# BFKL vs DGLAP in Mueller-Navelet jets at LHC

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*LHC Working Group on Forward Physics and Diffraction*,

April 21 - 24, 2015
Madrid, Spain

# Our goal

## Conclusions and future perspectives

**What did we do?**

- methods of optimization: **PMS**, **FAC** and **BLM**
- amplitude representations:
  - original definition
  - with squared impact factors
  - with collinear improvement
  - with collinear improvement & squared impact factors.

**What did we find?**

- Not good stability regions for FAC.
- Agreement between different amplitude representations using PMS and FAC **but** not with data.
- Two different choices for $\mu_R$ with BLM method: agreement with data in both cases at large energy.

**What's next?**

Could Mueller-Navelet jets discriminate between the BFKL and the DGLAP dynamics?

B. Murdaca (UNICAL & INFN) Mueller-Navelet jets at LHC April 15, 2014 31 / 31

[F. Caporale, D.Yu Ivanov, B.M., A. Papa, (2014)]

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# Outline

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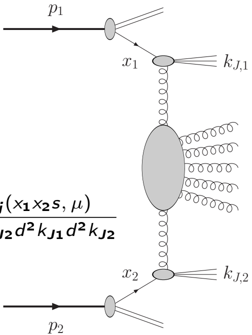
1. Introduction
   - The Mueller-Navelet jet production process
2. Theoretical setup
   - BFKL approach
   - The cross section in BFKL & DGLAP approaches
   - BLM method
3. Results
   - Our analysis
4. Conclusions

# Mueller-Navelet jets




$$\text{proton}(p_1) + \text{proton}(p_2) \to \text{jet}_1(k_1) + \text{jet}_2(k_2) + X$$

$$\frac{d\sigma}{dx_{J1}dx_{J2}d^2k_{J1}d^2k_{J2}} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2\, f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}_{i,j}(x_1 x_2 s,\mu)}{dx_{J1}dx_{J2}d^2k_{J1}d^2k_{J2}}$$

At the LHC anergies:

- Fixed-order perturbative approach
- BFKL approach:
  - large jet transverse momenta: $\vec{k}_{J,1}^2 \sim \vec{k}_{J,2}^2 \gg \Lambda_{\mathrm{QCD}}^2$
    $\longrightarrow$ perturbative QCD applicable
  - large rapidity gap between jets, $\Delta y = \ln \frac{x_{J,1} x_{J,2} s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$,

    $\longrightarrow$ $s = 2p_1 \cdot p_2 \gg \vec{k}_{J,1\,2}^2$

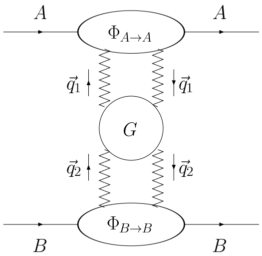
    $\longrightarrow$ BFKL resummation: $\sum_n c_n\, \alpha_s^n \ln^n s + d_n\, \alpha_s^n \ln^{n-1} s$

# Outline

# The BFKL approach

Total cross section $A + B \to X$, via the optical theorem, $\sigma = \frac{\mathrm{Im}_s \mathcal{A}}{s}$

- **Regge limit** ($s \to \infty$)
  $\Rightarrow$ BFKL factorization for $\mathrm{Im}_s \mathcal{A}$:
  convolution of the **Green's function** of two interacting Reggeized gluons and of the **impact factors** of the colliding particles.

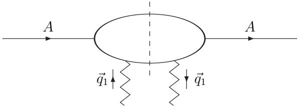



- Valid both in
  **LLA** (resummation of all terms $(\alpha_s \ln s)^n$)
  **NLA** (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$).

$$\mathrm{Im}_s \mathcal{A} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2}} \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2} q_2}{\vec{q}_2^{\,2}} \Phi_B(-\vec{q}_2, s_0) \int\limits_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

- The **Green's function** is **process-independent** and is determined through the **BFKL equation**. [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- **Impact factors** are **process-dependent**;




only very few have been calculated in the NLA:

- colliding partons

  [V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
  [M. Ciafaloni and G. Rodrigo (2000)]

- $\gamma^* \longrightarrow V$, with $V = \rho^0, \omega, \phi$, forward case

  [D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

- forward jet production

  [J. Bartels, D. Colferai, G.P. Vacca (2003)]
  [F. Caporale, D.Yu. Ivanov, B. M., A. Papa, A. Perri (2012)]
  [D.Yu. Ivanov, A. Papa (2012)]
  [D. Colferai, A. Niccoli (2015)]

- forward identified hadron production [D.Yu. Ivanov, A. Papa (2012)]

- $\gamma^* \longrightarrow \gamma^*$

  [J. Bartels *et al.* (2001) $\rightarrow$]
  [V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2002)-(2003)]
  [I. Balitsky, G.A. Chirilli (2011)-(2014)]
  [G.A. Chirilli, Yu.V. Kovchegov (2014)]

# Outline

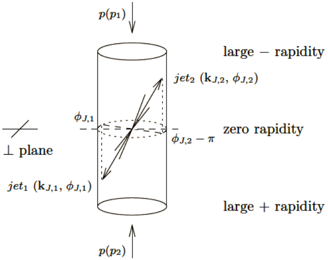
# The cross section

$$\frac{d\sigma}{dx_{J_1}\,dx_{J_2}\,d|\vec{k}_{J_1}|\,d|\vec{k}_{J_2}|d\phi_{J_1}\,d\phi_{J_2}} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\,\mathcal{C}_n\right]$$

Moments of the azimuthal decorrelations

$$\langle\cos\left[n\left(\phi_{J_1}-\phi_{J_2}-\pi\right)\right]\rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0},$$

$$\mathcal{R}_{m,n} = \frac{\langle\cos\left(m\Delta\phi\right)\rangle}{\langle\cos\left(n\Delta\phi\right)\rangle}$$




**Picture from**
[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]

Introducing the definitions

$$Y = \ln\frac{x_{J_1}x_{J_2}s}{|\vec{k}_{J_1}||\vec{k}_{J_2}|}, \qquad Y_0 = \ln\frac{s_0}{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$$

# ...in BFKL approach

$$\mathcal{C}_n^{BFKL} = \int_{-\infty}^{+\infty} d\nu \; e^{(Y-Y_0)\left[\bar{\alpha}_s(\mu_R)\chi(n,\nu)+\bar{\alpha}_s^2(\mu_R)K^{(1)}(n,\nu)\right]}\alpha_s^2(\mu_R)$$
$$\times c_1(n,\nu)\, c_2(n,\nu)\left[1+\alpha_s(\mu_R)\left(\frac{c_1^{(1)}(n,\nu)}{c_1(n,\nu)}+\frac{c_2^{(1)}(n,\nu)}{c_2(n,\nu)}\right)\right]$$

where

$$\chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right) - \psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)$$

$$K^{(1)}(n,\nu) = \bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c}\chi(n,\nu)\left(-\chi(n,\nu)+\frac{10}{3}+\imath\frac{d}{d\nu}\ln\left(\frac{c_1(n,\nu)}{c_2(n,\nu)}\right)+2\ln\left(\mu_R^2\right)\right)$$

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In the BFKL approach several NLA-equivalent expressions can be adopted for $\mathcal{C}_n$!

See for example [F. Caporale, D.Yu Ivanov, B.M., A. Papa, (2014)]

# Fixed-order DGLAP approach...

$$\mathcal{C}_n^{DGLAP} = \int_{-\infty}^{+\infty} d\nu \, \alpha_s^2 \left(\mu_R\right) c_1 \left(n, \nu\right) c_2 \left(n, \nu\right)$$

$$\times \left[1 + \bar{\alpha}_s \left(\mu_R\right) \left(Y - Y_0\right) \chi \left(n, \nu\right) + \alpha_s \left(\mu_R\right) \left( \frac{c_1^{(1)} \left(n, \nu\right)}{c_1 \left(n, \nu\right)} + \frac{c_2^{(1)} \left(n, \nu\right)}{c_2 \left(n, \nu\right)} \right)\right]$$

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# Optimization procedure

What renormalization scale do you have to choose when you work with a truncated series or resum only certain logarithms?

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- Optimization method
  - Principle of Minimal Sensitivity (PMS) [P.M. Stevenson (1981)]
  - Fast Apparent Convergence (FAC) [G. Grunberg (1980)-(1982)-(1984)]
  - BLM [ S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]

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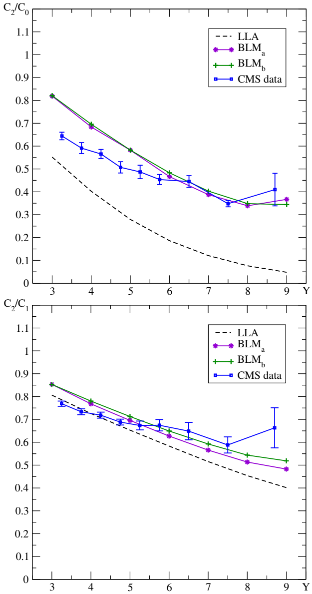
**...BLM method!**

# Why BLM?




[F. Caporale, D.Yu. Ivanov, B. M., A. Papa, (2014)]

# BLM method

- **Step 1**: change of renormalization scheme

$$\alpha_s^{\overline{\rm MS}}(\mu_R) \;=\; \alpha_s^{MOM}(\mu_R)\left(1+\frac{\alpha_s^{MOM}(\mu_R)}{\pi}T\right) \quad \text{with} \quad T = T^{\beta} + T^{conf} \,,$$

$$T^{\beta} \;=\; -\frac{\beta_0}{2}\left(1+\frac{2}{3}I\right) \quad \text{and} \quad T^{conf} \;=\; \frac{C_A}{8}\left[\frac{17}{2}I + \frac{3}{2}(1-I)\,\xi + \left(1-\frac{1}{3}I\right)\xi^2 - \frac{1}{6}\xi^3\right]$$

$$I = -2\int_0^1 dx\,\frac{\ln(x)}{[x^2-x+1]} \simeq 2.3439\,, \quad \xi = 0\,.$$

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$$\Downarrow$$

$$\mathcal{C}_n \simeq \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} d\nu\; e^{(Y-Y_0)\bar{\alpha}_s^{MOM}\chi(\nu,n)}\left(\alpha_s^{MOM}\right)^2 c_1 c_2 \left\{1+\bar{\alpha}_s^{MOM}\left(\frac{\bar{c}_1^{(1)}}{c_1}+\frac{\bar{c}_2^{(1)}}{c_2}\right)\right.$$

$$+\bar{\alpha}_s^{MOM}\left(\frac{\tilde{c}_1^{(1)}}{c_1}+\frac{\tilde{c}_2^{(1)}}{c_2}\right)+\bar{\alpha}_s^{MOM}\frac{2T^{conf}}{C_A}+\left(\bar{\alpha}_s^{MOM}\right)^2(Y-Y_0)\left(\frac{T^{conf}}{C_A}\chi(\nu,n)+\bar{\chi}(\nu,n)\right)$$

$$+\bar{\alpha}_s^{MOM}\left[\frac{2T^{\beta}}{C_A}+\bar{\alpha}_s^{MOM}(Y-Y_0)\left(\frac{T^{\beta}}{C_A}\chi(\nu,n)+\frac{\beta_0}{8C_A}\left(-\chi(\nu,n)+\frac{10}{3}\right.\right.\right.$$

$$\left.\left.\left.\left.+\; i\frac{d}{d\nu}\ln\left(\frac{c_1}{c_2}\right)+2\ln\left(\mu_R^2\right)\right)\right)\right]\right\}$$

- **Step 2**: choice of the BLM scale

$$\frac{\beta_0}{2C_A}\left\{\left[-2\left(1+\frac{2}{3}I\right)+\frac{5}{3}+f(\nu)+\ln\left(\frac{\mu_R^2}{k_1 k_2}\right)\right]+\bar{\alpha}_s^{MOM}(\mu_R)(Y-Y_0)\chi(\nu,n)\right.$$
$$\left.\times\left[-1-\frac{2}{3}I-\frac{1}{4}\chi(\nu,n)+\frac{5}{6}+f(\nu)+\frac{i}{4}\frac{d}{d\nu}\ln\left(\frac{c_1}{c_2}\right)+\frac{1}{2}\ln\left(\mu_R^2\right)\right]\right\}=0$$

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$$\Downarrow$$

$$\mu_R\equiv\mu_R\left(\nu,n,k_i\right)$$

**Partial BLM**:

a) $\left(\mu_R^{BLM}\right)^2=k_1 k_2\ \exp\left[2\left(1+\frac{2}{3}I\right)-f\left(\nu\right)-\frac{5}{3}\right]\sim 5^2 k_1 k_2$

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b) $$\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \ \exp\left[2\left(1+\frac{2}{3}I\right)-2\ f(\nu)-\frac{5}{3}+\frac{1}{2}\chi(\nu,n)\right] < (11.5)^2 k_1 k_2$$

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$f(\nu)$ is a function that depends on the considered process.

[F. Caporale, D.Yu. Ivanov, B. M., A. Papa, (2015)]

# Outline

# Our analysis

- **Observables:**

$$\langle\cos\left[n\left(\phi_{J_1}-\phi_{J_2}-\pi\right)\right]\rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0}\,, \quad \text{with} \;\; n=1,2,3$$

$$\frac{\langle\cos\left[2\left(\pi-\Delta\phi\right)\right]\rangle}{\langle\cos\left(\pi-\Delta\phi\right)\rangle}=\frac{\mathcal{C}_2}{\mathcal{C}_1}\,, \qquad \frac{\langle\cos\left[3\left(\pi-\Delta\phi\right)\right]\rangle}{\langle\cos\left[2\left(\pi-\Delta\phi\right)\right]\rangle}=\frac{\mathcal{C}_3}{\mathcal{C}_2}\,,$$

$$\text{with} \quad \Delta\phi=\phi_{J_2}-\phi_{J_1}\,.$$

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$$\frac{\langle \cos[2(\pi - \Delta\phi)]\rangle}{\langle \cos(\pi - \Delta\phi)\rangle} = \frac{\mathcal{C}_2}{\mathcal{C}_1}, \qquad \frac{\langle \cos[3(\pi - \Delta\phi)]\rangle}{\langle \cos[2(\pi - \Delta\phi)]\rangle} = \frac{\mathcal{C}_3}{\mathcal{C}_2},$$

$$\text{with} \quad \Delta\phi = \phi_{J_2} - \phi_{J_1} \;.$$

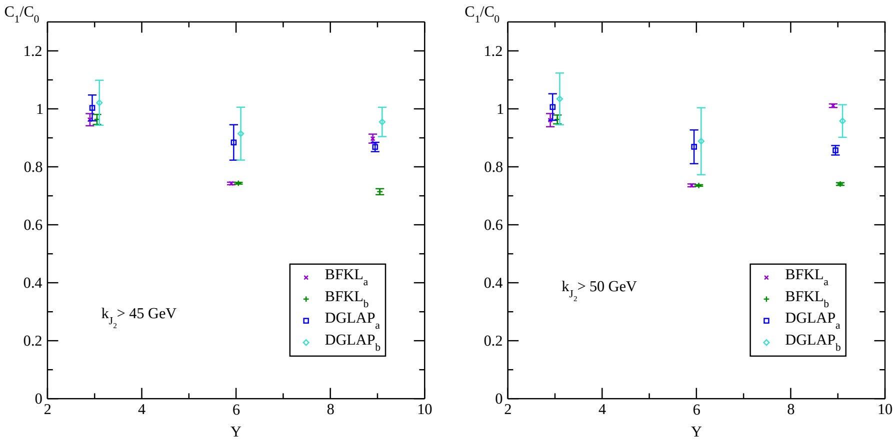
- **Kinematic settings:** $R = 0.5$ and $\sqrt{s} = 7$ TeV

- **Integrated coefficients**

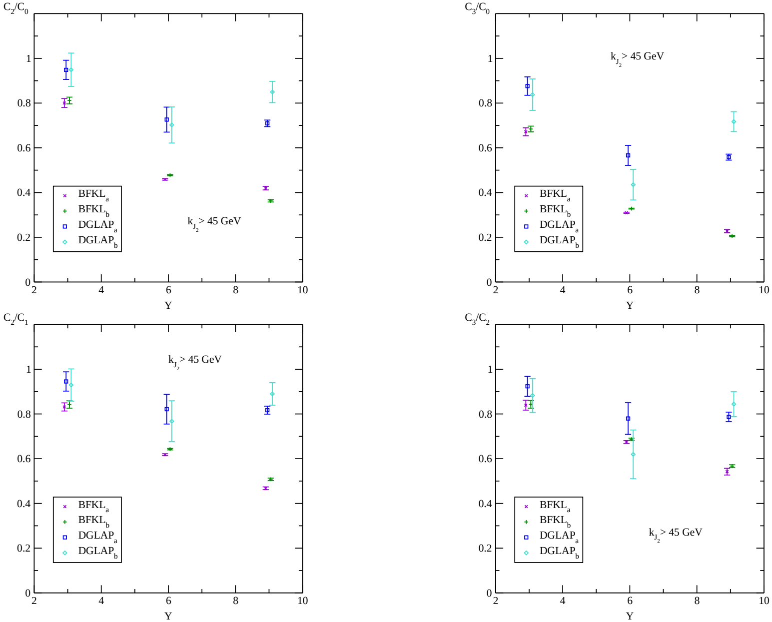
$$C_n = \int_{y_{1,\min}}^{y_{1,\max}} dy_1 \int_{y_{2,\min}}^{y_{2,\max}} dy_2 \int_{k_{J_1,\min}}^{\infty} dk_{J_1} \int_{k_{J_2,\min}}^{\infty} dk_{J_2} \delta\left(y_1 - y_2 - Y\right) \mathcal{C}_n\left(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2}\right)$$

- $-4.7 \le y_i \le 4.7$, with $i = 1, 2$
- 35 GeV $\le k_{J_1} \le$ 60 GeV
  - 45 GeV $\le k_{J_2} \le$ 60 GeV
  - 50 GeV $\le k_{J_2} \le$ 60 GeV

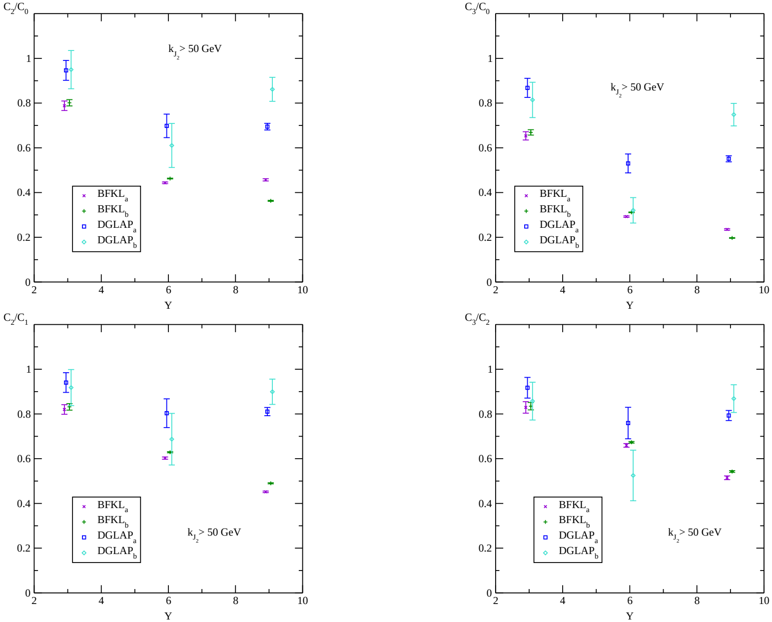
# $C_1/C_0$


C1/C0
1.2
1
0.8
0.6
0.4
0.2
0
2
4
6
8
10
Y
kJ2 > 45 GeV
kJ2 > 50 GeV
BFKLa
BFKLb
DGLAPa
DGLAPb

# $k_{J_2} > 45\,GeV$



# $k_{J_2} > 50\,GeV$



$C_2/C_0$
$k_{J_2} > 50$ GeV
BFKL$_a$
BFKL$_b$
DGLAP$_a$
DGLAP$_b$
Y
$C_3/C_0$
$k_{J_2} > 50$ GeV
BFKL$_a$
BFKL$_b$
DGLAP$_a$
DGLAP$_b$
Y
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BFKL$_b$
DGLAP$_a$
DGLAP$_b$
Y

# Conclusions

We have compared predictions for several $R_{nm}$ in full NLA BFKL approach and fixed-order NLO DGLAP.

- Implementation of a partial **BLM method** $\Rightarrow$ two different choices of scale!
- Asymmetric cuts for the transverse momenta of the detected jets:
  - $k_{J_1} > 35 GeV$ and $k_{J_2} > 45 GeV$
  - $k_{J_1} > 35 GeV$ and $k_{J_2} > 50 GeV$.

$\Rightarrow$ **Separation** between **BFKL** and **DGLAP** in particular at large energy!

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We suggest experimentalist collaborations to consider asymmetric cuts in jet transverse momenta!