Mueller-Navelet jets at LHC: matching NLL BFKL with fixed NLO calculations

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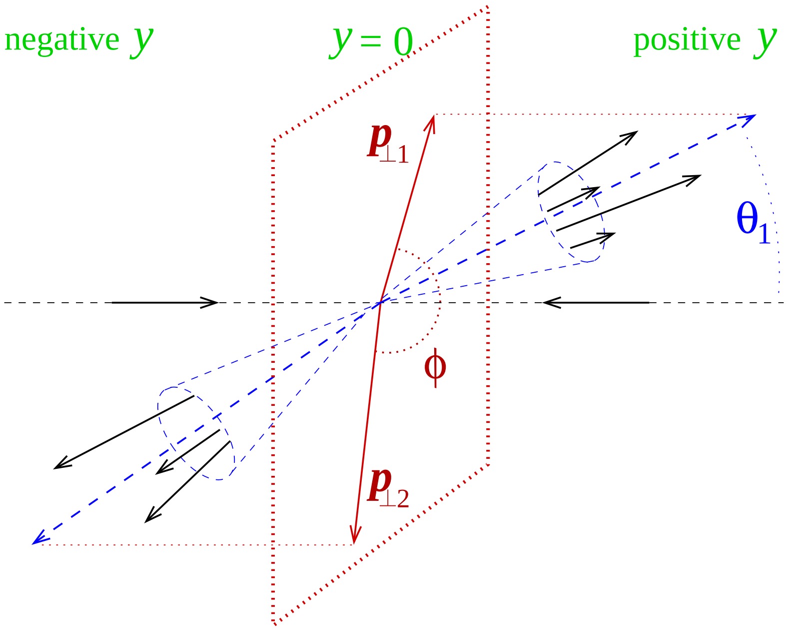
Motivation and Outline

- Motivations
  - One of the important longstanding theoretical questions: the behaviour of QCD in the high-energy (Regge) limit $s \gg -t$
  - We expect a new kind of dynamics (BFKL dynamics) beyond fixed order perturbative predictions, with amplitudes and cross section governed by power-like behaviour s^ω
  - For (semi-)hard processes $s \gg -t \gg \Lambda^2_{\mathrm{QCD}}$, P.Th still applicable with all-order resummation of logarithmic coefficients $(\alpha_{\mathrm{s}} \log s)^n$
- Outline
  - Process suited for study of high energy QCD: Mueller-Navelet dijets
  - Review the theoretical description of MN jets within the BFKL approach
  - CMS analysis (2012) $\to$ comparison with BFKL and with MonteCarlo
  - Improvement by matching fixed NLO with resummed BFKL: method and preliminary results
  - Importance of using the proper jet algorithm (in narrow-jet approx)

Mueller-Navelet jets

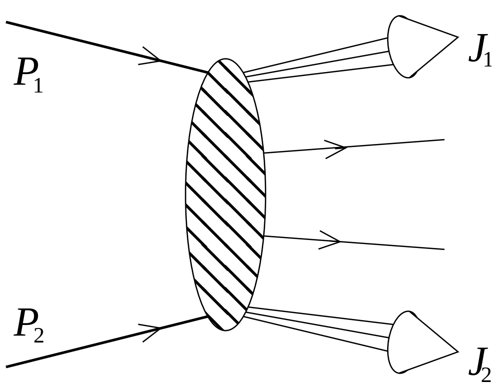
One of most famous testing processes for studying PT high-energy QCD at hadron colliders *[Mueller Navelet 1987]*



Final states with two jets with similar E_T and large rapidity separation

Rapidity: $y = \log(\cot(\theta/2))$

- Comparable hard scales (jet energies) limit the logarithms of collinear type $\log(E_1/E_2)$
- Big separation in rapidity $Y \equiv y_1 - y_2 \quad \Rightarrow \quad$ large $\log(s/E_J^2) \sim Y$



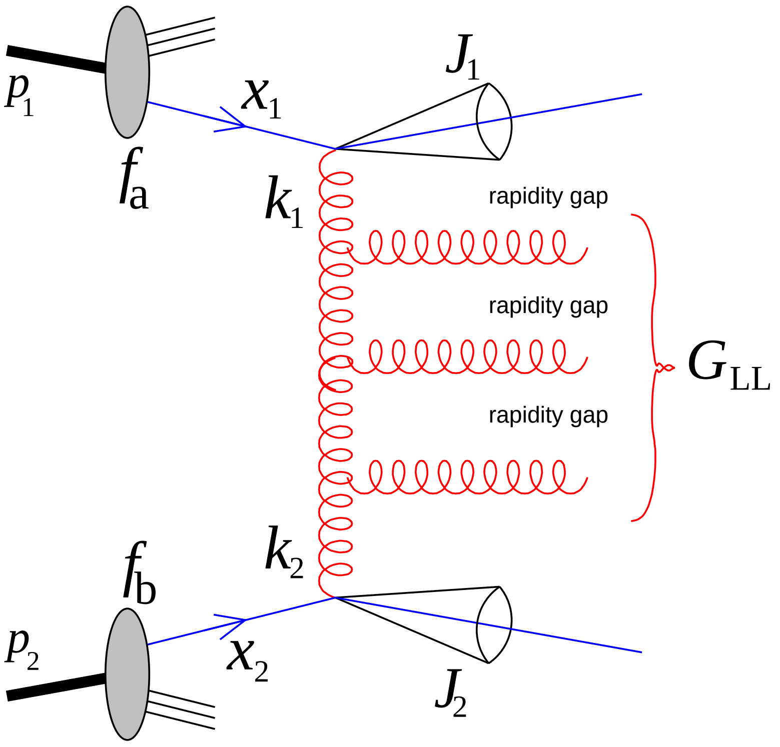
Anything can be emitted between the jets

MN Jets in LL approximation

MN jet factorization formula is a convolution of 5 objects

Starting from LL factorization formula $[J \equiv (y, E_T, \phi)]$

$$\frac{\mathrm{d}\sigma(s)}{\mathrm{d}J_1\mathrm{d}J_2} = \sum_{a,b}\int_0^1 \mathrm{d}x_1\mathrm{d}x_2 \int \mathrm{d}\boldsymbol{k}_1\mathrm{d}\boldsymbol{k}_2$$
$$\times\ f_a(x_1)$$
$$\times\ V_a^{(0)}(x_1,\boldsymbol{k}_1;J_1)$$
$$\times\ G_{\mathrm{LL}}(x_1x_2s,\boldsymbol{k}_1,\boldsymbol{k}_2)$$
$$\times\ V_b^{(0)}(x_2,\boldsymbol{k}_2;J_2)$$
$$\times\ f_b(x_2)$$



where $\frac{\partial}{\partial \log s}G(s,\boldsymbol{k}_1,\boldsymbol{k}_2) = \int \mathrm{d}\boldsymbol{k}\ K(\boldsymbol{k}_1,\boldsymbol{k})G(s,\boldsymbol{k},\boldsymbol{k}_2)\ ,\qquad K = \alpha_{\mathrm{s}}K_0$

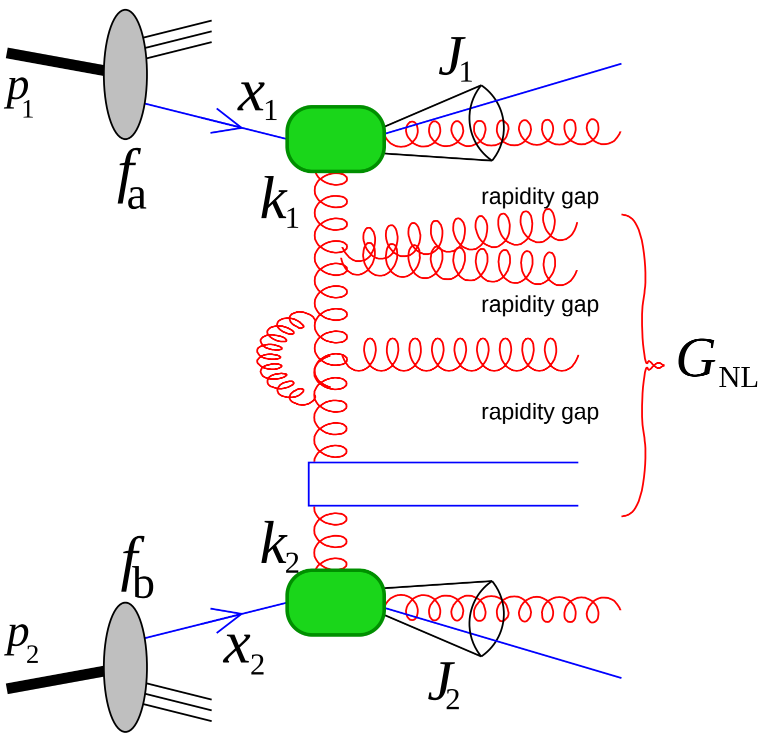
- Kinematics characterized by large rapidity gaps among particles
- At LL level the jet vertex condition is trivial (only 1 parton)

MN Jets in NLL approximation

[Bartels, DC, Vacca '02] computed NLL calculations of impact factors for Mueller-Navelet jets

Proved NLL factorization formula $[J \equiv (y, E_T, \phi)]$

$$\frac{\mathrm{d}\sigma(s)}{\mathrm{d}J_1\mathrm{d}J_2} = \sum_{a,b} \int_0^1 \mathrm{d}x_1\mathrm{d}x_2 \int \mathrm{d}\boldsymbol{k}_1\mathrm{d}\boldsymbol{k}_2 \times f_a(x_1) \times \boldsymbol{V_a^{(1)}}(x_1, \boldsymbol{k}_1; J_1) \times G_{\mathrm{NL}}(x_1x_2s, \boldsymbol{k}_1, \boldsymbol{k}_2) \times \boldsymbol{V_b^{(1)}}(x_2, \boldsymbol{k}_2; J_2) \times f_b(x_2)$$



where $\frac{\partial}{\partial \log s} G(s, \boldsymbol{k}_1, \boldsymbol{k}_2) = \int \mathrm{d}\boldsymbol{k}\ K(\boldsymbol{k}_1, \boldsymbol{k}) G(s, \boldsymbol{k}, \boldsymbol{k}_2)$, $\quad K = \alpha_s K_0 + \alpha_s^2 K_1$

- Pairs of particles can be emitted without rapidity gaps
- At NL level the jet vertex condition is non-trivial (e.g. depends on jet radius R and algorithm)

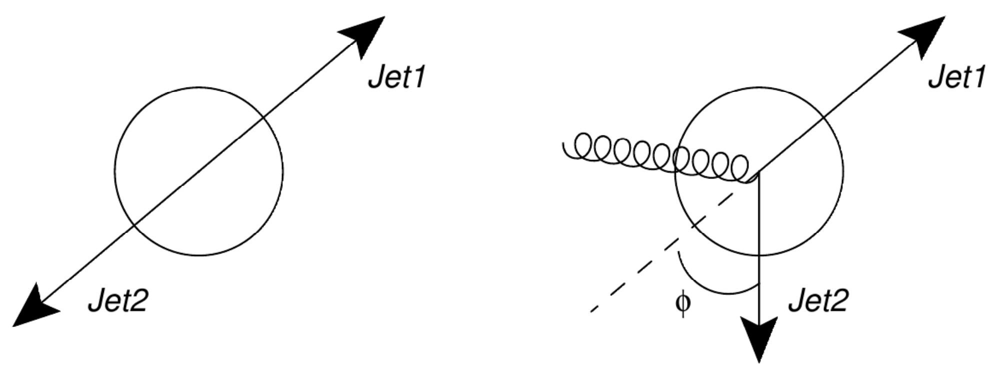
With LHC we can test these ideas!

CMS analysis of MN jets at 7 TeV

Analysis of the azimuthal decorrelation of the two jets *[CMS: FSQ-12-002-pas]*

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} \qquad \Big|\Big| \qquad \langle\cos(m\phi)\rangle = \frac{C_m(Y)}{C_0(Y)} \equiv \frac{\int \mathrm{d}\phi\, \frac{\mathrm{d}^2(\sigma\cos(m\phi))}{\mathrm{d}\phi \mathrm{d}Y}}{\mathrm{d}\sigma/\mathrm{d}Y}$$

- Distinguishes BFKL dynamics from fixed order one: they provide different amount of particle emissions between jets, which is responsible for their decorrelation
- $\langle\cos(m\phi)\rangle$ has reduced theoretical scale uncertainties being a ratio of differential cross sections



CMS analysis of MN jets at 7 TeV

Angular distribution $\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\phi}$ with $\phi \equiv |\pi - \phi_1 - \phi_2|$

Data selection: $E_{T1,2} > 35\mathrm{GeV}$, $|y_i| < 4.7$

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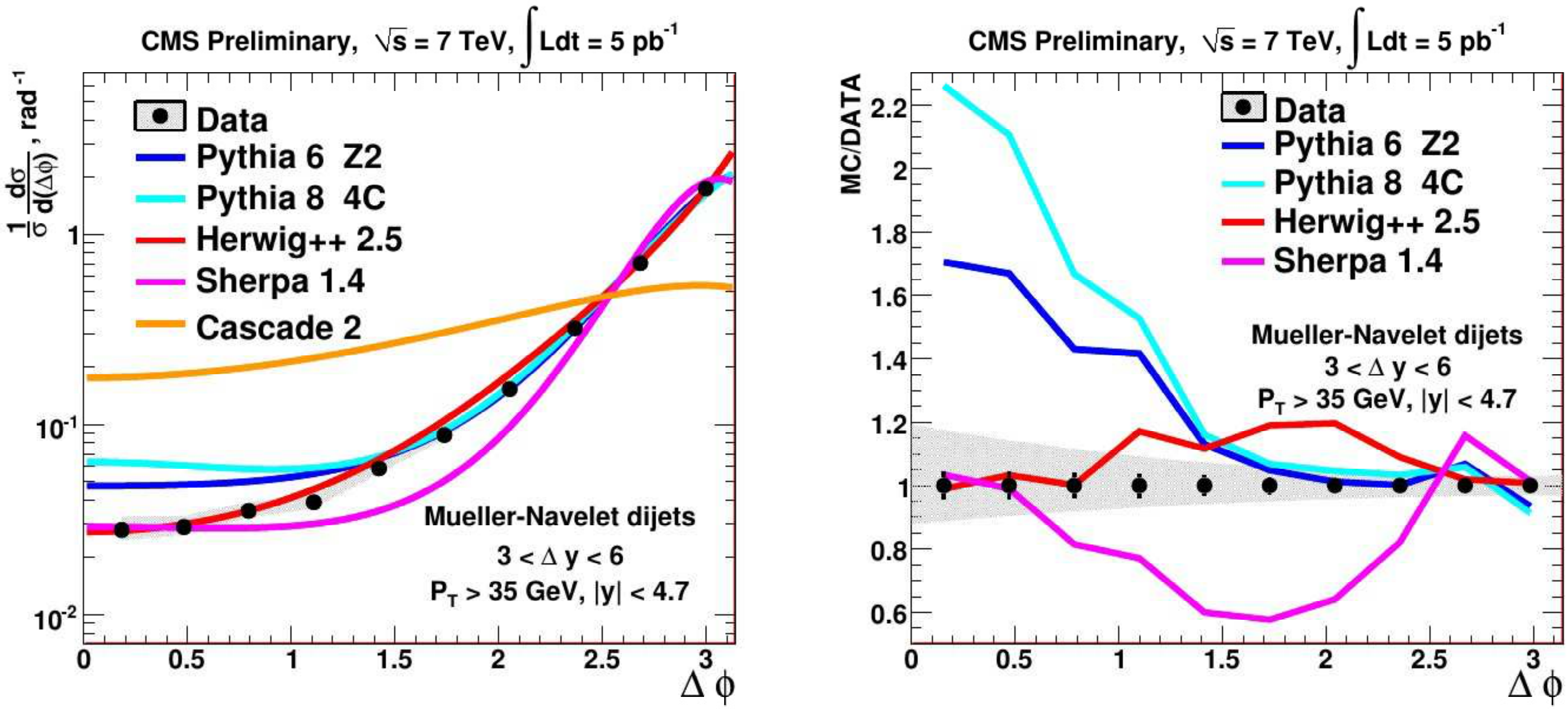
Data selection: $E_{T1,2} > 35\text{GeV}, |y_i| < 4.7$ $3 < \Delta y \equiv Y < 6$



Some MC are close to data somewhere in ϕ

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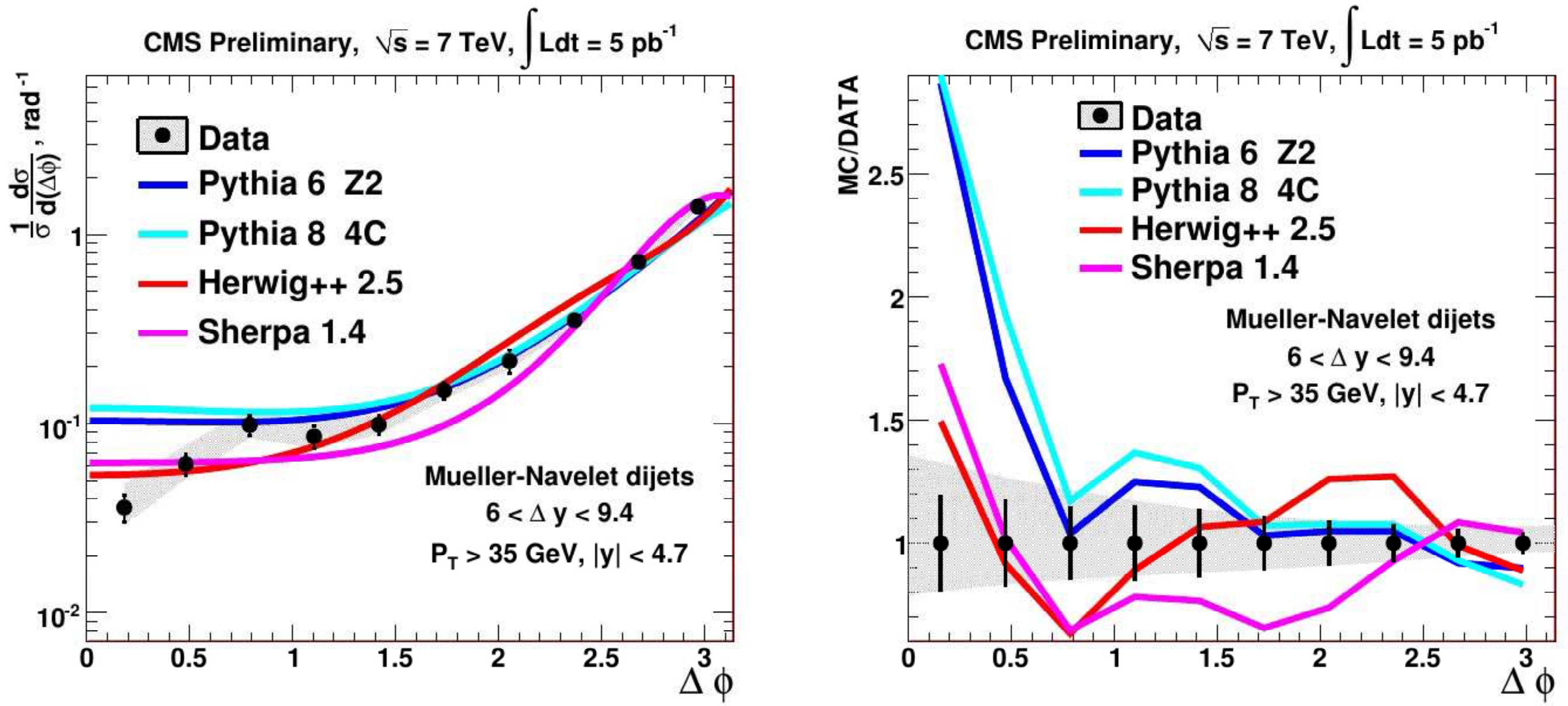


Some MC are close to data somewhere in ϕ

Overall description is not very good

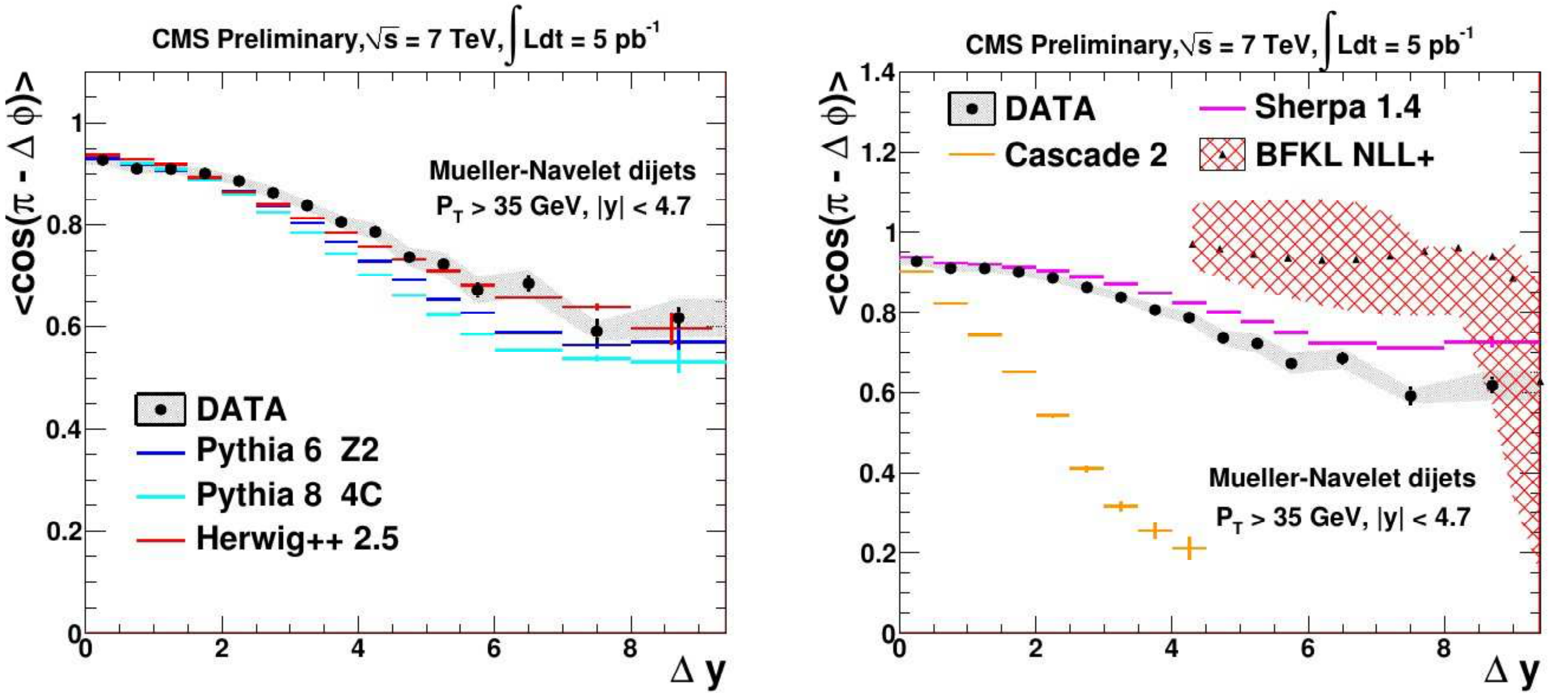
CMS analysis of MN jets at 7 TeV

Data: $E_{T1,2} > 35\text{GeV}$, $|y_i| < 4.7$ $\qquad$ $\Delta y \equiv Y \equiv |y_1 - y_2| < 9.4$

$$\langle\cos(m\phi)\rangle = \frac{C_m(Y)}{C_0(Y)} \equiv \frac{\int \mathrm{d}\phi \, \frac{\mathrm{d}^2(\sigma\cos(m\phi))}{\mathrm{d}\phi \mathrm{d}Y}}{\mathrm{d}\sigma/\mathrm{d}Y}$$

CMS analysis of MN jets at 7 TeV

Data: $E_{T1,2} > 35\text{GeV}$, $|y_i| < 4.7$ $\Delta y \equiv Y \equiv |y_1 - y_2| < 9.4$ $m = 1$



The larger Y, the more radiation and decorrelation

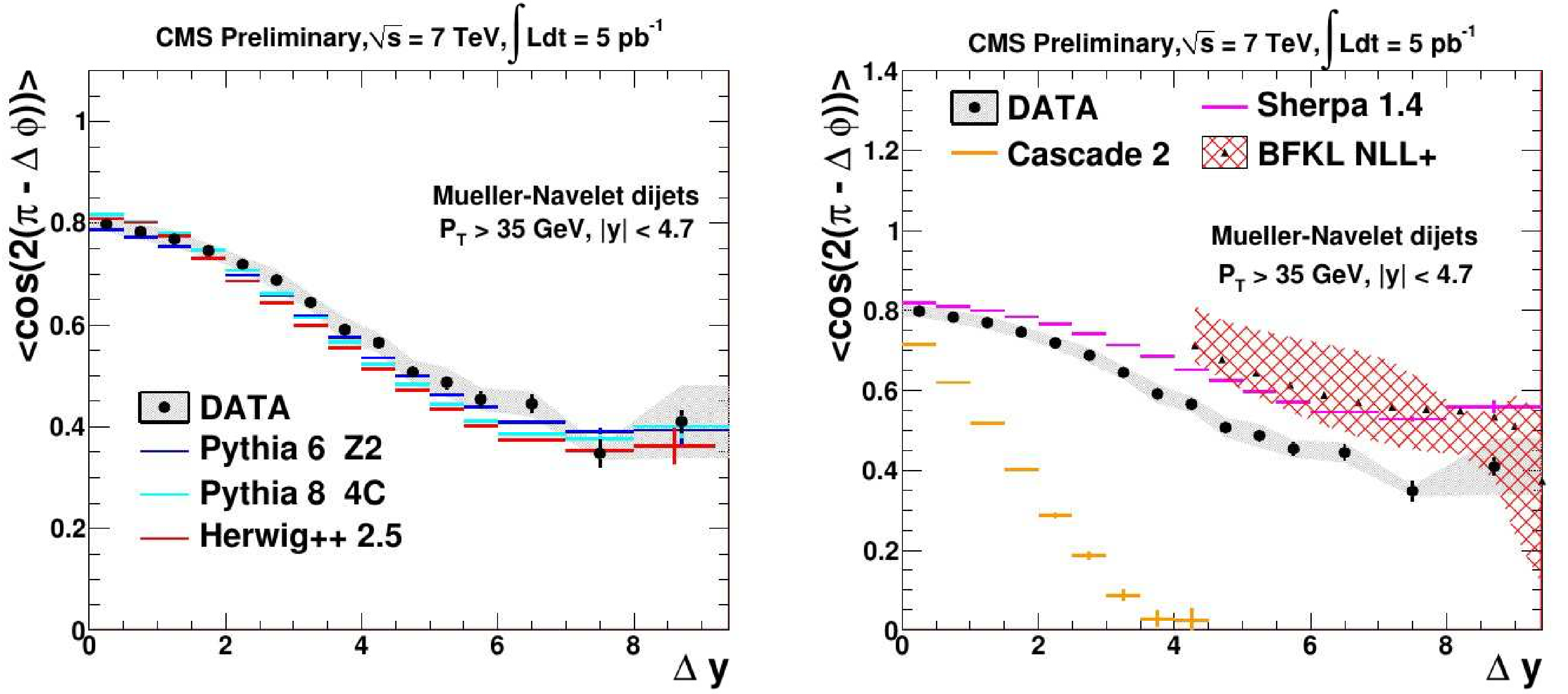
BFKL was expected to predict more radiation than fixed order $\Rightarrow$ more decorrelation

Some MC agree with data

NLL BFKL estimate has problems $\langle \cos \phi \rangle > 1$ for $\mu_R = \mu_F = E_T/2$

CMS analysis of MN jets at 7 TeV

Data: $E_{T1,2} > 35\text{GeV}$, $|y_i| < 4.7$ $\quad$ $\Delta y \equiv Y \equiv |y_1 - y_2| < 9.4$ $\quad$ $m = 2$



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NLL BFKL still unable to reproduce data

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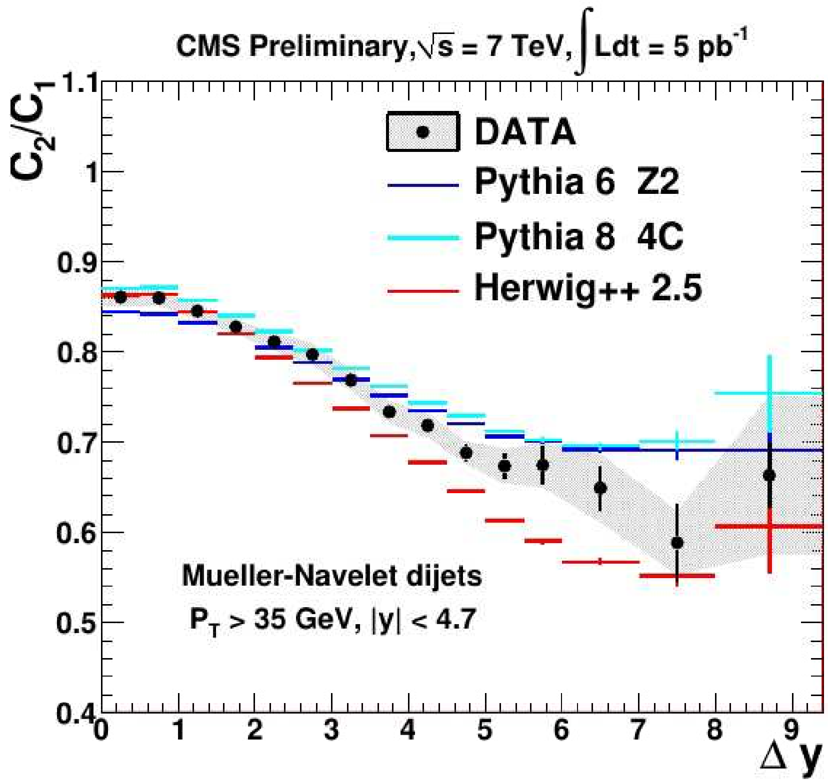

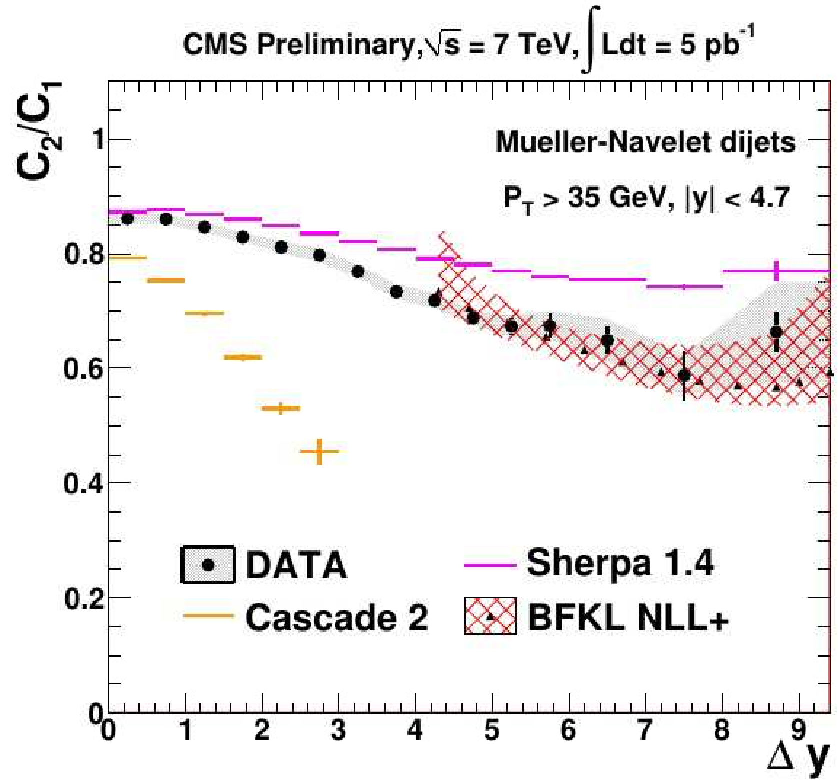

Ratio $\frac{C_2}{C_1} = \frac{\langle \cos(2\phi) \rangle}{\langle \cos \phi \rangle}$

MCs don't agree well with data

NLL BFKL in perfect agreement with data

- Neither BFKL NLL nor fixed order MC give a satisfactory description of data yet

- BFKL NLL still suffers from large scale uncertainties $\sim 10 \div 15\%$

NLL with BLM scale fixing

[Ducloué,Szymanowski,Wallon '13] proposed to tame large scale dependence of BFKL by fixing μ_R with BLM procedure

$$\mu_R^2 = \exp\left[\frac{1}{2}\chi_0 - \frac{5}{3} + 2\left(1 + \frac{2}{3}I\right)\right] E_{T1}\, E_{T2}$$

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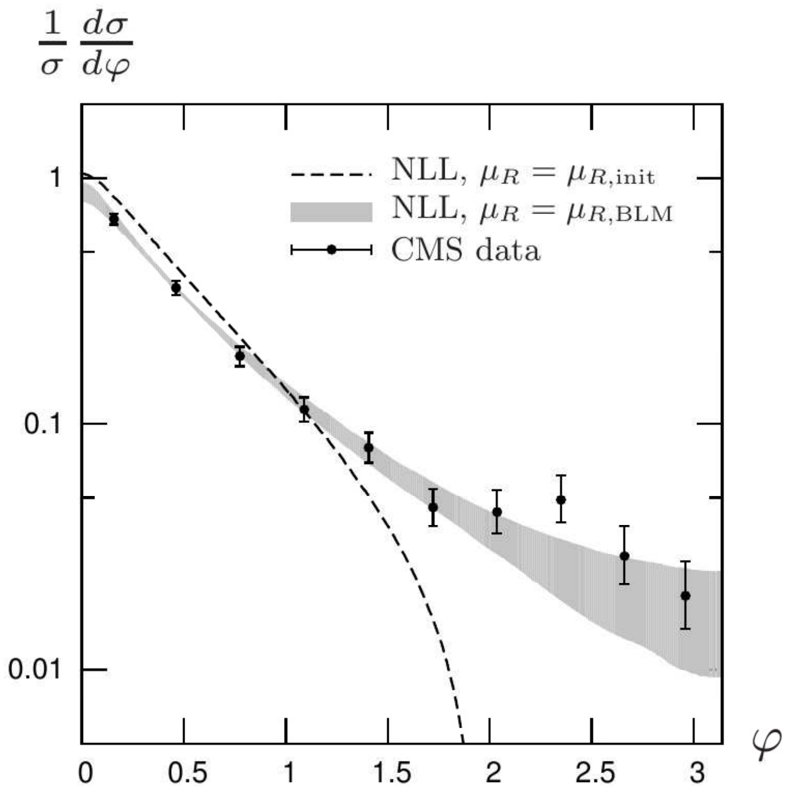

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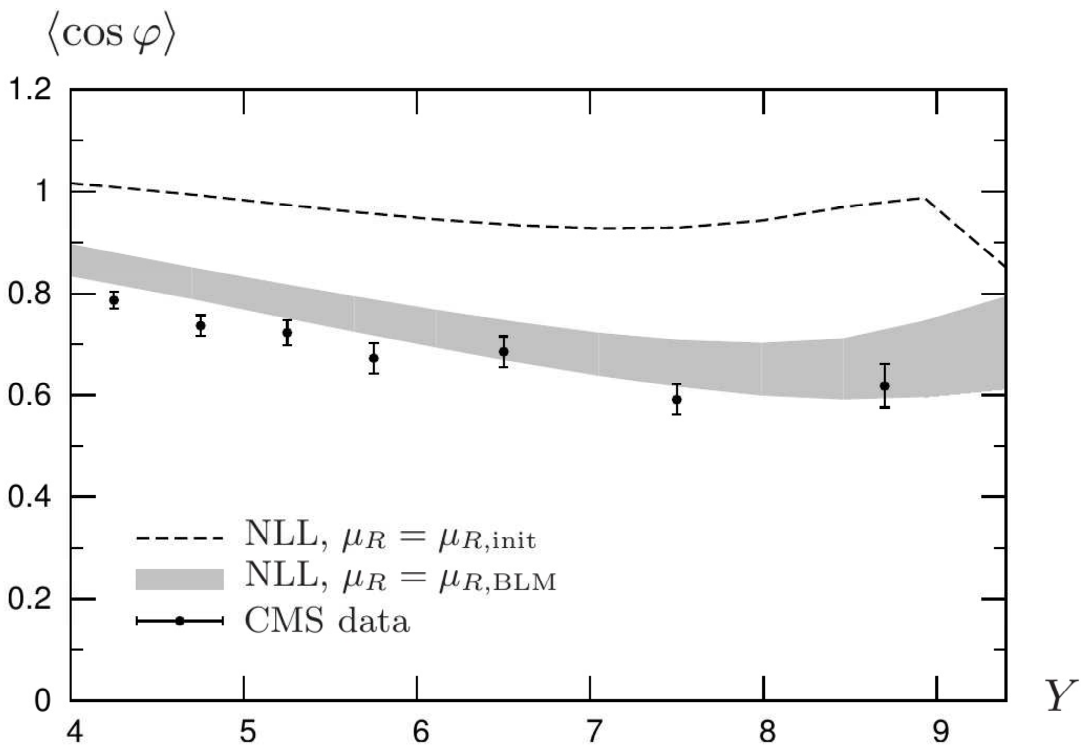

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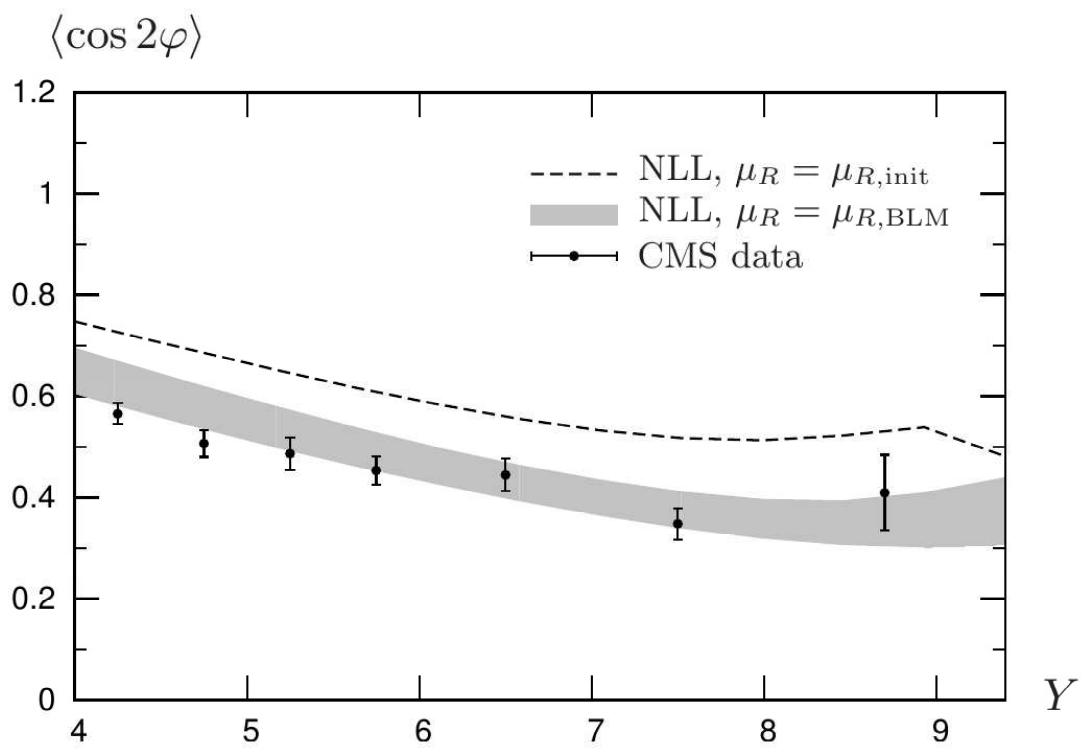

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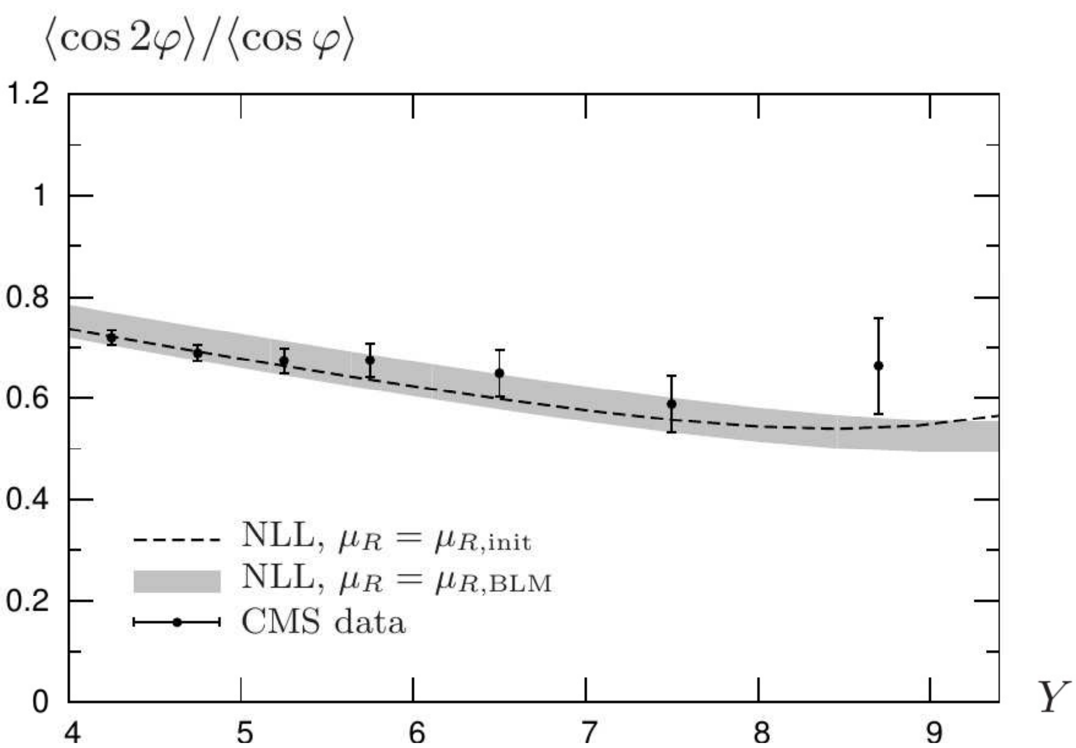
$$\mu_R^2 = \exp\left[\frac{1}{2}\chi_0 - \frac{5}{3} + 2\left(1 + \frac{2}{3}I\right)\right] E_{T1}\, E_{T2} \sim 20^2\, E_{T1}\, E_{T2}$$



Very large renorm. scale

NLL BFKL + BLM provides good description of data

Other methods

- *[Ducloué,Szymanowski,Wallon '14]*
  try to take into account energy-momentum conservation
  by using an effective rapidity $Y_{\rm eff}$, as suggested by *[Del Duca, Schmidt]*
- *[Caporale, Ivanov, Murdaca, Papa '14]*
  consider various representations of the NLL cross section
  by fixing energy scales with PMS, FAC, BLM

Underlying idea: to effectively include higher-orders

Why not including known NLO order?

Matching BFKL with Fixed NLO

Our aim is to merge fixed NL order and NLL BFKL resummation

- more reliable results $\Rightarrow$ improve description of data
- correctly reproduce not only ratios but absolute values

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Standard matching procedure:

- add to BFKL the full perturbative NLO result $\mathcal{O}\left(\alpha_s^3\right)$
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Results for cross section and C_m coefficients

- The implementation is still work in progess
- Preliminary results of central values (no error estimate yet)

Matching (sym. jets $E_1, E_2 > 35\text{GeV}$)

Cross section: NLL BFKL + NLO pert. $\mathcal{O}(\alpha_s)^3$ − BFKL $\mathcal{O}(\alpha_s^3)$

$$\frac{d\sigma(s)}{dJ_1 dJ_2} = \sum_{a,b} \int_0^1 dx_1 dx_2\; f_a(x_1) f_b(x_2) \Big\{$$

$$\int d\boldsymbol{k}_1 d\boldsymbol{k}_2 \Big[V_a^{(0+1)}(x_1, \boldsymbol{k}_1; J_1)\, G_{\rm NLL}(x_1 x_2 s, \boldsymbol{k}_1, \boldsymbol{k}_2)\, V_b^{(0+1)}(x_2, \boldsymbol{k}_2; J_2) \Big]$$

$$+ \frac{d\hat{\sigma}^{(NLO)}(x_1, x_2)}{dJ_1 dJ_2}$$

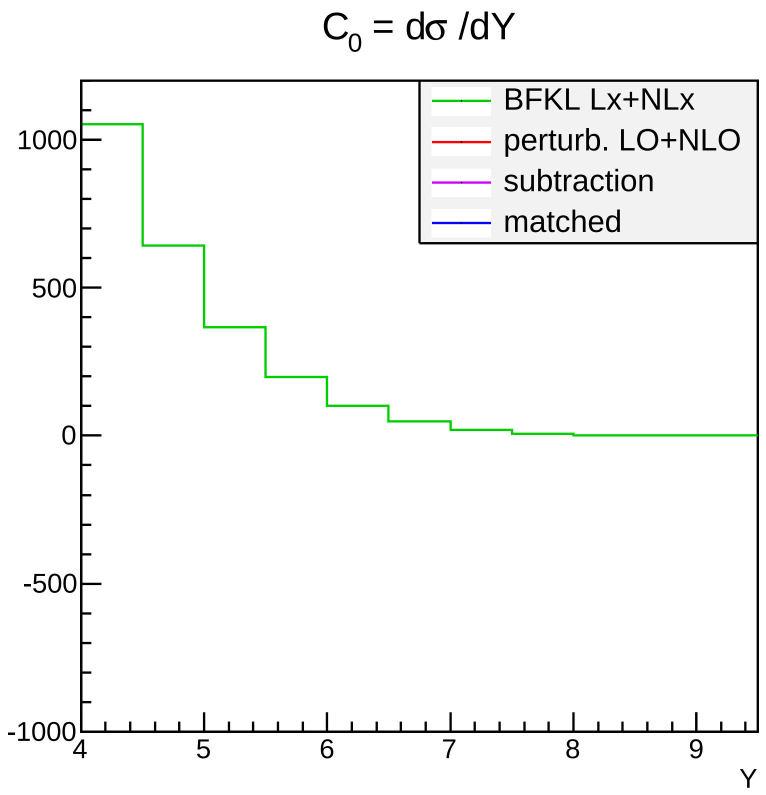
$$- \int d\boldsymbol{k}_1 d\boldsymbol{k}_2 \Big[V_a^{(0)}(x_1, \boldsymbol{k}_1; J_1)\, \delta^2(\boldsymbol{k}_1 - \boldsymbol{k}_2)\, V_b^{(0)}(x_2, \boldsymbol{k}_2; J_2) \Big]$$

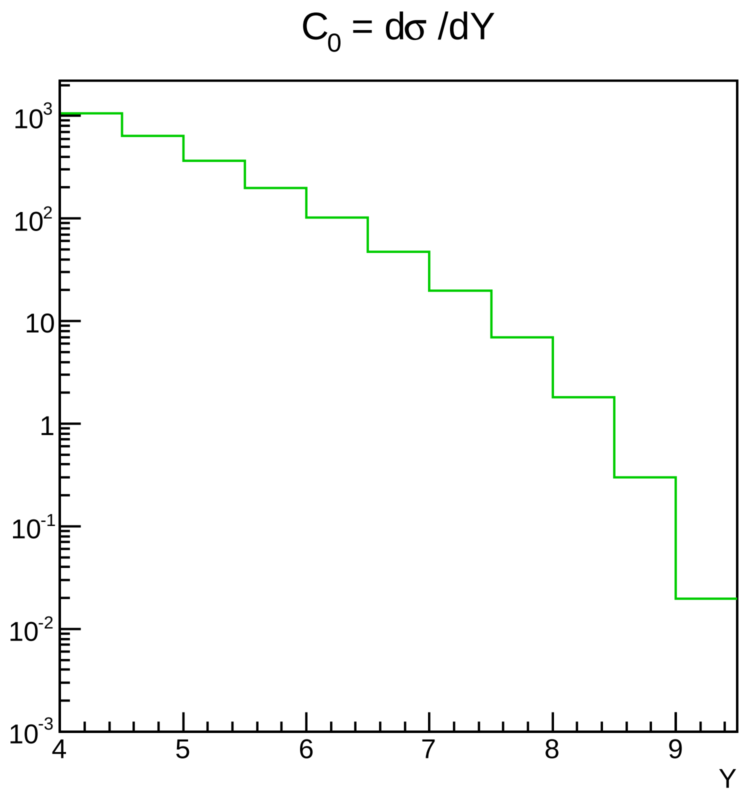
$$- \int d\boldsymbol{k}_1 d\boldsymbol{k}_2 \Big[V_a^{(1)}(x_1, \boldsymbol{k}_1; J_1)\, \delta^2(\boldsymbol{k}_1 - \boldsymbol{k}_2)\, V_b^{(0)}(x_2, \boldsymbol{k}_2; J_2) \Big]$$

$$- \int d\boldsymbol{k}_1 d\boldsymbol{k}_2 \Big[V_a^{(0)}(x_1, \boldsymbol{k}_1; J_1)\, \delta^2(\boldsymbol{k}_1 - \boldsymbol{k}_2)\, V_b^{(1)}(x_2, \boldsymbol{k}_2; J_2) \Big]$$

$$- \int d\boldsymbol{k}_1 d\boldsymbol{k}_2 \Big[V_a^{(0)}(x_1, \boldsymbol{k}_1; J_1)\, \alpha_s \log\frac{\hat{s}}{s_0} K_0(\boldsymbol{k}_1, \boldsymbol{k}_2)\, V_b^{(0)}(x_2, \boldsymbol{k}_2; J_2) \Big] \Big\}$$

Matching (sym. jets $E_{T1}, E_{T2} > 35\text{GeV}$)





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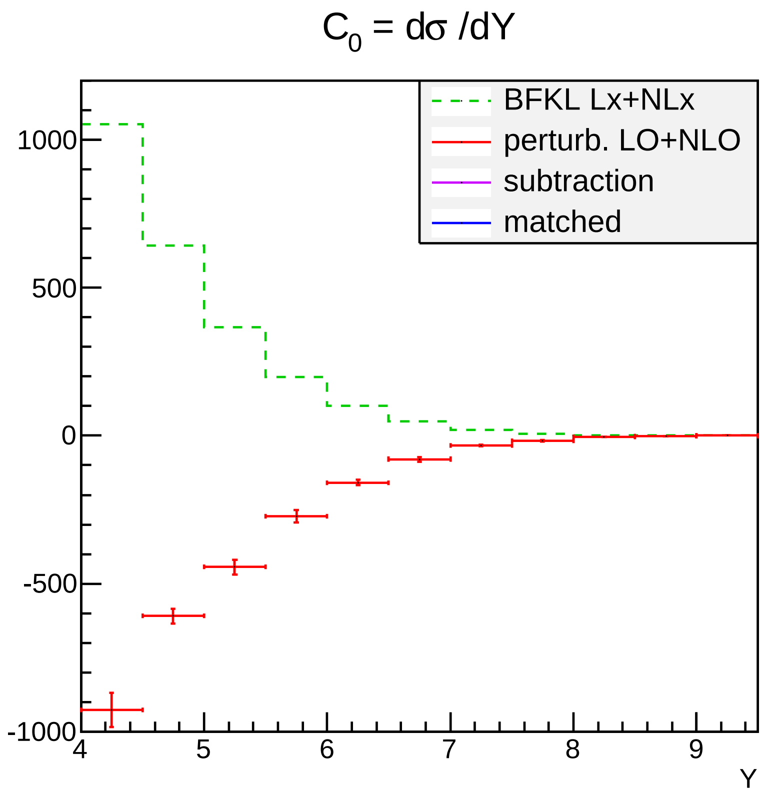

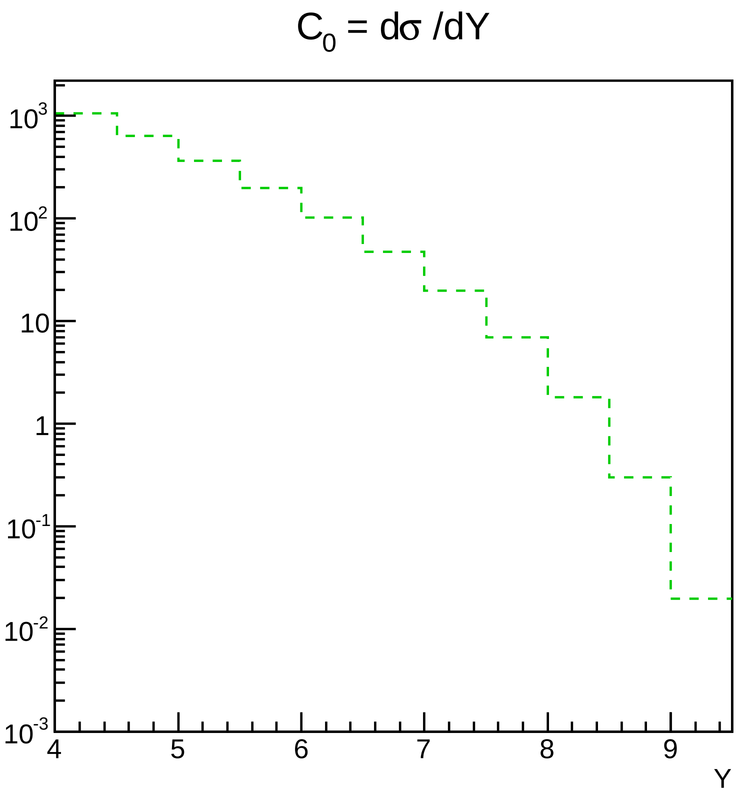

LO+NLO cross section obtained with NLOJET++ *[Nagy]* is negative!

Large errors due to very slow convergence in MC integration

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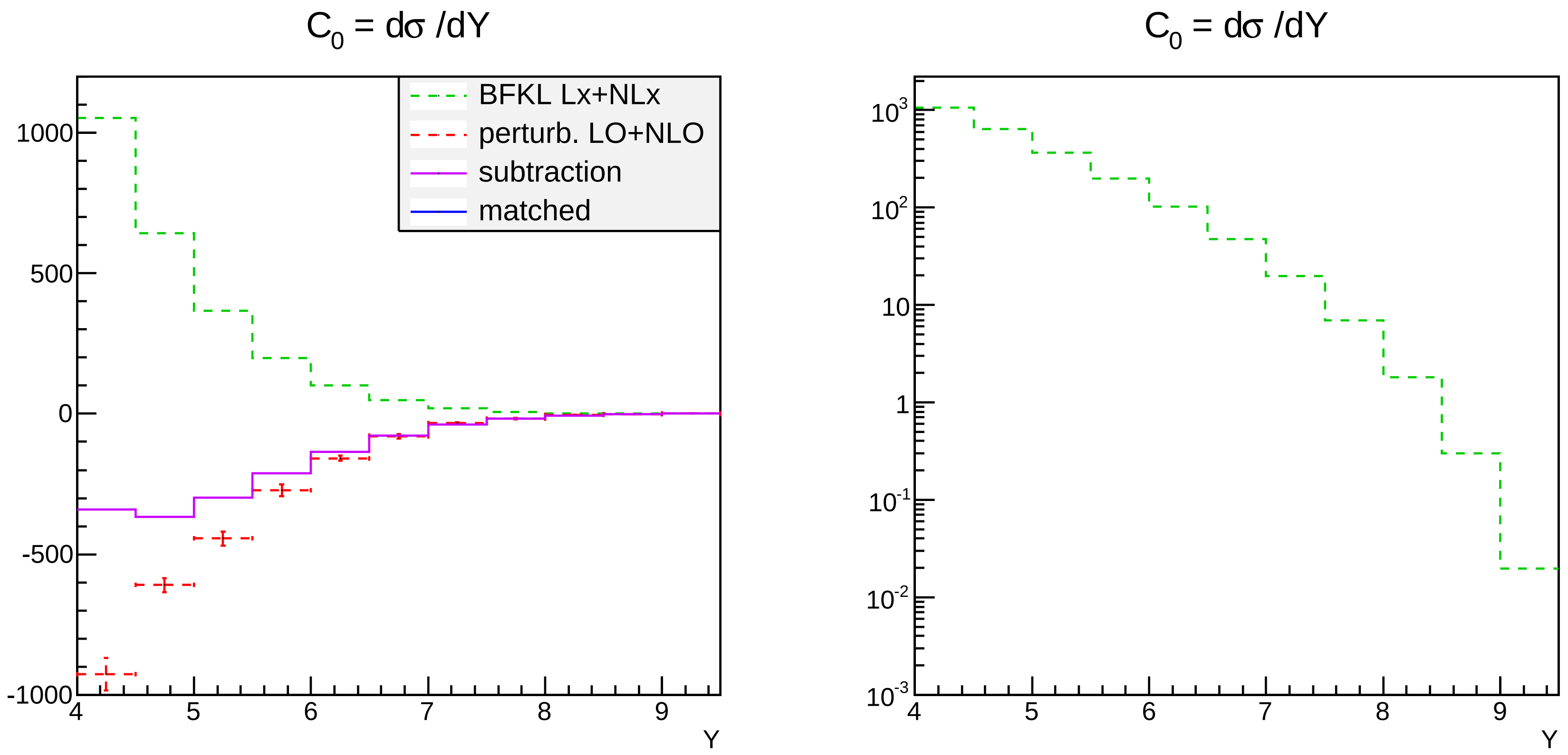

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However, also the subtraction is negative

Their difference is moderate

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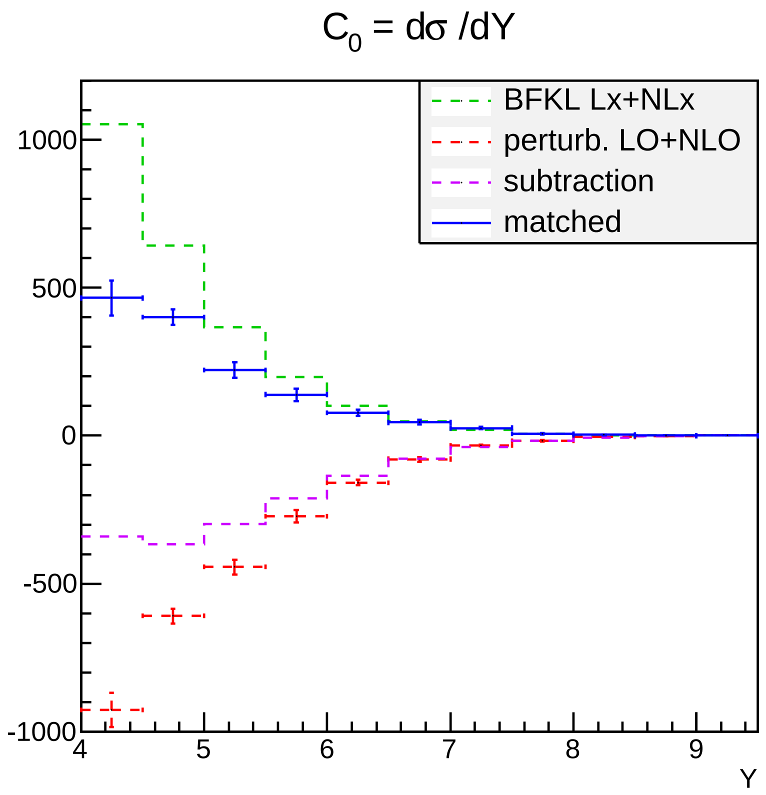

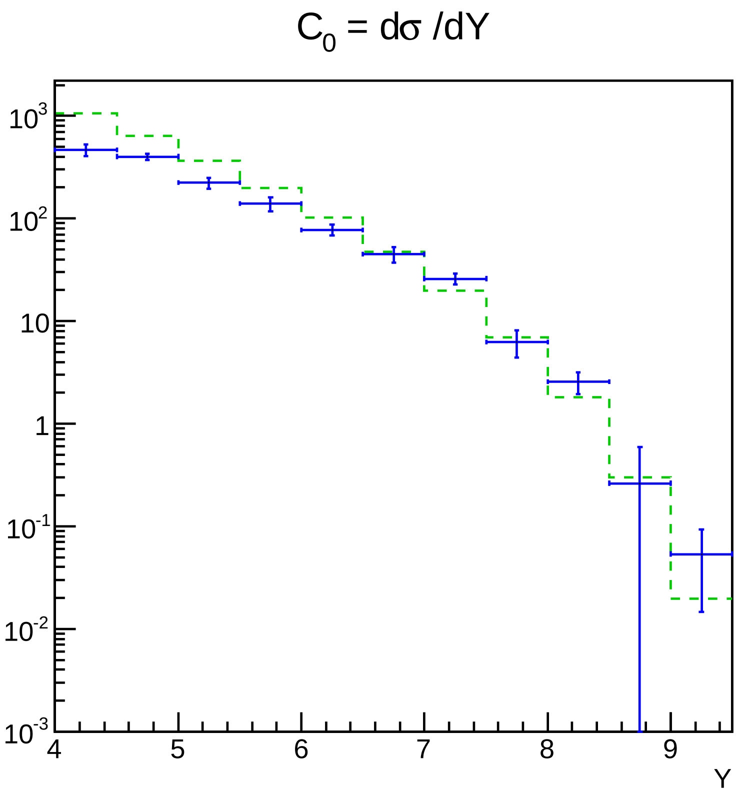

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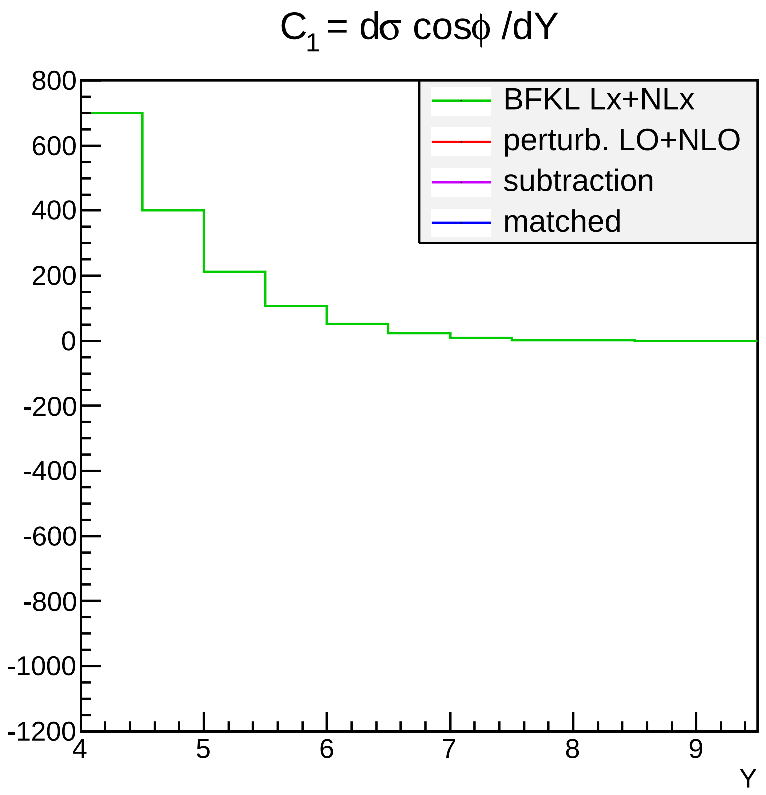
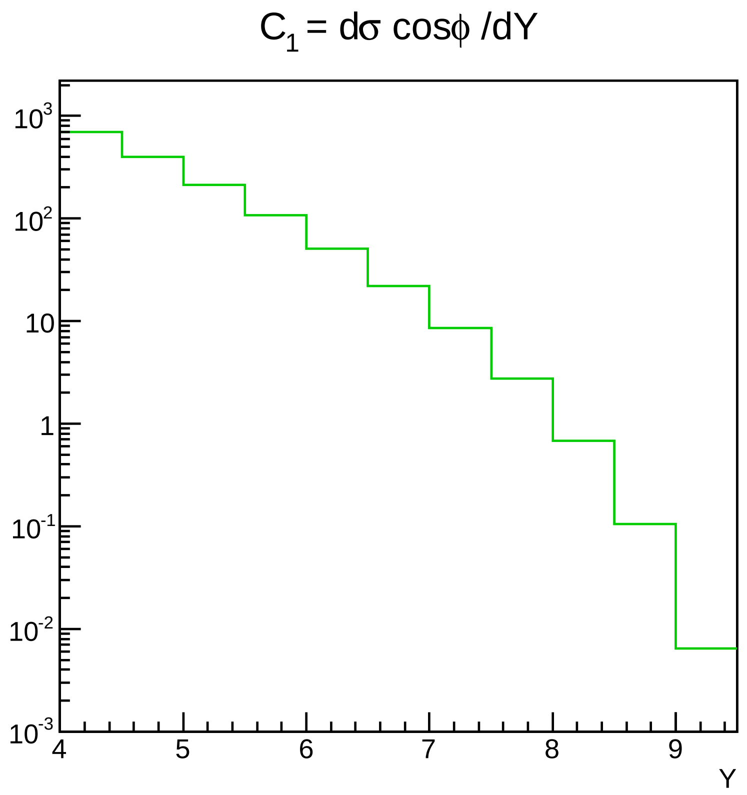
Matched cross section is positive, of the same magnitude of NLL BFKL prediction

Matching (azimuthal coeff. C_1)





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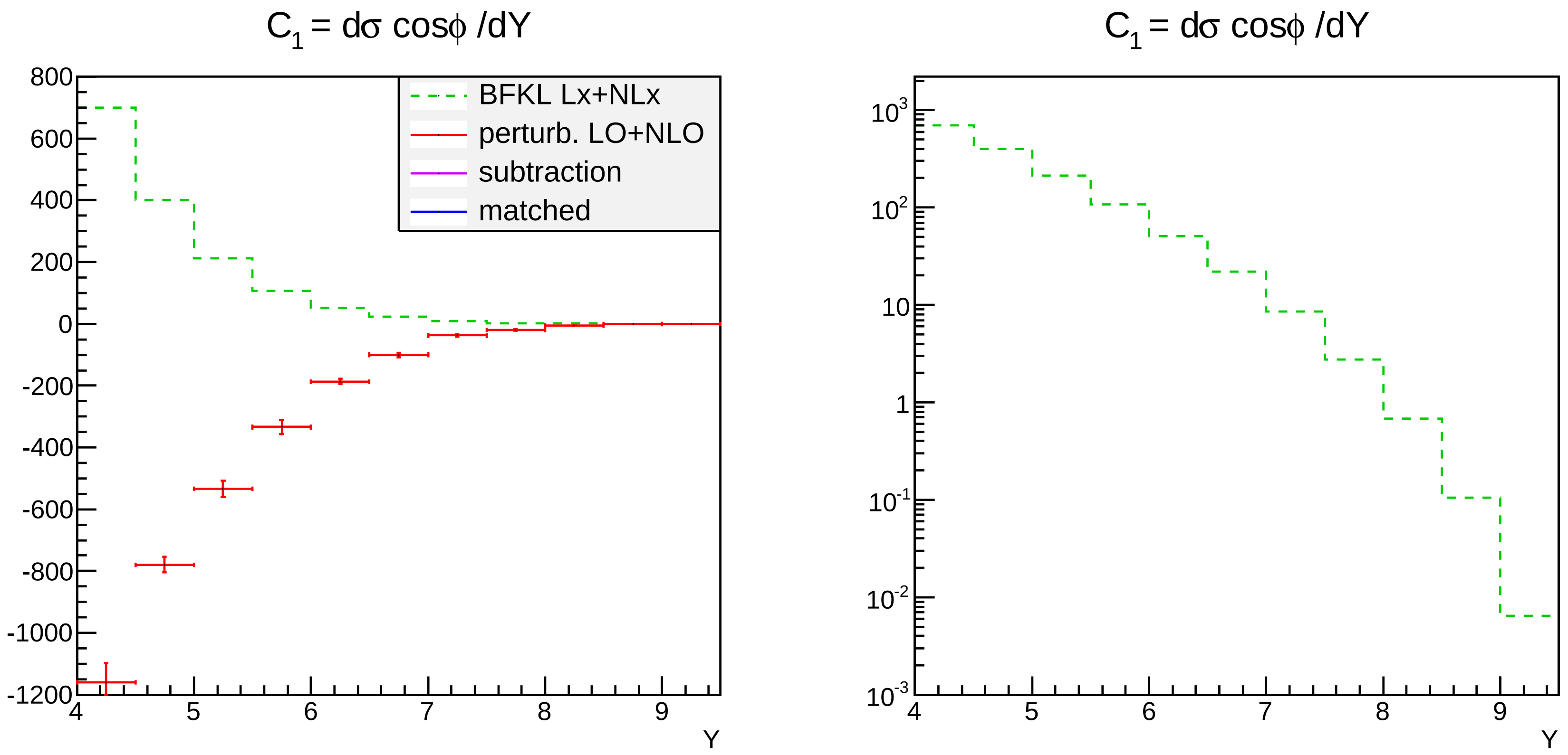

Large errors of NLO calculation due to very slow convergence in MC integration

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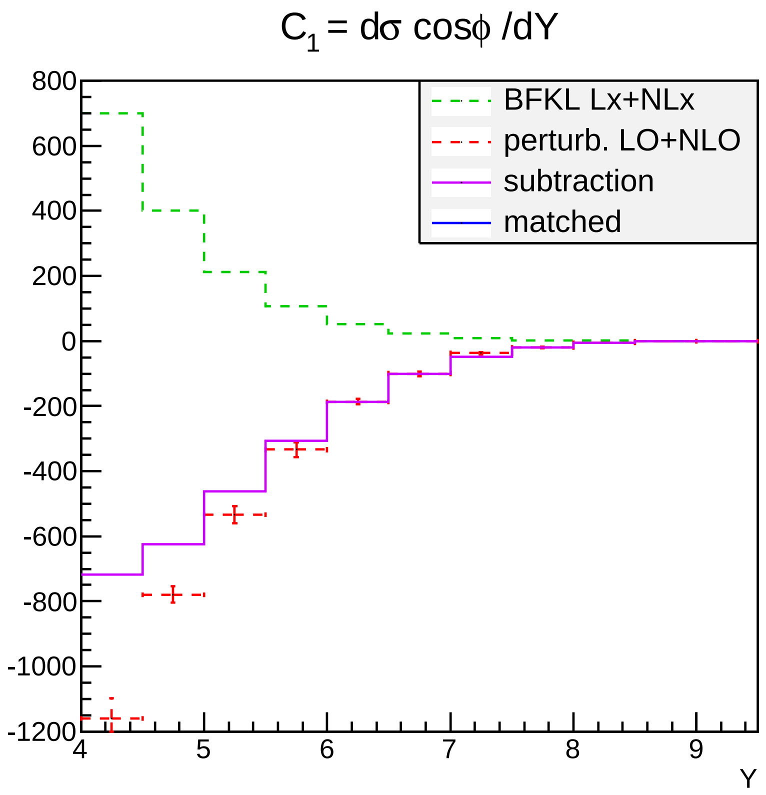

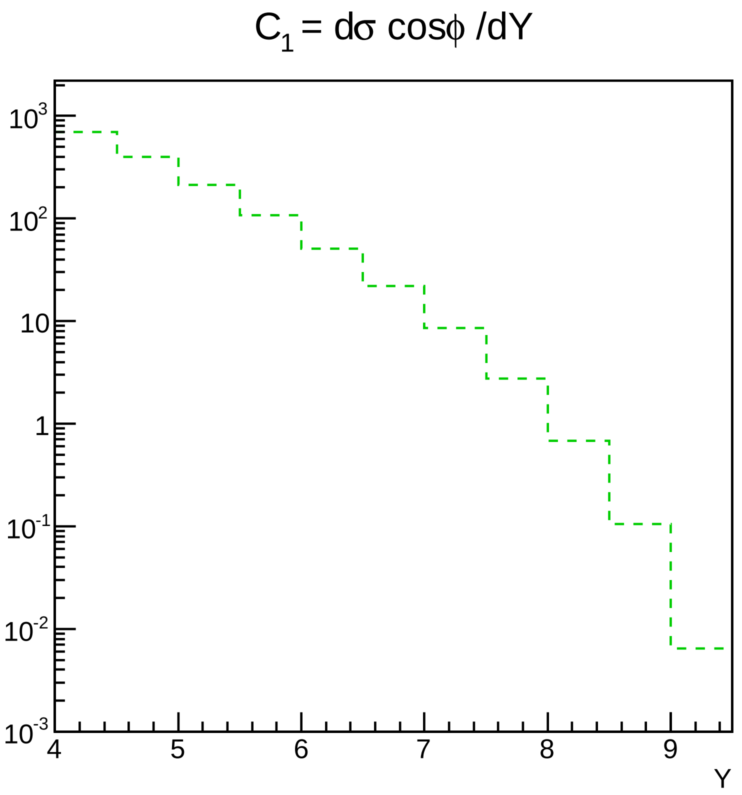

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Moderate difference between NLO and subtraction

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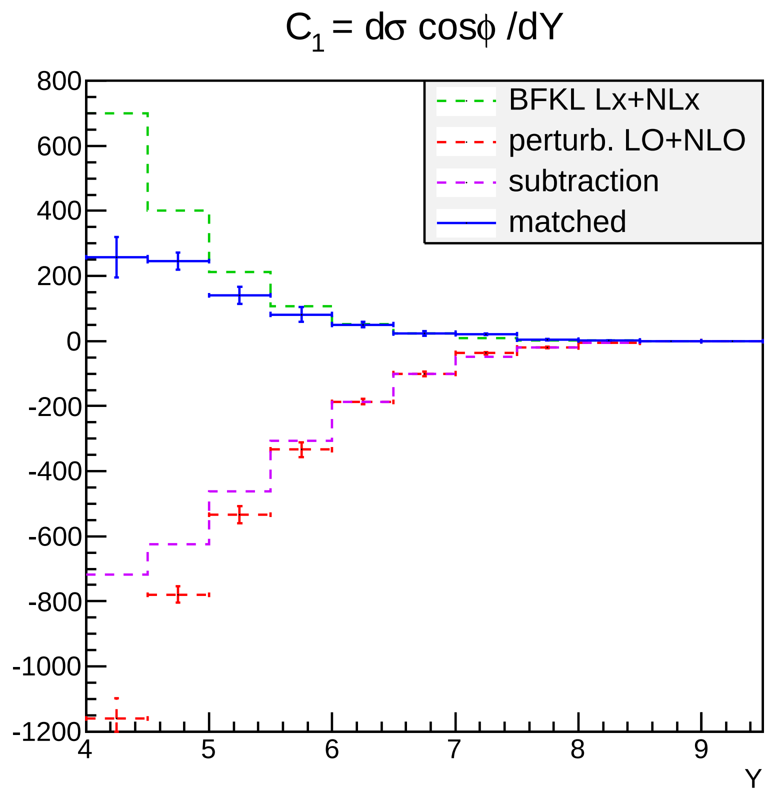

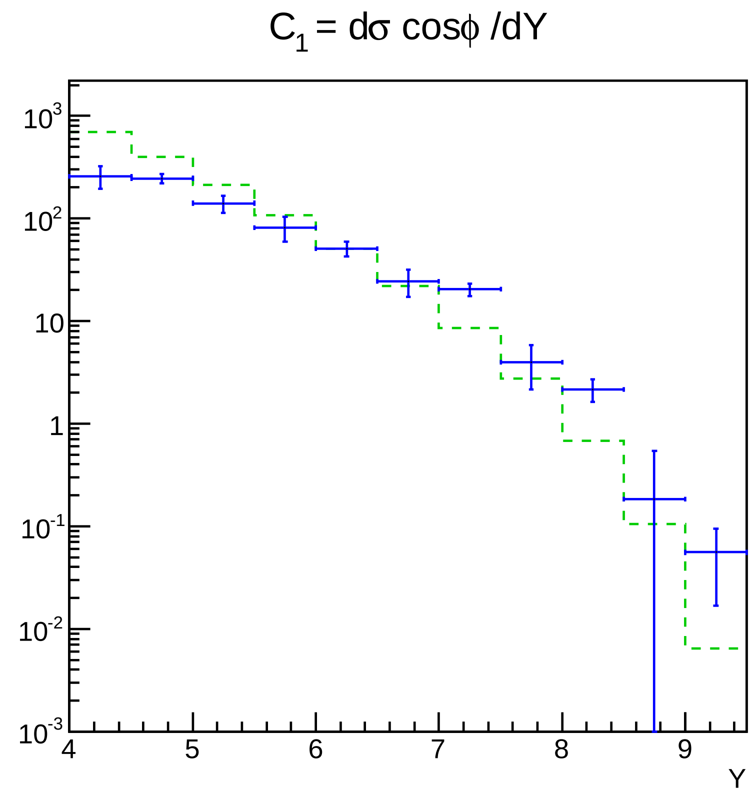

Large errors of NLO calculation due to very slow convergence in MC integration

Moderate difference between NLO and subtraction

Matched C_1 of the same magnitude of NLL BFKL prediction

but definitely different at intermediate $Y \simeq 4 \div 6$

PT instability of symmetric jets

It is well known that cross section of jets at NLO is very sensitive to the asymmetry parameter $\Delta = E_{T1} - E_{T2}$ *[Frixione,Ridolfi '97]*
The leading collinear singularity for real emission is given by

$$\sigma^{(r)} \propto \int \mathrm{d}\boldsymbol{k}_1 \mathrm{d}\boldsymbol{k}_2 \Theta(|\boldsymbol{k}_1| - E)\Theta(|\boldsymbol{k}_2| - (E+\Delta)) \frac{1}{(\boldsymbol{k}_1 + \boldsymbol{k}_2)^2 + \epsilon^2}$$
$$= A(\Delta, \epsilon) + B \log(\epsilon) - C\,(\Delta + \epsilon) \log(\Delta + \epsilon)$$

thus fixed order PTh is not reliable in this case (finite, but infinite deriv at $\Delta = 0$)

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$$= A(\Delta, \epsilon) + B \log(\epsilon) - C\,(\Delta + \epsilon) \log(\Delta + \epsilon)$$

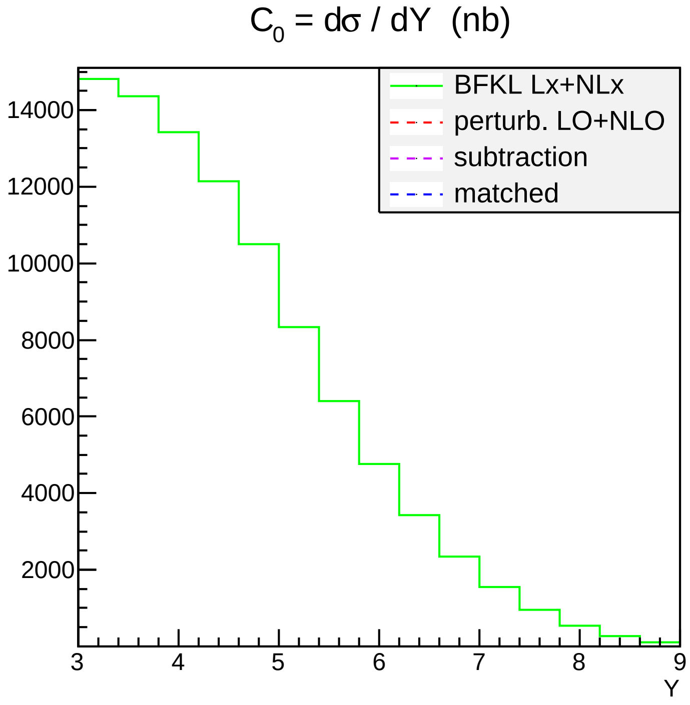
thus fixed order PTh is not reliable in this case (finite, but infinite deriv at $\Delta = 0$)

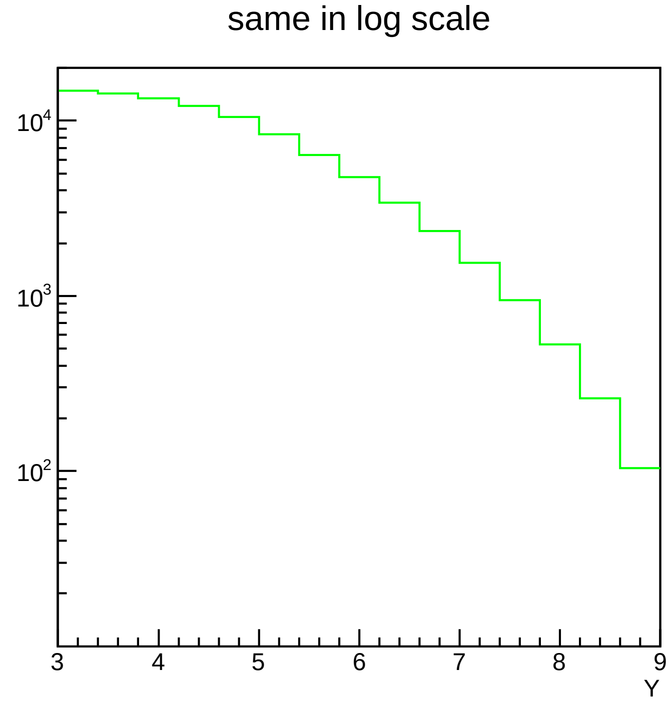
An analogous singularity occurs in the PT expansion of LL BFKL *[Andersen, Del Duca et al. '01]*

$$\sigma_{gg} \propto \frac{1}{(E+\Delta)^2}\left[1 - \alpha_s Y\left(\frac{2E\Delta + \Delta^2}{E^2} \log \frac{2E\Delta + \Delta^2}{(E+\Delta)^2} + 2\log\frac{E}{E+\Delta}\right)\right]$$

In the matching procedure such collinear $\Delta \log(\Delta)$ cancels out to a large extent, therefore the matching procedure should be safe

$\langle E_T \rangle$ cut: $\frac{1}{2}(E_{T1} + E_{T2}) > 35\mathrm{GeV}$





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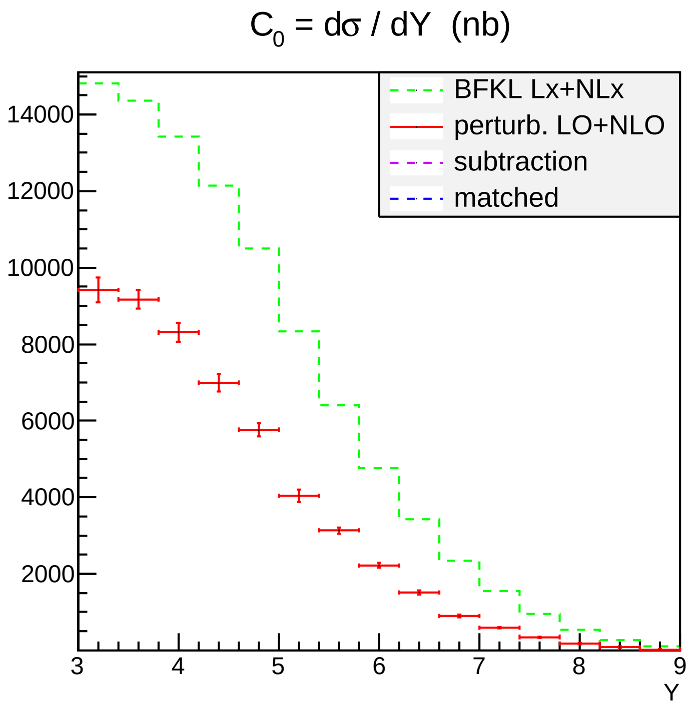

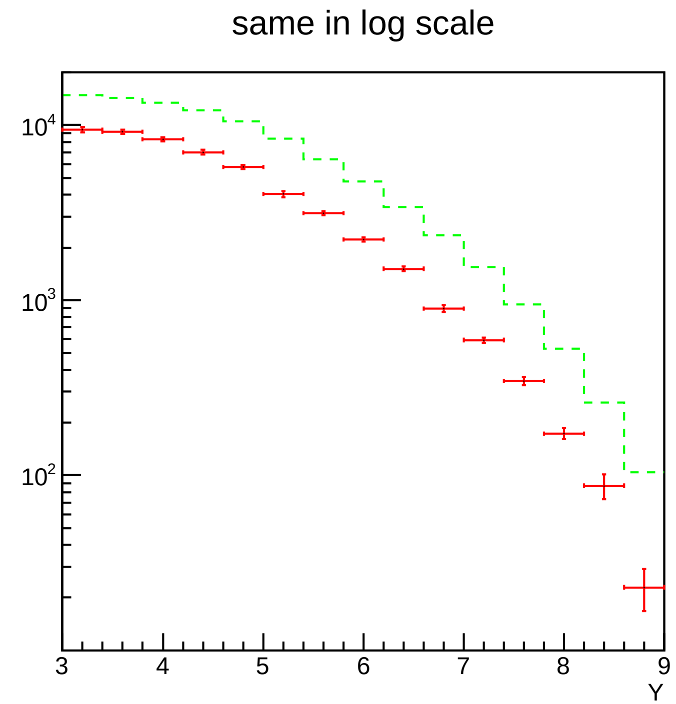

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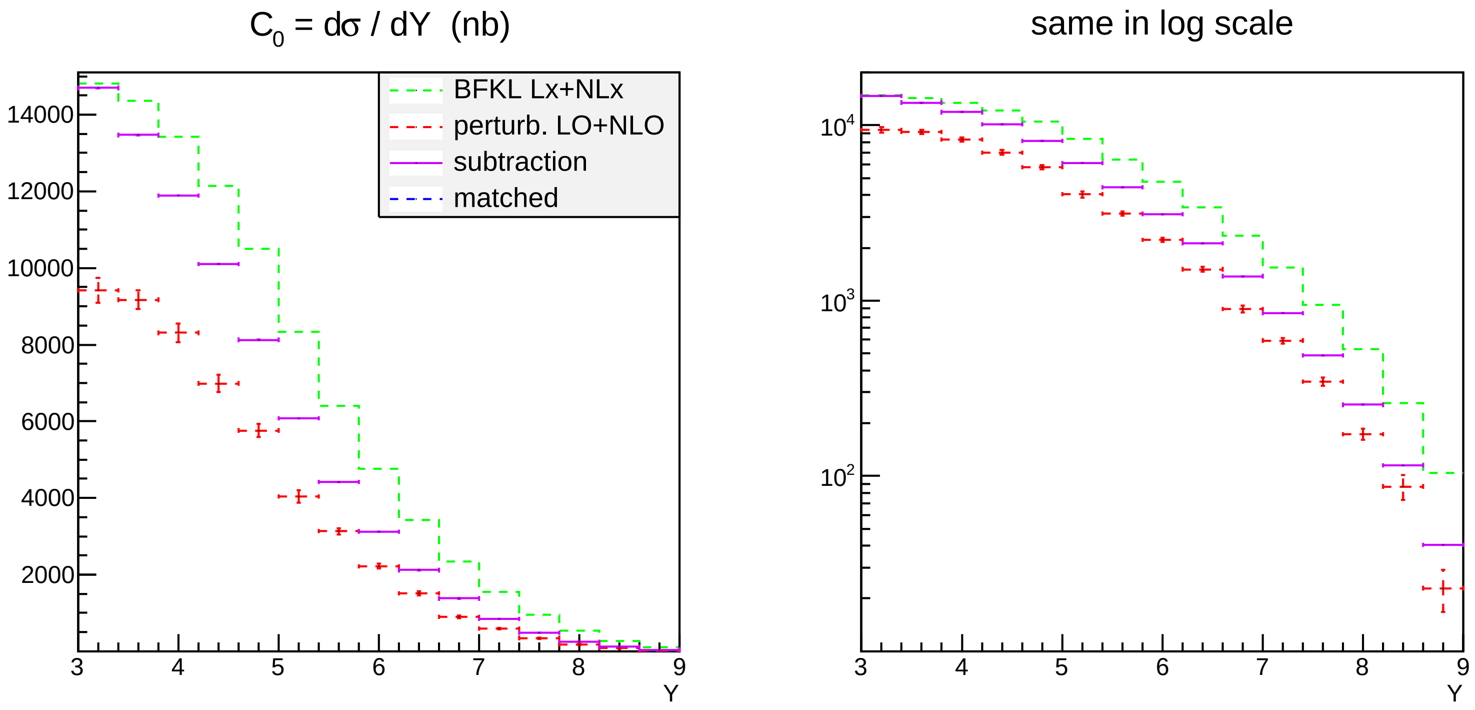

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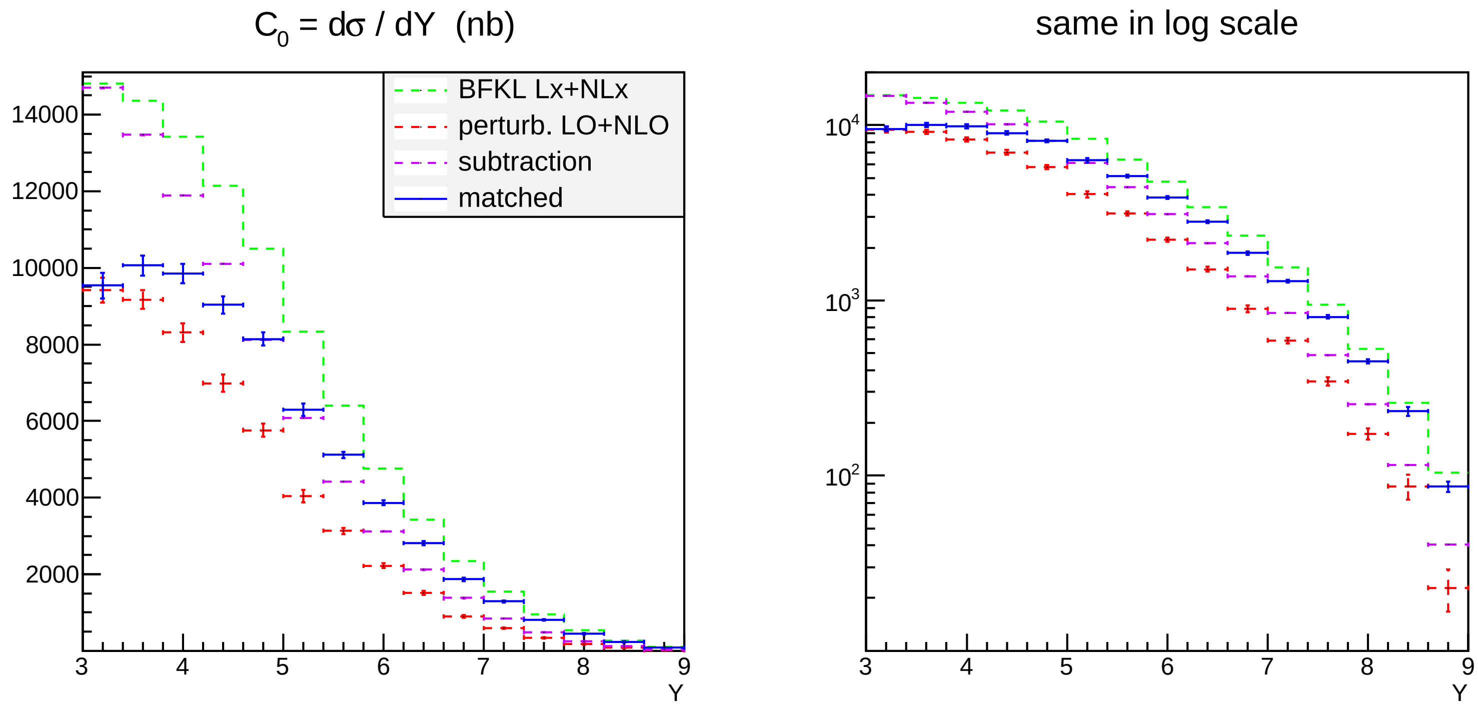

Procedure is more stable than that for symmetric jets

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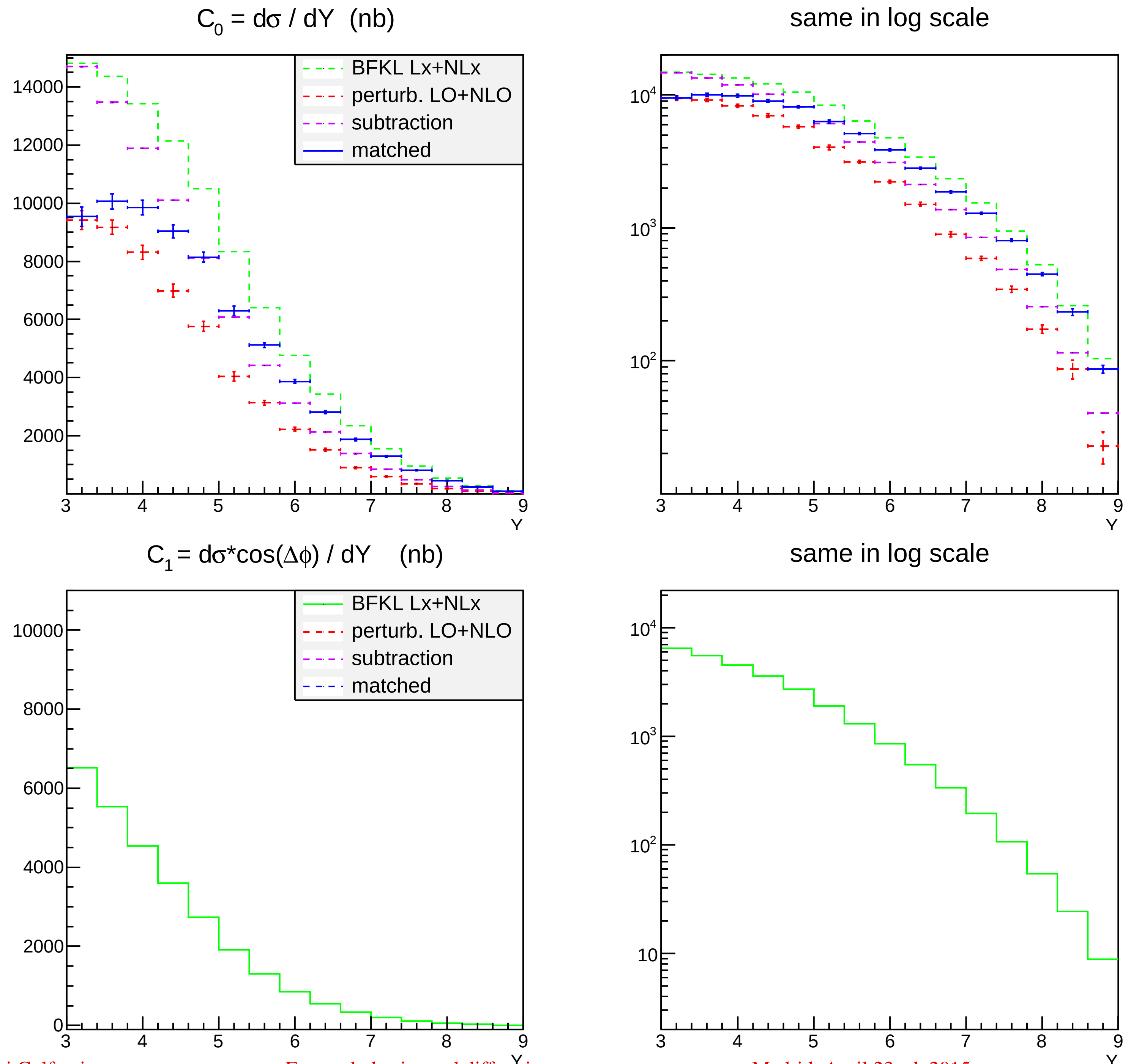

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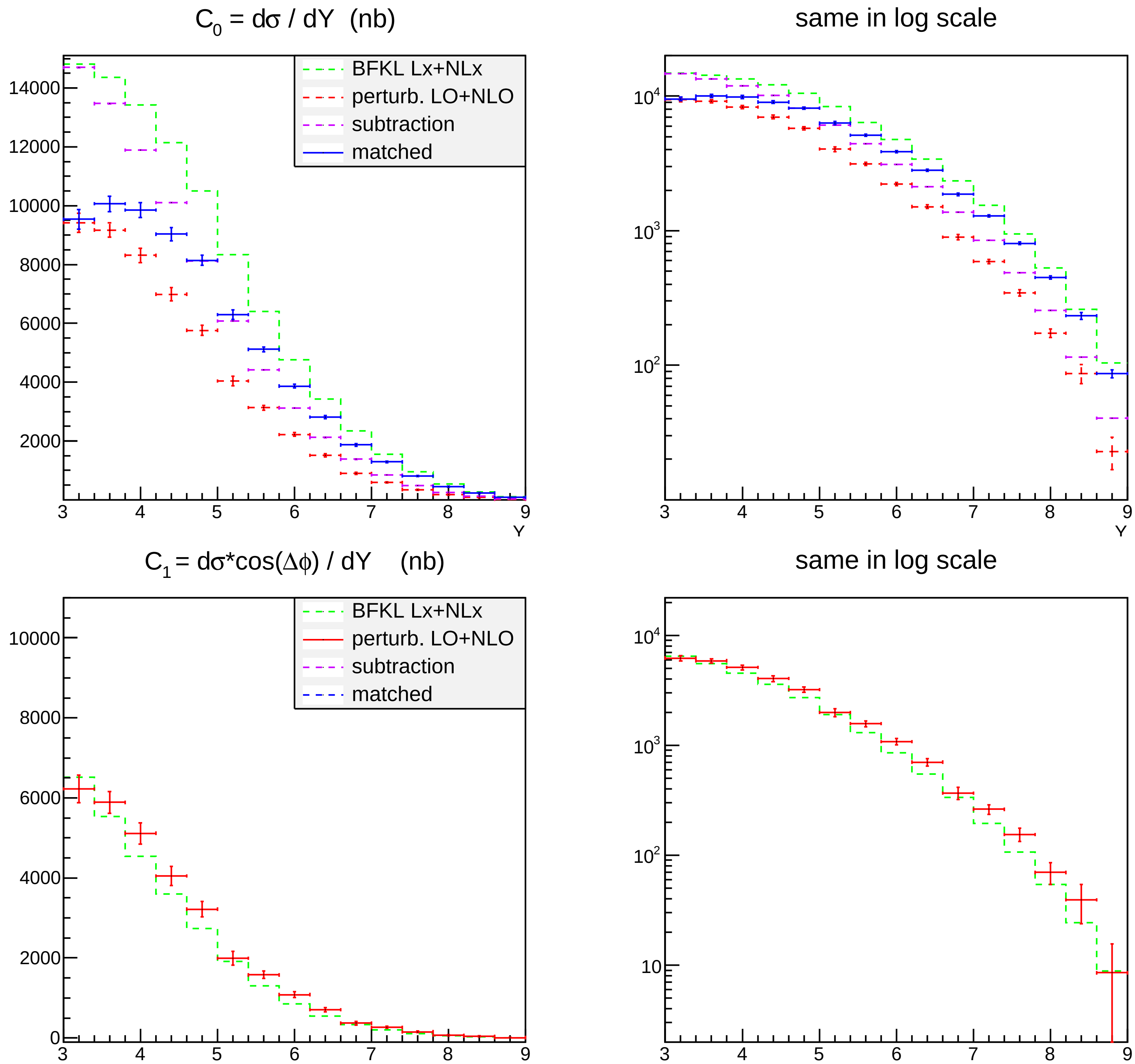

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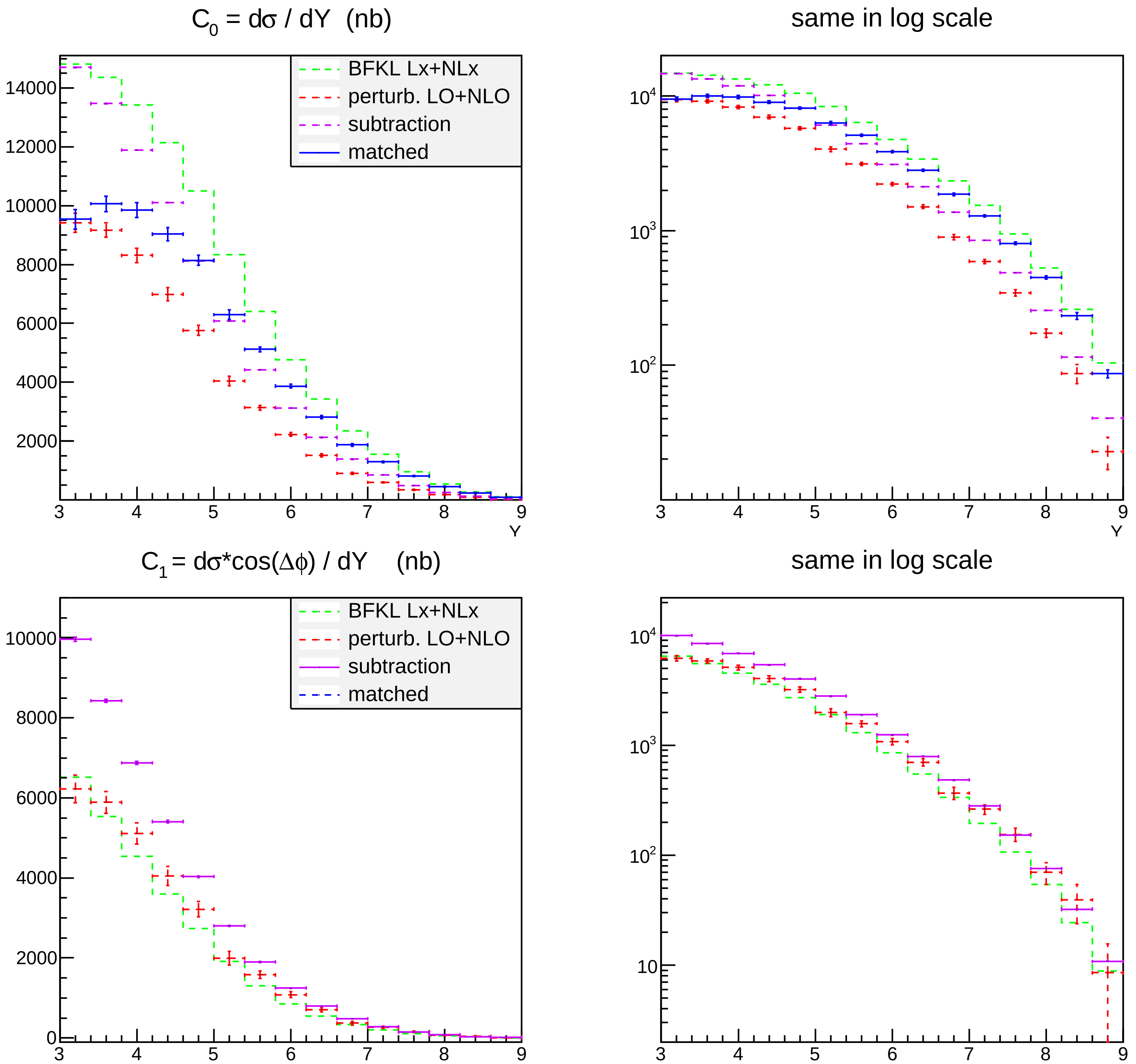

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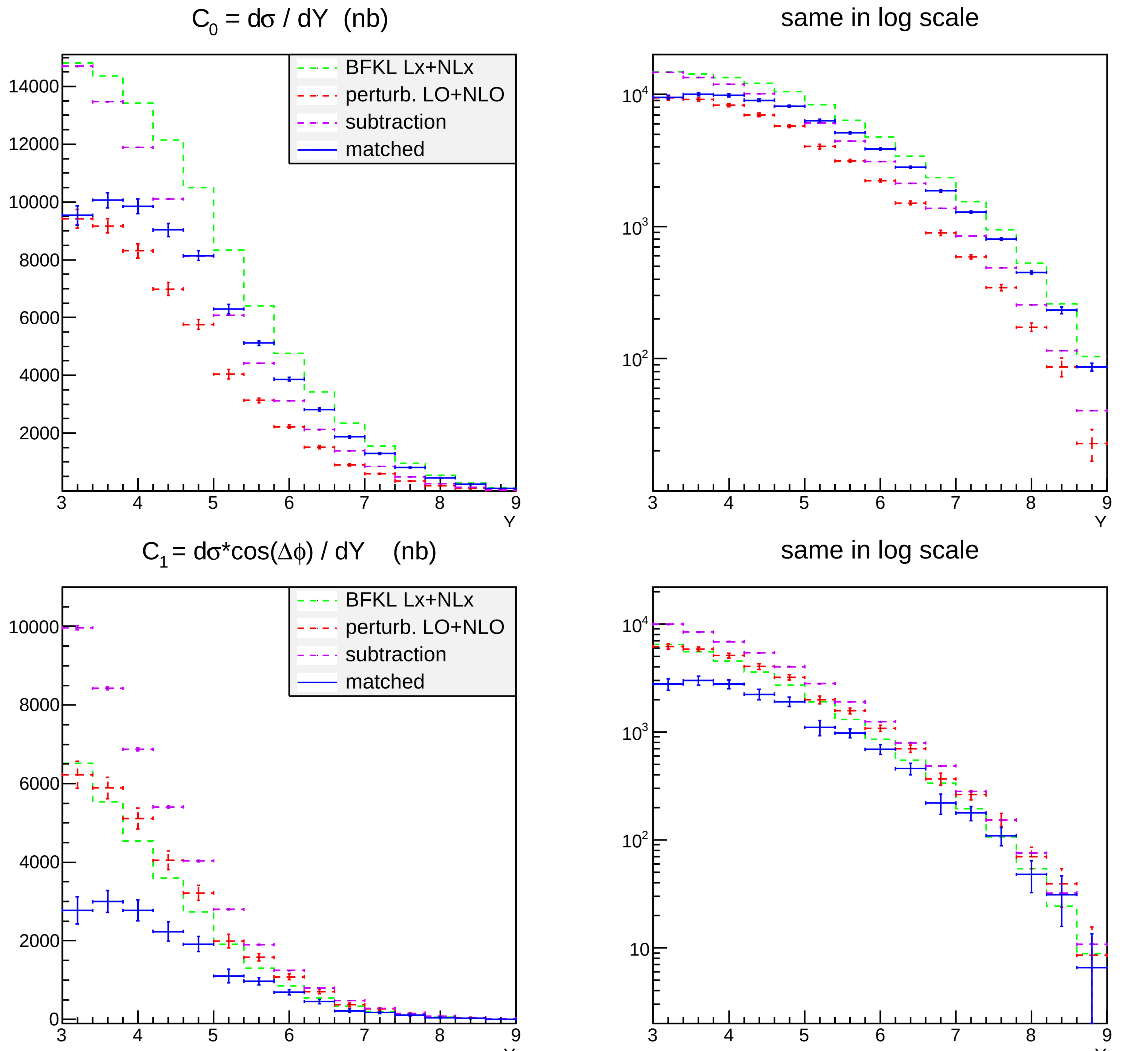

Future developments

- Increase "statistics" to reduce MC errors
- Estimate of errors due to variation of:
  - μ_R and μ_F scales
  - energy scale s_0
  - PDF uncertainties

- We strongly suggest experimentalists to perform MN jet analysis with *average* E_T *cut:* $\frac{1}{2}(E_{T1} + E_{T2}) > E_{\text{cut}}$
  in order to avoid perturbative sensitivity to phase space corner $E_{T1} = E_{T2} = E_{\text{cut}}$

  $\Longrightarrow$ smaller theoretical uncertainties

Jet algorithm and NJA

- Narrow-jet approximation (NJA):
  semi-analytic expansion of jet vertices at small "cone size" R *[Ivanov-Papa'12]*

$$f \otimes V = A \log(R) + B + \mathcal{O}\left(R^2\right)$$

- Useful approximation for fast (and accurate) computation of jet vertices

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- We recomputed NJA vertices with $k_\perp$-algorithm *[DC, Niccoli '15]*
- Sizeable impact of algorithm on jet observables:
  $\sim$15% on cross section and $\sim$6% on angular ratios

NJA in Furman and kT algorithms

$$
\begin{aligned}
I_q = {} & \frac{\alpha_{\rm s}}{2\pi}(k^2)^\gamma \mathrm{e}^{in\phi} \int_{x_J}^1 \frac{\mathrm{d}\zeta}{\zeta} \sum_{a=q,\bar{q}} f_a\left(\frac{x_J}{\zeta}\right) \Bigg\{ \left[P_{qq}(\zeta) + \frac{C_A}{C_F} P_{gq}(\zeta)\right] \log\frac{k^2}{\mu_F^2} + \\
& - 2\zeta^{-2\gamma}[P_{qq}(\zeta) + P_{gq}(\zeta)] \log\frac{R}{\langle\, \boldsymbol{\max(\zeta,\bar{\zeta})} \rangle_{\boldsymbol{C}}} - \frac{\beta_0}{2}\log\frac{k^2}{\mu_R^2}\delta(1-\zeta) \\
& + C_A \delta(1-\zeta)\left\{\chi_{n\nu}^{(0)} \log\frac{s_0}{k^2} + \frac{85}{18} + \frac{\pi^2}{2} + \frac{1}{2}\left[\psi'\left(1+\gamma+\frac{n}{2}\right) - \psi'\left(\frac{n}{2}-\gamma\right) - \chi_{n\nu}^{(0)\,2}\right]\right\} \\
& + (1+\zeta^2)\left\{ C_A \left[\frac{(1+\zeta^{-2\gamma})\chi_{n\nu}^{(0)}}{2(1-\zeta)_+} - \zeta^{-2\gamma}\left(\frac{\log(1-\zeta)}{1-\zeta}\right)_+\right]\right. \\
& \left. + \left(C_F - \frac{C_A}{2}\right)\left[\frac{\bar{\zeta}}{\zeta^2} I_2 - \frac{2\log\zeta}{\bar{\zeta}} + 2\left(\frac{\log(1-\zeta)}{1-\zeta}\right)_+\right]\right\} \\
& + \delta(1-\zeta)\left[C_F\left(3\log 2 - \frac{\pi^2}{3} - \frac{9}{2} + \left\langle \boldsymbol{3 - \frac{\pi^2}{3} - 3\log 2} \right\rangle_{\boldsymbol{K}}\right) - \frac{10}{9} n_f T_R\right] \\
& + C_A\zeta + C_F\bar{\zeta} + \frac{1+\bar{\zeta}^2}{\zeta}\left[C_A \frac{\bar{\zeta}}{\zeta} I_1 + 2C_A \log\frac{\bar{\zeta}}{\zeta} + C_F \zeta^{-2\gamma}(\chi_{n\nu}^{(0)} - 2\log\bar{\zeta})\right] \Bigg\},
\end{aligned}
$$

where $\gamma \equiv \mathrm{i}\nu - 1/2$, $\beta_0 \equiv (11C_A - 4n_f T_R)/3$

NJA in Furman and kT algorithms

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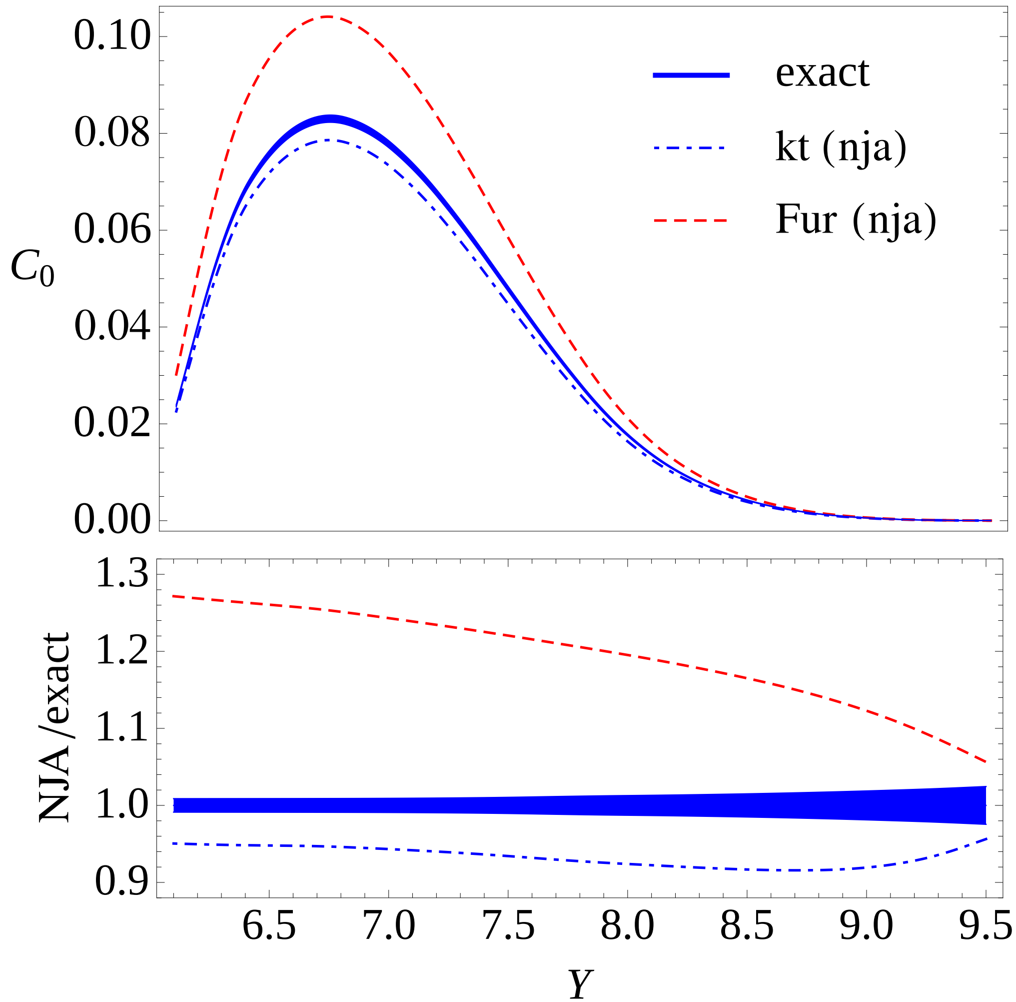
\begin{aligned}
I_g = {} & \frac{\alpha_s}{2\pi}(k^2)^\gamma e^{in\phi} \int_{x_J}^1 \frac{d\zeta}{\zeta} f_g\left(\frac{x_J}{\zeta}\right) \frac{C_A}{C_F} \Bigg\{ \left[P_{gg}(\zeta) + \frac{C_A}{C_F} 2n_F P_{qg}(\zeta)\right] \log\frac{k^2}{\mu_F^2} + \\
& - 2\zeta^{-2\gamma}[P_{gg}(\zeta) + 2n_f P_{qg}(\zeta)] \log\frac{R}{\langle \boldsymbol{\max(\zeta,\bar\zeta)} \rangle_{\boldsymbol{C}}} - \frac{\beta_0}{2}\log\frac{k^2}{4\mu_R^2}\delta(1-\zeta) \\
& + C_A \delta(1-\zeta) \left\{ \chi_{n\nu}^{(0)} \log\frac{s_0}{k^2} + \frac{1}{2}\left[\psi'\left(1+\gamma+\frac{n}{2}\right) - \psi'\left(\frac{n}{2}-\gamma\right) - \chi_{n\nu}^{(0)\,2}\right] \right. \\
& \left. \qquad + \frac{1}{12} + \frac{\pi^2}{6} + \left\langle \boldsymbol{\frac{131}{36} - \frac{\pi^2}{3} - \frac{11}{3}\log 2} \right\rangle_{\boldsymbol{K}} \right\} \\
& + 2C_A(1-\zeta^{-2\gamma})\left[\left(\frac{1}{\zeta} - 2 + \zeta\bar\zeta\right)\log\bar\zeta + \frac{\log(1-\zeta)}{1-\zeta}\right] \\
& + C_A\left[\frac{1}{\zeta} + \frac{1}{(1-\zeta)_+} - 2 + \zeta\bar\zeta\right]\left[(1+\zeta^{-2\gamma})\chi_{n\nu}^{(0)} - 2\log\zeta + \frac{\bar\zeta^2}{\zeta^2} I_2\right] \\
& + 2n_f T_R \left[2\frac{C_F}{C_A}\zeta\bar\zeta + (\zeta^2 + \bar\zeta^2)\left(\frac{C_F}{C_A}\chi_{n\nu}^{(0)} + \frac{\bar\zeta}{\zeta} I_3\right)\right. \\
& \left.\left. \qquad + \delta(1-\zeta)\left(-\frac{1}{12} + \left\langle \boldsymbol{\frac{2}{3}\log 2 - \frac{23}{36}} \right\rangle_{\boldsymbol{K}}\right)\right] \right\}.
\end{aligned}
$$

Furman VS kT algorithm in NJA

Differential cross section ($R = 0.5$)

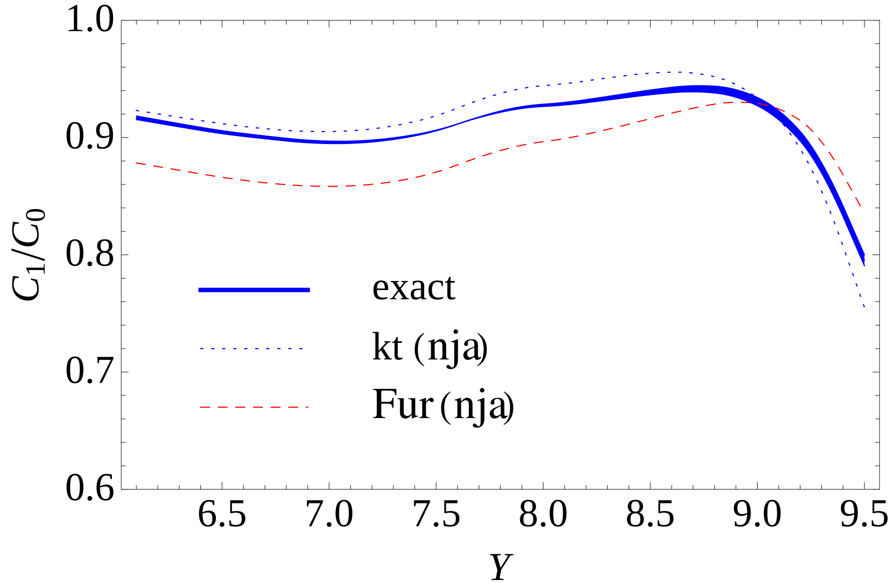


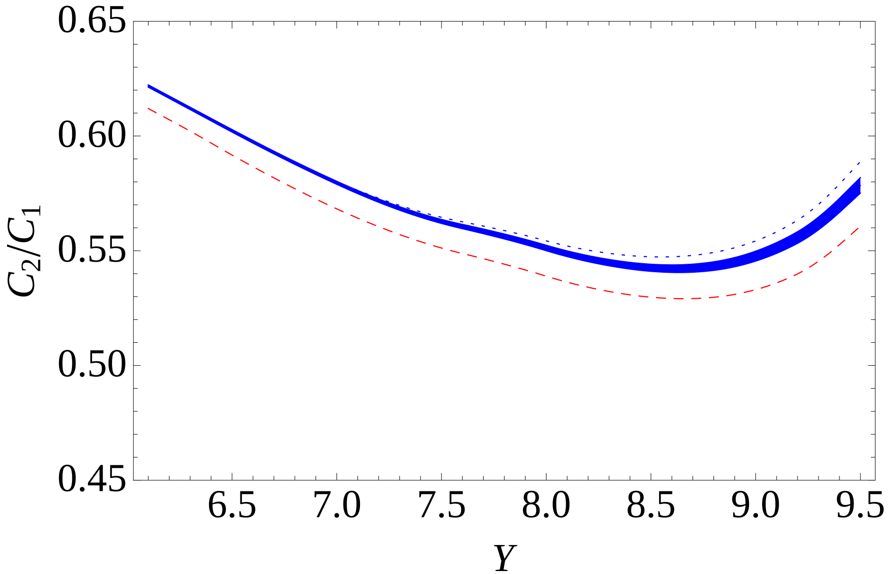
NJA within 4-6% of exact result

Wrong algorithm ⇒ discrepancy ∼20%

Furman VS kT algorithm in NJA

Angular ratios $C_m/C_n = \langle\cos(m\phi)\rangle \, / \, \langle\cos(n\phi)\rangle$ for $R = 0.5$





NJA within 2% of exact result

Wrong algorithm ⇒ discrepancy $\sim$5%

Choice of algorithm is important

Conclusions and outlook

- Mueller-Navelet jets appear to be a good observable for demonstrating presence of BFKL dynamics at high energy
- Fixed order MC and NLL BFKL quite different,
  in some cases close to data, but overall agreement is not good
- NLL predictions suffer scale uncertainties $\sim 15\%$
  Satisfactory phenomenology with a scale-fixing at very large scale $\mu_R \sim 20\, E_{TJ}$

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- NLL predictions suffer scale uncertainties $\sim 15\%$
  Satisfactory phenomenology with a scale-fixing at very large scale $\mu_R \sim 20\, E_{TJ}$
- We propose to match fixed order and resummed calculations
  in order to obtain more accurate and stable predictions
- Preliminary results are encouraging,
  in particular with asymmetric jets or E_T-sum cut
  $\longrightarrow$ new experimental analysis is required
- We provide the NJA for $k_\perp$ algorithm
- Full analysis with estimate of errors is in the way