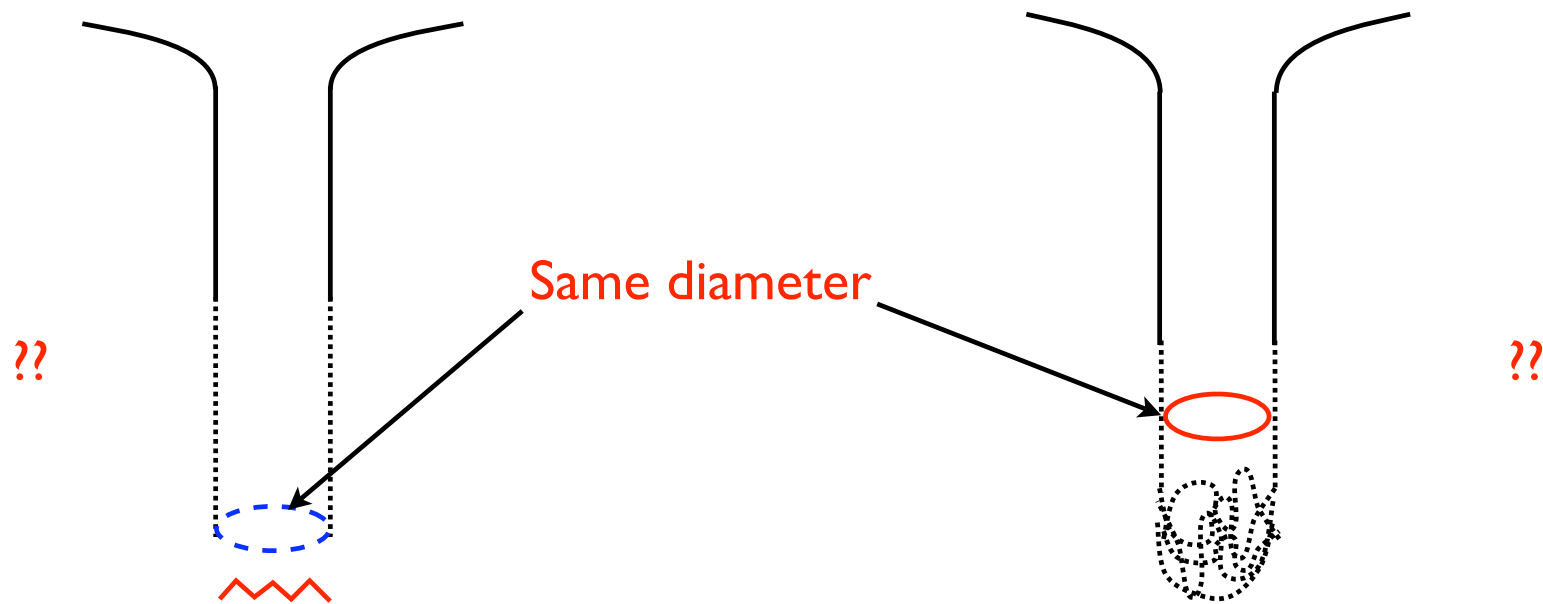
Each point on the ring is the center of a Kaluza-Klein monopole

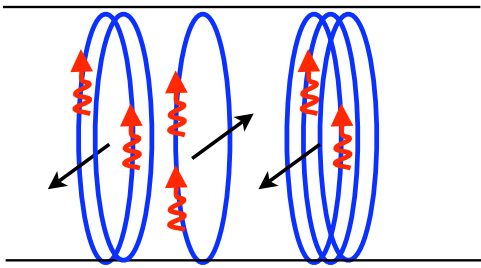
No net KK monopole charge :
KK is a *dipole charge*

2-charge extremal D1D5 :

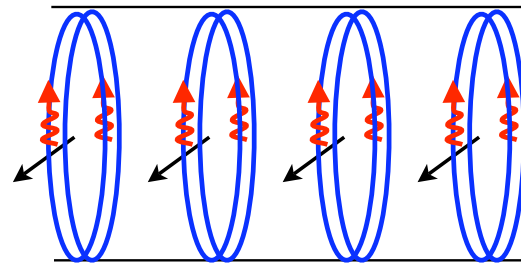


3-charge extremal D1D5 P ?





Generic D1D5P CFT state

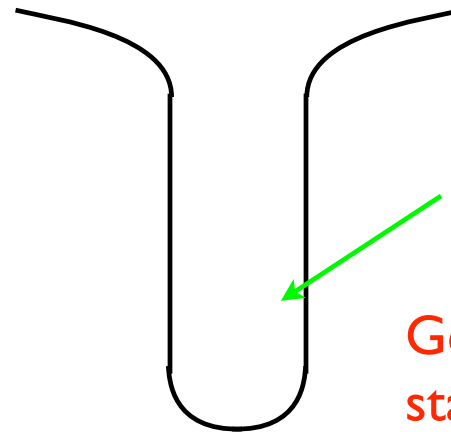


Simple states: all components the same,
excitations fermionic, spin aligned

$$|k\rangle^{total} = (J_{-(2k-2)}^{-,total})^{n_1 n_5} (J_{-(2k-4)}^{-,total})^{n_1 n_5} \dots (J_{-2}^{-,total})^{n_1 n_5} |1\rangle^{total}$$

Can make geometries for
these simple states :

$U(1) \times U(1)$ symmetry



$AdS_3 \times S^3 \times T^4$

Geometry for simple
state (winding = 1)

$$\begin{aligned}
ds^2 = & -\frac{1}{h}(dt^2 - dy^2) + \frac{Q_p}{hf}(dt - dy)^2 + hf \left(\frac{dr_N^2}{r_N^2 + a^2\eta} + d\theta^2 \right) \\
& + h \left(r_N^2 - na^2\eta + \frac{(2n+1)a^2\eta Q_1 Q_5 \cos^2 \theta}{h^2 f^2} \right) \cos^2 \theta d\psi^2 \\
& + h \left(r_N^2 + (n+1)a^2\eta - \frac{(2n+1)a^2\eta Q_1 Q_5 \sin^2 \theta}{h^2 f^2} \right) \sin^2 \theta d\phi^2 \\
& + \frac{a^2\eta^2 Q_p}{hf} (\cos^2 \theta d\psi + \sin^2 \theta d\phi)^2 \\
& + \frac{2a\sqrt{Q_1 Q_5}}{hf} [n \cos^2 \theta d\psi - (n+1) \sin^2 \theta d\phi] (dt - dy) \\
& - \frac{2a\eta\sqrt{Q_1 Q_5}}{hf} [\cos^2 \theta d\psi + \sin^2 \theta d\phi] dy + \sqrt{\frac{H_1}{H_5}} \sum_{i=1}^4 dz_i^2
\end{aligned}$$

$$\begin{aligned}
f &= r_N^2 - a^2\eta n \sin^2 \theta + a^2\eta (n+1) \cos^2 \theta \\
h &= \sqrt{H_1 H_5}, \quad H_1 = 1 + \frac{Q_1}{f}, \quad H_5 = 1 + \frac{Q_5}{f}
\end{aligned}$$

$$\eta \equiv \frac{Q_1 Q_5}{Q_1 Q_5 + Q_1 Q_p + Q_5 Q_p}$$

(Giusto SDM Saxena 04)

2-charges, $4+1$ dimensions, noncompact excitations: Lunin+SDM '01

2-charges, $4+1$ d, torus excitations: Lunin+Maldacena+Maoz '02, Skenderis
+Taylor '07

2-charges, $4+1$ d, fermionic excitations: Taylor '05

3-charges, $4+1$ d, one charge 'test quantum' wavefunction;
SDM+Saxena+Srivastava '03

3-charge, $4+1$ d, $U(1) \times U(1)$ axial symmetry: Giusto+SDM+Saxena '04,
Lunin '04

3-charge, $4+1$ d, $U(1)$ axial symmetry: Bena+Kraus '05,
Berglund+Gimon+Levi '05

3 charges, $3+1$ d, $U(1)$ axial symmetry: Bena+Kraus '05

4-charges, $3+1$ d, $U(1) \times U(1)$ symmetry: Saxena+Giusto+Potvin+Peet '05

4 charges, $3+1$ d, $U(1)$ symmetry: Balasubramanian+Gimon+Levi '06

Non-extremal geometries, 3 charges, $4+1$ d, $U(1) \times U(1)$ axial symmetry:
Jejjala+Madden+Ross+Titchener 05

Non-extremal geometries, 4 charges, $3+1$ d, $U(1) \times U(1)$ axial symmetry:
Giusto+Ross+Saxena 07

2-charges, $4+1$ d, K3 compactification: Skenderis+Taylor 07

2-charges, 1 -point functions: Skenderis+Taylor 06

General structure of extremal solutions: hyperkahler base + 2-d fiber
(Gauntlett+Gutowski+Hull+Pakis+Reall 02, Gutowski+Martelli+Reall 03)

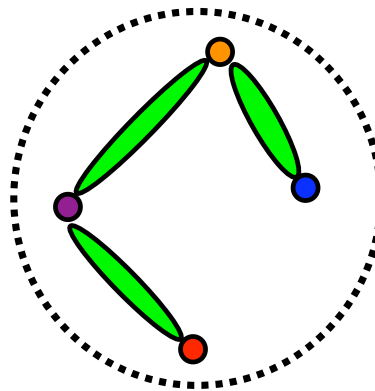
Decomposing known microstate solutions into base + fiber:
hyperkahler \longrightarrow psedo-hyperkahler
(Giusto+SDM 04)

Structure of general 3-charge and 4-charge geometries :

Bound states of branes is on Higgs branch. Dipole charges form, are held apart by fluxes ...

(Bena+Warner 05)

$g \rightarrow 0$



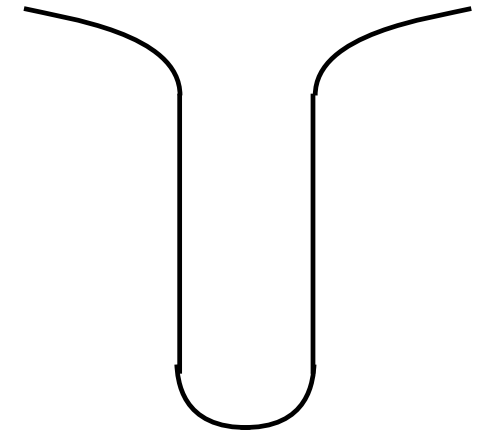
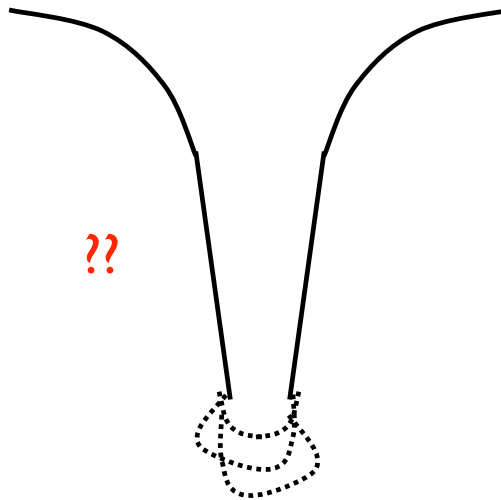
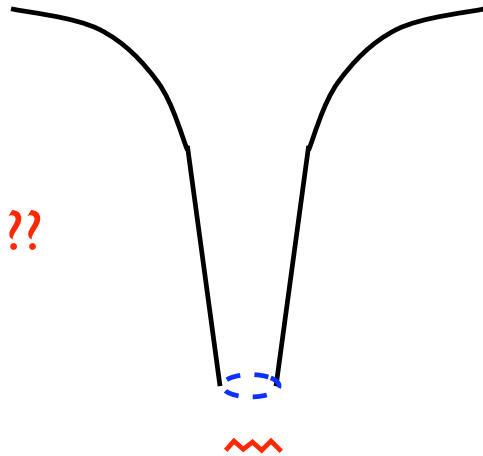
g nonzero

If we reduce to 3+1 dimensions, get metrics for 'branes at angles' (Denef '02, Balasubramanian+Gimon+Levi 05)

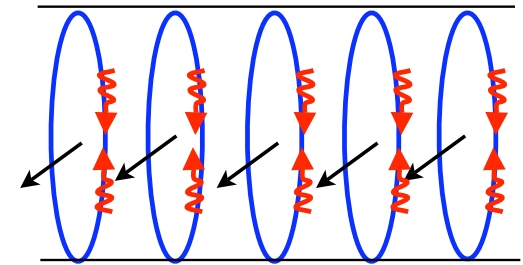
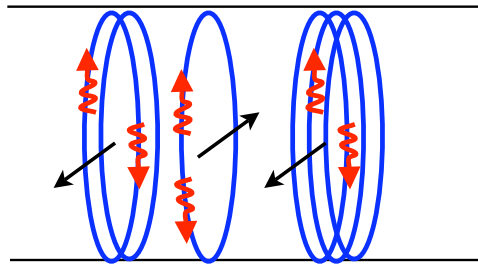
Recent work (Bena+Bobev+Ruef+Warner 08) ... supertubes in the 'throat' might give correct order for number of states ...

The Non-Extremal Hole :

(Jejalla, Madden,
Ross Titchener '05)



DI-D5 CFT has both
left and right moving
excitations



Gravity dual again has
no horizon or singularity

$$\begin{aligned}
ds^2 = & -\frac{f}{\sqrt{\tilde{H}_1 \tilde{H}_5}}(dt^2 - dy^2) + \frac{M}{\sqrt{\tilde{H}_1 \tilde{H}_5}}(s_p dy - c_p dt)^2 \\
& + \sqrt{\tilde{H}_1 \tilde{H}_5} \left(\frac{r^2 dr^2}{(r^2 + a_1^2)(r^2 + a_2^2) - Mr^2} + d\theta^2 \right) \\
& + \left(\sqrt{\tilde{H}_1 \tilde{H}_5} - (a_2^2 - a_1^2) \frac{(\tilde{H}_1 + \tilde{H}_5 - f) \cos^2 \theta}{\sqrt{\tilde{H}_1 \tilde{H}_5}} \right) \cos^2 \theta d\psi^2 \\
& + \left(\sqrt{\tilde{H}_1 \tilde{H}_5} + (a_2^2 - a_1^2) \frac{(\tilde{H}_1 + \tilde{H}_5 - f) \sin^2 \theta}{\sqrt{\tilde{H}_1 \tilde{H}_5}} \right) \sin^2 \theta d\phi^2 \\
& + \frac{M}{\sqrt{\tilde{H}_1 \tilde{H}_5}} (a_1 \cos^2 \theta d\psi + a_2 \sin^2 \theta d\phi)^2 \\
& + \frac{2M \cos^2 \theta}{\sqrt{\tilde{H}_1 \tilde{H}_5}} [(a_1 c_1 c_5 c_p - a_2 s_1 s_5 s_p) dt + (a_2 s_1 s_5 c_p - a_1 c_1 c_5 s_p) dy] d\psi \\
& + \frac{2M \sin^2 \theta}{\sqrt{\tilde{H}_1 \tilde{H}_5}} [(a_2 c_1 c_5 c_p - a_1 s_1 s_5 s_p) dt + (a_1 s_1 s_5 c_p - a_2 c_1 c_5 s_p) dy] d\phi \\
& + \sqrt{\frac{\tilde{H}_1}{\tilde{H}_5}} \sum_{i=1}^4 dz_i^2
\end{aligned}$$

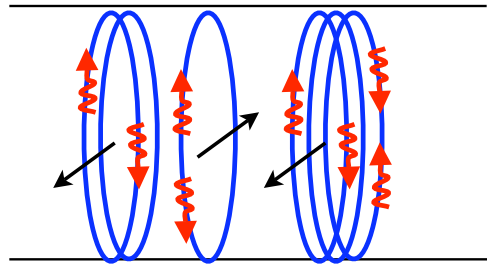
$$\begin{aligned}
Q_1 &= \frac{g\alpha'^3}{V} n_1 \\
Q_5 &= g\alpha' n_5 \\
Q_p &= \frac{g^2 \alpha'^4}{V R^2} n_p
\end{aligned}$$

(Jejalla, Madden, Ross
Titchener '05)

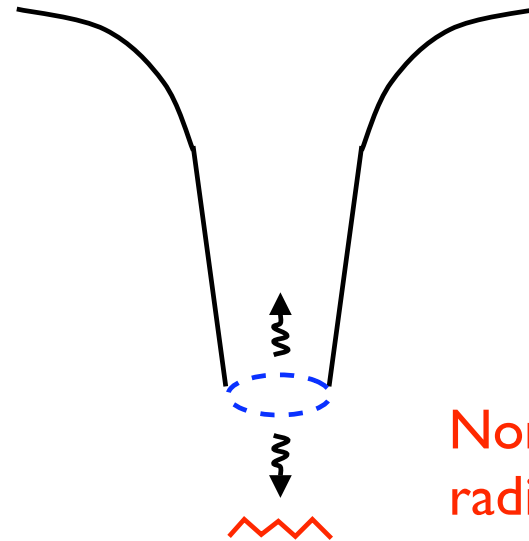
$$\tilde{H}_i = f + M \sinh^2 \delta_i, \quad f = r^2 + a_1^2 \sin^2 \theta + a_2^2 \cos^2 \theta$$

$$Q_1 = M \sinh \delta_1 \cosh \delta_1, \quad Q_5 = M \sinh \delta_5 \cosh \delta_5, \quad Q_p = M \sinh \delta_p \cosh \delta_p$$

Hawking radiation



Unitary radiation
process in CFT



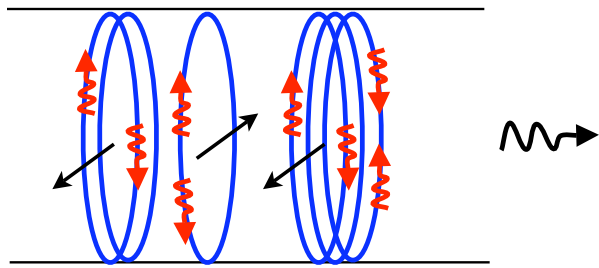
Non-Unitary
radiation from
semiclassical gravity

Radiation rates agree (Spins, greybody factors ...)

(Callan-Maldacena 96, Dhar-Mandal-Wadia 96, Das-Mathur 96, Maldacena-Strominger 96)

Can we get UNITARY radiation (information carrying) in the GRAVITY description ??

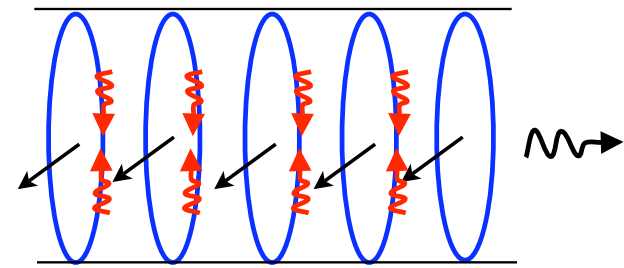
As in any statistical system, each microstate radiates a little differently



$$\Gamma_{CFT} = V \rho_L \rho_R$$

Emission
vertex

Occupation numbers
of left, right excitations
Bose, Fermi distributions
for generic state

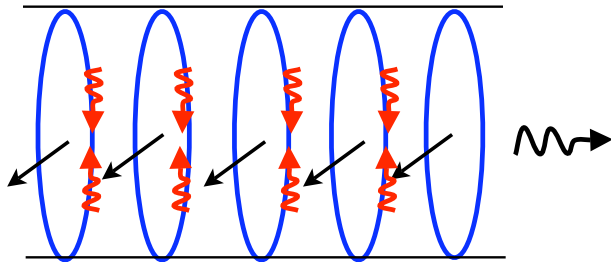


$$\Gamma_{CFT} = V \bar{\rho}_L \bar{\rho}_R$$

Occupation numbers
for this particular
microstate

Emission from the special microstate is peaked at definite frequencies and grows exponentially, like a laser

'Hawking radiation' from the special microstate'



The emitted frequencies are peaked at

$$\omega_R^{CFT} = \frac{1}{R}[-l - 2 - m_\psi m + m_\phi n]$$

$$m = n_L + n_R + 1, \quad n = n_L - n_R$$

Emission grows exponentially because after n de-excited strings have been created, the probability for creating the next one is Bose enhanced by $(n+1)$

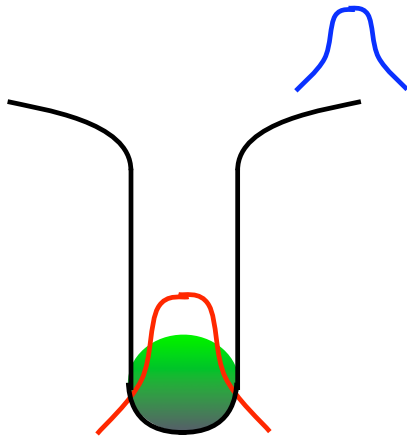
Emission grows as

$$\text{Exp}[\omega_I^{CFT} t]$$

Gravity description of emission :

This gravity solution has no horizon, no singularity , but it has an **ergoregion**

(all non-extremal states made so far are either time-dependent or have an ergoregion)



$$\omega = \omega_R^{gravity} + i\omega_I^{gravity}$$

(Cardoso, Dias, Jordan,
Hovdebo, Myers, '06)

Negative energy quanta collect in the ergoregion, positive energy quanta radiated to infinity

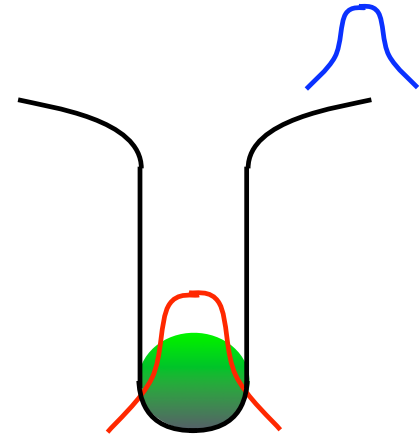
Radiation: The gravity calculation

$$M_{9,1} \rightarrow M_{4,1} \times T^4 \times S^1$$

Graviton with indices on the torus is a scalar in 6-d

$$h_{12} \equiv \Psi$$

$$\square \Psi = 0$$



$$M_{4,1} \rightarrow t, r, \theta, \psi, \phi$$

$$S^1 \rightarrow y \quad y : (0, 2\pi R)$$

$$\Psi = \exp(-i\omega t + i\lambda \frac{y}{R} + im_\psi \psi + im_\phi \phi) \chi(\theta) h(r)$$

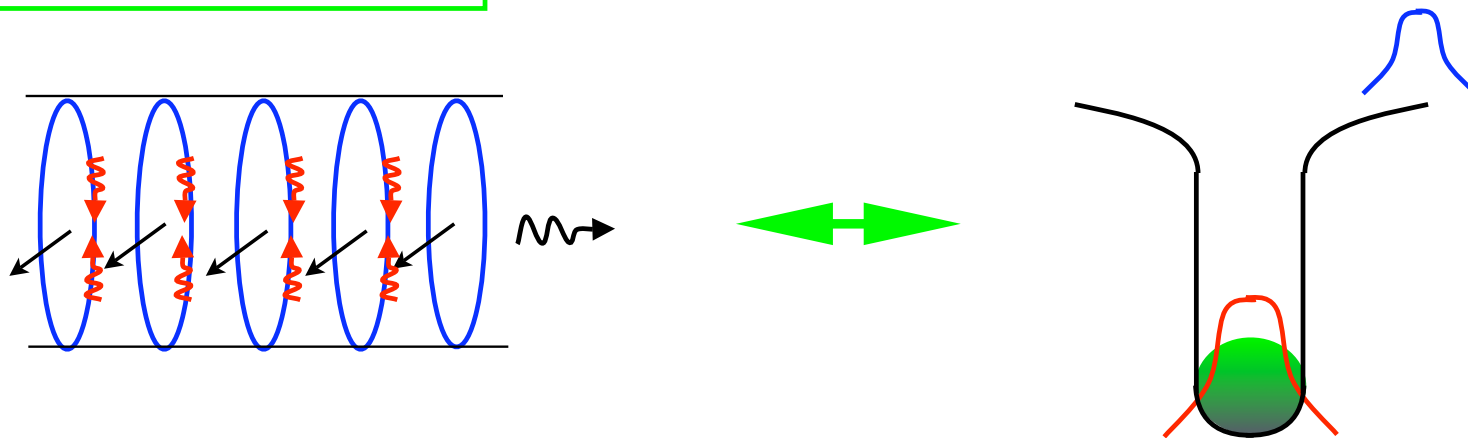
Solve by matching inner and outer region solutions

One finds :

$$\omega_R^{CFT} = \omega_R^{gravity}$$

$$\omega_I^{CFT} = \omega_I^{gravity}$$

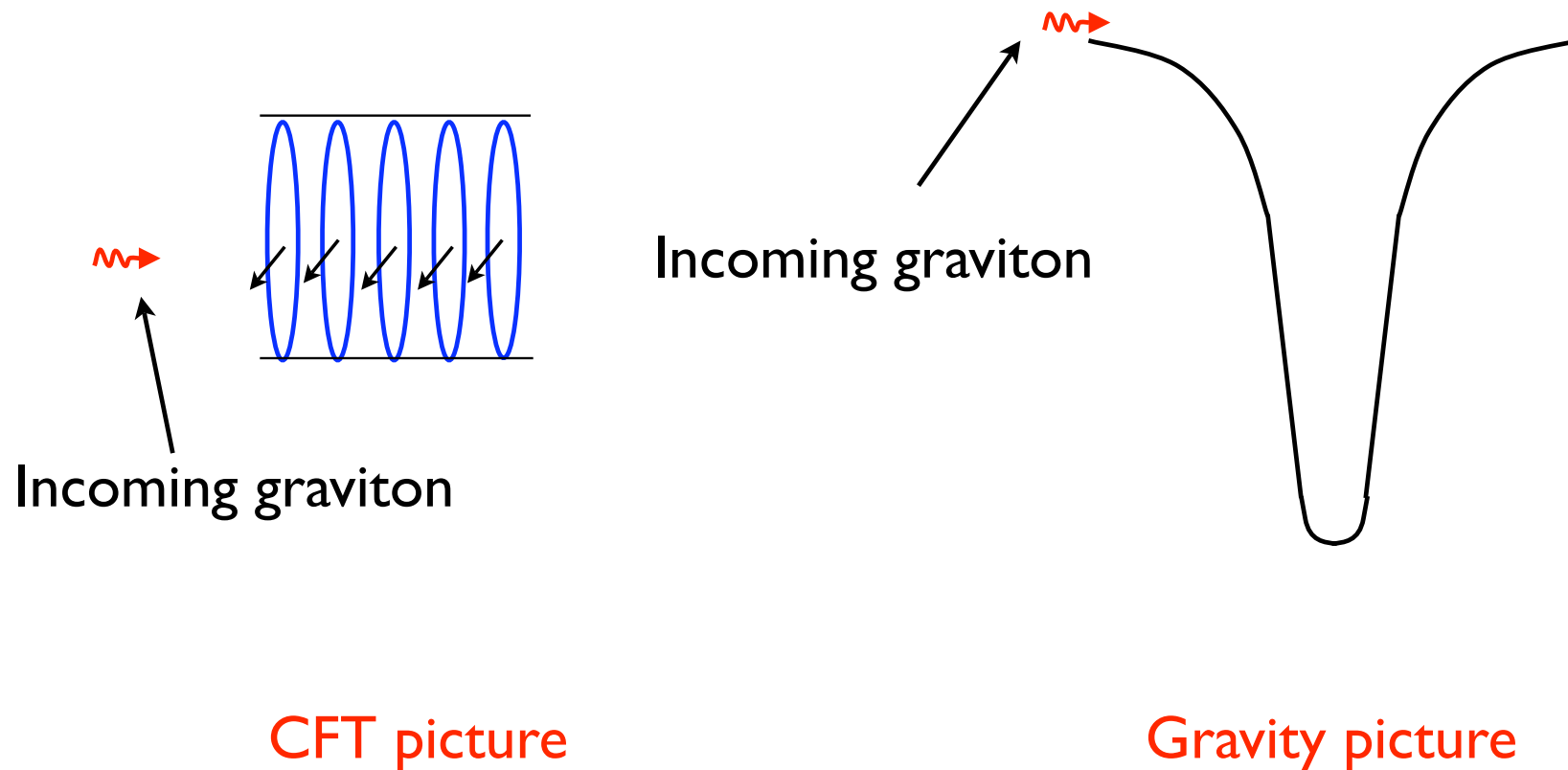
(Chowdhury + SDM 07, 08)



Thus for a set of (nongeneric) microstates we can explicitly see 'information carrying radiation' which is the 'Hawking radiation' for these microstates

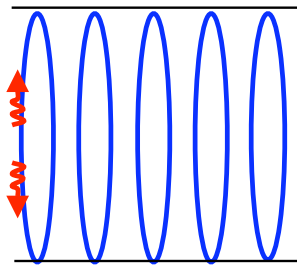
Dynamical questions

(A) An illustration of AdS/CFT duality

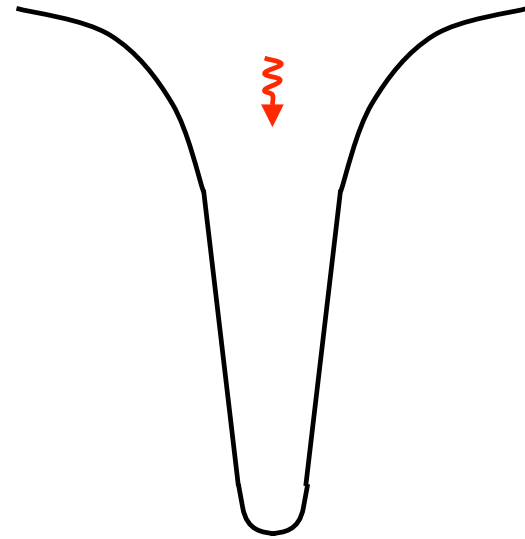


We want to study what happens in each picture ...

Absorption

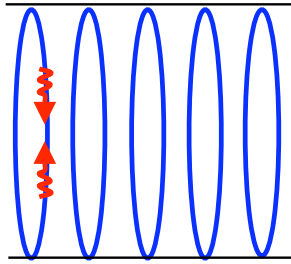


With an absorption probability P , the energy of the graviton gets converted to a pair of vibration modes on one of the pieces of the effective string



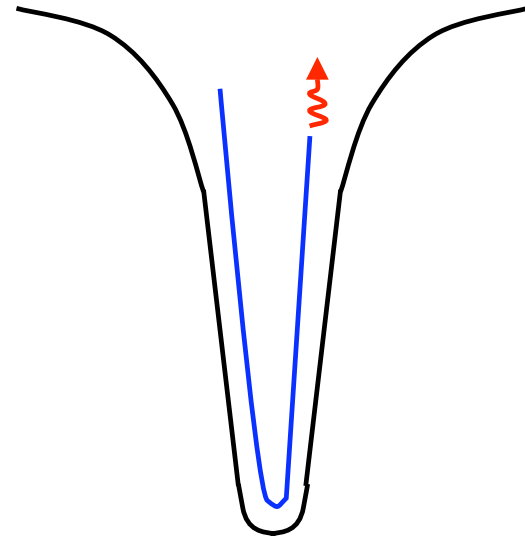
With probability P , the graviton enters the throat of the geometry

Time delay



The excitations travel around the loop in a time T and re-collide

The colliding excitations can lead to re-emission of the graviton with probability P



The graviton travels down the throat, bounces off the 'cap' and comes back up in a time T

It can re-emerge from the throat with probability P

B. Collapsing shell

Consider a shell that is collapsing to form a black hole ...

We have shown that eigenstates of the hole are fuzzballs.

But how does a collapsing shell turn into a fuzzball ?

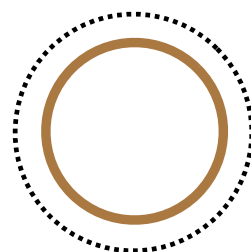
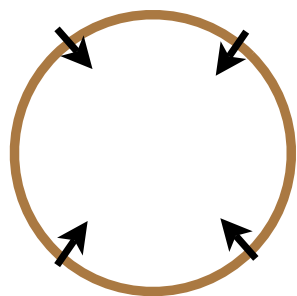
(a) The shell should be able to turn into a fuzzball

and

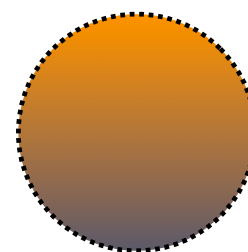
(b) This should happen in a time less than Hawking evaporation time, otherwise the fuzzball picture would not help with information loss

Suppose we make a black hole by collapsing a shell of matter

How can this shell change into a fuzzball ?

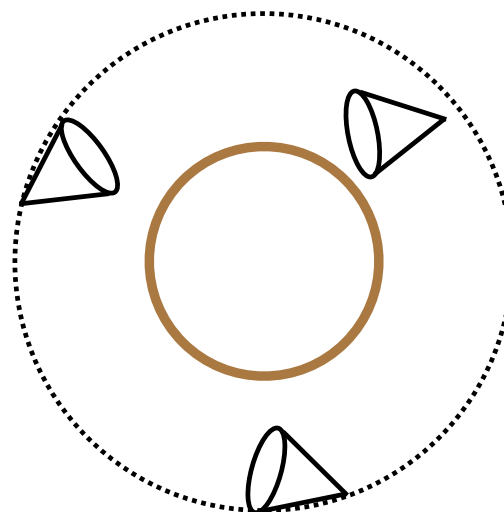


??



Light cones point inwards

How does data get out to horizon ?



(a) We cannot assume classical physics in the black hole, even though the hole is large

(i) Suppose we have a shell of radius of order the horizon radius, GM

(ii) A fuzzball state has a size of the same order

(iii) Let us ask if the shell state can tunnel into the fuzzball state

(iv) Both these states are large, heavy states, so the tunneling probability should be very very small

(v) Estimating the tunneling probability

The probability amplitude is e^{-S}

where the action is to be computed from the Einstein action

$$S = \frac{1}{16\pi G} \int R d^4x$$

The length scale for the solution is $L \sim GM$

Then $R \sim \frac{1}{L^2} \sim \frac{1}{(GM)^2}$ $d^4x \sim (GM)^2$

$$S \sim GM^2$$

$$e^{-S} \sim e^{-GM^2}$$

This is indeed a very small probability

(vi) But there are many different fuzzball states that we can tunnel to

The number of fuzzball states is

$$e^{S_{bek}} \sim e^{GM^2}$$

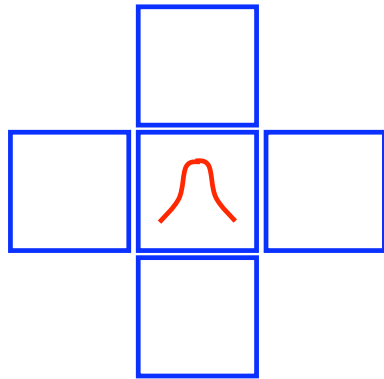
(vii) Thus we can see that this large number of states can cancel the

smallness of the tunneling amplitude

$$e^{-S} \sim e^{-GM^2}$$

What kind of state will such a cancellation generate ?

Toy model

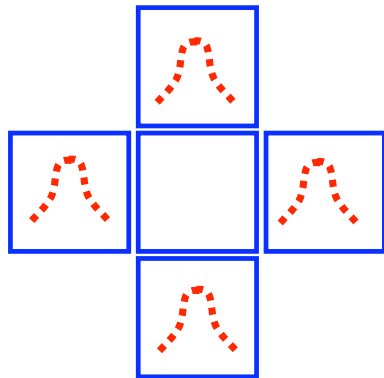


Put a quantum in a potential well

Tunneling probability is small

But there are many neighboring wells

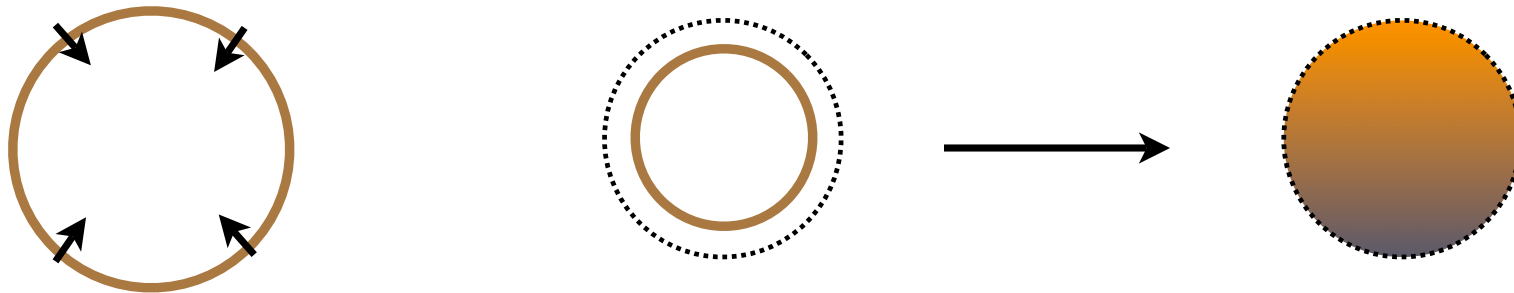
In a time of order unity, the quantum spreads to a linear combination of states in all potential wells



(SDM 08)

Thus we see that even though a collapsing shell looks classical, once it reaches order horizon size, the physics need not be classical.

Tunneling can spread its wavefunction to a linear combination of fuzzball states



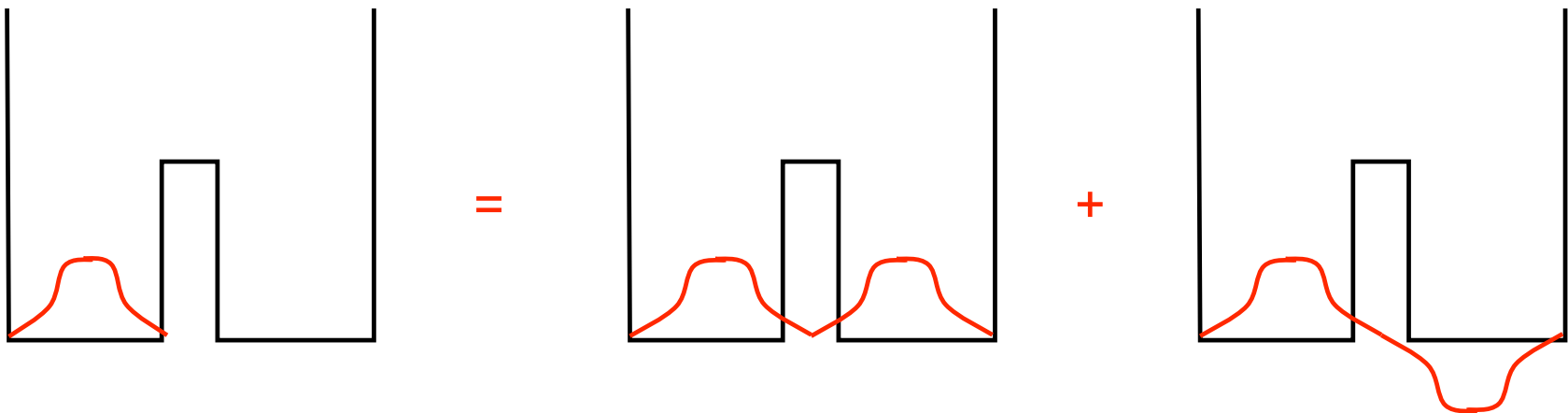
But how long will this process take ?

Tunneling is just 'de-phasing' of eigenstates :

Suppose we have two potential wells, separated by a barrier

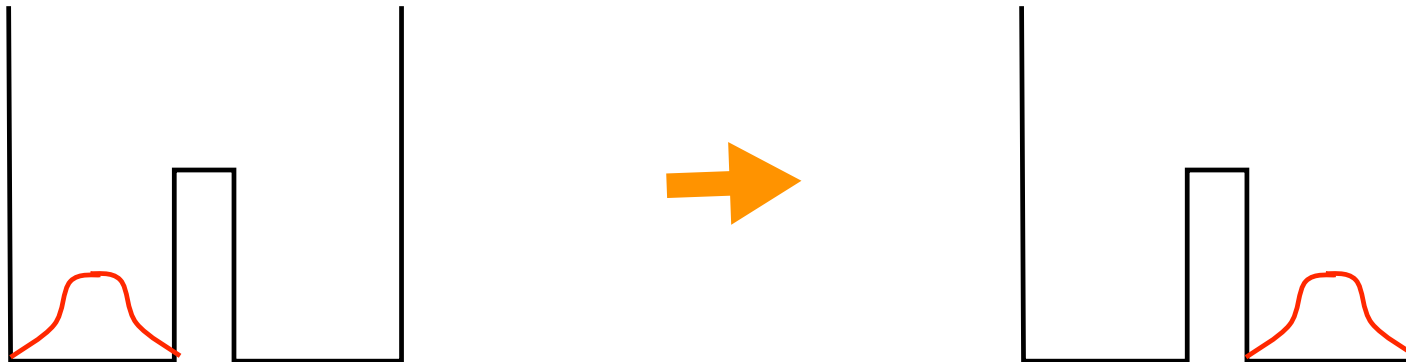
The energy eigenstates are symmetric and antisymmetric wavefunctions

The state in the left well is a superposition of these two eigenfunctions



The two eigenfunctions have slightly different eigenvalues, so after some time they go OUT of phase

$$|\psi\rangle = \frac{1}{2}|\psi_S\rangle + \frac{1}{2}|\psi_A\rangle \rightarrow \frac{1}{2}e^{-iE_S t}|\psi_S\rangle + \frac{1}{2}e^{-iE_A t}|\psi_A\rangle$$



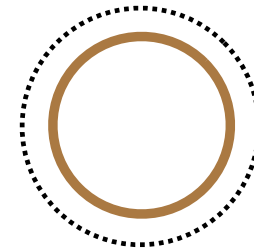
The wavefunction now ends up in the second well. This is tunneling.
The time for tunneling is thus

$$t_{\text{tunnel}} \sim t_{\text{dephase}} \sim 1/\Delta E$$

(b) How long does it take for the shell to become a general linear combination of fuzzballs ?

If it takes more than Hawking evaporation time, fuzzballs dont help !

(i) Since the fuzzballs form a complete set of eigenstates, we can write the state of the shell as a linear combination of fuzzball states



$$|\psi\rangle = \sum_k c_k |E_k\rangle$$

(ii) Let the horizon radius be R . Since the shell has to fit inside the horizon, the uncertainty principle gives

$$\Delta P_r \gg \frac{1}{R}$$

(iii) Then the spread in energy will be

$$\Delta E \sim \frac{P_r \Delta P_r}{M} \gg \frac{(\Delta P_r)^2}{M} \gg \frac{1}{MR^2}$$

(iv) Thus

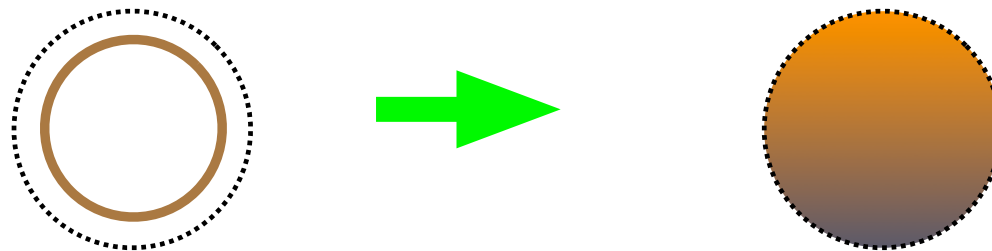
$$t_{dephase} \sim \frac{1}{\Delta E} \ll MR^2$$

(v) Note that $t_{evap} \sim MR^2$

So

$$t_{dephase} \ll t_{evap}$$

That is, the state becomes a linear combination of fuzzballs much before the hole evaporates



Resolving the information paradox

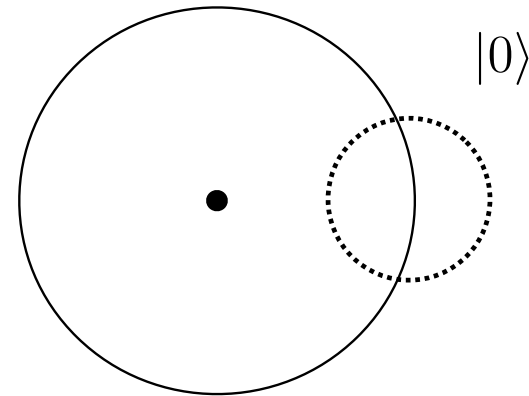
The information paradox :

The Hawking 'theorem' : If

(a) All quantum gravity effects are confined to within a given distance like planck length or string length

(b) The vacuum is unique

Then there **WILL** be information loss



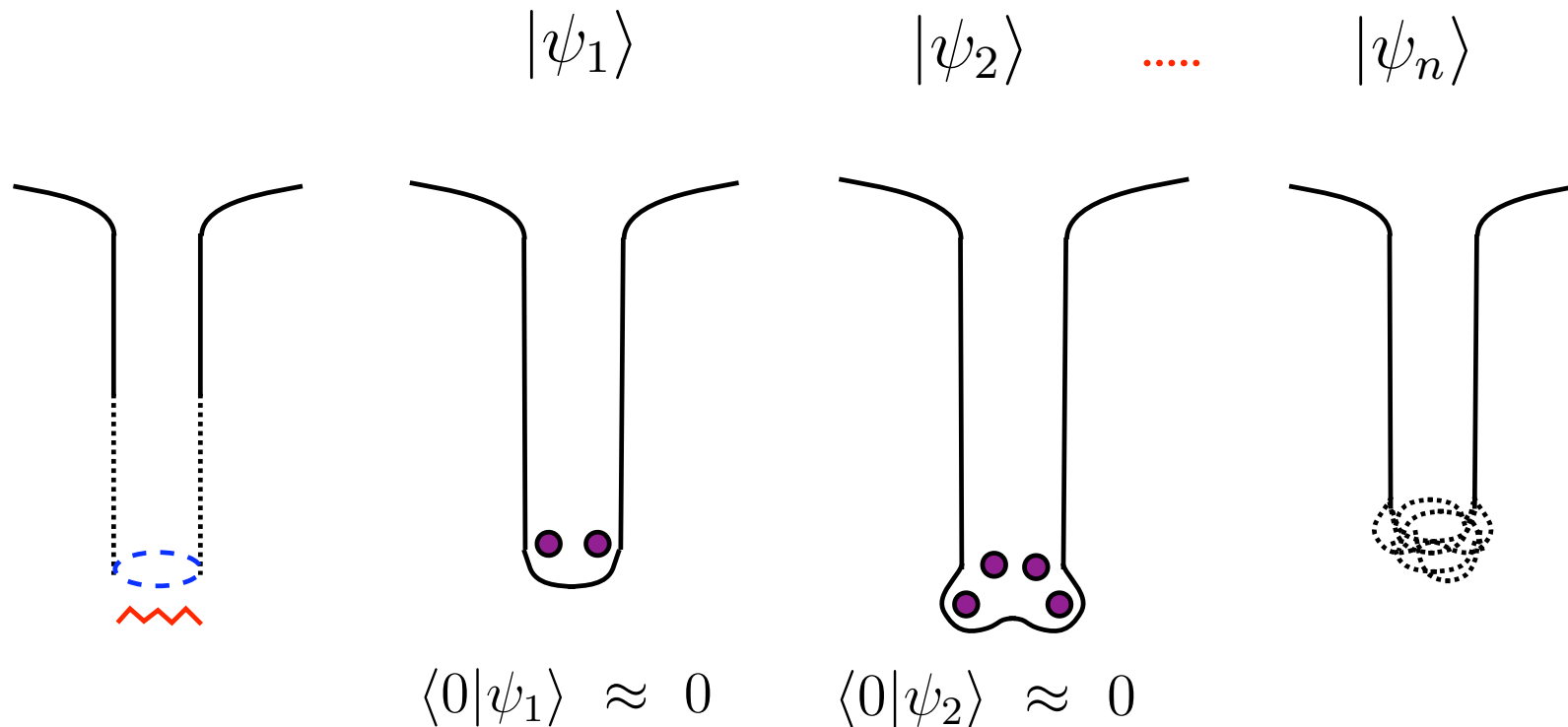
Hawking gives an explicit construction of the evolution of the vacuum state near the horizon, and shows that it gives entangled pairs

If we can show that the state is not $|0\rangle$, then we resolve the problem

$$|0\rangle \rightarrow |\psi\rangle$$

$$\langle 0|\psi\rangle \approx 0$$

We explicitly construct microstates starting with simple ones ...



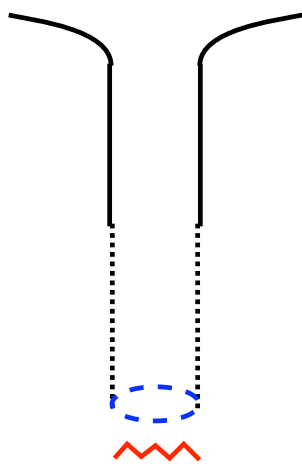
We are resolving a *paradox*. All we have to show is that there is a physical way out of the Hawking construction.

We do not need to make all states in all detail.

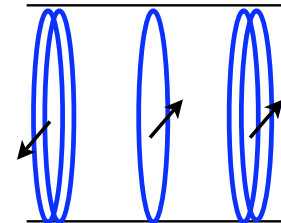
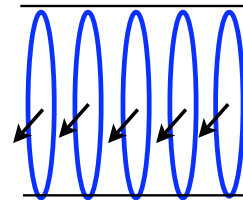
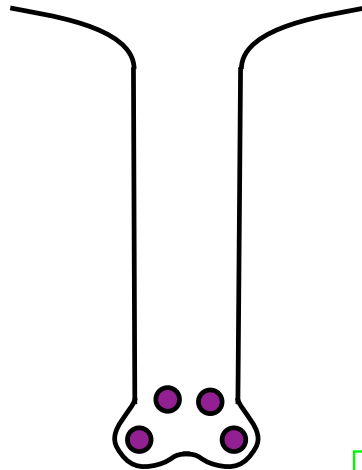
If someone wants to still argue there is a paradox, then he has to show that other states will *not* behave this way

Earlier attempts to construct hair were trying perturbative deformations, while the actual constructions turn out to be nonperturbative.

String theory gives us a new expansion: since we can catalog all states, we can start with states which have 'many excitations in the same mode', and then move to more generic states ...

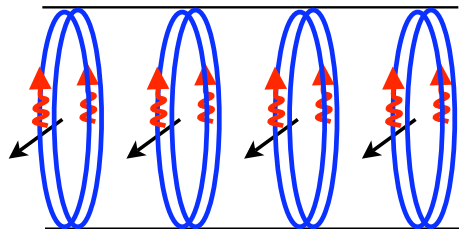
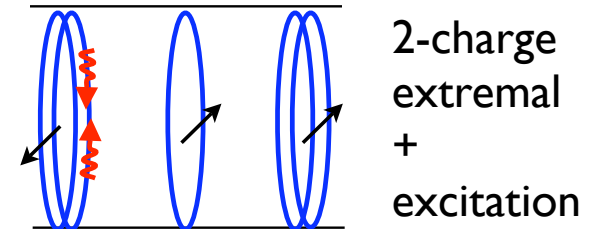
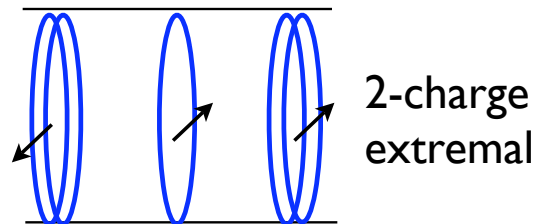
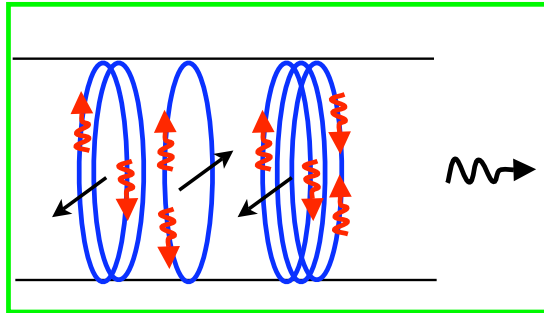


No
perturbative
deformations

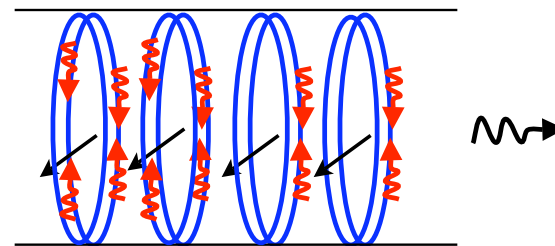
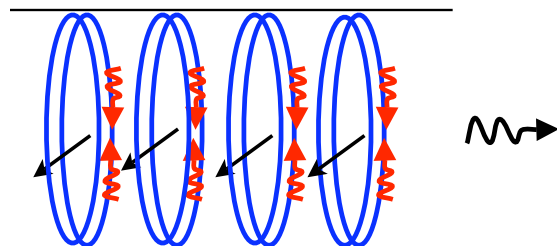


2-charge extremal, 3-charge extremal,
some nonextremal, Hawking radiation ...

Summary : All microstates of black holes made so far are ‘fuzzballs’



3-charge extremal: Large classes also known with CFT state not yet identified



Nonextremal: Some families known, radiation agrees