

High-energy resummation effects in Mueller-Navelet jets production at the LHC

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in collaboration with

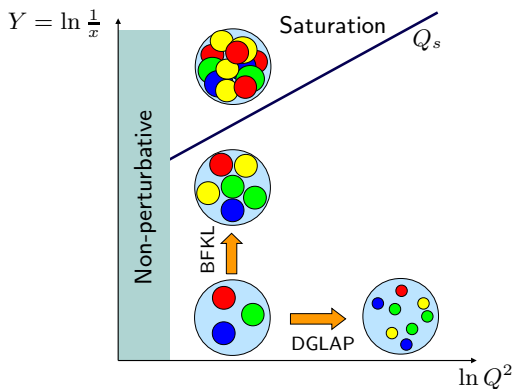
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JHEP **1305** (2013) 096 [arXiv:1302.7012 [hep-ph]]

PRL **112** (2014) 082003 [arXiv:1309.3229 [hep-ph]]

PLB **738** (2014) 311 [arXiv:1407.6593 [hep-ph]]

- One of the important longstanding theoretical questions raised by QCD is its behaviour in the perturbative **Regge** limit $s \gg -t$
- We want to identify and test suitable observables in order to test these peculiar dynamics



\Rightarrow select semi-hard processes with $s \gg p_{T_i}^2 \gg \Lambda_{QCD}^2$ where $p_{T_i}^2$ are typical transverse scales, **all of the same order**

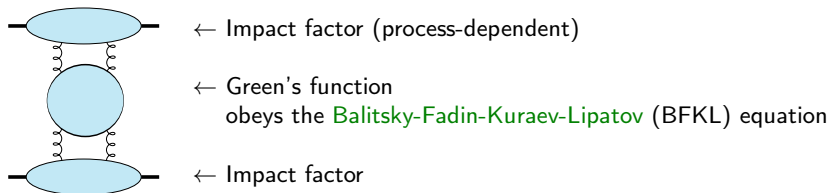
At leading logarithmic (LL) accuracy (resumming terms $(\alpha_s \ln s)^n$), the scattering amplitude can be written as:

$$\mathcal{A} = \underbrace{\text{Diagram 1}}_{\sim s} + \left(\underbrace{\text{Diagram 2}}_{\sim s} + \underbrace{\text{Diagram 3}}_{\sim s} + \dots \right) + \left(\underbrace{\text{Diagram 4}}_{\sim s} + \dots \right) + \dots$$

The diagrams are:

- Diagram 1: Two light blue ovals connected by two vertical wavy lines.
- Diagram 2: Two light blue ovals connected by two vertical wavy lines, with a horizontal wavy line connecting the two vertical lines.
- Diagram 3: Two light blue ovals connected by two vertical wavy lines, with a circular loop of wavy lines between the two vertical lines.
- Diagram 4: Two light blue ovals connected by two vertical wavy lines, with a square loop of wavy lines between the two vertical lines.

this can be put in the following form :



Often LL calculations don't describe experimental data very well

⇒ What about higher orders?

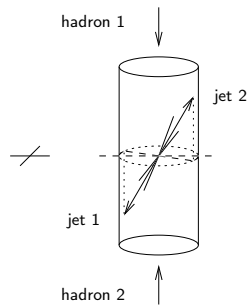
- The next-to-leading logarithmic (NLL) corrections to the BFKL kernel are known (Lipatov, Fadin; Camici, Ciafaloni)
Corresponds to resumming also $\alpha_s (\alpha_s \ln s)^n$ terms
- Impact factors are known in some cases at NLL
 - $\gamma^* \rightarrow \gamma^*$ at $t = 0$ (Bartels, Colferai, Gieseke, Kyrieleis, Qiao; Balitski, Chirilli)
 - Forward jet production (Bartels, Colferai, Vacca; Caporale, Ivanov, Murdaca, Papa, Perri)
small cone approximation (Ivanov, Papa; Colferai, Niccoli)
 - Forward jet with rapidity gap (Hentschinski, Madrigal, Murdaca, Sabio Vera)
 - Inclusive production of a pair of hadrons separated by a large interval of rapidity (Ivanov, Papa)
 - $\gamma_L^* \rightarrow \rho_L$ in the forward limit (Ivanov, Kotsky, Papa)

Mueller-Navelet jets were proposed as a possible test of BFKL dynamics at hadron colliders

Consider two jets separated by a large interval rapidity, i.e. each of them almost fly in the direction of the hadron “close” to it, and with similar transverse momenta

In a pure LO collinear treatment, these two jets should be emitted exactly back to back: $\varphi = 0$ ($\varphi = \phi_{J,1} - \phi_{J,2} - \pi$)

A BFKL calculation predicts some decorrelation because of the emission of soft gluons in the rapidity interval



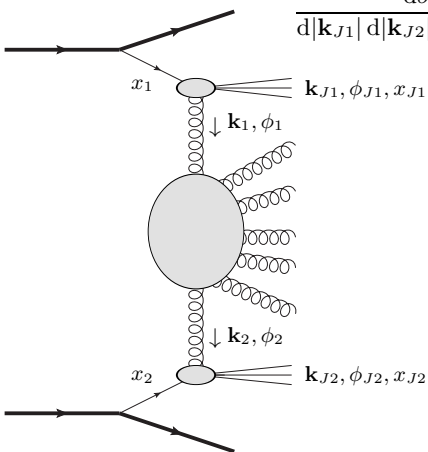
k_T -factorized differential cross section

$$\frac{d\sigma}{d|\mathbf{k}_{J1}| d|\mathbf{k}_{J2}| dy_{J1} dy_{J2}} = \int d\phi_{J1} d\phi_{J2} \int d^2\mathbf{k}_1 d^2\mathbf{k}_2$$

$$\times \Phi(\mathbf{k}_{J1}, x_{J1}, -\mathbf{k}_1)$$

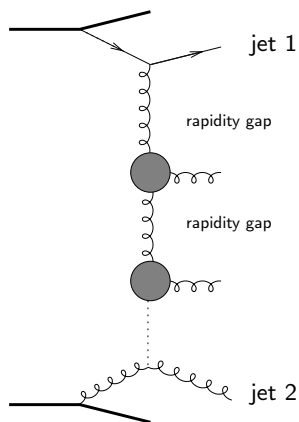
$$\times G(\mathbf{k}_1, \mathbf{k}_2, \hat{s})$$

$$\times \Phi(\mathbf{k}_{J2}, x_{J2}, \mathbf{k}_2)$$



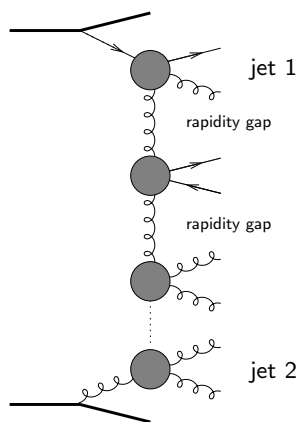
$$\text{with } \Phi(\mathbf{k}_{J2}, x_{J2}, \mathbf{k}_2) = \int dx_2 f(x_2) V(\mathbf{k}_2, x_2) \quad f \equiv \text{PDF} \quad x_J = \frac{|\mathbf{k}_J|}{\sqrt{s}} e^{y_J}$$

LL BFKL



$$\sum (\alpha_s \ln s)^n$$

NLL BFKL



$$\sum (\alpha_s \ln s)^n + \alpha_s \sum (\alpha_s \ln s)^n$$

It is convenient to define the coefficients \mathcal{C}_n as

$$\mathcal{C}_n \equiv \int d\phi_{J1} d\phi_{J2} \cos(n(\phi_{J1} - \phi_{J2} - \pi)) \\ \times \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 \Phi(\mathbf{k}_{J1}, x_{J1}, -\mathbf{k}_1) G(\mathbf{k}_1, \mathbf{k}_2, \hat{s}) \Phi(\mathbf{k}_{J2}, x_{J2}, \mathbf{k}_2)$$

- $n = 0 \implies$ differential cross-section

$$\mathcal{C}_0 = \frac{d\sigma}{d|\mathbf{k}_{J1}| d|\mathbf{k}_{J2}| dy_{J1} dy_{J2}}$$

- $n > 0 \implies$ azimuthal decorrelation

$$\frac{\mathcal{C}_n}{\mathcal{C}_0} = \langle \cos(n(\phi_{J,1} - \phi_{J,2} - \pi)) \rangle \equiv \langle \cos(n\varphi) \rangle$$

- sum over $n \implies$ azimuthal distribution

$$\frac{1}{\sigma} \frac{d\sigma}{d\varphi} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \langle \cos(n\varphi) \rangle \right\}$$

Comparison with data

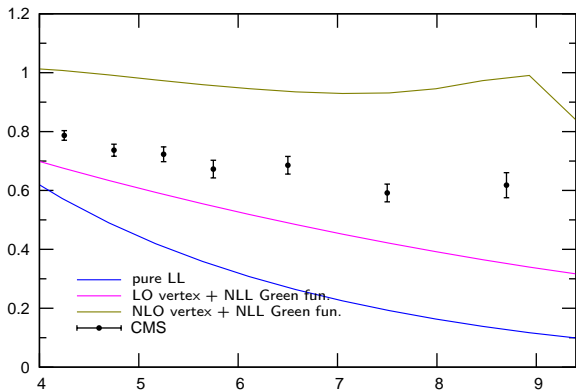
The following results are for

- $\sqrt{s} = 7 \text{ TeV}$
- $35 \text{ GeV} < |\mathbf{k}_{J1}|, |\mathbf{k}_{J2}| < 60 \text{ GeV}$
- $0 < |y_1|, |y_2| < 4.7$

And we compare these with experimental data on the azimuthal correlations of Mueller-Navelet jets at the LHC from CMS (CMS-PAS-FSQ-12-002)

Azimuthal correlation $\langle \cos \varphi \rangle$

$$\frac{c_1}{c_0} = \langle \cos \varphi \rangle \equiv \langle \cos(\phi_{J1} - \phi_{J2} - \pi) \rangle$$



$$35 \text{ GeV} < |\mathbf{k}_{J1}| < 60 \text{ GeV}$$

$$35 \text{ GeV} < |\mathbf{k}_{J2}| < 60 \text{ GeV}$$

$$0 < |y_1| < 4.7$$

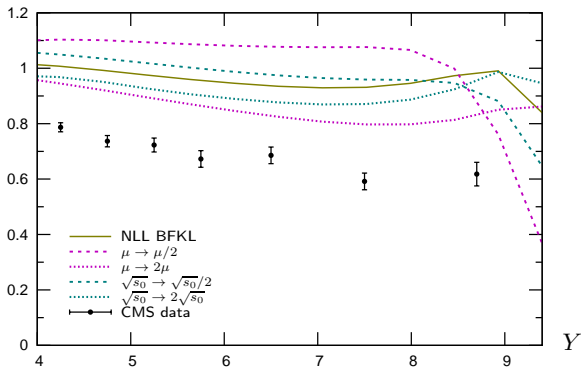
$$0 < |y_2| < 4.7$$

$$Y \equiv |y_1 - y_2|$$

The NLO corrections to the jet vertex lead to a large increase of the correlation

Azimuthal correlation $\langle \cos \varphi \rangle$

$$\langle \cos \varphi \rangle \equiv \langle \cos(\phi_{J1} - \phi_{J2} - \pi) \rangle$$



$$35 \text{ GeV} < |\mathbf{k}_{J1}| < 60 \text{ GeV}$$

$$35 \text{ GeV} < |\mathbf{k}_{J2}| < 60 \text{ GeV}$$

$$0 < |y_1| < 4.7$$

$$0 < |y_2| < 4.7$$

- NLL BFKL predicts a too small decorrelation
- The NLL BFKL calculation is still rather dependent on the scales, especially the renormalization / factorization scale

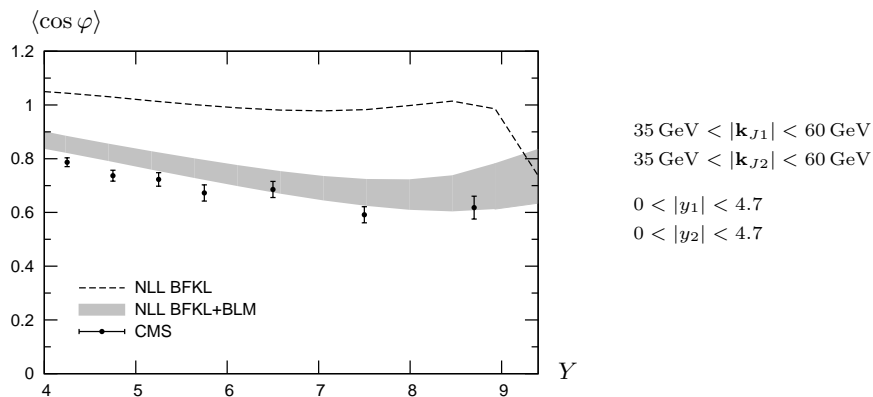
- A LL calculation cannot describe the experimental data
- A NLL calculation does not really provide a better agreement
- The NLL calculation still depends strongly on the choice of the renormalization scale μ_R
- An all-order calculation would be independent of the choice of μ_R . This feature is lost if we truncate the perturbative series
⇒ How to choose the renormalization scale?

We used the **Brodsky-Lepage-Mackenzie** (BLM) procedure to fix the renormalization scale

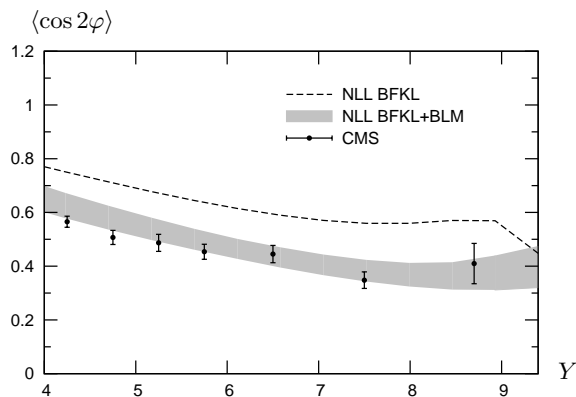
The **Brodsky-Lepage-Mackenzie** (BLM) procedure resums the self-energy corrections to the gluon propagator at one loop into the running coupling.

First attempts to apply BLM scale fixing to BFKL processes lead to problematic results. **Brodsky, Fadin, Kim, Lipatov and Pivovarov** suggested that one should first go to a physical renormalization scheme like MOM and then apply the 'traditional' BLM procedure, i.e. identify the β_0 dependent part and choose μ_R such that it vanishes.

We follow this prescription for the full amplitude at NLL.

Azimuthal correlation $\langle \cos \varphi \rangle$ 

Using the BLM scale setting, the agreement with data becomes much better

Azimuthal correlation $\langle \cos 2\varphi \rangle$ 

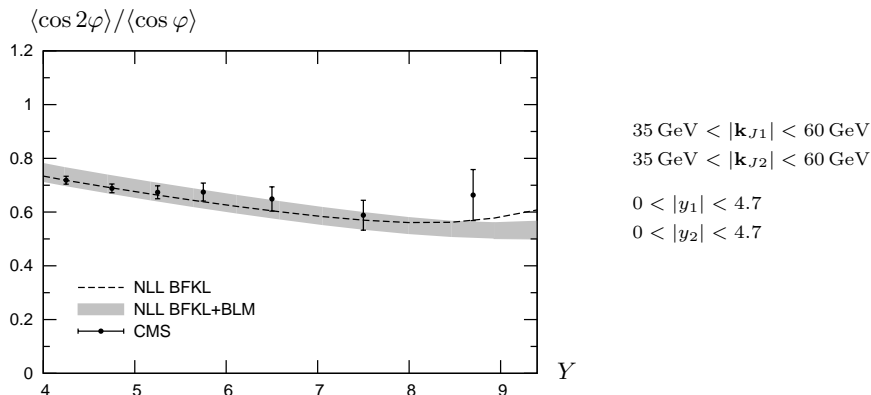
$$35 \text{ GeV} < |\mathbf{k}_{J1}| < 60 \text{ GeV}$$

$$35 \text{ GeV} < |\mathbf{k}_{J2}| < 60 \text{ GeV}$$

$$0 < |y_1| < 4.7$$

$$0 < |y_2| < 4.7$$

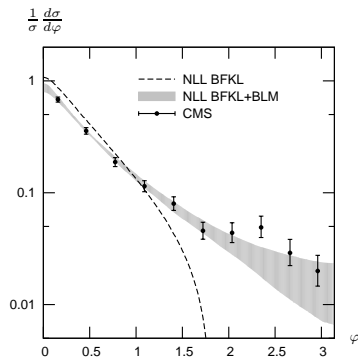
Using the BLM scale setting, the agreement with data becomes much better

Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ 

This observable is much less dependant on μ than $\langle \cos n\varphi \rangle$

Already the case at LL (Sabio Vera, Schwennsen)

The good agreement with data is not affected by the BLM procedure

Azimuthal distribution (integrated over $6 < Y < 9.4$)

With the BLM scale setting the azimuthal distribution is in good agreement with the data across the full φ range.

Using the BLM scale setting:

- The agreement of $\langle \cos n\varphi \rangle$ with the data becomes much better
- The agreement for $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ is still good and unchanged as this observable is weakly dependent on μ_R
- The azimuthal distribution is in much better agreement with the data

But the configuration chosen by **CMS** with $\mathbf{k}_{J_{\min 1}} = \mathbf{k}_{J_{\min 2}}$ does not allow us to compare with a **fixed-order** $\mathcal{O}(\alpha_s^3)$ treatment (i.e. without resummation)

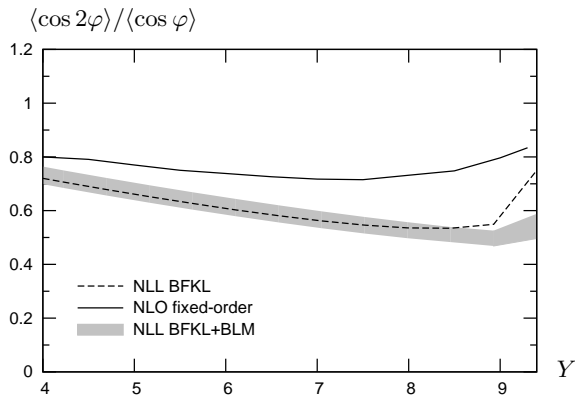
These calculations are unstable when $\mathbf{k}_{J_{\min 1}} = \mathbf{k}_{J_{\min 2}}$ because the cancellation of some divergencies is difficult to obtain numerically

Results for an asymmetric configuration

In this section we choose the cuts as

- $35 \text{ GeV} < |\mathbf{k}_{J1}|, |\mathbf{k}_{J2}| < 60 \text{ GeV}$
- $50 \text{ GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$
- $0 < |y_1|, |y_2| < 4.7$

And we compare our results with the NLO fixed-order code Dijet ([Aurenche, Basu, Fontannaz](#)) in the same configuration

Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ 

$35 \text{ GeV} < |\mathbf{k}_{J1}| < 60 \text{ GeV}$
 $35 \text{ GeV} < |\mathbf{k}_{J2}| < 60 \text{ GeV}$
 $50 \text{ GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$
 $0 < |y_1| < 4.7$
 $0 < |y_2| < 4.7$

This observable is very stable in a BFKL calculation and shows a sizable difference between with a fixed-order treatment

It is necessary to have $\mathbf{k}_{J_{\min 1}} \neq \mathbf{k}_{J_{\min 2}}$ for comparison with fixed order calculations but this could be problematic for BFKL because of energy-momentum conservation: there is no strict energy-momentum conservation in BFKL.

This was studied at LL accuracy by [Del Duca & Schmidt](#). They found that, at this order, a BFKL calculation strongly overestimates the cross section compared to an exact calculation when $|\mathbf{k}_{J_1}|$ and $|\mathbf{k}_{J_2}|$ are not very similar.

One could expect that after taking into account NLL corrections, the violation of energy-momentum conservation should be less severe.

In practice, **Del Duca & Schmidt** introduced an effective rapidity Y_{eff} defined as

$$Y_{\text{eff}} \equiv Y \frac{\sigma^{2 \rightarrow 3}}{\sigma^{\text{BFKL}, \mathcal{O}(\alpha_s^3)}}$$

where $\sigma^{2 \rightarrow 3}$ is the exact $\mathcal{O}(\alpha_s^3)$ contribution to the $gg \rightarrow ggg$ process, and $\sigma^{\text{BFKL}, \mathcal{O}(\alpha_s^3)}$ is obtained by expanding the BFKL result in powers of α_s and truncating to order $\mathcal{O}(\alpha_s^3)$.

If one replaces Y by Y_{eff} in the BFKL calculation, expands in powers of α_s and truncates to order α_s^3 , the exact result is recovered.

A value of Y_{eff} significantly different from Y means that the BFKL approximation is a too strong assumption in the kinematics under study.

We follow the same general procedure, but we also take into account **NLL** corrections

The partonic cross section schematically reads:

$$\begin{array}{ccccc}
 \text{upper vertex} & & \text{Green's function} & & \text{lower vertex} \\
 [V^{(0)} + \alpha_s V^{(1)}] & \otimes & \text{Exp}[\bar{\alpha}_s \chi_0 Y + \bar{\alpha}_s^2 \chi_1 Y] & \otimes & [V^{(0)} + \alpha_s V^{(1)}]
 \end{array}$$

Or, expanding in powers of α_s :

$$[V^{(0)} + \alpha_s V^{(1)}] \otimes [1 + \bar{\alpha}_s \chi_0 Y + \bar{\alpha}_s^2 \chi_1 Y + \dots] \otimes [V^{(0)} + \alpha_s V^{(1)}]$$

There are three $\mathcal{O}(\alpha_s^3)$ contributions ($V^{(0)}$ and $V^{(1)}$ both contain an α_s factor):

- The LL one, studied by Del Duca and Schmidt

$$\left[V^{(0)} + \alpha_s V^{(1)} \right] \otimes \left[1 + \bar{\alpha}_s \chi_0 Y + \bar{\alpha}_s^2 \chi_1 Y + \dots \right] \otimes \left[V^{(0)} + \alpha_s V^{(1)} \right]$$

Two jets at LO and one emission from the Green's function

- The one coming from the NLO jet vertex

$$\left[V^{(0)} + \alpha_s V^{(1)} \right] \otimes \left[1 + \bar{\alpha}_s \chi_0 Y + \bar{\alpha}_s^2 \chi_1 Y + \dots \right] \otimes \left[V^{(0)} + \alpha_s V^{(1)} \right]$$

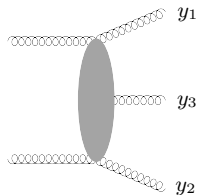
$$\left[V^{(0)} + \alpha_s V^{(1)} \right] \otimes \left[1 + \bar{\alpha}_s \chi_0 Y + \bar{\alpha}_s^2 \chi_1 Y + \dots \right] \otimes \left[V^{(0)} + \alpha_s V^{(1)} \right]$$

One jet at NLO and no emission from the Green's function

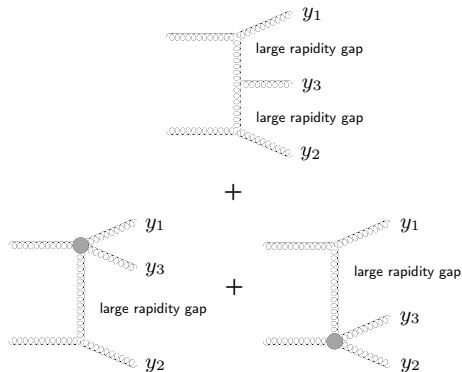
(there is no contribution from the NLL corrections to the Green's function at this order)

Thus we compare:

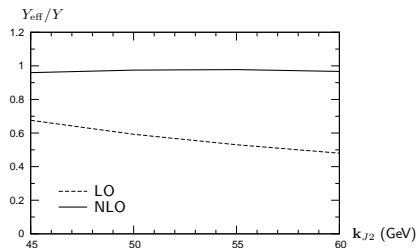
exact $2 \rightarrow 3$



BFKL



Variation of Y_{eff}/Y as a function of k_{J2} for fixed $k_{J1} = 35$ GeV (with $\sqrt{s} = 7$ TeV, $Y = 8$):



- With the **LO** jet vertex, Y_{eff} is much smaller than Y when k_{J1} and k_{J2} are significantly different
- This is the region important for comparison with fixed order calculations
- The improvement coming from the **NLO** jet vertex is very large in this region
- For $k_{J1} = 35$ GeV and $k_{J2} = 50$ GeV, typical of the values we used for comparison with fixed order, we get $\frac{Y_{\text{eff}}}{Y} \simeq 0.98$ at NLO vs. ~ 0.6 at LO

Originally Mueller and Navelet proposed to study the cross section for this process to get access the partonic cross section

The LL differential cross section reads

$$\frac{d\sigma}{d|\mathbf{k}_{J1}| d|\mathbf{k}_{J2}| dy_{J1} dy_{J2}} = \left(\frac{\alpha_s C_A}{\mathbf{k}_{J1} \mathbf{k}_{J2}} \right)^2 x_{J,1} f(x_{J,1}) x_{J,2} f(x_{J,2}) \int d\nu \left(\frac{\mathbf{k}_{J1}^2}{\mathbf{k}_{J2}^2} \right)^{i\nu} e^{\omega(0,\nu)Y}$$

with $f(x) \equiv f_g(x) + \frac{C_F}{C_A} f_q(x)$, $x_J = \frac{\mathbf{k}_J}{\sqrt{s}} e^{y_J}$, $\omega(0,\nu) = \bar{\alpha} \chi(\nu)$ and $\chi(\nu) = 2\Psi(1) - \Psi(\frac{1}{2} + i\nu) - \Psi(\frac{1}{2} - i\nu)$

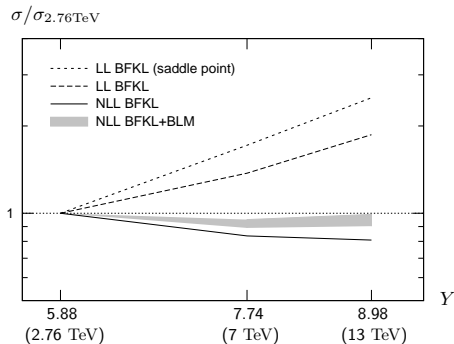
If we vary \sqrt{s} together with $y_{J,1}$ and $y_{J,2}$ while keeping $x_{J,1}$ and $x_{J,2}$ fixed, we get (for $|\mathbf{k}_{J1}| = |\mathbf{k}_{J2}|$):

$$\frac{\sigma_{\sqrt{s_1}}}{\sigma_{\sqrt{s_2}}} = \frac{\int d\nu e^{\omega(0,\nu)Y_1}}{\int d\nu e^{\omega(0,\nu)Y_2}}$$

Using the saddle-point approximation to perform the ν integration they obtained

$$\tilde{\sigma}(Y) \equiv \int d\nu e^{\omega(0,\nu)Y} \approx e^{\frac{\alpha_s N_c}{\pi} \ln(16) Y} \frac{\pi}{\sqrt{\alpha_s N_c 14 \zeta(3) Y}}$$

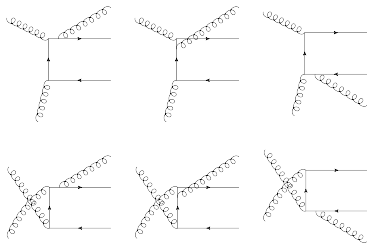
Variation of the cross section as a function of Y for fixed $x_{J,1}, x_{J,2}$
 ($|\mathbf{k}_{J1}| = |\mathbf{k}_{J2}| = 35 \text{ GeV}$):



The inclusion of the NLL corrections change the picture dramatically

To study BFKL dynamics one could also study similar processes where one of the two jets (or both) is replaced with something else, for example a J/ψ

The J/ψ vertex is given by the following diagrams (leading color-singlet contribution):



- The cross section seems to be very small
- We can't go to low p_{\perp} for the J/ψ , else BFKL should not be reliable ($p_{\perp jet} \sim 20 - 30$ GeV)
- Importance of color octet contributions?
- Importance of NLO corrections to the J/ψ vertex?

- We studied Mueller-Navelet jets at full (vertex + Green's function) NLL accuracy and compared our results with the first data from the LHC
- The agreement with CMS data at 7 TeV is greatly improved by using the BLM scale fixing procedure
- A measurement with asymmetric p_T cuts would be useful to compare with a fixed-order treatment
- Energy-momentum conservation seems to be less severely violated at NLL accuracy