DIFFRACTION CROSS SECTIONS AS SATURATION SIGNATURES

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S-CHANNEL UNITARITY

The simplest $a_{el}(s,b)$ **s-channel unitarity bound is obtained from a diagonal re-scattering matrix, where repeated elastic scatterings secure s-channel unitarity:** $2Im a_{el}(s,b) = |a_{el}(s,b)|^2 + G^{in}(s,b).$**, i.e.** $\sigma_{tot}(s,b) = \sigma_{el}(s,b) + \sigma_{in}(s,b).$

Its solution is:

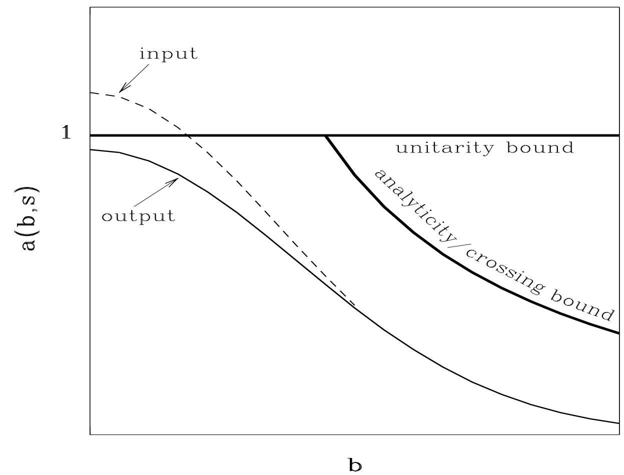
$a_{el}(s,b) = i\left(1 - e^{-\Omega(s,b)/2}\right),\ G^{in}(s,b) = 1 - e^{-\Omega(s,b)}.$ Ω **is model dependant.**

In a Glauber/Gribov eikonal approximation, the input opacity $\Omega(s,b)$ **is real. It equals the imaginary part of the input model Born term, a Pomeron exchange in our context. The output** $a_{el}(s,b)$ **is imaginary.**

The consequent bound is $\mid a_{el}(s,b) \mid \leq 1,$ **which is the black disc bound.**

In a single channel eikonal model, the screened cross sections are:

$$\sigma_{tot} = 2\int d^2b\left(1 - e^{-\Omega(s,b)/2}\right), \quad \sigma_{el} = \int d^2b\left(1 - e^{-\Omega(s,b)/2}\right)^2, \quad \sigma_{in} = \int d^2b\left(1 - e^{-\Omega(s,b)}\right).$$



The figure shows the s-channel black bound, the analyticity/crossing bound implied by the $ln^2(s)$ expanding amplitude radius and the input non screened and the output screened elastic b-space amplitudes. The consequent Froissart-Martin bound is: $\sigma_{tot} \leq Cln^2(s/s_0), s_0 = 1GeV^2, C \propto 1/2m_\pi^2 \simeq 30mb.$ **C is much too large to be relevant even at the TeV-scale. s-unitarity implies:** $\sigma_{el} \leq \frac{1}{2}\sigma_{tot}$ **and** $\sigma_{in} \geq \frac{1}{2}\sigma_{tot}$. **At saturation,** $\sigma_{el} = \sigma_{in} = \frac{1}{2}\sigma_{tot}$.

GOOD-WALKER DECOMPOSITION

Consider a system of two orthonormal states in an hadron-hadron interaction: an hadronic state Ψ_h and a diffractive state Ψ_D.

Ψ_h and Ψ_D do not diagonalize the 2x2 interaction matrix T.

This observation was originally made by Feinberg and Pomeranchuk (1956) and, independantly, by Good and Walker (GW) (1960) in a nore elegant form.

Let Ψ_1 and Ψ_2 be eigen states of T : $\Psi_h = \alpha\Psi_1 + \beta\Psi_2, \quad \Psi_D = -\beta\Psi_1 + \alpha\Psi_2, \quad \alpha^2 + \beta^2 = 1.$

The eigen states initiate 4 $A_{i,k}$ elastic GW amplitudes $(\psi_i + \psi_k \rightarrow \psi_i + \psi_k)$. **i,k=1,2.**

For initial $p(\bar{p}) - p$ we have $A_{1,2} = A_{2,1}$.

The Elastic, SD and DD amplitudes in a 2 channel screened GW model are:

$$a_{el}(s,b) = i\{\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}\},$$

$$a_{sd}(s,b) = i\alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2 - \beta^2)A_{1,2} + \beta^2 A_{2,2}\},$$

$$a_{dd}(s,b) = i\alpha^2\beta^2\{A_{1,1} - 2A_{1,2} + A_{2,2}\},$$

$$A_{i,k}(s,b) = \left(1 - e^{\frac{1}{2}\Omega_{i,k}(s,b)}\right) \leq 1.$$

As we shall see, Diffraction has 2 components induced by different dynamics.

The predominantly low mass, GW sector, is screened by eikonalisation.

The high mass, non GW, is screened by the survival probability.

In the GW sector:

- **We obtain the Pumplin bound:** $\sigma_{el} + \sigma_{diff}^{GW} \leq \frac{1}{2}\sigma_{tot}$.

  σ_{diff}^{GW} **is the sum of the GW soft diffractive cross sections.**

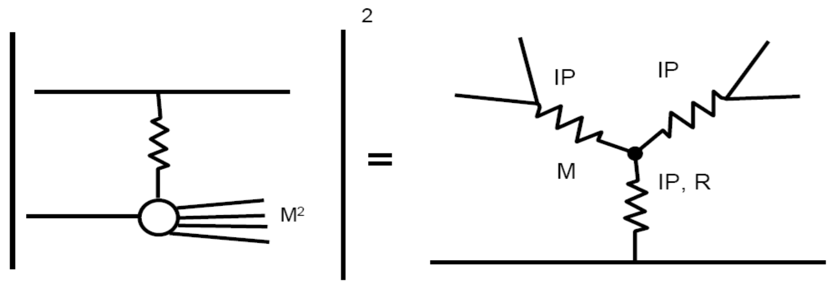
- **Below saturation,** $\sigma_{el} < \frac{1}{2}\sigma_{tot} - \sigma_{diff}^{GW}$ **and** $\sigma_{in} > \frac{1}{2}\sigma_{tot} + \sigma_{diff}^{GW}$.

- $a_{el}(s,b) = 1$, **when and only when,** $A_{1,1}(s,b) = A_{1,2}(s,b) = A_{2,2}(s,b) = 1$.

- **When** $a_{el}(s,b) = 1$, **all diffractive amplitudes at the same (s,b) vanish.**

- **The saturation signature,** $\sigma_{el} = \sigma_{in} = \frac{1}{2}\sigma_{tot}$, **in a multi channel calculation is coupled to** $\sigma_{diff} = 0$. **Consequently, prior to saturation the diffractive cross sections stop growing and start to decrease with energy.**

- **The comment above does not hold in a single channel model.**

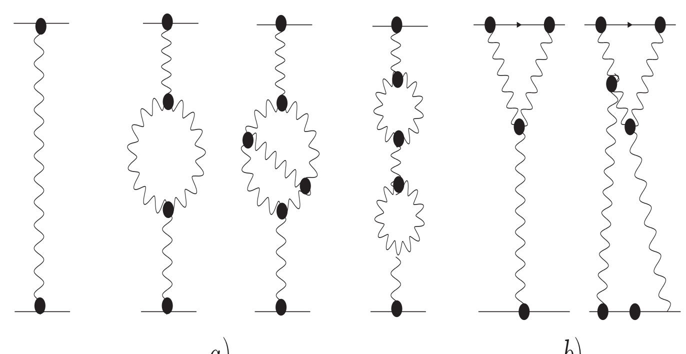


CROSSED CHANNELED UNITARITY

Translating the concepts presented into a viable phenomenology requires a specification of $\Omega(s,b)$, for which Regge Pomeron ($I\!P$) theory is a powerful tool.

Mueller(1971) applied 3 body unitarity to equate the cross section of $a + b \to M^2_{sd} + b$ to the triple Regge diagram $a + b + \bar{b} \to a + b + \bar{b}$, with a leading $3I\!P$ vertex term.

The $3I\!P$ approximation is valid when $M^2_p/M_{sd} << 1$ and $M^2_{sd}/s << 1$.



Mueller's 3$I\!P$ approximation for non GW diffraction is the lowest order of t-channel multi $I\!P$ interactions, compatible with t-channel unitarity.

The figure shows the $I\!P$ Green function in which Multi $I\!P$ interactions induce high mass diffraction.

a)Presents the propagator renormalization (Enhanced diagrams).

b)Presents the coupling renormalization.(Semi-Enhanced diagrams).

Most of GLM diffraction is GW, which extends up to the kinematic high mass bound. Non GW is high mass only. Most of KMR diffraction is non GW high mass. KMR associate GW with low mass and non GW with high mass.

THE PARTONIC POMERON

Current $I\!P$ models differ in details, but have in common a relatively large adjusted input $\Delta_{I\!P}$ and a diminishing $\alpha'_{I\!P}$. Recall: $\alpha_{I\!P}(t) = 1 + \Delta_{I\!P} + \alpha'_{I\!P}t$.

Traditionally, $\Delta_{I\!P}$ determines the energy dependence of the Total, Elastic and Diffractive cross sections, while $\alpha'_{I\!P}$ determines the forward slopes.

This picture is modified in updated $I\!P$ models in which s and t unitarity screenings induce a smaller $I\!P$ intercept at t=0, which gets smaller with energy.

The exceedingly small fitted $\alpha'_{I\!P}$ implies a partonic description of the $I\!P$ which leads to a pQCD interpretation.

Gribov's partonic Regge theory provides the microscopic sub structure of the $I\!P$, where the slope of the $I\!P$ trajectory is related to the mean transverse momentum of the partonic dipoles constructing the Pomeron.

$$\alpha'_{I\!P} \propto 1/<p_t>^2, \quad \textbf{accordingly:} \quad \alpha_S(QCD) \propto \pi/ln\left(<p_t^2>/\Lambda_{QCD}^2\right) << 1.$$

We obtain a $I\!\!P$ with hardness changing continuesly from hard (BFKL like) to soft (Regge like). This is a non trivial relation as the soft $I\!\!P$ is a moving pole in J-plane, while, the BFKL hard $I\!\!P$ is a branch cut, approximated, some times, as a simple pole with $\Delta_{I\!\!P} = 0.2 - 0.3$, $\alpha'_{I\!\!P} \simeq 0$.

GLM and KMR models are rooted in Gribov's partonic $I\!\!P$ theory with a hard pQCD $I\!\!P$ input. It is softened by unitarity screening (GLM), or the dependence of its partons transverse momenta on their rapidity (KMR).

GLM and KMR have a bound of validity, at 60(GLM) and 100(KMR) TeV, implied by their approximations. Consequently, as attractive as updated Pomeron models are, we can not utilize them above 100 TeV at the most.

To this end, the only relevant models are single channeled, most of which have a logarithmic parametrization input such as $\sigma_{tot}(s) = Aln(s) + Bln^2(s)$.

UPDATED POMERON MODELS

Any analysis relating to phenomenological updated Pomeron models, has to distinguish between pre LHC and post LHC data.

To an extent, we observe a case in which a theoretical prejudice distorted the phenomenological interpratation of Fermilab raw data.

Consider $\sigma_{tot}(p\bar{p})$ at W=1.8 TeV:

Fermilab E710 measurement (PRD 1990) reported value was 72.1 $\pm$ 3.3 mb.

This value was supported by E81 (PLB 2002) who got 72.42 $\pm$ 1.55 mb.

CDF published (PRD 1994) a considerably higher value of 80.03 $\pm$ 2.24 mb.

The CDF number was rejected because its value was not consistent with the popular DL and COMPETE models. The 1.8 TeV under estimated cross sections, were supported by all updated $I\!\!P$ models, which predicted that LHC soft pp cross sections would be considerably smaller than the actual data.

Most models which reproduce TOTEM, ATLAS and CMS total, elastic and inelastic cross sections, reproduce also CDF 1.8 TeV.

TOTEM 7 TeV

$\sigma_{tot}(pp)$**=98.3±2.8mb**

$\sigma_{el}(pp)$**=25.8±2.8mb**

$\sigma_{in}(pp)$**=73.2±4.0mb**

ATLAS 7 TeV

$\sigma_{tot}(pp)$**=95.4±1.4mb**

$\sigma_{el}(pp)$**=24.0±0.6mb**

$\sigma_{in}(pp)$**=71.4±1.52mb**

TOTEM-CMS at 8 TeV supports the 7 TeV data.

TOTEM, ATLAS and CMS results force significant improvements of the updated $I\!P$ models.

REVISED UPDATED POMERON MODELS

The desired improvement of the updated $I\!P$ models can be achieved by either improving the data fitting, or re-formulating the theoretical model, or both.

In the following I shall compare 6 updated Pomeron models, 3 by KMR and one each by GLM, Ostapchenko (OSTAP) and Kaidalov-Poghosian (KP). Note that, none of these models in their pre LHC version reproduced the TOTEM-ATLAS-CMS p-p cross sections. OSTAP and KP outputs are pre LHC. They had the largest cross sections predictions, which are, though, not large enough to describe the TOTEM-ATLAS-CMS data.

- GLM (Gotsman, Levin, Maor) operate with a single hard BFKL $I\!P$ input, in a two channel eikonal model. The hard input is softened by unitarity screenings. In the revised model the fitting procedure has 2 steps: In the first we fix the secondary Regge parameters from the low energy data base. We then fit the $I\!P$ parameters from the over all data base. The output changes of the fitted parameters are not severe. $\Delta_{I\!P}$ was changed from 0.21 to 0.23.

$\alpha'_{I\!P}$ changed from 0.0 to 0.028 GeV^{-2}.

These small changes enabled us to obtain an excellent reproduction of soft cross sections data. GLM diffraction will be discussed latter.

- KMR (Khoze, Martin, Ryskin) produced 3 single $I\!P$ models:

  One is a 2 channel eikonal model with $\Delta_{I\!P} = 0.11$, and $\alpha'_{I\!P} = 0.06 GeV^{-2}$.

  The second is a 3 channel eikonal with $\Delta_{I\!P} = 0.14$, and $\alpha'_{I\!P} = 0.1 GeV^{-2}$.

  The third model is an effective $I\!P$ model, based on non-enhanced eikonal which suppresses the growth of the soft cross sections.

  To this end KMR fix $\Delta_{I\!P} = 0.12$, and $\alpha'_{I\!P} = 0.05 GeV^{-2}$.

  KMR SD output is compatible with TOTEM. They ignor Elice diffractive results. Note that ALICE diffraction values correspond to GW + non GW.

- **Ostapchenko has made (pre LHC) a comprehensive calculation in the framework of Reggeon Field Theory based on the resummation of both enhanced and semi enhanced $I\!\!P$ diagrams. To fit the elastic and diffractive cross sections he assumed 2 Pomerons (set C):** $\alpha^{soft} = 1.14 + 0.14t$ **and** $\alpha^{hard} = 1.31 + 0.085t$**.**

- **KP (Kaidalov and Poghosyan) model is based on Reggeon calculus. They describe the soft diffraction data taking all non enhanced absorptive corrections to the 3 Reggeon vertices and loop diagrams. It is a single** $I\!\!P$ **model with secondary Regge poles. Their** $I\!\!P$ **trajectory fitted parameters are** $\Delta_{I\!\!P} = 0.12$ **and** $\alpha'_{I\!\!P} = 0.22 GeV^{-2}$**.**

- **The issue of diffractive scattering will be discussed further on.**

UNITARITY SATURATION

Unitarity saturation is coupled to 3 experimental signatures:

$\frac{\sigma_{in}}{\sigma_{tot}} = \frac{\sigma_{el}}{\sigma_{tot}} = 0.5,\ \frac{\sigma_{tot}}{B_{el}} = 9\pi = 28.27,\ \sigma_{diff}$**=0 (in a multi-channel model).**

Following is p-p TeV-scale data relevant to the assessment of saturation:

CDF(1.8 TeV): $\sigma_{tot} = 80.03 \pm 2.24mb,\ \sigma_{el} = 19.70 \pm 0.85mb,\ B_{el} = 16.98 \pm 0.25GeV^{-2}$.

TOTEM(7 TeV): $\sigma_{tot} = 98.3 \pm 0.2(stat) \pm 2.8(sys)mb,\ \sigma_{el} = 24.8 \pm 0.2(stat) \pm 2.8(sys)mb,$
$B_{el} = 20.1 \pm 0.2(stat) \pm 0.3(sys)GeV^{-2}$.

ATLAS(7 TeV): $\sigma_{tot} = 95.4 \pm 1.4mb,\quad \sigma_{el} = 24.0 \pm 0.6mb$.

AUGER(57 TeV): $\sigma_{tot} = 133 \pm 13(stat) \pm^{17}_{20} sys \pm 16(Glauber)mb,$
$\sigma_{in} = 92 \pm 7(stat) \pm^{9}_{11} (sys) \pm 16(Glauber)mb.$

We get: $\frac{\sigma_{in}}{\sigma_{tot}}$**=0.754(CDF), 0.748(TOTEM,ATLAS), 0.692(AUGER).**

The numbers above suggest a very slow approach toward saturation, well above the TeV-scale. Consequently, the study of pp saturation depends on information above the TeV-scale.

There are 2 sources from which we may obtain the desired information:

- Cosmic Rays data. Recall that p-p cross sections obtained from p-Air data have relatively large margin of errore. AUGER p-p cross sections are a good example.

- Since updated $I\!P$ models are confined to the TeV-scale, p-p cross sections at higher energies can be calculated only in single channeled models, the deficiencies of which have been stated before.

Out of a few single channeled nodels, I shall quote Block and Halzen (BH), which reproduce well the inelastic and total cross sections at the TeV-scale. The BH model can be applied at exceedingly high energies.

The prediction of BH at the Planck-scale ($\mathbf{1.22} \cdot 10^{16} TeV$) is:

$$\sigma_{in}/\sigma_{tot} = 1131mb/2067mb = \mathbf{0.547.}$$

It implies that saturation will be attained, if at all, at un measurable energies.

The predicted multi channel vanishing of the diffractive cross sections at saturation implies that σ_{sd}, which up to the TEVATRON grows slowly with energy, will eventually start to reduce.

This may serve as an early signature that saturation is being approached.

Specifically, the preliminary TOTEM output is:

$$\sigma_{sd} = 6.5 \pm 1.3 mb$$

$$3.4 < M_{sd} < 1100 GeV$$

$$2.4 \cdot 10^{-7} < \xi < 0.025$$

The results above are preliminary as the upper mass limit of σ_{sd} corresponds to 0.025s, compatible with KMR GW bound.

Note that ALICE diffractive cross sections $\sigma_{sd} = 14.9^{+3.4}_{-5.9} mb$ and $\sigma_{dd} = 9.0 \pm 2.6 mb$ are significantly different from TOTEM's data.

They are compatible with GLM predictions based on 0.05s diffractive mass bound.

TOTEM SD results may indicate a radical change in the energy dependence of σ_{sd}/σ_{in} which is much smaller than ALICE value:

σ_{sd}/σ_{in}**=0.151(CDF), 0.20(ALICE), 0.088(TOTEM).**

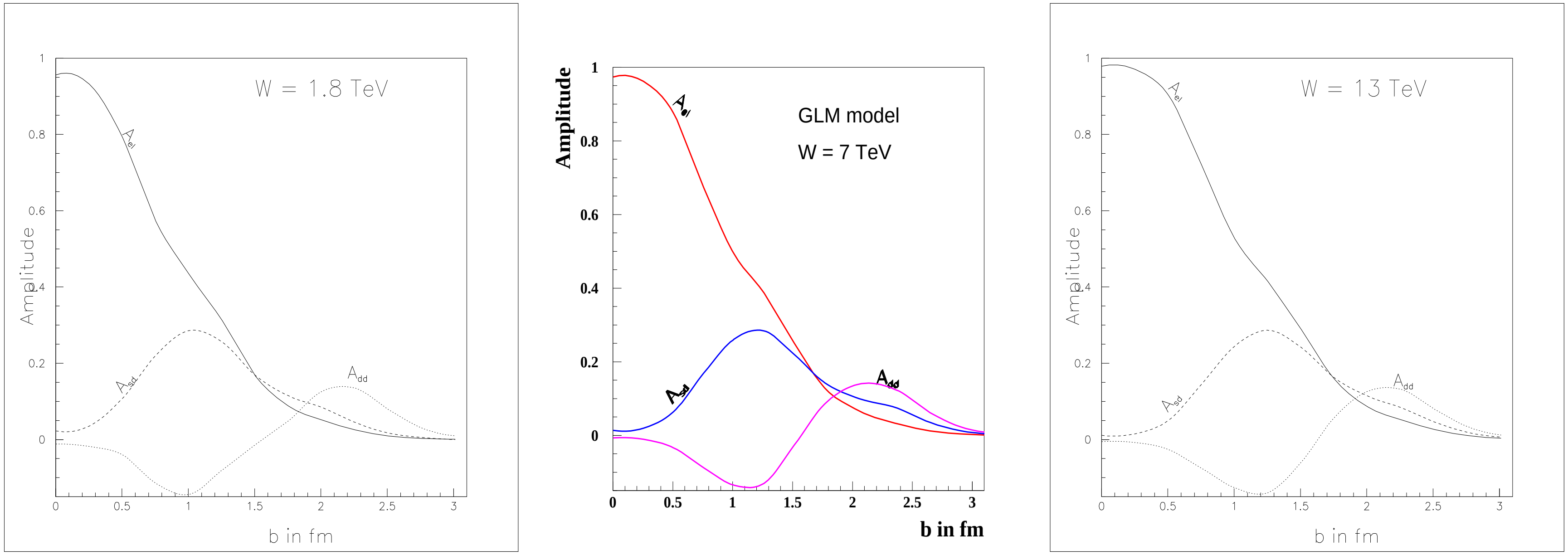
TOTEM results may imply a much faster approach toward unitarity saturation than suggested by CDF anf ALICE.

TOTEM diffractive data is very preliminary. Regardless, the compatibility between the information derived from different channels of soft scattering deserves a very careful study!

The figures next page show the GLM Elastic, SD and DD b-amplitudes at 1.8, 7 and 14 TeV. The difference between our output and competing models is not dramatic.

The GLM SD cross sections (in mb) are:

$\sigma_{sd}(W) = \sigma_{sd}^{GW} + \sigma_{sd}^{nonGW} = 9.2 + 1,95(1.8),$ **10.7+4.18(7),** $11.5 + 5.81(14).$



Recall that, EL, SD and DD cross section values are obtained from a b^2 integration of the corresponding amplitude square. The growth of σ_{sd}, as a function of W, is mainly a consequence of $a_{sd}(s,b)$ moving slowly to higher b values. The net result is a continuation of SD moderate increase with W. Consequently, I do not expect a suppression of σ_{sd} at an energy of 7 TeV, as inplied by TOTEM SD data and recent KMR papers. An early reduction of the diffractive channels at relatively low energies, will

require, thus, a fundamental change in our interpretation of soft scattering at the TeV-scale.

A FINAL NOTE:

In a recent GLM study based on CDC (Color Glass Condensate) saturation, we have compared a single versus a double amplitude models. In a single amplitude model we do not have a GW like cancelations, whereas, in the double amplitude model we have incorporated such cancelations. The output of the double amplitude model is better than the output of the single model.