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Exclusive BFKL observables with a Monte Carlo approach

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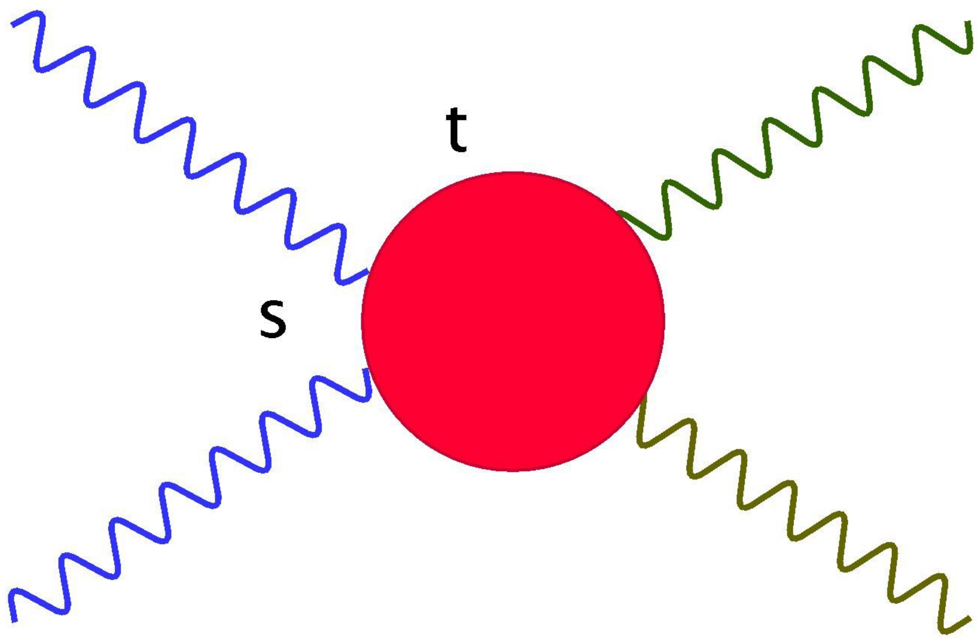
Outline

- For yet another time, a few words on the BFKL equation
- The need for a Monte Carlo approach
- The LO/NLO BFKL equation in the color octet and color singlet (the usual) representation
- Looking into the core of the BFKL ladder
- The BKP equation – The Odderon
- Outlook

SAME OLD NECESSARY STUFF TO SET UP THE STAGE

High energy limit in QCD

- We want the elastic amplitude of the process with Mandelstam variables s, t in pQCD
- We need to have a hard scale $Q^2 \gg \Lambda_{QCD}^2$
- We want the amplitude in the high energy (Regge) limit where $s \gg |t|, Q^2$
- The hard scale ensures that $\alpha_s(Q^2) \ll 1$
- The problem then becomes a problem of resumming terms of the form $(\alpha_s \ln s)^n$

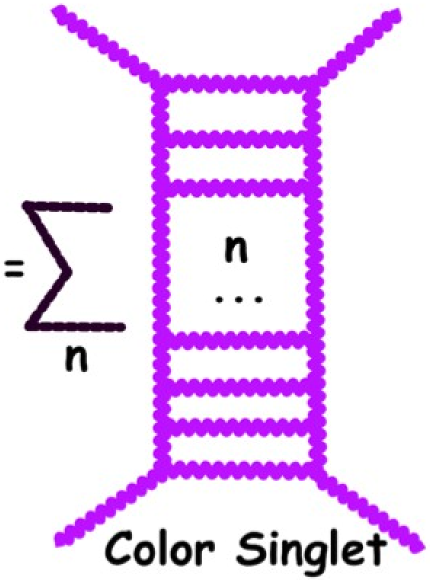


Keep in mind that we want to work entirely in momentum space.

Ladder diagrams

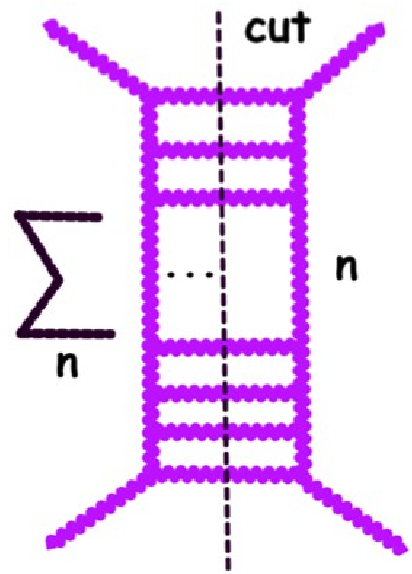
All-orders resummation of $\alpha_s(Q^2)\log\left(\frac{s}{Q^2}\right)$ terms: How? Ladder structure

$A_{\text{elastic}}(s,t) =$



Optical Theorem :

$$\sigma_{\text{TOT}} \simeq \frac{1}{s}\text{Im}\mathcal{A}_{\text{elastic}}(s,t=0) = \frac{1}{s}$$



$$= \frac{1}{s}\sum_n |A_n(s,t)|^2$$

The BFKL equation and Multi-Regge kinematics I

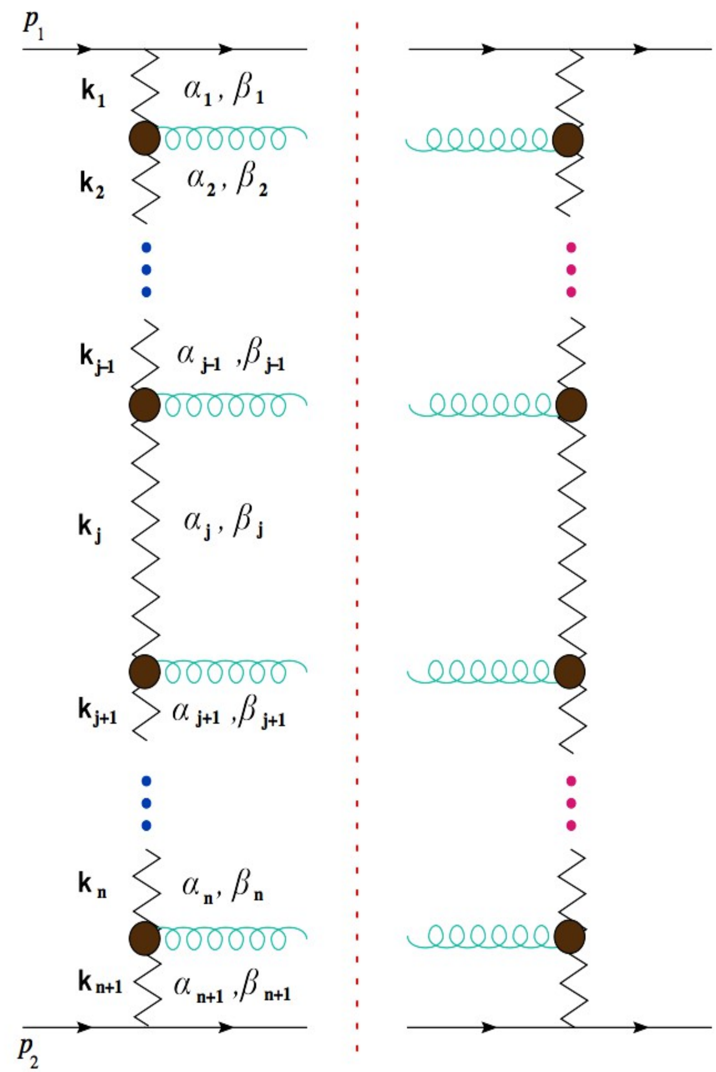
Decompose into Sudakov variables, e.g.

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1\perp}$$

Tracking leading logarithms only suggests a restriction of the kinematical configuration to the so-called
Multi-Regge kinematics (MRK):

$$\mathbf{k}_1^2 \simeq \mathbf{k}_2^2 \simeq \ldots \mathbf{k}_i^2 \simeq \mathbf{k}_{i+1}^2 \ldots \simeq \mathbf{k}_n^2 \simeq \mathbf{k}_{n+1}^2 \gg \mathbf{q}^2 \simeq s_0,$$
$$1 \gg \alpha_1 \gg \alpha_2 \gg \ldots \alpha_i \gg \alpha_{i+1} \gg \alpha_{n+1} \gg \frac{s_0}{s},$$
$$1 \gg |\beta_{n+1}| \gg |\beta_n| \gg \ldots \gg |\beta_2| \gg |\beta_1| \gg \frac{s_0}{s}.$$

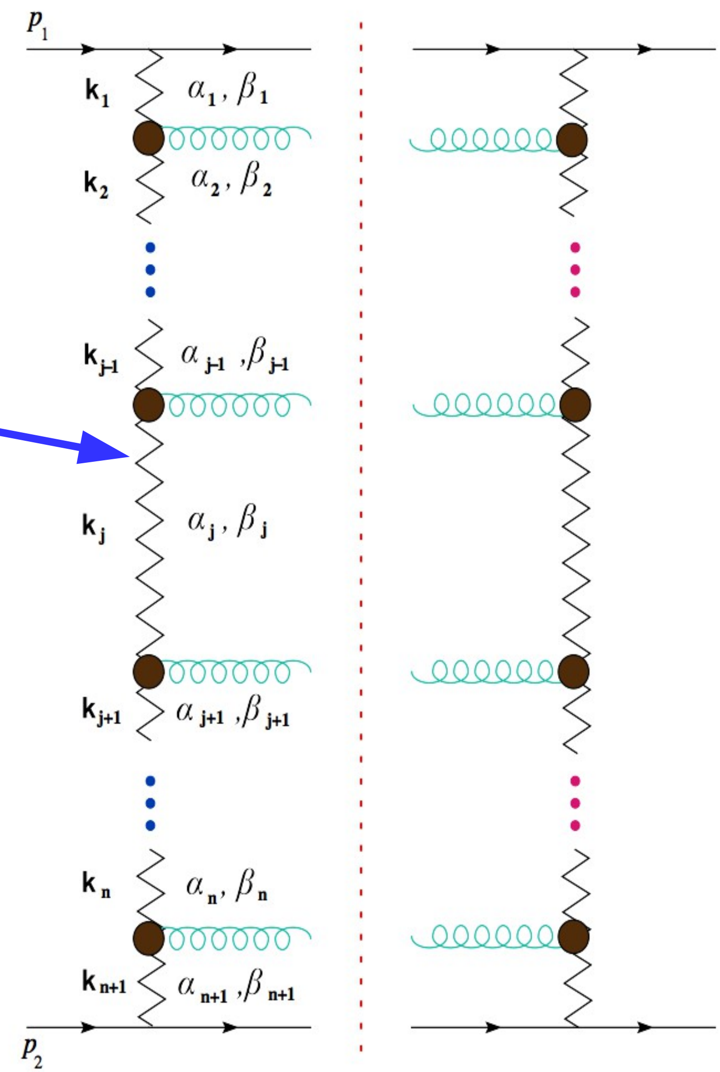
s_0 is a typical normalization scale for the BFKL equation



The BFKL equation and Multi-Regge kinematics II

There are also virtual (loop) corrections which are encoded into modifying the gluon propagators in the t-channel such that they become the so-called Reggeized gluon propagators.

Reggeized gluon



The propagator of a reggeized gluon is:

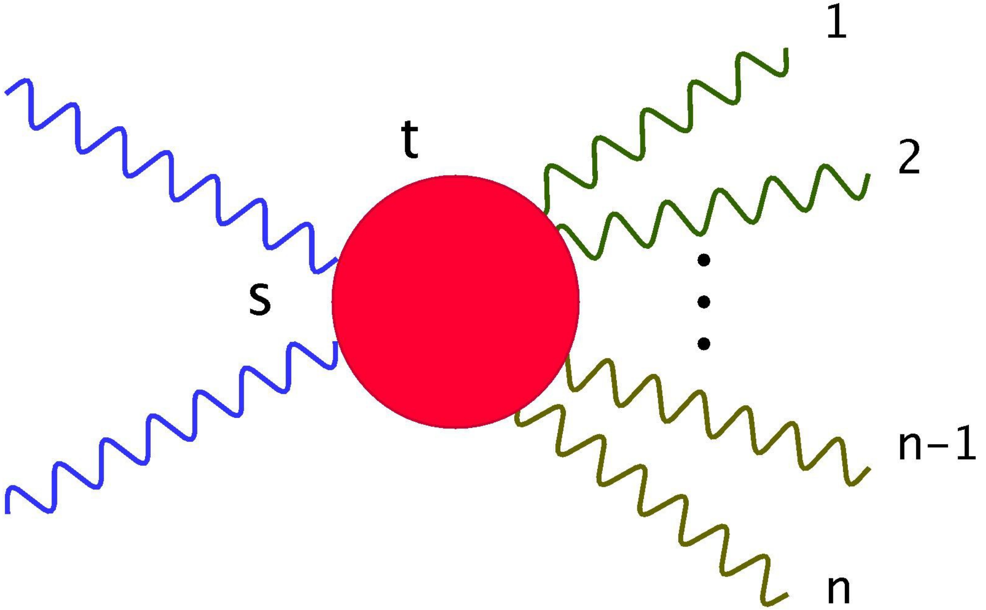
$$D_{\mu\nu}(s, q^2) = -i\frac{g_{\mu\nu}}{q^2}\left(\frac{s}{\mathbf{k}^2}\right)^{\epsilon(q^2)}$$

$$\epsilon(t) = \frac{N_c \alpha_s}{4\pi^2}\int -\mathbf{q}^2 \frac{d^2\mathbf{k}}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$

with $\mathbf{k}^2 \ll s$ a typical momentum scale and $1 + \epsilon(q^2)$ the gluon trajectory

The LL BFKL equation and the Multi-Regge kinematics schematically

$$\mathbf{k}_1^2 \simeq \mathbf{k}_2^2 \simeq \ldots \mathbf{k}_i^2 \simeq \mathbf{k}_{i+1}^2 \ldots \simeq \mathbf{k}_n^2 \simeq \mathbf{k}_{n+1}^2 \gg \mathbf{q}^2 \simeq s_0,$$
$$1 \gg \alpha_1 \gg \alpha_2 \gg \ldots \alpha_i \gg \alpha_{i+1} \gg \alpha_{n+1} \gg \frac{s_0}{s},$$
$$1 \gg |\beta_{n+1}| \gg |\beta_n| \gg \ldots \gg |\beta_2| \gg |\beta_1| \gg \frac{s_0}{s}.$$



Multi-Regge limit:
Regge limit in all sub-channels, strong ordering in rapidity:

$$s \gg s_i \gg t_i \sim Q^2$$

$$Y \sim \ln(s),\ y_i \sim \ln(s_i),\ y_i \gg y_{i-1}$$

LO BFKL:
Fadin, Kuraev, Lipatov (1977),
Balitsky, Lipatov (1978)

Quasi-Multi-Regge kinematics I

To have the BFKL equation to NNL accuracy, resum terms of the form: $\alpha_s(\alpha_s \ln s)^n$

NLO BFKL: Fadin, Lipatov (1998)
Ciafaloni, Gamici (1998)

- The ways to obtain a term of the type above is by either losing a logarithm of s starting from an amplitude at LL or by including loop corrections to the vertices.
- For the real emission corrections, the key feature that generates these logarithmic terms is the strong ordering in rapidity.
- Thus, if we allow for a state where two of the emitted particles are close to each other, we are in the Quasi-Multi-Regge-kinematics (QMRK):

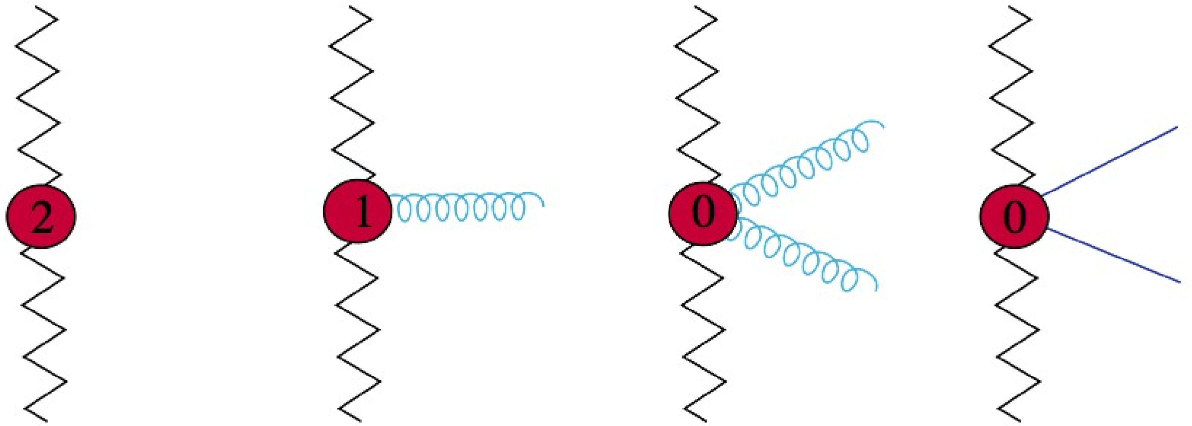
$$\mathbf{k}_1^2 \simeq \mathbf{k}_2^2 \simeq \ldots \mathbf{k}_i^2 \simeq \mathbf{k}_{i+1}^2 \ldots \simeq \mathbf{k}_n^2 \simeq \mathbf{k}_{n+1}^2 \gg \mathbf{q}^2 \simeq s_0,$$
$$1 \gg \alpha_1 \gg \alpha_2 \gg \ldots \alpha_i \gg \alpha_{i+1} \gg \alpha_{n+1} \gg \frac{s_0}{s},$$
$$1 \gg |\beta_{n+1}| \gg |\beta_n| \gg \ldots \gg |\beta_2| \gg |\beta_1| \gg \frac{s_0}{s}.$$

- The relations above still hold with the exception of a pair of particles. The pair can be a pair of gluons or a quark anti-quark pair.

Quasi-Multi-Regge kinematics II

To have the BFKL equation to NNL accuracy, resum terms of the form: $\alpha_s (\alpha_s \ln s)^n$

NLO BFKL: Fadin, Lipatov (1998)
Ciafaloni, Gamici (1998)



Why a Monte Carlo approach?

- We don't always know the analytic solution
- Even if we know it, we still want to store and analyze information about “differential” quantities (e.g. rapidities, transverse momenta, angles) that will be lost once we perform the integrations analytically. We want this for two reasons:
    1. Because then we can compare theoretical predictions to a greater set of observables
    2. Because there are lots of things we can still learn about concepts we use every day and maybe we don't fully understand
- We want to have a common language with people that work and are familiar with fixed order calculations and who are the majority in the “pheno” community – the interaction will help both sides
- We want to work in momentum space

DIFFERENT COLOR REPRESENTATIONS OF THE SYSTEM OF TWO REGGEIZED GLUONS

Some generic statements on the BFKL dynamics

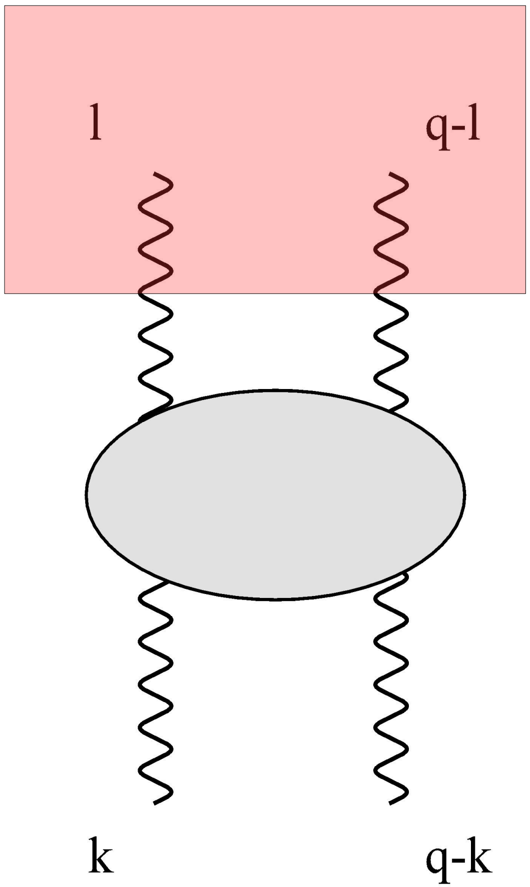
- Usually one has in mind the BFKL equation for the case of forward scattering (momentum transfer t=0 and vacuum quantum numbers exchanged in the t-channel (color singlet, Pomeron)
- The BFKL equation though, was from the beginning developed for arbitrary t and for all possible t-channel color states. The BFKL kernel for the latter case is know to NLO

Fadin, Gorbachev (2000)
Fadin, Fiore (2005)

A few words on color

This -Nc is a color factor, assuming that the color state of the two gluons in the graph is the color singlet. If this is the case, then the kernel is IR finite!

Color state of the two gluons



For QCD, the possible states are:

$$\mathbf{1},\ \mathbf{8_A},\ \mathbf{8_S},\ \mathbf{10}+\overline{\mathbf{10}},\ \mathbf{27}$$

with color factors:

$$-3,\ -\frac{3}{2},\ -\frac{3}{2},\ 0,\ 1$$

$$K_{\rm BFKL}(\mathbf{l},\mathbf{q}-\mathbf{l};\mathbf{k},\mathbf{q}-\mathbf{k}) = -N_c g^2\left[\mathbf{q}^2-\frac{\mathbf{k}^2(\mathbf{q}-\mathbf{l})^2}{(\mathbf{k}-\mathbf{l})^2}-\frac{(\mathbf{q}-\mathbf{k})^2\mathbf{l}^2}{(\mathbf{k}-\mathbf{l})^2}\right] + (2\pi)^3\mathbf{k}^2(\mathbf{q}-\mathbf{k})^2\left[\beta(\mathbf{k})+\beta(\mathbf{q}-\mathbf{k})\right]\delta^{(2)}(\mathbf{k}-\mathbf{l})\,.$$

Why the color octet representation is important

Symmetric octet

- It was in a generalized leading logarithmic approximation, and by iterating the BFKL kernel in the s-channel, where the Bartels-Kwiecinski-Praszalowicz (BKP) equation was proposed

Bartels (1980)

Kwiecinski, Praszalowicz (1980)

- BKP was found to have a hidden integrability being equivalent to a periodic spin chain of a XXX Heisenberg ferromagnet. This was the first example of the existence of integrable systems in QCD

Lipatov (1986, 1990, 1993)

- **It will be directly connected to any numerical solution of the BKP, if any such work is to be done with the aim to perform phenomenological studies for the Odderon**

Antisymmetric octet

- Corrections to the Bern-Dixon-Smirnov (BDS) iterative ansatz (Bern, Dixon, Smirnov, 2005) for the n-point maximally helicity violating (MHV) and planar amplitudes were found in MRK in the six-point amplitude at two loops

Bartels, Lipatov, Sabio Vera (2009, 2010)

in other words, it is a fundamental ingredient of the finite remainder of scattering amplitudes with arbitrary number of external legs and internal loops

Let us rewrite the BFKL equation in a different format

$$\left\{\omega + (c_{\mathcal{R}} - 1)\frac{\bar{\alpha}_s}{2}\left[\frac{2}{\epsilon} - \log\left(\frac{\mathbf{q}_1^2}{\mu^2}\right) - \log\left(\frac{\mathbf{q}_1'^2}{\mu^2}\right)\right]\right.$$
$$\left. + c_{\mathcal{R}}\frac{\bar{\alpha}_s}{2}\left[\log\left(\frac{\mathbf{q}_1^2}{\lambda^2}\right) + \log\left(\frac{\mathbf{q}_1'^2}{\lambda^2}\right)\right]\right\}\mathcal{G}_\omega(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}) = \delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2)$$
$$+ c_{\mathcal{R}}\int \frac{d^2\mathbf{k}}{\pi \mathbf{k}^2}\theta\left(\mathbf{k}^2 - \lambda^2\right)\frac{\bar{\alpha}_s}{2}\left[1 + \frac{\mathbf{q}_1'^2(\mathbf{q}_1 + \mathbf{k})^2 - \mathbf{q}^2\mathbf{k}^2}{(\mathbf{q}_1' + \mathbf{k})^2\mathbf{q}_1^2}\right]\mathcal{G}_\omega(\mathbf{q}_1 + \mathbf{k}, \mathbf{q}_2; \mathbf{q})$$

C_R = ½ for octet
C_R = 1 for singlet

This can now be iterated and, performing the Mellin transform,

Gluon Green's Function →

$$\mathcal{F}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y}) = \int \frac{d\omega}{2\pi i} e^{\omega \mathrm{Y}} \mathcal{G}_\omega(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q})$$

we finally obtain

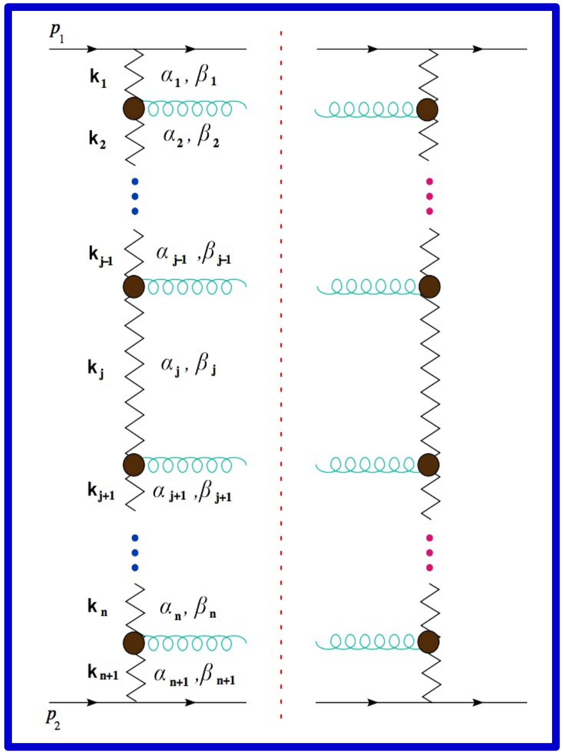
$$\mathcal{F}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y}) = \exp\left\{\omega^{(\epsilon;\lambda)}(\mathbf{q}_1; \mathbf{q})\mathrm{Y}\right\}\left\{\delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2)\right.$$
$$+ \sum_{n=1}^{\infty}\prod_{i=1}^{n} c_{\mathcal{R}}\int \frac{d^2\mathbf{k}_i}{\pi \mathbf{k}_i^2}\theta(\mathbf{k}_i^2 - \lambda^2)\,\xi\left(\mathbf{q}_1 + \sum_{l=1}^{i-1}\mathbf{k}_l, \mathbf{k}_i; \mathbf{q}\right)\delta^{(2)}\left(\mathbf{q}_1 + \sum_{l=1}^{n}\mathbf{k}_l - \mathbf{q}_2\right)$$
$$\left. \times \int_0^{y_{i-1}} dy_i \exp\left\{\left[\omega^{(\epsilon;\lambda)}\left(\mathbf{q}_1 + \sum_{l=1}^{i}\mathbf{k}_l; \mathbf{q}\right) - \omega^{(\epsilon;\lambda)}\left(\mathbf{q}_1 + \sum_{l=1}^{i-1}\mathbf{k}_l; \mathbf{q}\right)\right] y_i\right\}\right\},$$

} Monte Carlo

MONTE CARLO SOLUTION TO THE BFKL EQUATION

In a manner, we are back to our old diagram:

$$\mathcal{F}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y}) = \exp\left\{\omega^{(\epsilon;\lambda)}(\mathbf{q}_1;\mathbf{q})\mathrm{Y}\right\}\left\{\delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2)\right.$$
$$+\sum_{n=1}^{\infty}\prod_{i=1}^{n} c_{\mathcal{R}}\int \frac{d^2\mathbf{k}_i}{\pi\mathbf{k}_i^2}\theta(\mathbf{k}_i^2-\lambda^2)\,\xi\left(\mathbf{q}_1+\sum_{l=1}^{i-1}\mathbf{k}_l, \mathbf{k}_i; \mathbf{q}\right)\delta^{(2)}\left(\mathbf{q}_1+\sum_{l=1}^{n}\mathbf{k}_l-\mathbf{q}_2\right)$$
$$\left.\times\int_0^{y_{i-1}} dy_i \exp\left\{\left[\omega^{(\epsilon;\lambda)}\left(\mathbf{q}_1+\sum_{l=1}^{i}\mathbf{k}_l;\mathbf{q}\right)-\omega^{(\epsilon;\lambda)}\left(\mathbf{q}_1+\sum_{l=1}^{i-1}\mathbf{k}_l;\mathbf{q}\right)\right]y_i\right\}\right\},$$



Solving BFKL with Monte Carlo integration techniques

- Many people have worked on it, the origin goes back to the late 90's:

Kwiecinski, Lewis, Martin (1996), Schmidt (1996), Orr, Stirling (1998)

Effective Feynman Rules:
simplest case, $t = 0$, leading order

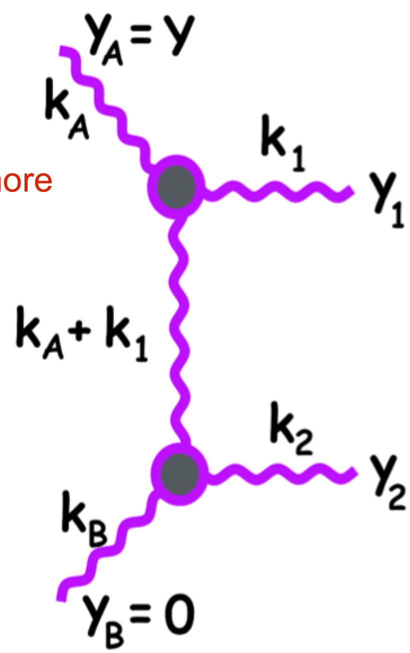
Note the change in the naming of the gluon Regge trajectory once more

Gluon Regge trajectory:

$$\omega\left(\vec{q}\right) = -\frac{\alpha_s N_c}{\pi}\log\frac{q^2}{\lambda^2}$$

Modified t-channel propagators:

$$\left(\frac{s_i}{s_0}\right)^{\omega(t_i)} = e^{\omega(t_i)(y_i - y_{i+1})}$$



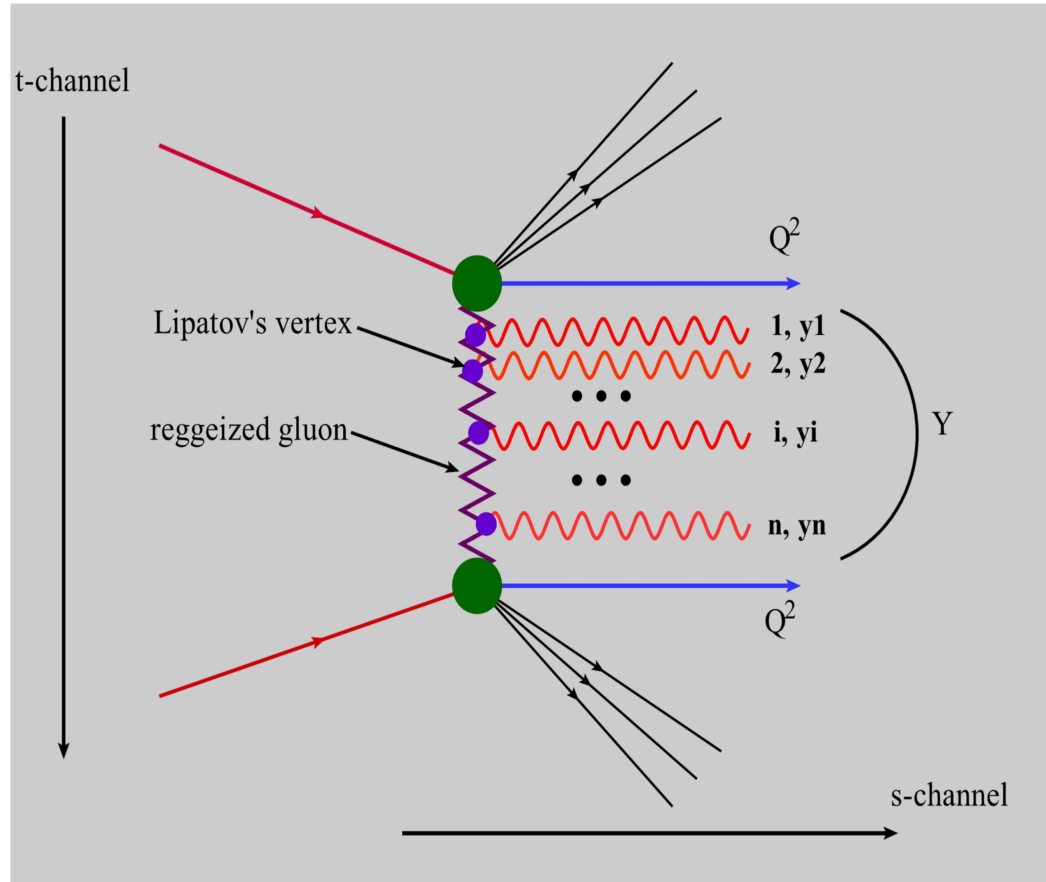
$$\left(\frac{\alpha_s N_c}{\pi}\right)^2 \int d^2\vec{k}_1 \frac{\theta\left(k_1^2 - \lambda^2\right)}{\pi k_1^2} \int d^2\vec{k}_2 \frac{\theta\left(k_2^2 - \lambda^2\right)}{\pi k_2^2} \delta^{(2)}\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2 - \vec{k}_B\right)$$

$$\times \int_0^Y dy_1 \int_0^{y_1} dy_2 e^{\omega\left(\vec{k}_A\right)(Y - y_1)} e^{\omega\left(\vec{k}_A + \vec{k}_1\right)(y_1 - y_2)} e^{\omega\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2\right)y_2}$$

Very simplified -pictorial- view, the main elements and ideas are here though

The ladder in a different depiction:

- Two projectiles collide and produce two hard jets (Q^2 is a hard scale so that we can use perturbation theory) and n gluons "flying" in the s-channel (gluon-1, gluon-2,...,gluon-n).
- In the t-channel, a reggeized gluon is exchanged. The purple blobs are the so-called Lipatov effective vertices.
- Could numerical methods help us tackle the problem of calculating the process?
- We can use Monte Carlo methods to obtain a numerical solution to the BFKL equation.



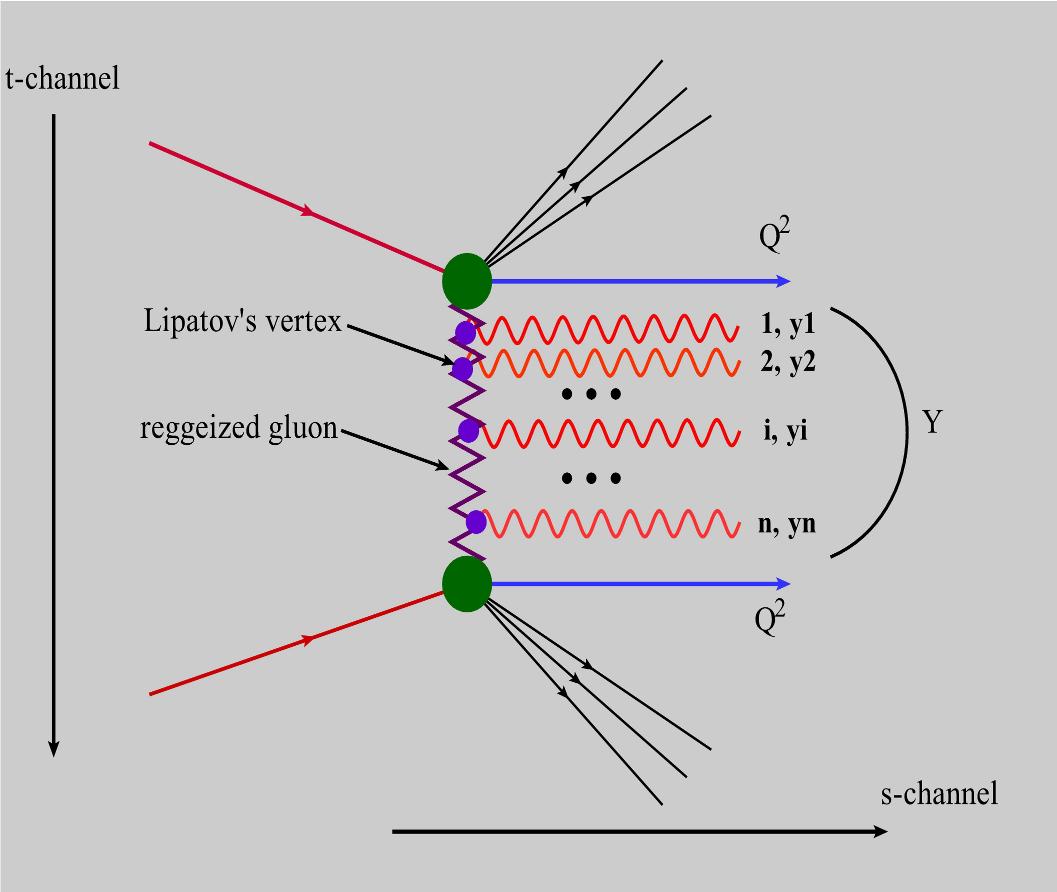
Any sort of explicit information can be stored during an MC run.
Jets may be "tagged", **experimental cuts can be imposed**, **angular information may be monitored** and **differential distributions can be produced**. This makes it a whole lot easier to compare against experimental data or to get a deeper insight when the aim is more theoretical.

The ladder in a different depiction:

To calculate this process at LO, one has to solve the BFKL Eq. and obtain the gluon Green's function (GGF) $\mathcal{F}(q_1, q_2; Y)$.

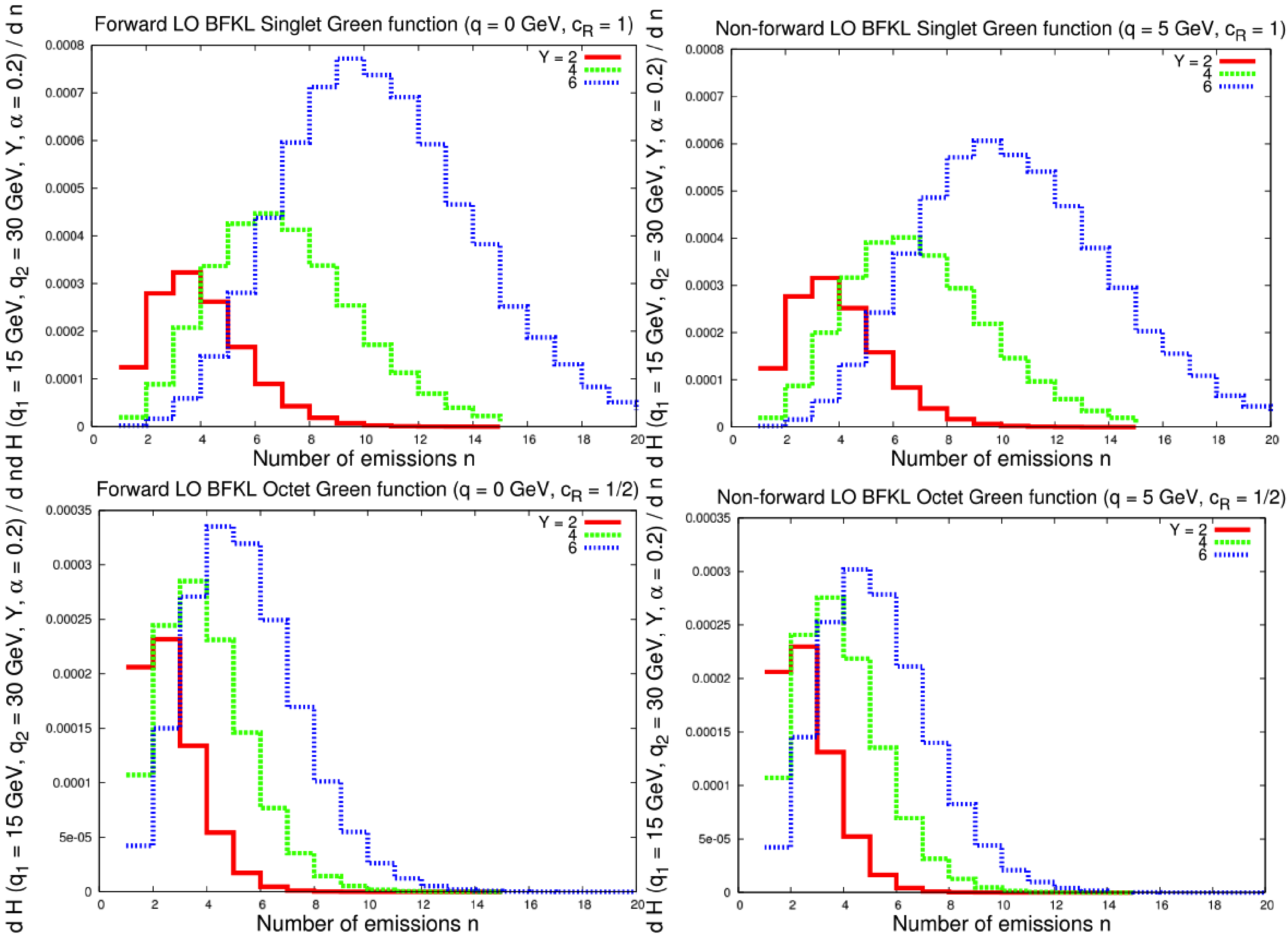
q_1 and q_2 are the momenta of the reggeized gluons above gluon-1 and below gluon-n respectively. Y is the rapidity span from gluon-1 to gluon-n. In principle, one needs to consider an infinite sum of terms:
the 1st one with no gluon (n=0),
the 2nd one with one gluon emission (n=1),
the 3rd one with two gluon emission, etc.

Every term is an integral over the emitted gluon momenta and their individual rapidities. Depending on Y, one can truncate the sum to a finite N (max n=N) in order to have a "numerically acceptable" result.



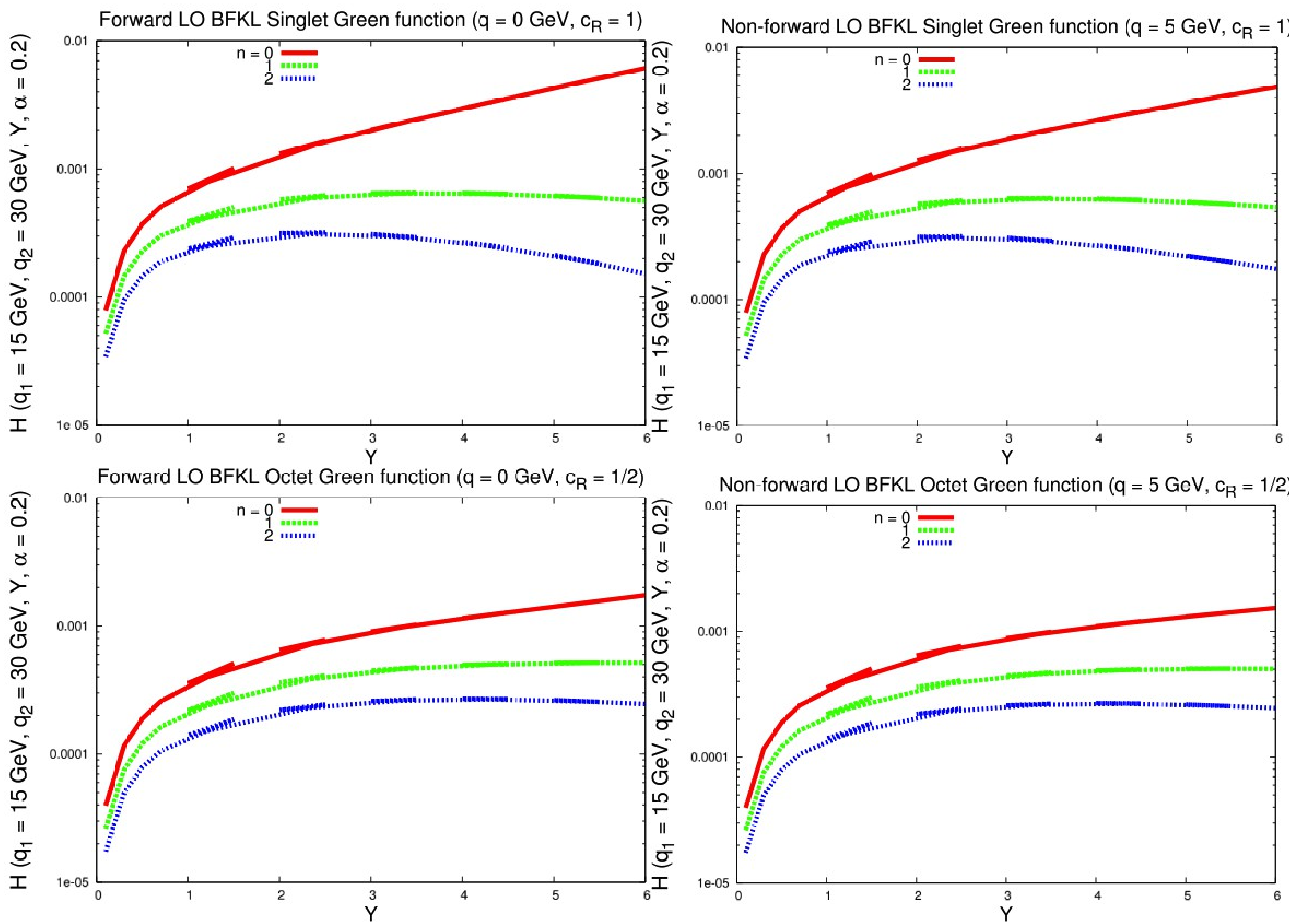
Numerical results

$$\mathcal{H}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y}) \equiv \mathcal{F}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y}) \left(\frac{e^{\frac{1}{\epsilon}} \mu^2}{\sqrt{\mathbf{q}_1^2 \mathbf{q}_1'^2}} \right)^{\bar{\alpha}_s (c_{\mathcal{R}} - 1)\mathrm{Y}}$$



Numerical results

$$\mathcal{H}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q};\mathrm{Y}) \equiv \mathcal{F}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q};\mathrm{Y})\left(\frac{e^{\frac{1}{\epsilon}}\mu^2}{\sqrt{\mathbf{q}_1^2\mathbf{q}_1'^2}}\right)^{\bar{\alpha}_s(c_{\mathcal{R}}-1)\mathrm{Y}}$$



The NLO BFKL equation in the color octet

Fadin, Lipatov (2012)

$$\mathcal{F}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y}) = \left(\frac{\mathbf{q}^2\lambda^2}{\mathbf{q}_1^2\mathbf{q}_1'^2}\right)^{\frac{\bar{\alpha}}{2}\left(1-\frac{\zeta_2}{2}\bar{\alpha}\right)\mathrm{Y}} e^{\frac{3}{4}\zeta_3\bar{\alpha}^2\mathrm{Y}} \Bigg\{ \delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2)$$

$$+ \sum_{n=1}^{\infty}\prod_{i=1}^{n}\left[\int d^2\mathbf{k}_i \frac{\bar{\alpha}}{4}\left(1-\frac{\zeta_2}{2}\bar{\alpha}\right)\frac{\theta(\mathbf{k}_i^2-\lambda^2)}{\pi\mathbf{k}_i^2}\left(1+\frac{\left(\mathbf{q}_1'+\sum_{l=1}^{i-1}\mathbf{k}_l\right)^2(\mathbf{q}_1+\sum_{l=1}^{i}\mathbf{k}_l)^2-\mathbf{q}^2\mathbf{k}_i^2}{(\mathbf{q}_1'+\sum_{l=1}^{i}\mathbf{k}_l)^2\left(\mathbf{q}_1+\sum_{l=1}^{i-1}\mathbf{k}_l\right)^2}\right)\right.$$

$$\left. + \Phi\left(\mathbf{q}_1+\sum_{l=1}^{i-1}\mathbf{k}_l, \mathbf{q}_1+\sum_{l=1}^{i}\mathbf{k}_l\right)\right]\delta^{(2)}\left(\mathbf{q}_1+\sum_{l=1}^{n}\mathbf{k}_l-\mathbf{q}_2\right)$$

$$\times \int_0^{y_{i-1}} dy_i \left(\frac{\left(\mathbf{q}_1+\sum_{l=1}^{i-1}\mathbf{k}_l\right)^2}{\left(\mathbf{q}_1+\sum_{l=1}^{i}\mathbf{k}_l\right)^2}\right)^{1+\frac{\bar{\alpha}y_i}{2}\left(1-\frac{\zeta_2}{2}\bar{\alpha}\right)} \left(\frac{\left(\mathbf{q}_1'+\sum_{l=1}^{i-1}\mathbf{k}_l\right)^2}{\left(\mathbf{q}_1'+\sum_{l=1}^{i}\mathbf{k}_l\right)^2}\right)^{\frac{\bar{\alpha}y_i}{2}\left(1-\frac{\zeta_2}{2}\bar{\alpha}\right)} \Bigg\}$$

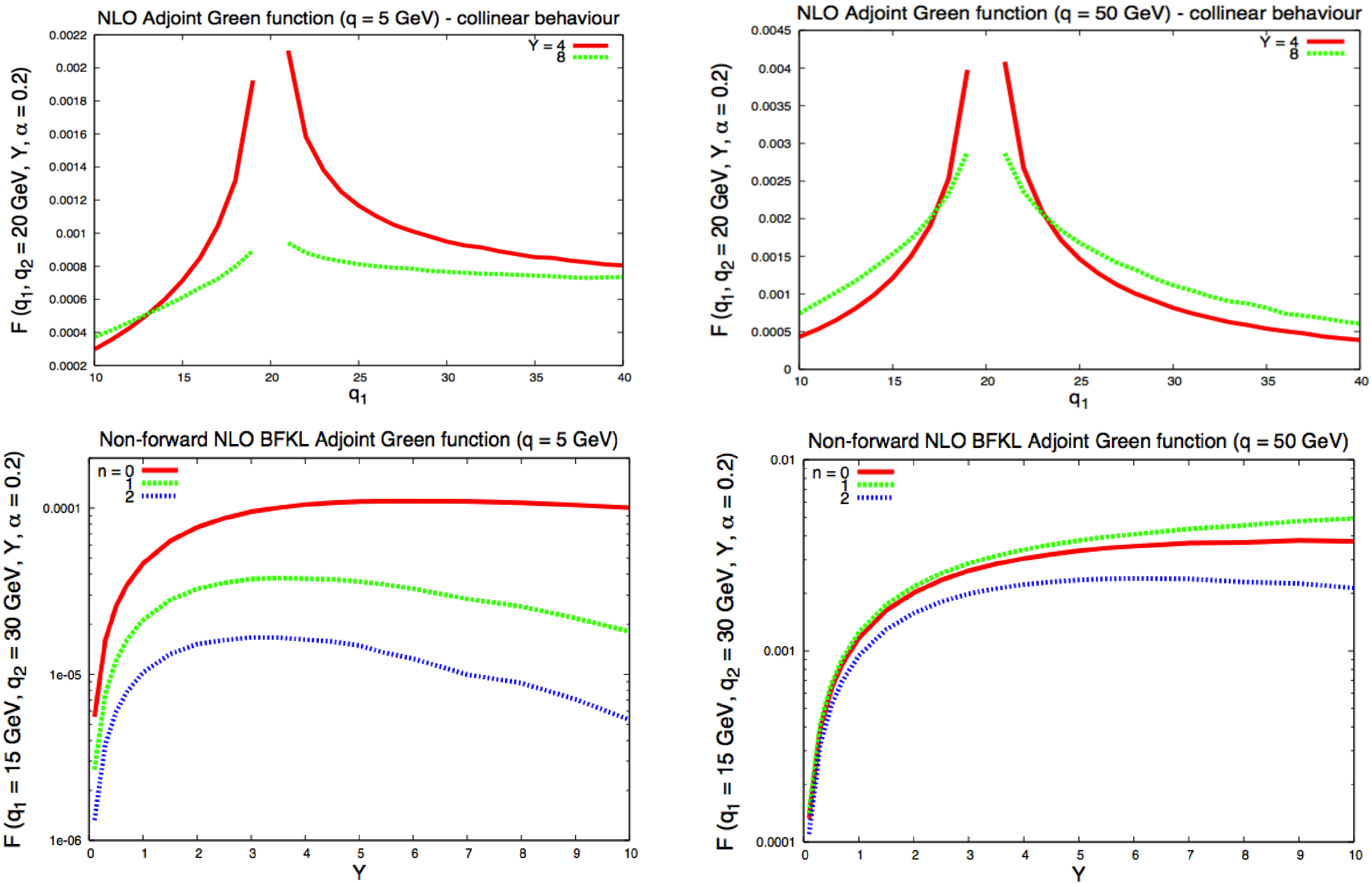
where

$$
\begin{aligned}
\Phi\left(\mathbf{q}_1,\mathbf{q}_1+\mathbf{k}\right) \quad = \\
& \frac{\bar{\alpha}^2}{32\pi}\frac{1}{\mathbf{q}_1^2(\mathbf{k}+\mathbf{q}_1')^2}\left\{\mathbf{q}^2\left[\ln\left(\frac{\mathbf{q}_1^2}{\mathbf{q}^2}\right)\ln\left(\frac{\mathbf{q}_1'^{\,2}}{\mathbf{q}^2}\right)+\ln\left(\frac{\left(\mathbf{q}_1+\mathbf{k}\right)^2}{\mathbf{q}^2}\right)\ln\left(\frac{\left(\mathbf{q}_1'+\mathbf{k}\right)^2}{\mathbf{q}^2}\right)\right.\right. \\
& +\frac{1}{2}\ln^2\left(\frac{\mathbf{q}_1^2}{\left(\mathbf{q}_1+\mathbf{k}\right)^2}\right)+\frac{1}{2}\ln^2\left(\frac{\mathbf{q}_1'^2}{\left(\mathbf{q}_1'+\mathbf{k}\right)^2}\right)\Bigg]+\frac{1}{2}\frac{\left(\mathbf{q}_1^2\left(\mathbf{q}_1'+\mathbf{k}\right)^2-\mathbf{q}_1'^2\left(\mathbf{q}_1+\mathbf{k}\right)^2\right)}{\mathbf{k}^2} \\
& \quad\times\left[\ln\left(\frac{\mathbf{q}_1'^2}{\left(\mathbf{q}_1'+\mathbf{k}\right)^2}\right)\ln\left(\frac{\mathbf{q}_1'^2\left(\mathbf{q}_1'+\mathbf{k}\right)^2}{\mathbf{k}^4}\right)-\ln\left(\frac{\mathbf{q}_1^2}{\left(\mathbf{q}_1+\mathbf{k}\right)^2}\right)\ln\left(\frac{\mathbf{q}_1^2\left(\mathbf{q}_1+\mathbf{k}\right)^2}{\mathbf{k}^4}\right)\right] \\
& -\frac{\left(\mathbf{q}_1^2\left(\mathbf{q}_1'+\mathbf{k}\right)^2+\mathbf{q}_1'^2\left(\mathbf{q}_1+\mathbf{k}\right)^2\right)}{\mathbf{k}^2}\left[\ln^2\left(\frac{\mathbf{q}_1^2}{\left(\mathbf{q}_1+\mathbf{k}\right)^2}\right)+\ln^2\left(\frac{\mathbf{q}_1'^2}{\left(\mathbf{q}_1'+\mathbf{k}\right)^2}\right)\right] \\
& +\left[\mathbf{q}^2\left(\mathbf{k}^2-\mathbf{q}_1^2-\left(\mathbf{q}_1+\mathbf{k}\right)^2\right)+2\mathbf{q}_1^2\left(\mathbf{q}_1+\mathbf{k}\right)^2-\mathbf{q}_1^2\left(\mathbf{q}_1'+\mathbf{k}\right)^2-\mathbf{q}_1'^2\left(\mathbf{q}_1+\mathbf{k}\right)^2\right. \\
& \quad\left.+\frac{\left(\mathbf{q}_1^2\left(\mathbf{q}_1'+\mathbf{k}\right)^2-\mathbf{q}_1'^2\left(\mathbf{q}_1+\mathbf{k}\right)^2\right)}{\mathbf{k}^2}\left(\mathbf{q}_1^2-\left(\mathbf{q}_1+\mathbf{k}\right)^2\right)\right]\mathcal{I}\left(\mathbf{q}_1^2,\left(\mathbf{q}_1+\mathbf{k}\right)^2,\mathbf{k}^2\right) \\
& +\left[\mathbf{q}^2\left(\mathbf{k}^2-\mathbf{q}_1'^2-\left(\mathbf{q}_1'+\mathbf{k}\right)^2\right)+2\mathbf{q}_1'^2\left(\mathbf{q}_1'+\mathbf{k}\right)^2-\mathbf{q}_1'^2\left(\mathbf{q}_1+\mathbf{k}\right)^2-\mathbf{q}_1^2\left(\mathbf{q}_1'+\mathbf{k}\right)^2\right. \\
& \quad\left.\left.+\frac{\left(\mathbf{q}_1'^2\left(\mathbf{q}_1+\mathbf{k}\right)^2-\mathbf{q}_1^2\left(\mathbf{q}_1'+\mathbf{k}\right)^2\right)}{\mathbf{k}^2}\left(\mathbf{q}_1'^2-\left(\mathbf{q}_1'+\mathbf{k}\right)^2\right)\right]\mathcal{I}\left(\mathbf{q}_1'^2,\left(\mathbf{q}_1'+\mathbf{k}\right)^2,\mathbf{k}^2\right)\right\}
\end{aligned}
$$

with

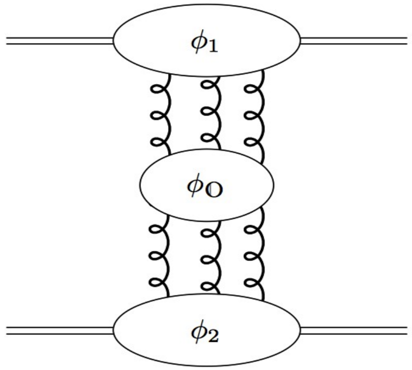
$$
\mathcal{I}\left(\mathbf{p}^2,\mathbf{q}^2,\mathbf{r}^2\right) \quad = \quad \int_0^1 \frac{dx}{\mathbf{p}^2(1-x)+\mathbf{q}^2x-\mathbf{r}^2x(1-x)}\ln\left(\frac{\mathbf{p}^2(1-x)+\mathbf{q}^2x}{\mathbf{r}^2x(1-x)}\right).
$$

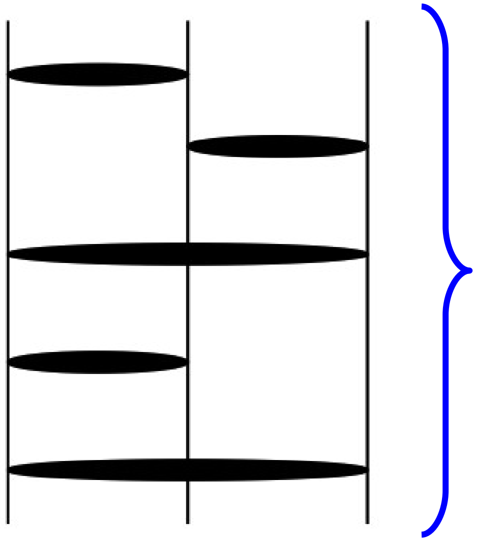
Numerical results



BKP – The Odderon

- Pomeron is the state of two interacting reggeized gluons in the t-channel in the color singlet. It has the quantum numbers of the vacuum
- Odderon is the state of three interacting gluons exchanged in the t-channel in the color singlet but with C =-1 and P=-1
- Any pair of two gluons in the Odderon forms symmetric color octet subsystems





Ladder structure of the Odderon. BKP resums term of the form $\alpha_s(\alpha_s \log s)^n$

NLO corrections recently available

Bartels, Fadin, Lipatov, Vacca (2012)

BKP – The Odderon

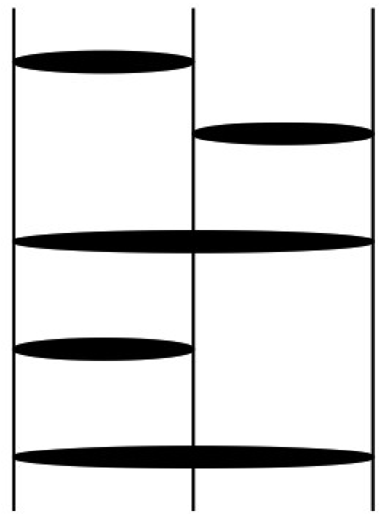
In the BKP framework, the usual ladder picture has to be modified.

There are now three reggeized gluons exchanged in the t-channel and they can interact, **locally in rapidity**, in pairs through the exchange of ordinary gluons.

The ladder is no more one with two rails but rather one with three rails and each rung can connect any two of them.

The whole system is in the color singlet representation whereas any subset of two reggeized gluons is in the symmetric color octet representation.

There would not be hardly any hope to pursue an iterative solution through the Monte Carlo approach if it were not for the fact that any pair of the three reggeized gluons is in the color octet representation which, as we saw previously, leads to a much faster convergence compared to the color singlet state.



Conclusions – Outlook

- We have studied the LO and NLO BFKL equation using Monte Carlo techniques. By using numerical methods, it is possible to probe regions that analytic work cannot access
- The experience gained will be used for having a numerical solution of the BKP, the LO BKP project is currently underway.We would like to have a solid phenomenological study program for Odderon searches

Project in collaboration with

Agustin Sabio Vera

- First NLO study in momentum space of the angular decorrelation of Mueller-Navelet jets at hadron colliders

Project in collaboration with

F. Caporale,

B. Murdaca

A. Sabio Vera

Backup slides

$$f\left(\vec{k}_a, \vec{k}_b, \mathrm{Y}\right) = \sum_{n=-\infty}^{\infty} f_n\left(|\vec{k}_a|, |\vec{k}_b|, \mathrm{Y}\right) e^{in\theta}$$

$$f_n\left(|\vec{k}_a|, |\vec{k}_b|, \mathrm{Y}\right) = \frac{1}{\pi|\vec{k}_a||\vec{k}_b|} \int \frac{d\gamma}{2\pi i} \left(\frac{\vec{k}_a^2}{\vec{k}_b^2}\right)^{\gamma-\frac{1}{2}} e^{\omega_n(a,\gamma)\mathrm{Y}}$$

$$f_n\left(|\vec{k}_a|, |\vec{k}_b|, \mathrm{Y}\right) = \int_0^{2\pi} \frac{d\theta}{2\pi} f\left(\vec{k}_a, \vec{k}_b, \mathrm{Y}\right) \cos(n\theta)$$

$$\chi(n,\gamma) = 2\Psi(1) - \Psi\left(\gamma + \frac{n}{2}\right) - \Psi\left(1 - \gamma + \frac{n}{2}\right)$$

Backup slides

$$\int_{virtual} + \int_{real} = \int_{virtual+real,unres.} + \int_{real,res}$$