Resumming Double Logarithms in the Balitsky-Kovchegov Equation

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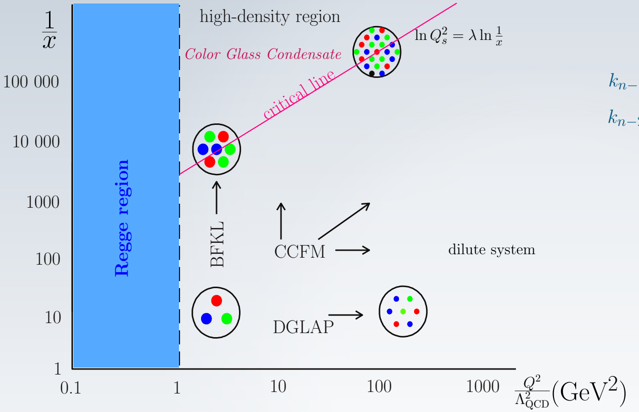
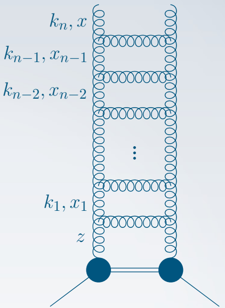
Introduction

DGLAP and BFKL Evolutions

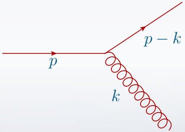




[Gribov & Lipatov '72; Dokshitzer '77; Altarelli & Parisi '77]

[Fadin, Kuraev & Lipatov '75,76,77]

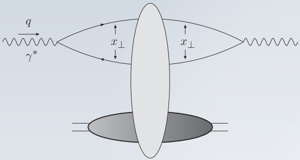
[Lipatov'76; Balitsky & Lipatov '78]



$k_z = xp \gg k_\perp$
$0 < x < 1$

$$dP_{\text{Brems}} \simeq \frac{\alpha_s\{C_A, C_F\}}{\pi^2} \frac{d^2k_\perp}{k_\perp^2} \frac{dx}{x}$$

High-Energy Scattering & the B-JIMWLK Equation



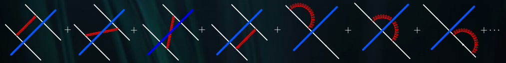
For a gluon crossing a shockwave target, the background field propagator is essentially a Wilson line

$$U^{\dagger}_{\boldsymbol{x}} = \mathcal{P}\exp\left[ig\int dx^{+} A^{-}_{a}(x^{+},\boldsymbol{x})T^{a}\right]$$

and then $\left(\int dp^{+}/p^{+} \to \ln(1/x)\right)$

$$\Delta H = \ln\frac{1}{x} H_{\mathrm{JIMWLK}}$$

$$H_{\mathrm{JIMWLK}} = \frac{1}{(2\pi)^3}\int \mathcal{K}_{\boldsymbol{xyz}}(U^{\dagger}_{\boldsymbol{x}} - U^{\dagger}_{\boldsymbol{z}})^{ab}(U^{\dagger}_{\boldsymbol{y}} - U^{\dagger}_{\boldsymbol{z}})^{ac}R^{b}_{\boldsymbol{x}}R^{c}_{\boldsymbol{y}}$$



$$R^{a}_{\boldsymbol{u}}U^{R\dagger}_{\boldsymbol{x}} = ig\delta_{\boldsymbol{ux}}U^{R\dagger}_{\boldsymbol{x}}T^{a}_{R}$$

$$\mathcal{K}_{\boldsymbol{xyz}} = \mathcal{K}^{i}_{\boldsymbol{xz}}\mathcal{K}^{i}_{\boldsymbol{yz}}$$

$$\int\frac{d^2\boldsymbol{k}}{(2\pi)^2}e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{z})}$$

$$\underbrace{}_{=2gt^{a}\frac{\varepsilon_{\lambda}\cdot\boldsymbol{k}}{\boldsymbol{k}^2}} = \frac{ig}{\pi}t^{a}\boldsymbol{\varepsilon}^{i}_{\lambda}\underbrace{\frac{(\boldsymbol{x}-\boldsymbol{z})^{i}}{(\boldsymbol{x}-\boldsymbol{z})^{2}}}_{\equiv\mathcal{K}^{i}_{\boldsymbol{xz}}}$$

Dipole Picture [Nikolaev & Zakharov '91; Mueller '94]

$$S_{\boldsymbol{xy}} = \frac{\mathrm{tr}}{N_c}\left[V^{\dagger}_{\boldsymbol{x}}V_{\boldsymbol{y}}\right]$$

☐ Mixed representation $\{x_{\perp}, k^{+}\}$ well-suited for high-energy scattering (diagonalizes SW interaction)

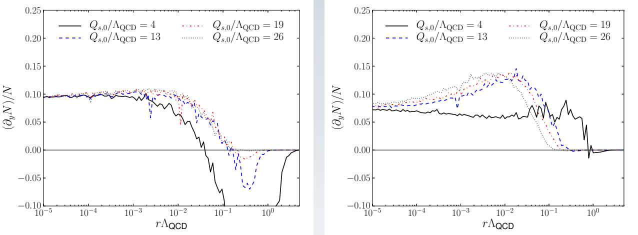
Hierarchy of equations which at large-N_c simplify to **Balitsky-Kovchegov (BK) equation** $\partial_Y S_{\boldsymbol{xy}} = \frac{\bar{\alpha}_s}{2\pi}\int_{\boldsymbol{z}}\mathcal{M}_{\boldsymbol{xyz}}[S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}]$

[Balitsky '96; Kovchegov '98]

[Jalilian-Marian, Kovner, McLerran & Weigert '97; J.-Marian, Kovner, Leonidov & Weigert '98; Iancu, Leonidov & McLerran '01]

NLO Corrections & Resummation of Collinear Logarithms

- *Tour-de-force* computations of NLO corrections to BFKL [Fadin & Lipatov '98; Camici & Ciafaloni '98], BK [Balitsky & Chirilli '08] and JIMWLK [Balitsky & Chirilli '13; Kovner, Lublinsky & Mulian '14] equations.
- Large size of the NLO corrections found in BFKL equation, that would deprive it of its predictive power and lead to instabilities [Ross '98].
- No reason to expect lack-of-convergence problems to be attenuated by non-linear terms in BK-JIMWLK equation [Triantafyllopoulos '03; Avsar, Staśto, Triantafyllopoulos & Zaslavsky '11].
- Origin of large NLO corrections identified to come from large transverse logarithms. Several procedures devised for all-order resummation of large logs and stabilization of the kernel [Salam '98; Ciafaloni, Colferai, Salam & Staśto '03; Sabio Vera '05].



(a) $\gamma = 0.6$

(b) $\gamma = 0.8$

Large corrections and instabilities in NLO BK traced back to double transverse logs [Lappi & Mantysäari '15]:

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\} = {} & \frac{\alpha_s}{2\pi^2}\int d^2z \frac{(x-y)^2}{X^2Y^2}\Big\{1+\frac{\alpha_s}{4\pi}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2} + \Big(\frac{67}{9}-\frac{\pi^2}{3}\Big)N_c - \frac{10}{9}n_f \\
& - 2N_c\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big]\Big\}\big[\mathrm{Tr}\{\hat U_x\hat U_z^\dagger\}\mathrm{Tr}\{\hat U_z\hat U_y^\dagger\} - N_c\,\mathrm{Tr}\{\hat U_x\hat U_y^\dagger\}\big] \\
& + \frac{\alpha_s^2}{16\pi^4}\int d^2z d^2z'\Big[\Big(-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2+X'^2Y^2-4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2-X'^2Y^2]} + \frac{(x-y)^4}{X^2Y'^2-X'^2Y^2} \\
& \times\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big] + \frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}-\frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big)\big[\mathrm{Tr}\{\hat U_x\hat U_z^\dagger\}\mathrm{Tr}\{\hat U_z\hat U_{z'}^\dagger\}\mathrm{Tr}\{\hat U_{z'}\hat U_y^\dagger\} \\
& - \mathrm{Tr}\{\hat U_x\hat U_z^\dagger\hat U_{z'}U_y^\dagger\hat U_z\hat U_{z'}^\dagger\} - (z'\to z)\big] + \Big\{\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big] - \frac{(x-y)^4}{X^2Y'^2X'^2Y^2}\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2} \\
& \times\mathrm{Tr}\{\hat U_x\hat U_z^\dagger\}\mathrm{Tr}\{\hat U_z\hat U_{z'}^\dagger\}\mathrm{Tr}\{\hat U_{z'}\hat U_y^\dagger\} + 4n_f\Big\{\frac{4}{(z-z')^4} - 2\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\} \\
& \times\mathrm{Tr}\{t^a\hat U_x t^b\hat U_y^\dagger\}\big[\mathrm{Tr}\{t^a\hat U_z t^b\hat U_{z'}^\dagger\} - (z'\to z)\big]\Big].
\end{aligned}
$$

Our Goals

1. Identify the diagrammatic origin of double logarithmic corrections and its relation to the 'kinematic constraint' [Ciafaloni '88; Andersson, Gustafson & Samuelsson '96; Kwieciński, Martin & Sutton '31; Beuf '14].
2. Implement directly the collinear resummation in coordinate space, as required by non-linear structure of BK equation.
3. Express the resummed evolution equation in terms of a local (energy-independent) kernel, as compared to non-local in rapidity proposals [Motyka & Staśto '09; Beuf '14]

The DLA Limit of the BFKL Equation

(Naive) DLA Limit of the BFKL Equation

BFKL Equation $(T = 1 - S, T \ll 1)$

$$\partial_Y T_{\boldsymbol{xy}}(Y) = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2\boldsymbol{z} \mathcal{M}_{\boldsymbol{xyz}} [T_{\boldsymbol{xz}}(Y) + T_{\boldsymbol{zy}}(Y) - T_{\boldsymbol{xy}}(Y)];$$

$$\mathcal{M}_{\boldsymbol{xyz}} = \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}$$

The $\boldsymbol{z}$-integration becomes logarithmic when daughter dipoles are much larger than the original one $(|\boldsymbol{x}-\boldsymbol{z}| \simeq |\boldsymbol{z}-\boldsymbol{y}| \gg r \equiv |\boldsymbol{x}-\boldsymbol{y}|)$ since $\mathcal{M}_{\boldsymbol{xyz}} \simeq r^2/(\boldsymbol{x}-\boldsymbol{z})^4$ and $T_{\boldsymbol{xz}} \simeq T_{\boldsymbol{zy}} \propto \boldsymbol{z}^2$. In this region, virtual term is negligible. Writing $T_{\boldsymbol{xy}}(Y) \equiv r^2 Q_0^2 \mathcal{A}_{\boldsymbol{xy}} \to r^2 Q_0^2 \mathcal{A}(Y, r^2)$

$$\mathcal{A}(Y, r^2) = \mathcal{A}(0, r^2) + \bar{\alpha}_s \int_0^Y \mathrm{d}Y_1 \int_{r^2}^{1/Q_0^2} \frac{\mathrm{d}z^2}{z^2} \mathcal{A}(Y_1, z^2)$$

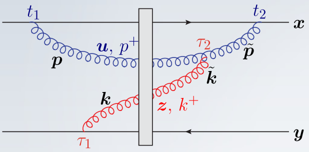
(Naive) DLA Equation (resums powers of $\bar{\alpha}_s Y \rho$, $\rho \equiv \ln[1/r^2 Q_0^2]$ to all orders)

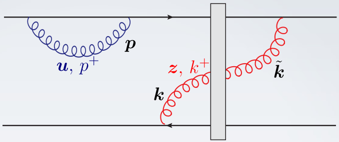
$$\mathcal{A}(Y, \rho) = I_0(2\sqrt{\bar{\alpha}_s Y \rho})$$

The Diagrammatic Origin of the DLA Equation

Computation of Time-Ordered Diagrams

- Lifetime of gluon fluctuation $\tau_p \equiv 2p^+/\boldsymbol{p}^2 = 1/p^-$
- Eikonal approximation $p^+ \gg k^+$





$$-\frac{g^4 N_c^2}{(2\pi)^2} \int_{\boldsymbol{uz}} S_{\boldsymbol{xu}} S_{\boldsymbol{uz}} S_{\boldsymbol{zy}}$$

$$\times \int_{\boldsymbol{p}\tilde{\boldsymbol{p}}\boldsymbol{k}\tilde{\boldsymbol{k}}} e^{i\boldsymbol{p}\cdot(\boldsymbol{u}-\boldsymbol{x})} e^{i\tilde{\boldsymbol{p}}\cdot(\boldsymbol{x}-\boldsymbol{u})} e^{i\boldsymbol{k}\cdot(\boldsymbol{z}-\boldsymbol{y})} e^{i\tilde{\boldsymbol{k}}\cdot(\boldsymbol{u}-\boldsymbol{z})} \frac{\boldsymbol{p}\cdot\tilde{\boldsymbol{p}}}{\boldsymbol{p}^2\tilde{\boldsymbol{p}}^2} \frac{\boldsymbol{k}\cdot\tilde{\boldsymbol{k}}}{\boldsymbol{k}^2\tilde{\boldsymbol{k}}^2}$$

$$\times \int_{q_0^+}^{q^+} \frac{dk^+}{k^+} \int_{k^+}^{q^+} \frac{dp^+}{p^+} \boxed{\frac{p^+}{p^+ + k^+\frac{\boldsymbol{p}^2}{\boldsymbol{k}^2}} \frac{p^+}{p^+ + k^+\frac{(\tilde{\boldsymbol{p}}-\tilde{\boldsymbol{k}})^2}{\tilde{\boldsymbol{k}}^2}}}.$$

$$\frac{p^+}{p^+ + k^+\frac{\boldsymbol{p}^2}{\boldsymbol{k}^2}} = \frac{\tau_p}{\tau_p + \tau_k} \simeq \begin{cases} 1 & \text{in BFKL} \\ \Theta(\tau_p - \tau_k) & \text{in DLA} \end{cases}$$

Real-Real Contribution

$$\left(\frac{\bar{\alpha}_s}{2\pi}\right)^2 \int_{q_0^+}^{q^+} \frac{\mathrm{d}k^+}{k^+} \int_{k^+}^{q^+} \frac{\mathrm{d}p^+}{p^+} \int_{\boldsymbol{uz}} \mathcal{M}_{\boldsymbol{xyu}}[\mathcal{M}_{\boldsymbol{uyz}} S_{\boldsymbol{xu}} S_{\boldsymbol{uz}} S_{\boldsymbol{zy}} + \mathcal{M}_{\boldsymbol{xuz}} S_{\boldsymbol{xz}} S_{\boldsymbol{zu}} S_{\boldsymbol{uy}}]$$

$$\times\Theta(p^+ \bar{u}^2 - k^+ \bar{z}^2), \qquad \bar{u} = \max(|\boldsymbol{u}-\boldsymbol{x}|, |\boldsymbol{u}-\boldsymbol{y}|);\ \bar{z} = \max(|\boldsymbol{z}-\boldsymbol{x}|, |\boldsymbol{z}-\boldsymbol{y}|)$$

Virtual-Real Contribution

$$-\left(\frac{\bar{\alpha}_s}{2\pi}\right)^2 \int_{q_0^+}^{q^+} \frac{\mathrm{d}k^+}{k^+} \int_{k^+}^{q^+} \frac{\mathrm{d}p^+}{p^+} \int_{\boldsymbol{uz}} \mathcal{M}_{\boldsymbol{xyu}} \mathcal{M}_{\boldsymbol{xyz}} S_{\boldsymbol{xz}} S_{\boldsymbol{zy}} \Theta(p^+ \bar{u}^2 - k^+ \bar{z}^2)$$

To DLA accuracy $\mathcal{M}_{\boldsymbol{uyz}} \mathcal{M}_{\boldsymbol{xyu}} \simeq \frac{r^2}{\bar{u}^2 \bar{z}^4}$ and $1 - S_{\boldsymbol{xu}} S_{\boldsymbol{uz}} S_{\boldsymbol{zy}} \simeq T_{\boldsymbol{uz}} + T_{\boldsymbol{zy}} \simeq 2T(\bar{z}^2)$ and we generate logarithmic phase space

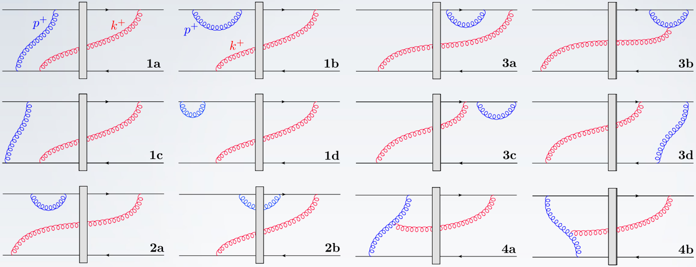
$$\int_{r^2}^{\bar{z}^2} \frac{\mathrm{d}\bar{u}^2}{\bar{u}^2} \int_{k^+ \frac{\bar{z}^2}{\bar{u}^2}}^{q^+} \frac{\mathrm{d}p^+}{p^+} = \int_{r^2}^{\bar{z}^2} \frac{\mathrm{d}\bar{u}^2}{\bar{u}^2} \left(\ln \frac{q^+}{k^+} - \ln \frac{\bar{z}^2}{\bar{u}^2}\right) = Y\rho - \frac{\rho^2}{2}$$

$$Y = \ln \frac{q^+}{k^+}; \qquad \rho = \ln \frac{\bar{z}^2}{r^2}$$

Cancellation of Anti-Time-Ordered Diagrams in DLA

Anti-time ordered graphs, involving factors $\frac{p^-}{p^-+k^-} \simeq \Theta(\tau_k - \tau_p)$ are also potentially enhanced by double transverse logs

$$\int_{r^2}^{\bar{z}^2} \frac{\mathrm{d}\bar{u}^2}{\bar{u}^2} \int_{k^+ \frac{\bar{z}^2}{\bar{u}^2}}^{q^+} \frac{\mathrm{d}p^+}{p^+} = \int_{r^2}^{\bar{z}^2} \frac{\mathrm{d}\bar{u}^2}{\bar{u}^2} \ln \frac{\bar{z}^2}{\bar{u}^2} = \frac{\rho^2}{2}$$



However, double logs cancel in the sum of all ATO diagrams. This also explains the peculiar way double logs arise in [Balitsky & Chirilli '08].

DLA Evolution for the Scattering Amplitude and the Lifetime Ordering Constraint

We conclude that perturbative corrections enhanced by double logarithms $Y\rho$ or ρ^2 can be resummed to all orders by solving a modified DLA equation involving manifest time-ordering

$$\mathcal{A}(q^+, r^2) = \mathcal{A}(0, r^2) + \bar{\alpha}_s \int_{r^2}^{1/Q_0^2} \frac{\mathrm{d}z^2}{z^2} \int_{q_0^+}^{q^+ \frac{r^2}{z^2}} \frac{\mathrm{d}k^+}{k^+} \mathcal{A}(k^+, z^2)$$

As it stands, this equation is non-local in rapidity

$$\partial_Y \mathcal{A}(Y, \rho) = \bar{\alpha}_s \int_0^{\rho} \mathrm{d}\rho_1 \mathcal{A}(Y - \rho + \rho_1, \rho)$$

Resummed Kernel for DLA, BFKL, and BK Evolutions

Towards a Resummed Rapidity-Independent Kernel

- By direct iteration of the modified DLA equation, we get

$$\mathcal{A}(Y,\rho) = \int_0^\rho \mathrm{d}\rho_1 f(Y,\rho-\rho_1)\mathcal{A}(0,\rho_1),$$

$$f(Y,\rho) = \delta(\rho) + \Theta(Y-\rho) \underbrace{\sum_{k=1}^{\infty} \frac{\bar{\alpha}_s^k (Y-\rho)^k \rho^{k-1}}{k!(k-1)!}}_{=\sqrt{\frac{\bar{\alpha}_s(Y-\rho)}{\rho}} I_1(2\sqrt{\bar{\alpha}_s(Y-\rho)\rho})}$$

- This can be written in integral representation: $f(Y,\rho) = \Theta(Y-\rho)\tilde{f}(Y,\rho)$;

$$\tilde{f}(Y,\rho) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{\mathrm{d}\xi}{2\pi i} \exp\left[\frac{\bar{\alpha}_s}{1-\xi}(Y-\rho) + (1-\xi)\rho\right]$$

The Local Kernel within DLA Approximation

A change of variables brings this as usual Mellin representation

$$\tilde{f}(Y,\rho) = \int_{\mathcal{C}} \frac{\mathrm{d}\gamma}{2\pi i} J(\gamma) \exp[\bar{\alpha}_s \chi_{\mathrm{DLA}}(\gamma) Y + (1-\gamma)\rho]$$

$$\bar{\alpha}_s \chi_{\mathrm{DLA}}(\gamma) = \frac{1}{2}\left[-(1-\gamma) + \sqrt{(1-\gamma)^2 + 4\bar{\alpha}_s}\right] = \frac{\bar{\alpha}_s}{(1-\gamma)} - \frac{\bar{\alpha}_s^2}{(1-\gamma)^3} + \cdots$$

$$J(\gamma) = 1 - \bar{\alpha}_s \chi'_{\mathrm{DLA}}(\gamma) = 1 - \frac{\bar{\alpha}_s}{(1-\gamma)^2} + \cdots$$

Mellin representation and exponentiation in Y ensures the existence of an evolution equation for f (and thus for $\mathcal{A}$) with an energy-independent kernel $\mathcal{K}_{\mathrm{DLA}}(\rho)$ defined as inverse Mellin of $\chi_{\mathrm{DLA}}(\gamma)$

$$\tilde{\mathcal{A}}(Y,\rho) = \tilde{\mathcal{A}}(0,\rho) + \bar{\alpha}_s \int_0^Y \mathrm{d}Y_1 \int_0^\rho \mathrm{d}\rho_1 \mathcal{K}_{\mathrm{DLA}}(\rho-\rho_1) \tilde{\mathcal{A}}(Y_1,\rho_1), \quad Y > \rho$$

$$\mathcal{K}_{\mathrm{DLA}}(\rho) = \frac{J_1(2\sqrt{\bar{\alpha}_s \rho^2})}{\sqrt{\bar{\alpha}_s \rho^2}} = 1 - \frac{\bar{\alpha}_s \rho^2}{2} + \frac{(\bar{\alpha}_s \rho^2)^2}{12} + \cdots$$

Similar approaches in [Salam '98; Sabio Vera '05; Motyka & Staśto '09]

The Change in the Initial Condition

Jacobian of Mellin transform induces also resummation in the initial condition ($\sim$ impact factor):

$$\tilde{\mathcal{A}}(0,\rho) = \int_0^\rho \mathrm{d}\rho_1 \tilde{f}(0,\rho-\rho_1)\mathcal{A}(0,\rho_1),$$
$$\tilde{f}(0,\rho) = \delta(\rho) - \sqrt{\bar{\alpha}_s} J_1(2\sqrt{\bar{\alpha}_s\rho^2}).$$

[$\tilde{\mathcal{A}}(Y,\rho)$ coincides with physical amplitude $\mathcal{A}(Y,\rho)$ for $Y > \rho$]

$$\tilde{\mathcal{A}}(0,\rho) = \begin{cases} \frac{1}{2}\left[1 + \mathrm{J}_0(\bar{\rho})\right] & \text{for} \quad \mathcal{A}(0,\rho) = 1, \\ \frac{\rho}{2}\left[1 + \mathrm{J}_0(\bar{\rho}) + \frac{\pi}{2}\mathbf{H}_0(\bar{\rho})\mathrm{J}_1(\bar{\rho}) - \frac{\pi}{2}\mathbf{H}_1(\bar{\rho})\mathrm{J}_0(\bar{\rho})\right] & \text{for} \quad \mathcal{A}(0,\rho) = \rho, \end{cases}$$

Resummed Kernel for BFKL/BK Evolution

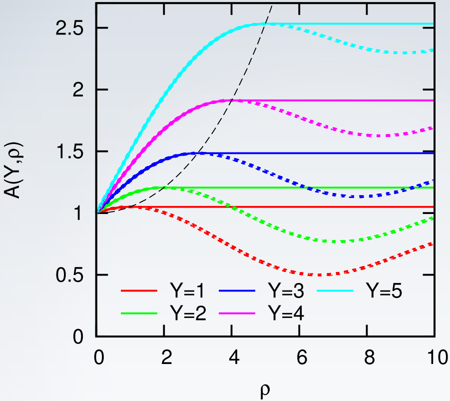
We can now easily promote our local DLA equation to easily include NLL BFKL/BK:

1. $\tilde{T}(Y,\rho) = \mathrm{e}^{-\rho}\tilde{\mathcal{A}}(\mathrm{Y},\rho)$
2. Return to transverse coordinates: $\rho = \ln(1/r^2 Q_0^2)$; $\rho - \rho_1 = \ln(z^2/r^2)$; $\tilde{T}(Y,\rho) = \tilde{T}_{\boldsymbol{xy}}(Y)$; $2\tilde{T}(Y,z^2) \to \tilde{T}_{\boldsymbol{xz}}(Y) + \tilde{T}_{\boldsymbol{zy}}(Y)$
3. Restore full dipole kernel $\frac{r^2}{z^4}\mathrm{d}z^2 \to \frac{1}{\pi}\mathcal{M}_{\boldsymbol{xyz}}\mathrm{d}^2\boldsymbol{z}$
4. Introduce the virtual term and temove IR and UV cutoffs in the $\boldsymbol{z}$ integration
5. Replace the argument of $\mathcal{K}_{\mathrm{DLA}}$ by $\ln\frac{z^2}{r^2} \to \sqrt{L_{\boldsymbol{xzr}}L_{\boldsymbol{yzr}}}$, with $L_{\boldsymbol{xzr}} \equiv \ln[(\boldsymbol{x}-\boldsymbol{z})^2/(\boldsymbol{x}-\boldsymbol{y})^2]$

$$\frac{\partial \tilde{T}_{\boldsymbol{xy}}(Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2 \boldsymbol{z} \mathcal{M}_{\boldsymbol{xyz}} \, \mathcal{K}_{\mathrm{DLA}} \left(\sqrt{L_{\boldsymbol{xz}r} L_{\boldsymbol{yz}r}} \right) \times \left[\tilde{T}_{\boldsymbol{xz}}(Y) + \tilde{T}_{\boldsymbol{zy}}(Y) - \tilde{T}_{\boldsymbol{xy}}(Y) - \tilde{T}_{\boldsymbol{xz}}(Y) \tilde{T}_{\boldsymbol{zy}}(Y) \right]$$

Numerical Results

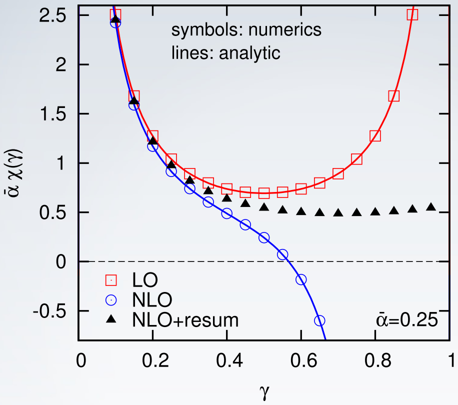
Identity of Local and Non-Local Solutions



$\mathcal{A}(Y,\rho)$: full lines; $\tilde{A}(Y,\rho)$: dashed lines

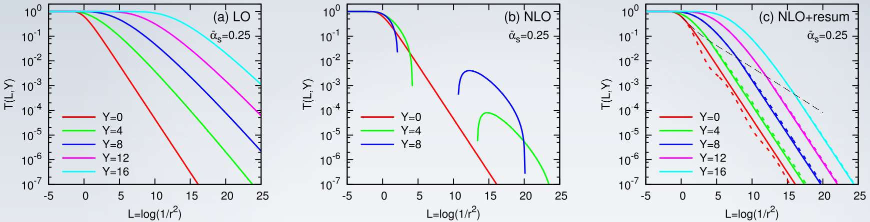
For $Y > \rho$ both funcions coincide; for $Y < \rho$, $\tilde{A}(Y,\rho)$ shows unphysical oscillations

Resummed Characteristic Function



All-orders resummation ensures smooth behavior near $\gamma = 1$. For $\bar{\alpha}_s = 0{,}25$, $\chi(\gamma)$ is essentially flat for $\gamma \gtrsim 0{,}5$

Numerical Solution of Resummed BK

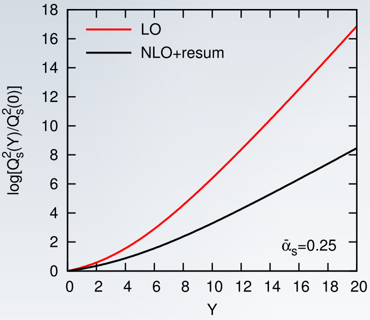


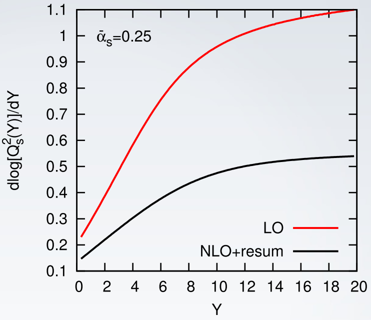
Initial condition of MV type $\mathcal{A}(0,\rho) = 1$

Reduction of phase-space coming from time-ordering and giving rise to collinear double logs leads to a considerable reduction in the speed of the evolution

For $\rho > Y$, expected physical behavior $T \propto \mathrm{e}^{-\rho}$

Modification of the Rapidity Dependence of the Saturation Momentum





The growth of the saturation scale with Y is considerably reduced by the resummation: for sufficiently large Y, the saturation exponent $\lambda_s \equiv \frac{d\rho_s}{dY}$ smaller by factor 2 compared to LO BFKL (asymptotically, $\lambda_s \sim 0{,}55$).

Conclusions & Outlook

Conclusions

1. We established clearly through a diagrammatic analysis the origin of double logs as coming from reduction of phase space due to time ordering
2. We were able to give an evolution equation with all-orders resummation of double logs in terms of an energy-independent kernel very convenient for numerical implementation
3. Our resummation is formulated directly in coordinate space allowing us its application to BK equation
4. Collinear resummation stabilizes and slows down the evolution. Very important phenomenological consequences expected

Outlook

- Applications to phenomenology
- Study and resummation of single logs
- Consequences of resummation for initial condition/impact factor

Back-Up Slides

Collinear Resummation à la Salam

Double Mellin Representation for BFKL Green's function

$$G(k,k_0,Y) = \frac{1}{k^2}\int_{a-i\infty}^{a+i\infty}\frac{\mathrm{d}\omega}{2\pi i}\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{\mathrm{d}\gamma}{2\pi i}\left(\frac{s}{kk_0}\right)^{\omega}\mathrm{e}^{\gamma\rho}\frac{1}{\omega-\kappa(\omega,\gamma)},$$

$$\rho = \ln(k^2/k_0^2); \qquad \kappa(\omega,\gamma) = \bar{\alpha}_s\chi(\gamma) + \bar{\alpha}_s^2\chi_1(\omega,\gamma) + \cdots$$

Matching with DGLAP through identification of relevant evolution variable for $k^2 > k_0^2$ and viceversa: ω-shift

$$G(k,k_0,Y) = \frac{1}{k^2}\int_{a-i\infty}^{a+i\infty}\frac{\mathrm{d}\omega}{2\pi i}\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{\mathrm{d}\gamma}{2\pi i}\left(\frac{s}{k^2}\right)^{\omega}\mathrm{e}^{(\gamma+\omega/2)\rho}\frac{1}{\omega-\kappa(\gamma,\omega)}$$

$$= \frac{1}{k_0^2}\int_{a-i\infty}^{a+i\infty}\frac{\mathrm{d}\omega}{2\pi i}\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{\mathrm{d}\gamma}{2\pi i}\left(\frac{s}{k_0^2}\right)^{\omega}\mathrm{e}^{(1-\gamma+\omega/2)(-\rho)}\frac{1}{\omega-\kappa(\omega,\gamma)}$$

Dipole Scattering Amplitude

Glauber-Mueller Formula for Dipole S-Matrix

$$S(r,Y) = \exp\left[-\frac{r^2 Q_s^2(Y)}{4}\right]$$

($T(r) \sim 1$ for $r \gg \frac{1}{Q_s}$ (black disk limit); $T(r) \sim 0$ for $r \ll \frac{1}{Q_s}$ (color transparency)

GBW Model for Dipole Cross Section

$$\sigma^{\mathrm{dip}} = \sigma_0 \left[1 - \exp\left(-\frac{r^2 Q_s^2(x)}{4}\right)\right]; \quad Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^{\lambda}$$

AAMQS Parametrization

$$T(r,b) = 1 - \exp\left[-\frac{(r^2 Q_{s0}^2(b))^{\gamma}}{4} \ln\left(\frac{1}{\Lambda r} + e\right)\right]$$

Saturation Momentum

Gribov-Levin-Ryskin Estimate

$$Q_s \sim \alpha_s^2 \Lambda_{\mathrm{QCD}} \left(\frac{1}{x}\right)^{\alpha_P - 1}$$

DLA Estimate of Rapidity Dependence of Dipole Scattering Amplitude $(r \ll 1/Q_{s0})$

$$T(r, Y) \sim (rQ_{s0})^2 (\bar{\alpha}_s Y)^{1/4} \rho^{-3/4} \exp[2\sqrt{2\bar{\alpha}_s Y \rho}]$$