Follow-up of the impedance of the crab cavities

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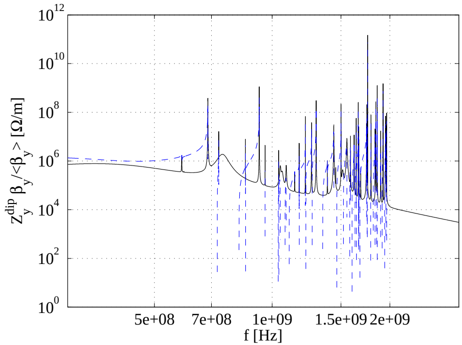
Outline

Motivation: We want to evaluate the effect of crab cavities focusing on their single bunch and coupled bunch kick factor in comparison with already existing equipment.



f (GHz)	Q	R (Ohm/m) per half cavity	plane	Deltaf in kHz
0.400	6.66E+09	5.98E+12	V	-
0.685	1880	4.67E+05	H	365
0.927	8600	1.38E+06	H	108
1.30	6540	3.73E+05	V	200
1.50	10800	2.67E+05	H	140
1.66	24200	3.14E+05	H	69
1.75	5800	2.57E+05	H	301
1.75	4160000	1.81E+08	H	0.4
1.84	9990	3.34E+05	V	185
1.86	26400	4.17E+05	H	70
1.86	88200	1.56E+06	V	21
1.92	102000	1.85E+06	H	18
1.96	54400	1.15E+05	H	36

Reminder:

- Crab cavities exhibit HOMs from ≃500 MHz up to 2 GHz in a location where the transverse β is ≃ 3000.
- Some HOM reach the level of $R_s = 10\,\mathrm{G\Omega/m} \rightarrow$ it might drive transverse CB instabilities!

Kick factor from Crab Cavities

- We define the transverse kick factor as:

$$k_t = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Z_t(\omega)h(\omega)\mathrm{d}\omega, \tag{1}$$

where $h(\omega) = \lambda(\omega)^2$ is the power spectrum of the current distribution , Z_t the transverse impedance (dipolar + quadrupolar). Given in units $[k_t]$=V/(mm pC).

- For a Gaussian bunch we have:

$$h(\omega) = e^{-\omega^2\sigma_t^2}. \tag{2}$$

- The kick factor is related to the transverse kick $\Delta y'$ a particle would get due to an impedance:

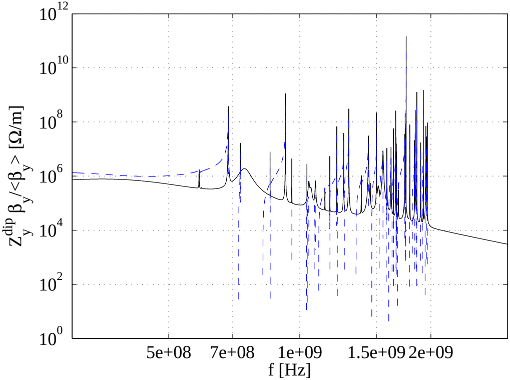
$$\Delta y' = -\frac{N_b q^2 y_0}{\beta^2 E}k_t. \tag{3}$$

with N_b bunch intensity, q, v and m_p, proton particle charge, velocity and rest mass, y_0 the closed orbit position at the impedance location.

- From the loss factor we can recover the usual tune shift formula (same as Sacherer for single bunch, azimuthal mode $m = 0$):

$$\Delta Q_y^{m=0} = \frac{1}{4\pi}\beta_k\Delta y' = -\frac{I_b q T_0}{4\pi\beta^2 E}\bar{\beta}_y k_t', \quad \text{with} \quad k_t' = \frac{\beta_k}{\bar{\beta}_y}k_t. \tag{4}$$

- Considering all the Crab Cavities (8 x plane x beam) we have



- $k_t' = 1.4$ V/mm pC.

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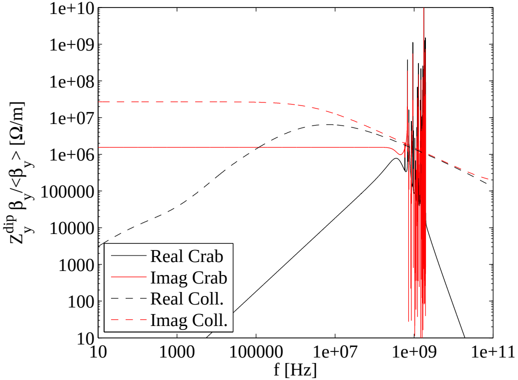

- $k_t' = 1.4$ V/mm pC.
- The TCP.D6L7.B1 with half gap at $\simeq 1$ mm would $k_t' = 3.14$ V/mm pC.

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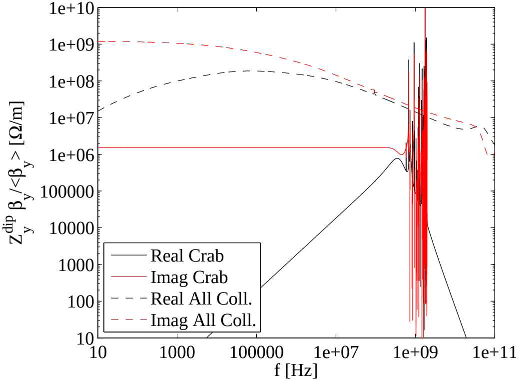


- $k_t' = 1.4$ V/mm pC.
- The TCP.D6L7.B1 with half gap at $\simeq 1$ mm would $k_t' = 3.14$ V/mm pC.
- All the collimators would give $k_t' = 45.3$ V/mm pC.

Single bunch:

- For a single bunch displaced by $y_0 = 1mm$ at the Crab cavities location, with $N_b = 2.2 \cdot 10^{11}$ ppb, the induced voltage is $V = N_b q k_t' y_0 = 50\,kV$. This is the voltage seen by the bunch passing through the impedance. The change in transverse energy $\Delta E_t = qV$ will determine the bunch oscillations and tune shift.
- The effect looks negligible comparing with usual equipments like collimators.

Coupled bunch:

- An HOM with shut impedance R_s, merit factor Q and resonant frequency f_r can drive unstable modes → the M bunches start oscillating coherently.
- We suppose rigid bunch oscillations (azimuthal $m = 0$) and derive the unstable frequency as $f_p = \left(n_x + k_p M + Q_{y_0}\right) f_0$ with $n_x \in (0..M-1)$ coupled bunch number, k_p line number.
- We choose the most unstable mode ($f_p \simeq f_r$).
- The kick factor can be defined extending the single bunch case as:

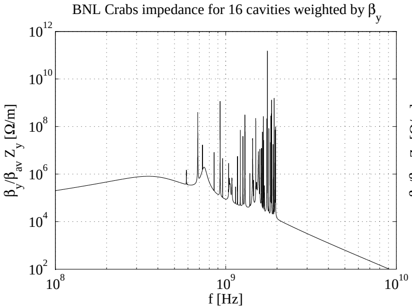
$$k_t = \omega_0 \sum_{p=-\infty}^{+\infty} Z_t(\omega_p) h(\omega_p), \quad (5)$$

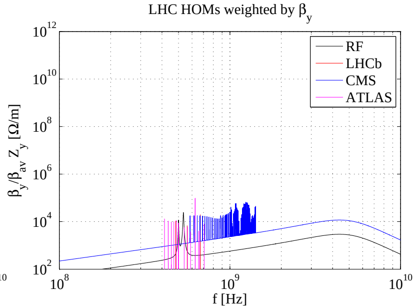
where $\omega_p = \left(n_x + k_p M + Q_{y_0}\right)\omega_0$.

- A transverse HOM can be characterized by shut impedance R_s, merit factor Q and resonant frequency f_r:

$$Z(f) = \frac{f_r}{f} \frac{R_s}{1 - jQ\left(\frac{f_r}{f} - \frac{f}{f_r}\right)};$$

- When falling on a CB line we have $k_t = \omega_0 R_s h(\omega_r)$.





- Comparison of HLLHC (round $\beta^* = 15$cm) BNL-HOMs Vs main HOMs from nominal LHC ($\beta^* = 60$cm).
- Several orders of magnitude difference → not negligible effect (DELPHI).

Conclusions

- We gave an overview of the effect of crab cavities in single bunch and coupled bunch.
- The single bunch effect of crab cavities can be compared with the one of a single LHC collimator.
- The single bunch effect of crab cavities is negligible comparing to the total LHC collimators contribution.
- The coupled bunch effect of crab cavities is driven by the equipment HOM → orders of magnitude above other machine equipment.

Outlook

- Continuing improving the crab cavities design in order to damp the HOM.
- Studying possible configurations for safe and stable machine operation with this equipment (collide and squeeze).
- ...

Thanks for your attention!

Backup

- Each CB line can be driven unstable in presence of an impedance.
- The rise-time and frequency shift can be approximately calculated by means of the Sacherer formula:

$$\Delta\omega_m^{x,y} = \frac{1}{|m|+1} \frac{jq\beta I_b}{2\, m_0 \gamma Q_{x_0,y_0} \Omega_0 L_b} \left(Z_{x,y}^{eff}\right)_m$$

where $\Delta\omega_m^{x,y}$ the CB line complex frequency shift, q is the proton charge, m_0 the proton rest mass, $I_b = e\, N_b/T_0$ the beam current, T_0 the revolution frequency,ω_0 revolution radial frequency, Q_{x_0,y_0} the machine unperturbed tune, β and γ relativistic factors, $L_b = 4\sigma_z$ with σ_z the rms bunch length, $Z_{x,y}^{eff}$ the impedance weighted by the sinusoidal modes with

$$h(\omega) = \frac{8\tau_b^2}{\pi^4} (|m|+1)^2 \frac{1 + (-1)^{|m|} \cos(\omega\, 4\tau_b)}{\left[(\omega\, 4\tau_b/\pi)^2 - (|m|+1)^2\right]^2}$$

and

$$Z_{x,y}^{eff} = \frac{\sum_{p=-\infty}^{+\infty} Z(\omega_p)_{x,y} h(\omega_p)}{\sum_{p=-\infty}^{+\infty} h(\omega_p)}$$

- The chromatic frequency $\omega_\xi = \dfrac{\omega\xi}{\eta}$ shifts the sinusoidal modes (replace $\omega \to \omega - \omega_\xi$).
- NB: No damper is considered here.

Backup

- A transverse HOM can be characterized by shut impedance R_s, merit factor Q and resonant frequency f_r:

$$Z(f) = \frac{f_r}{f} \frac{R_s}{1 - jQ\left(\frac{f_r}{f} - \frac{f}{f_r}\right)};$$

- If the bandwidth $\Delta f = \frac{f_r}{Q} \geq f_0$, the mode covers one or more CB lines that are driven unstable.
- If the bandwidth $\Delta f = \frac{f_r}{Q} \ll f_0$, the mode can fall between the CB lines → complicated situation due to the many azimuthal modes spaced by f_s.
  - If falling on the CB line we can simplify the Sacherer formula

$$\Delta\omega_m^{x,y} = \frac{1}{|m| + 1} \frac{jq\beta I_b}{2\, m_0 \gamma Q_{x_0,y_0} \Omega_0 L_b} R_s \frac{h(\omega_r)}{\sum_{p=-\infty}^{+\infty} h(\omega_p)}$$

  - If not falling on the CB line we are in principle stable.