Twist decomposition of forwad DY cross-sections

Leszek Motyka
Jagellonian University, Krakow

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Plan: to make full use of forward Drell-Yan process at the LHC to measure higher twist contributions

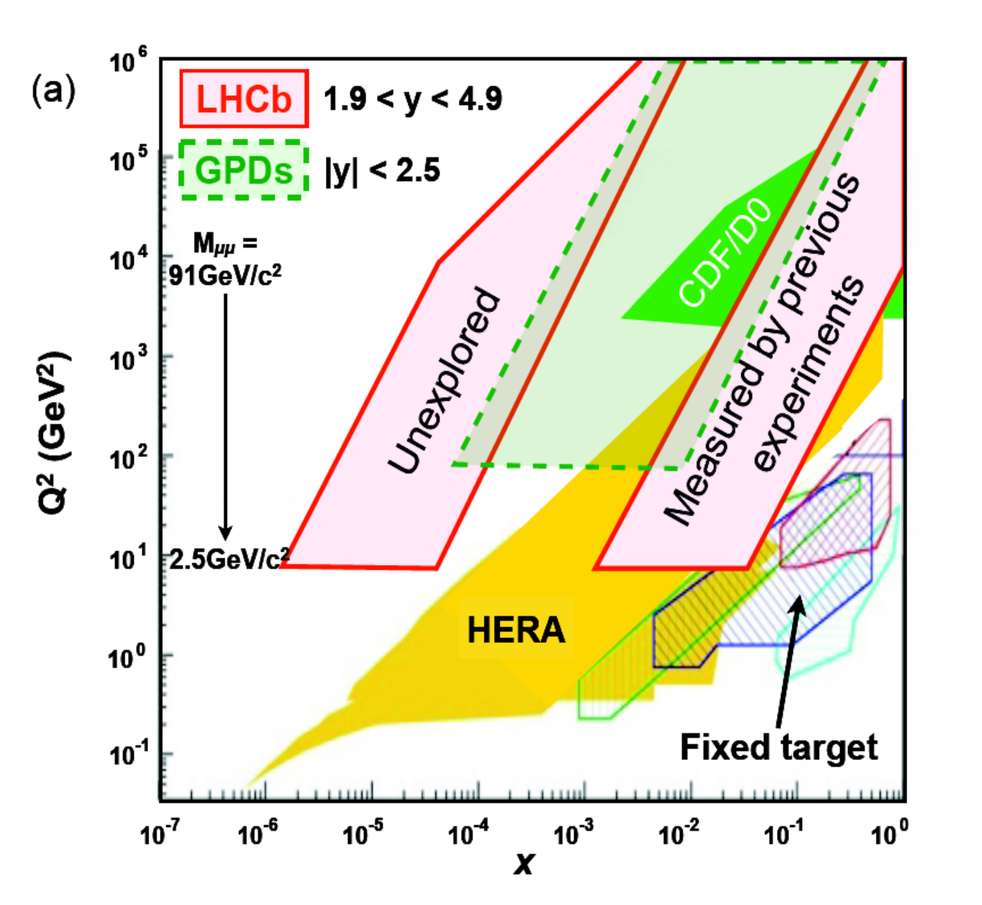
- Forward Drell-Yan: kinematics, observables
- Drell-Yan structure functions
- Dipole picture of forward Drell-Yan
- Twist decomposition: inclusive case
- Differential cross-sections: impact factors, Mellin representations
- Results on twist decomposition and Lam-Tung relation
- Conclusions

Work done with

Mariusz Sadzikowski and Tomasz Stebel

Forward Drell-Yan at LHC: kinematical reach and use

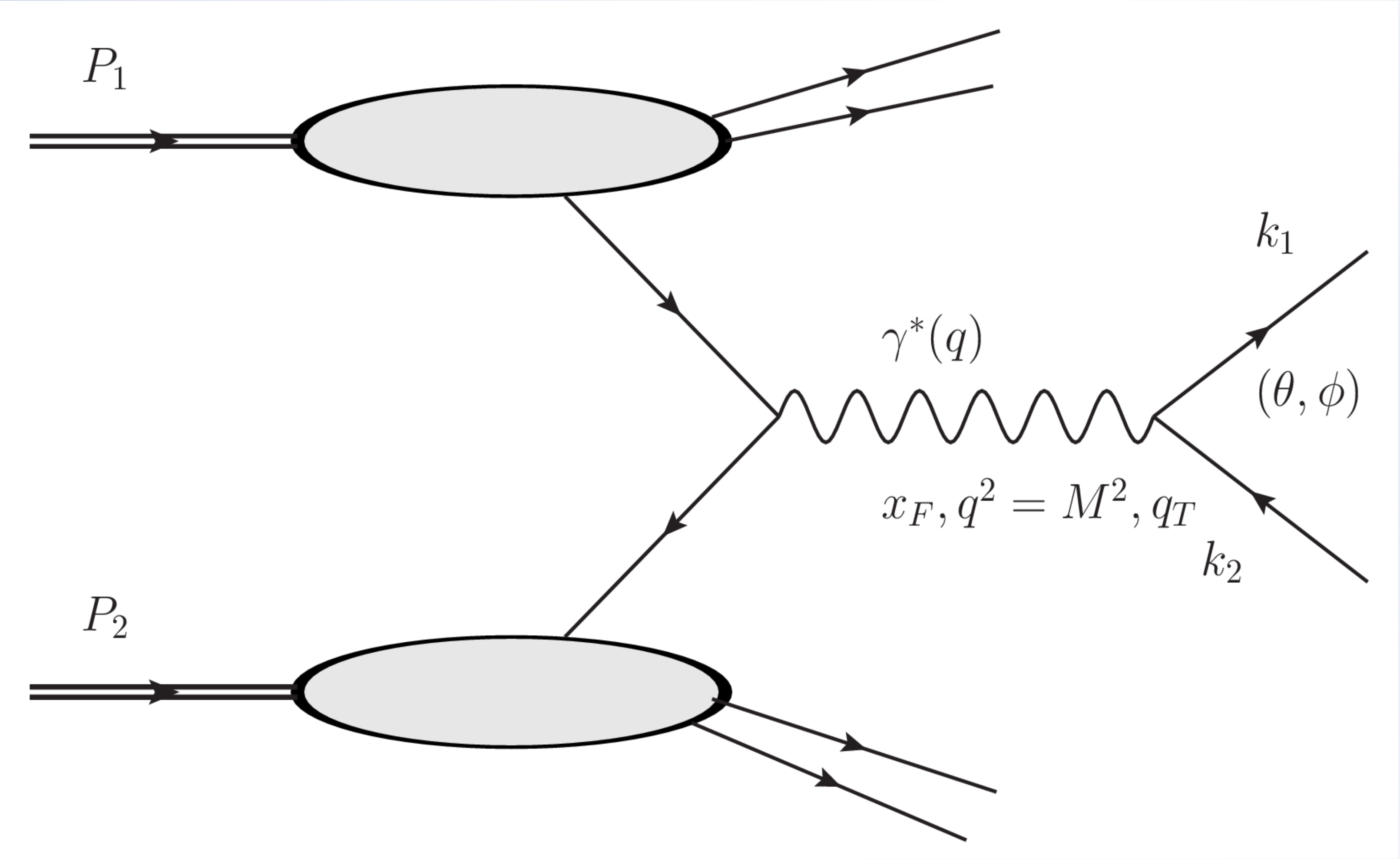
- Forward Drell-Yan may be used to measure parton densties down to $x < 10^{-6}$ at $M^2 \sim 10$ GeV^2



- Possible effects of multiple scattering and higher twists (small x enhancement of multiple gluon exchange): competition of $1/M^2$ and $x^{-\lambda}$ terms
- Needed to be controlled theoretically to avoid systematic errors of parton determination
- Potentially → measurement of higher twists.

  Advantage: 4 independent structure functions

Drell-Yan kinematics



Drell-Yan structure functions:

- Lepton angular distributions: 4 Drell-Yan structure functions (W_a – frame dependent)

$$\frac{d\sigma}{dx_F dM^2 d\Omega d^2 q_T} = \frac{\alpha_{\rm em}^2}{2(2\pi)^4 M^4}\left[(1-\cos^2\theta)W_L + (1+\cos^2\theta)W_T + (\sin^2\theta\cos 2\phi)W_{TT} + (\sin 2\theta\cos\phi)W_{LT}\right]$$

- Invariant structure functions:

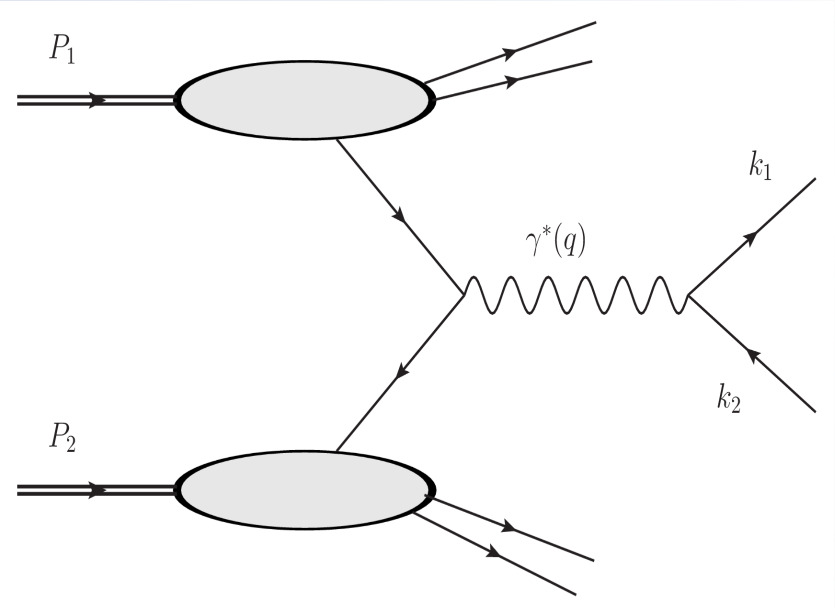
$$W^{\mu\nu} = -T_1\,\tilde{g}^{\mu\nu} + T_2\,\tilde{P}^\mu\tilde{P}^\nu - T_3\,\frac{1}{2}\left(\tilde{P}^\mu\tilde{p}^\nu + \tilde{p}^\mu\tilde{P}^\nu\right) + T_4\,\tilde{p}^\mu\tilde{p}^\nu$$

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu/q^2,\ \tilde{P}^\mu = \tilde{g}^{\mu\nu}P_\nu/\sqrt{S},\ \tilde{p}^\mu = \tilde{g}^{\mu\nu}p_\nu/\sqrt{S}$$

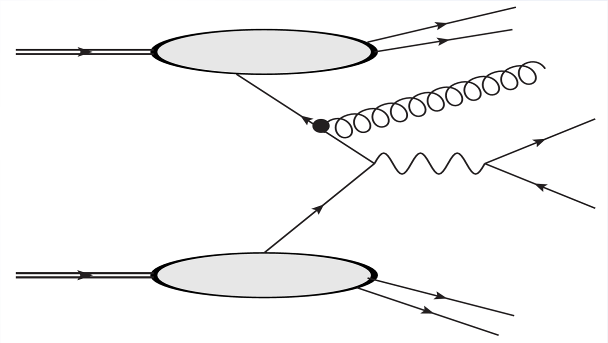
$$P = P_1 + P_2,\ p = P_1 - P_2$$

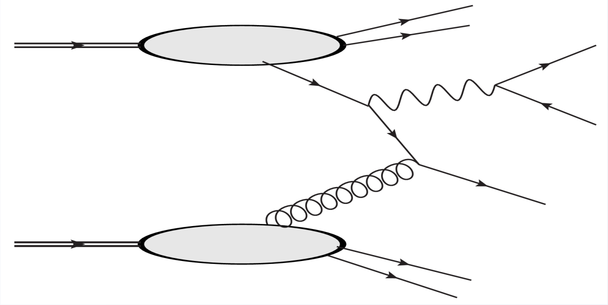
Leading diagrams of Drell-Yan

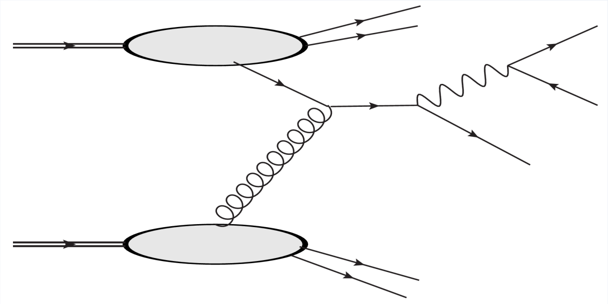
- Leading Order



- NLO

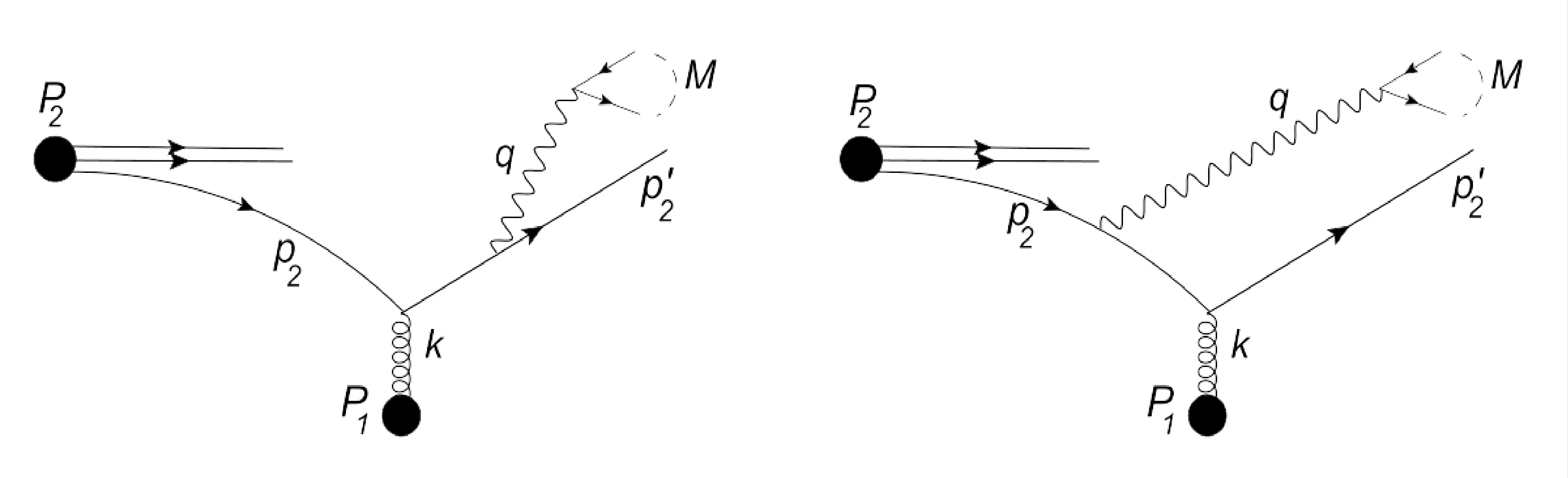






Leading diagrams of forward Drell-Yan

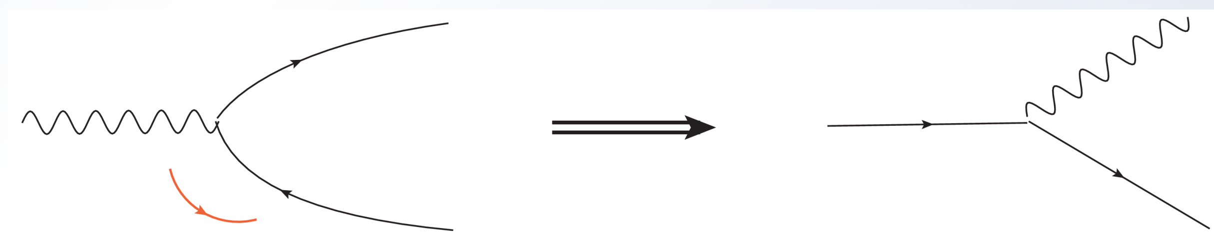
- Asymmetric kinematics: x_2 >> x1
- Dominance of the quark see → driven by gluon evolution
- Good approximation: gluon evolution followed by splitting to quark (anti-quark) in the last step



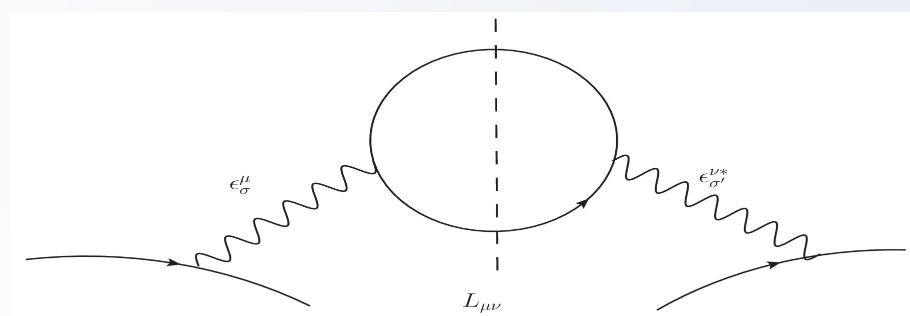
Forward Drell-Yan in dipole formulation

- Large energy limit: conservation of transverse positions in scattering
- "Effective color dipole" emerges from interference of photon emission before and after scattering, γ^* carries fraction z of p^+ of incident quark

- "Crossed" photon wave function:



- Interference of photon helicity states through leptonic tensor



Forward Drell-Yan in dipole formulation

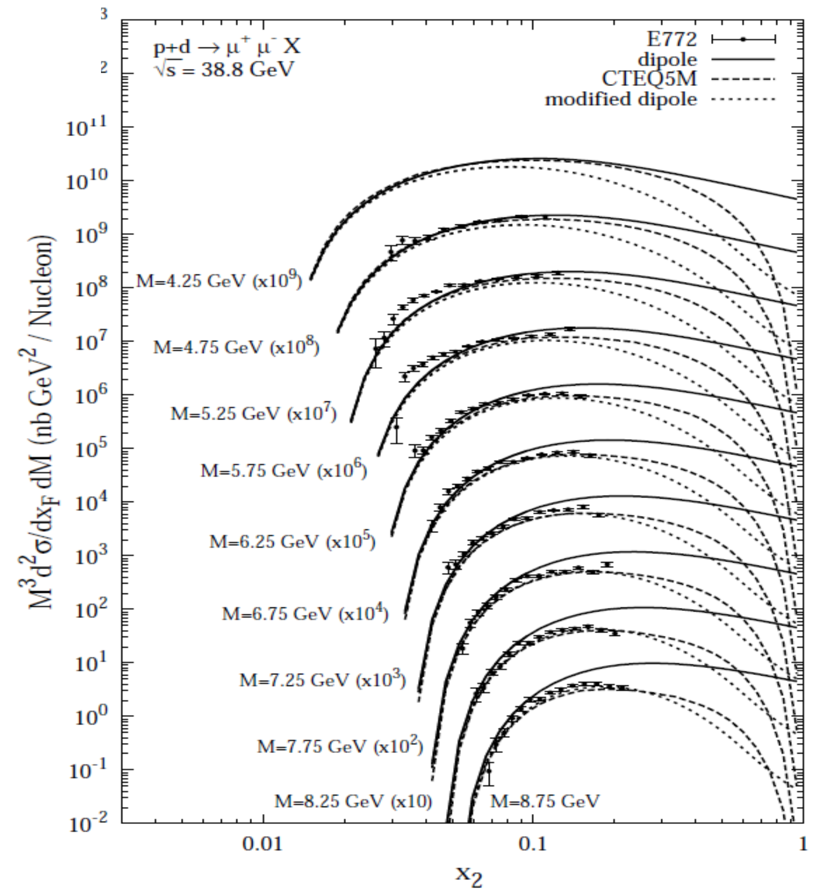
$$\sigma^f_{T,L}(qp \to \gamma^* X) = \int d^2r \, W^f_{T,L}(z, r, M^2, m_f) \, \sigma_{q\bar{q}}(x_2, zr)$$

$$W^f_T = \frac{\alpha_{em}}{\pi^2} \{[1 + (1-z)^2] \, \eta^2 K_1^2(\eta r) + m_f^2 \, z^4 K_0^2(\eta r)\}$$

$$W^f_L = \frac{2\alpha_{em}}{\pi^2} M^2 (1-z)^2 K_0^2(\eta r) \, ,$$

Formalism proposed and developed by:

- Brodsky, Hebecker, Quack (1997)
- B. Z. Kopeliovich, J. Raufeisen, A. V. Tarasov (2001)
- Gelis,Jalilian-Marian (2002)
- Raufeisen, Peng, Nayak (2002): plot →



Twists in a nutshell (1)

- Higher twists effects: power suppressed by hard scale:

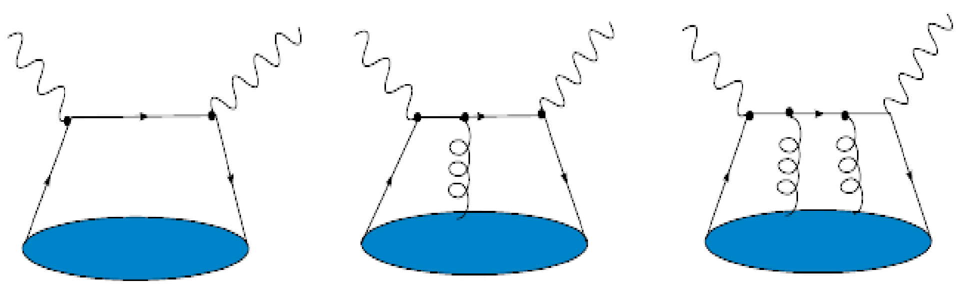
$$W^{\mu\nu} = \sum_{\tau} \left(\frac{\Lambda}{Q}\right)^{\tau-2} \sum_{i} C^{\mu\nu}_{\tau,i} \otimes f_{\tau,i}(Q^2/\Lambda^2)$$

- Typical operators:

$$\langle p|\bar{q}\gamma_{\{\mu_1} D_{\mu_2} \ldots D_{\mu_n\}} q|p\rangle = \langle x^n\rangle_q \, p_{\mu_1} \cdots p_{\mu_n}$$

- What is known on higher twists in proton?

Complete twist 4 analysis o $q\bar{q}gg$ evolution [Ellis, Furmanski and Petronzio, 1983]



- Understanding of twist-4 gluonic (gggg) operators – not complete
- However – dominant contribution should come from quasipartonic operators (twist = number of free partons in t-channel)

$$(\partial_{.} A^{\perp}_{\alpha})^2 (\partial_{.} A^{\perp}_{\beta})^2, \; \bar{\psi}\psi\bar{\psi}\psi$$

Twists in a nutshell (2)

- Evolution of quasi-partonic operators: n-tchannel partons + pairwise (non-forward) DGLAP interactions

- More rapid QCD evolution of higher twists with x

$$\frac{\text{Twist } 4}{\text{Twist } 2} \sim \frac{1}{Q^2 R^2} \exp\left(\sqrt{b \log(Q^2) \log(1/x)}\right)$$

- Significant corrections to precise parton determination, dependent on x and Q^2

- Quasi-partonic operators: relation of higher twists to multiple scattering, multiple parton densities and parton correlations

- At the LHC region of very small x may be probed for perturbative scales ~ 10 GeV^2

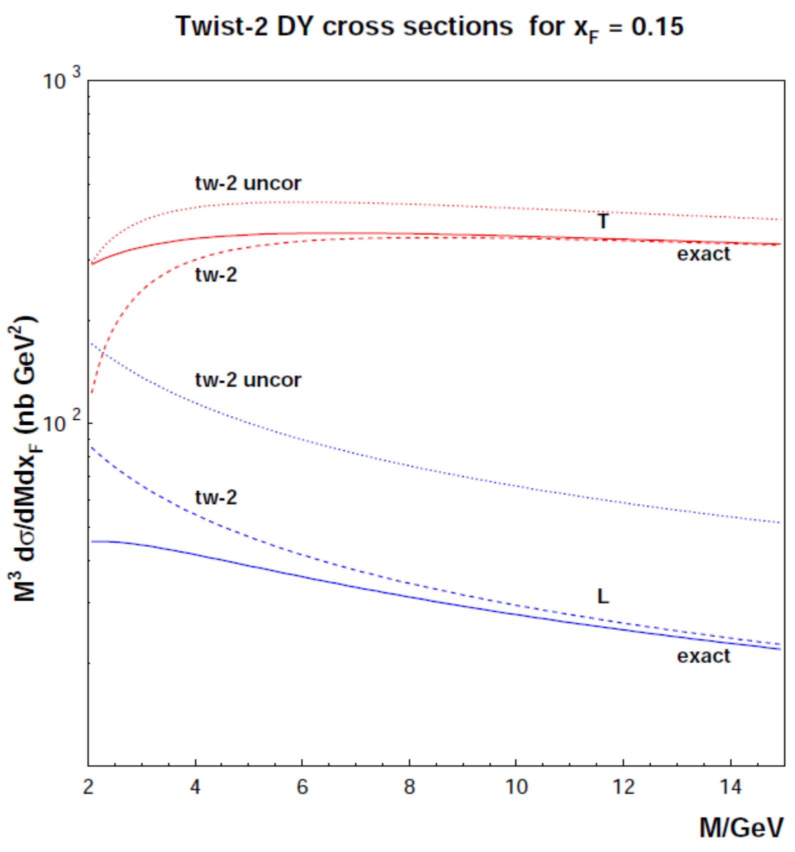
Difficulties in rigorous treatment of higher twists

- First-principle theory of higher twists: highly involved, few studies done within decades, not complete
- To provide reliable predictions: a lot of input from measurements is necessary – missing so far
- So → rely upon simplified picture: QCD guided model of rescattering with unitarity constraints
- Most advanced QCD studies of rescattering provided so far in the high energy limit, in kT-factorisation approach and small-x resummations (of logs(1/x))
- Efficient tool to treat multiple scattering: QCD guided saturation model

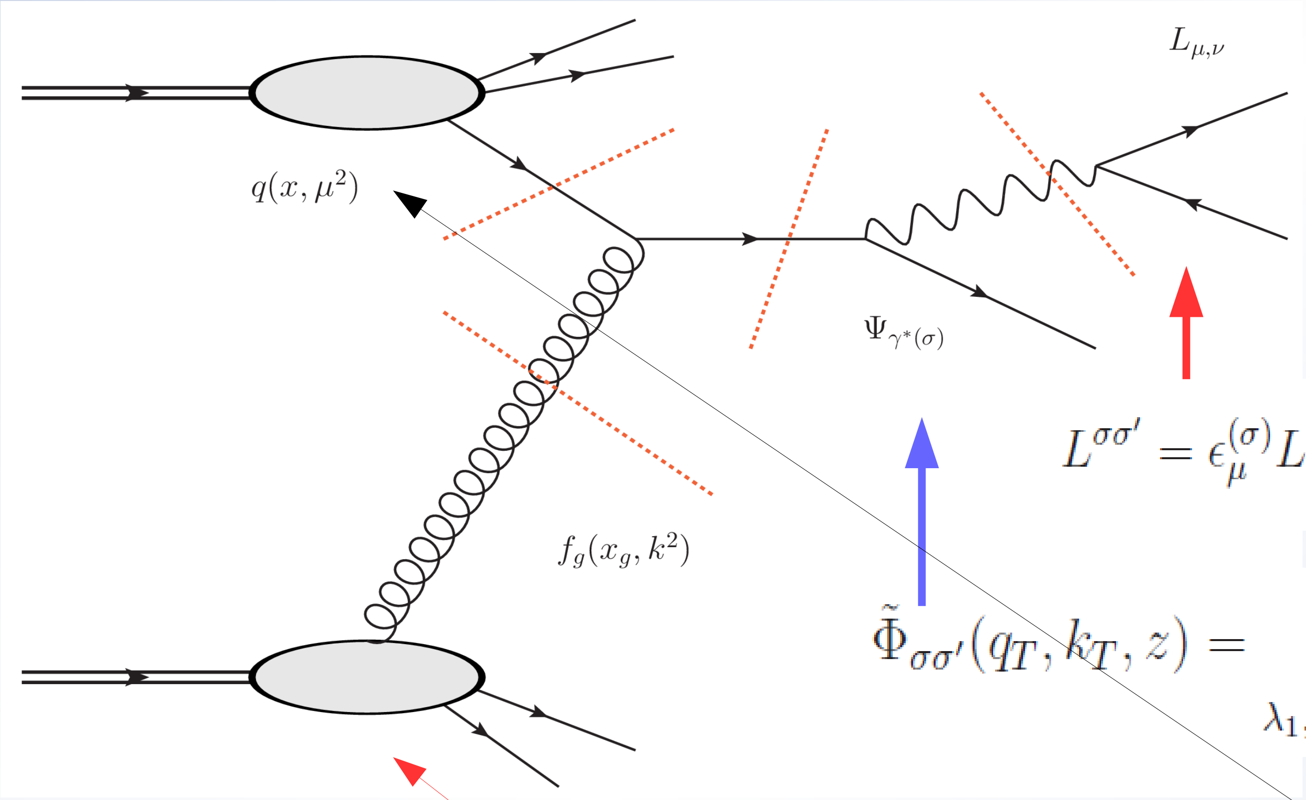
Inclusive forward Drell-Yan at LHC: higher twist corrections

- Golec-Biernat, Lewandowska, Staśto, 2010 (plot): first analysis of twist content of forward Drell-Yan within the GBW saturation model for dipole cross-section, using the technique of Bartels, Golec-Biernat and Peters done for the inclusive cross-section (in qT and the lepton azimuthal angle)

- Predictions for the LHC (plot) large higher twist corrections within kinematical range of LHC (LHCb)



Forward Drell-Yan cross-section in kT factorisation



$$L^{\sigma\sigma'} = \epsilon_\mu^{(\sigma)} L^{\mu\nu}\, \epsilon_\nu^{(\sigma')\dagger}, \quad L^{\mu\nu} = -g^{\mu\nu} + \frac{\kappa^\mu \kappa^\nu}{\kappa^2}$$

$$\tilde{\Phi}_{\sigma\sigma'}(q_T, k_T, z) = \sum_{\lambda_1,\lambda_2=+,-} A^{(\sigma)}_{\lambda_1,\lambda_2}(\vec{q}_T)^\dagger A^{(\sigma')}_{\lambda_1,\lambda_2}(\vec{q}_T)$$

$$\frac{d\sigma}{dx_F dM^2 d\Omega d^2 q_T} = \frac{\alpha_{\rm em}}{(2\pi)^2 (P_1\cdot P_2)^2\, M^2\, x_F^2 (1-z)} L^{\sigma\sigma'}(\Omega) \int_{x_F}^1 dz\; \wp(x_F/z)$$
$$\times \int d^2 k_T \frac{2\pi\alpha_s}{3} \frac{f(x_g,k_T^2)}{k_T^4} \tilde{\Phi}_{\sigma\sigma'}(q_T,k_T,z)$$

Mellin representation of forward Drell-Yan structure functions:

- Standard procedure: r-space → Mellin moments space

$$W_i = \int_{x_F}^{1} dz\; \wp(x_F/z) \int_{\mathcal{C}} \frac{ds}{2\pi i}\; \tilde{\sigma}(-s) \left(\frac{z^2 Q_0^2}{\eta_z^2}\right)^s \hat{\Phi}_i(q_T, s, z)$$

$$\eta_z^2 = M^2(1-z)$$

$$\hat{\Phi}_i(q_T, s, z) = \frac{2(2\pi)^4 M^4}{\alpha_{\rm em}^2} \int d^2r \; \left(\frac{\eta_z^2}{4z^2}\, r\right)^s \Phi_i(q_T, r, z)$$

$$\tilde{\sigma}(-s) = \int_0^\infty \frac{d\rho^2}{\rho^2} \, (\rho^2)^{-s} \;\; \hat{\sigma}(\vec{\rho})$$

Mellin representation of DY impact factors

- Mellin transforms of impact factors for all DY structure functions found, e.g.:

$$\hat{\Phi}_L(q_T,s,z) = \frac{2}{z^2}\left\{\frac{2\Gamma^2(s+1)}{1+q_T^2/\eta_z^2}\ {}_2F_1\left(s+1,s+1,1,-\frac{q_T^2}{\eta_z^2}\right) - \Gamma(s+1)\Gamma(s+2)\ {}_2F_1\left(s+1,s+2,1,-\frac{q_T^2}{\eta_z^2}\right)\right\}$$

$$\begin{aligned}\hat{\Phi}_{TT}(q_T,s,z) = \frac{1}{2z^2}\Bigg\{ & \frac{2\pi}{\Gamma(1-s)\sin\pi s\ q_T^2/\eta_z^2}\left(1+\frac{q_T^2}{\eta_z^2}\right)^{-s-3}\Gamma(s+2) \\ & \left[\left(1+\frac{q_T^2}{\eta_z^2}\right)\left(1+\frac{q_T^2}{\eta_z^2}(s+2)\right)\ {}_2F_1\left(-s+1,s+1,1,\frac{q_T^2}{q_T^2+\eta_z^2}\right)\right. \\ & \left. -\left(1+2\frac{q_T^2}{\eta_z^2}(s+1)\right){}_2F_1\left(-s+1,s+2,1,\frac{q_T^2}{q_T^2+\eta_z^2}\right)\right] \\ & -\frac{4q_T^2/\eta_z^2}{1+q_T^2/\eta_z^2}\Gamma(s+1)\Gamma(s+2)\ {}_2F_1\left(s+1,s+2,2,-\frac{q_T^2}{\eta_z^2}\right)\Bigg\}\end{aligned}$$

- Necessary for twist analysis, but useful also in BFKL approach

Twist decomposition of helicity structure functions (1)

- Necessary to assume certain form of dipole cross section. Choice to start with: Golec-Biernat Wuesthoff dipole cross-section, with saturation scale Q_0 (x)
- Problem of 2 hard scales M^2 and q_T → solution Q_0 expansion
- At twist 2:

$$W_L^{(2)} = \sigma_0 \frac{Q_0^2}{M^2} \int_{x_F}^{1} dz\ \wp(x_F/z) \frac{4M^6\ q_T^2 (1-z)^2}{\left[q_T^2 + M^2(1-z)\right]^4}$$

$$W_T^{(2)} = \sigma_0 \frac{Q_0^2}{M^2} \int_{x_F}^{1} dz\ \wp(x_F/z) \left[1+(1-z)^2\right] \frac{M^4 \left[q_T^4 + M^4(1-z)^2\right]}{2\left[q_T^2 + M^2(1-z)\right]^4}$$

$$W_{TT}^{(2)} = \sigma_0 \frac{Q_0^2}{M^2} \int_{x_F}^{1} dz\ \wp(x_F/z) \frac{2M^6\ q_T^2 (1-z)^2}{\left[q_T^2 + M^2(1-z)\right]^4}$$

$$W_{LT}^{(2)} = \sigma_0 \frac{Q_0^2}{M^2} \int_{x_F}^{1} dz\ \wp(x_F/z)(2-z) \frac{M^5\ q_T \left[-q_T^2 + M^2(1-z)\right](1-z)}{\left[q_T^2 + M^2(1-z)\right]^4}$$

Twist decomposition of helicity structure functions (2):

- Twist 4:

$$W_L^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z) z^2 \times \\ \times \frac{4M^8 \left[7q_T^2 - 10M^2 q_T^2(1-z) + M^4(1-z)^2\right](1-z)^2}{\left[q_T^2 + M^2(1-z)\right]^6}$$

$$W_T^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z) \left[1 + (1-z)^2\right] z^2 \times \\ \times \frac{M^6 \left[q_T^2 - 2M^2(1-z)\right]\left[q_T^4 - 4M^2 q_T^2(1-z) + M^4(1-z)^2\right]}{\left[q_T^2 + M^2(1-z)\right]^6}$$

$$W_{TT}^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z) z^2 \frac{12M^8 q_T^2 \left[q_T^2 - 2M^2(1-z)\right](1-z)^2}{\left[q_T^2 + M^2(1-z)\right]^6}$$

$$W_{LT}^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z)(2-z)\ z^2 \times \\ \times \frac{2M^7\ q_T \left[-2q_T^2 + M^2(1-z)\right]\left[q_T^2 - 5M^2(1-z)\right](1-z)}{\left[q_T^2 + M^2(1-z)\right]^6}$$

Mellin representation of qT-integrated DY structure functions

$$\tilde{W}_i = \frac{1}{2\pi M^2} \int W_i \, d^2 q_T$$

- Explicit integration → cross-checks with results known in the inclusive case

$$\tilde{W}_L = \int_{\mathcal{C}} \frac{ds}{2\pi i} \int_{x_F}^{1} dz \, \wp(x_F/z) \frac{1-z}{z^2} \left(\frac{z^2 Q_0^2}{4\eta_z^2} \right)^s \tilde{\sigma}(-s) \left\{ \frac{\sqrt{\pi}\, \Gamma^3(s+1)}{\Gamma\left(s+\frac{3}{2}\right)} \right\}$$

$$\tilde{W}_T = \int_{\mathcal{C}} \frac{ds}{2\pi i} \int_{x_F}^{1} dz \, \wp(x_F/z) \frac{1+(1-z)^2}{z^2} \left(\frac{z^2 Q_0^2}{4\eta_z^2} \right)^s \tilde{\sigma}(-s) \times \left\{ \frac{\sqrt{\pi}\, \Gamma(s)\Gamma(s+1)\Gamma(s+2)}{4\Gamma\left(s+\frac{3}{2}\right)} \right\}$$

Twist decomposition of integrated DY structure functions

- At twist-2:

$$\tilde{W}_L^{(2)} = \sigma_0 \frac{Q_0^2}{3M^2} \int_{x_F}^1 dz\ \wp(x_F/z), \quad \tilde{W}_{TT}^{(2)} = \sigma_0 \frac{Q_0^2}{6M^2} \int_{x_F}^1 dz\ \wp(x_F/z), \quad \tilde{W}_{LT}^{(2)} = 0$$

$$\tilde{W}_T^{(2)} = \sigma_0 \frac{Q_0^2}{4M^2} \Big\{ \wp(x_F) \Big[-1 + \frac{4}{3}\gamma_E + \frac{2}{3} \ln\left(\frac{4M^2(1-x_F)}{Q_0^2}\right) + \frac{2}{3}\psi(5/2) \Big] + \frac{2}{3} \int_{x_F}^1 dz\ \frac{\wp(x_F/z)[1+(1-z)^2] - \wp(x_F)}{1-z} \Big\}$$

- QT integration generates twist-3 contribution in W_{LT}:

$$\tilde{W}_{LT}^{(3)} = \sigma_0 \frac{\sqrt{\pi}\,[2 - \chi_1(3/2) - \chi_2(3/2)]}{6} \frac{Q_0^3}{M^3} \wp(x_F)$$

Twist decomposition of integrated DY structure functions:

- Twist-4 contributions also obtained, e.g.:

$$\tilde{W}_L^{(4)} = \frac{2}{15}\sigma_0 \frac{Q_0^4}{M^4}\left\{ \wp(x_F)\left[3 - 2\gamma_E - \ln\left(\frac{4M^2(1-x_F)}{Q_0^2}\right) - \psi(7/2)\right] - \int_{x_F}^{1} dz\, \frac{\wp(x_F/z)z^2 - \wp(x_F)}{1-z} \right\}$$

- General structure:

$$\mathrm{W}_a^{(\tau)} = \sigma_0 \left(\frac{Q_0}{M}\right)^{\tau} \left[\tilde{A}_1^{(\tau)} \ln\left(\frac{4M^2(1-x_F)}{Q_0^2}\right) + \tilde{A}_0^{(\tau)} + \tilde{B}^{(\tau)}\right]$$

Lam-Tung relation in dipole model

- Lam-Tung relation (1980, 1982): vanishing combination of DY structure functions at leading twist up to NNLO:

$$W_L - 2W_{TT} = 0$$

- Holds in the dipole model at twist 2
- At twist 4 – non-zero contribution → enhanced higher twist contributions

$$W_L^{(4)} - 2W_{TT}^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z) z^2 \frac{4M^8(1-z)^2}{[q_T^2 + M^2(1-z)]^4}$$

- qT-integrated cross-section also shows breaking of Lam-Tung relation

$$\int \left(W_L^{(4)} - 2W_{TT}^{(4)}\right) d^2q_T = 2\pi\sigma_0 M^2 \left(\tilde{W}_L^{(4)} - 2\tilde{W}_{TT}^{(4)}\right)$$
$$= 2\pi\sigma_0 \frac{Q_0^4}{M^2} \left\{ \frac{1}{18}\wp(x_F) \left[-19 + 12\gamma_E + 12\ln\left(\frac{M^2(1-x_F)}{Q_0^2}\right)\right] + \right.$$
$$\left. + \frac{2}{3}\int_{x_F}^1 dz\ \frac{\wp(x_F/z)z^2 - \wp(x_F)}{1-z} \right\}$$

Conclusions

- We computed Mellin representation of forward Drell-Yan impact factors – suitable for twist and BFKL analysis of forward Drell-Yan structure functions

- Assuming saturation model explicit form was found of twist expansion of forward DY structure functions: differential and integrated

- Lam-Tung relation preserved at twist-2, broken beyond → Lam-Tung combination of DY structure functions may be used to measure higher twist terms → need to measure forward DY leptons angular distributions at low masses!

- Phenomenological analysis is on the way


Thank you!