New MC methods for neutrino telescopes

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Goals

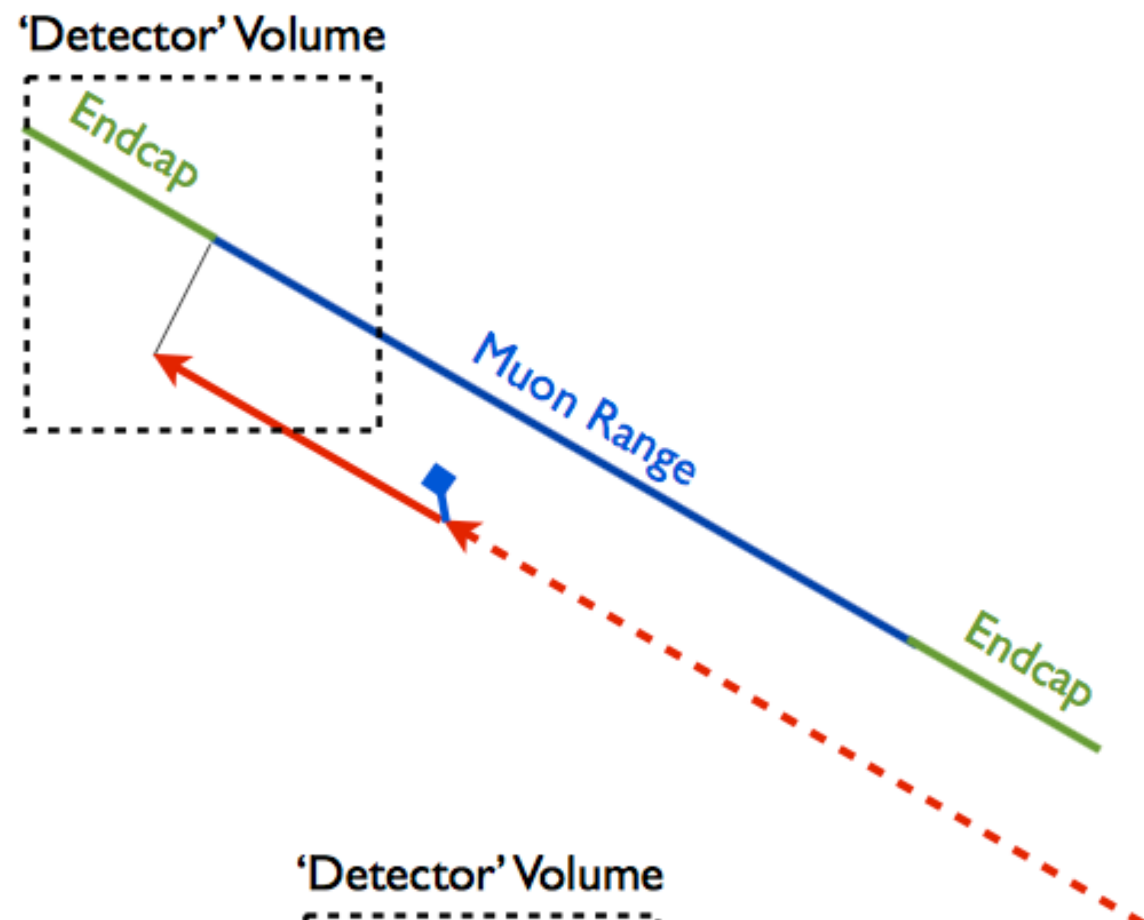
Generate neutrino events in a manner which does not tie the user to the details of how the neutrino got to the detector

- ➡ **Neutrino Flux**
- ➡ **Neutrino-Nucleon Cross Section**
- ➡ **Earth Composition**
- ➡ **New Neutrino Physics**

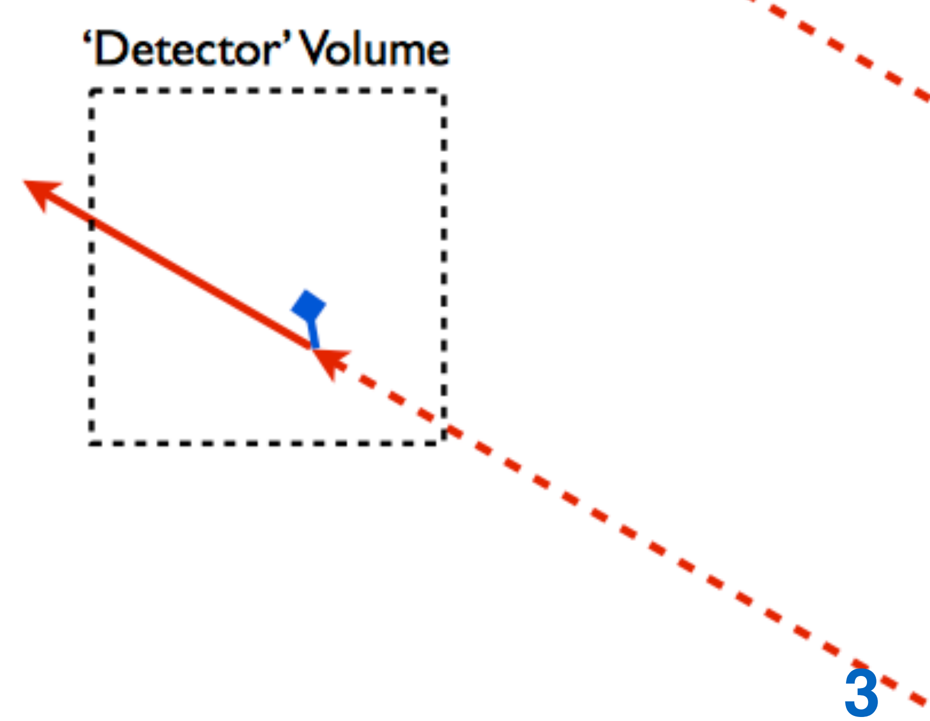
New Lepton Injection

Rather than injecting a neutrino on the other side of the Earth and propagating it, **just posit a neutrino interacting near the detector**.

➡ **Ranged** mode: sample a vertex point (uniformly in column depth!) some distance away from the detector, up to the **muon range**.



➡ **Volume** mode: sample a **vertex point** inside a cylinder (assume uniform density!).

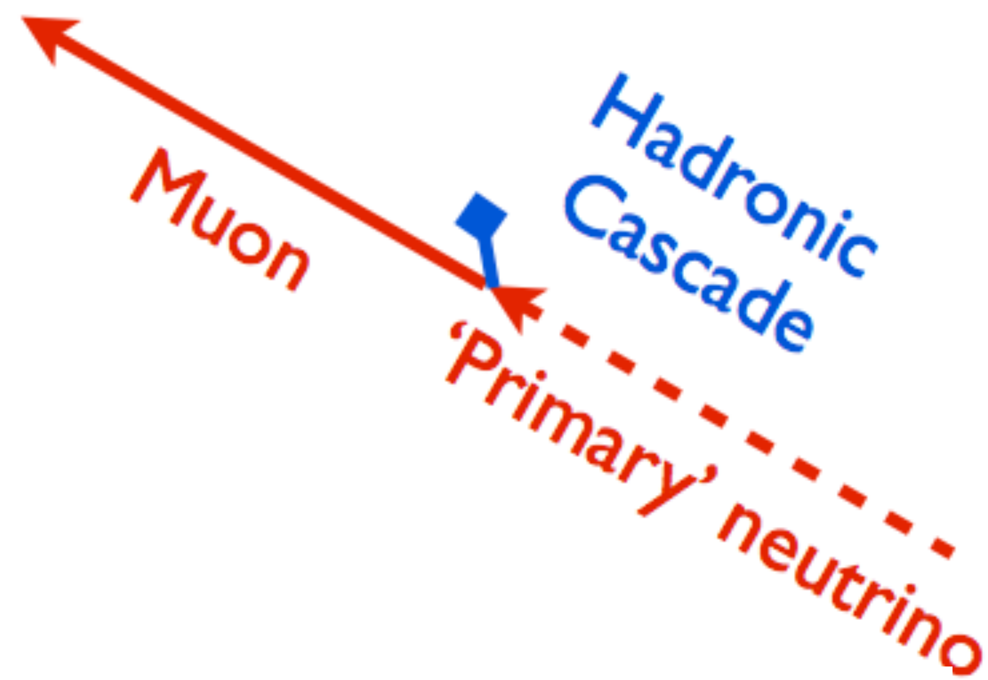


New Lepton Injection, cont'd

- Sample the properties of the daughter particles from the differential cross section

- Does not have to exactly the right cross section, just close enough that **reweighting** will be efficient

- Need to treat each process separately:



CC	NC	GR	
e^-, (hadrons)	ν_e, (hadrons)	$e^-, \bar{\nu}_e$	
e^+, (hadrons)	$\bar{\nu}_e$, (hadrons)	e^+, ν_e	
μ^-, (hadrons)	ν_μ, (hadrons)	$\mu^-, \bar{\nu}_\mu$	(hadrons), (hadrons)
μ^+, (hadrons)	$\bar{\nu}_\mu$, (hadrons)	μ^+, ν_μ	
τ^-, (hadrons)	ν_τ, (hadrons)	$\tau^-, \bar{\nu}_\tau$	
τ^+, (hadrons)	$\bar{\nu}_\tau$, (hadrons)	τ^+, ν_τ	

New Neutrino Propagation

- We **do not** solve the neutrino evolution by propagating each neutrino (**MC**)
- We **solve the *master* equation** that evolves the flux from the atmosphere to the detector

The Master Equation

$$\frac{dF_\nu(E)}{dx} = -i[H, F_\nu(E)] - \sum_\alpha \frac{1}{2\lambda^\alpha(E)}\{\Pi_\alpha, F_\nu(E)\}$$
$$+ \int_{E'} \frac{1}{\lambda_{\mathrm{NC}}(E')} \sum_\alpha \{\Pi_\alpha, F_\nu(E')\} NC(E', E)$$
$$+ \int_{E'} \frac{1}{\lambda^\tau(E')} F_\tau(E') CC_\tau(E', E) \Pi_\tau$$
$$+ \mathrm{Br}_\mu \int_{E'} \frac{1}{\lambda^\tau(E')} \bar{F}_\tau(E') CC_\tau(E', E) \Pi_\mu$$
$$+ \mathrm{Br}_e \int_{E'} \frac{1}{\lambda^\tau(E')} \bar{F}_\tau(E') CC_{\bar{\tau}(E', E)} \Pi_e$$

$$\frac{dF_\tau(E)}{dx} = -\frac{1}{\lambda^\tau(E)} F_\tau(E) + \int_{E'} \frac{1}{\lambda^\tau_{\mathrm{CC}}(E')} \mathrm{Tr}[\Pi_\tau, F_\nu(E')] CC_\tau(E_\nu, E_\tau)$$

Gonzalez-Garcia et al. Phys.Rev. D71 (2005) 093010
Argüelles et al. arXiv:1412.3832, CPC 2015.06.022.

Advantages of the *Master* Equation

✓ **Fast** and **accurate**.

✓ **Neutrino oscillations** incorporated.

✓ Easy to incorporate **new neutrino physics**

✓ **No** MC **statistic limitation.**

The Master Equation

$$\frac{dF_\nu(E)}{dx} = -i[H, F_\nu(E)] - \sum_\alpha \frac{1}{2\lambda^\alpha(E)}\{\Pi_\alpha, F_\nu(E)\}$$
$$+\int_{E'} \frac{1}{\lambda_{\mathrm{NC}}(E')}\sum_\alpha \{\Pi_\alpha, F_\nu(E')\} NC(E', E)$$
$$+\int_{E'} \frac{1}{\lambda^\tau(E')} F_\tau(E') CC_\tau(E', E)\Pi_\tau$$
$$+\mathrm{Br}_\mu \int_{E'} \frac{1}{\lambda^\tau(E')} \bar{F}_\tau(E') CC_\tau(E', E)\Pi_\mu$$
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How do we solve the *Master* Equation efficiently?

We can **expand** the ***F*** as a function of the **SU(3) generators + Identity** (λ_i):

$$F_\nu(E,x) = \sum_i f_i(E,x)\lambda_i$$

Advantages

- **QM operations** (commutation, anticommutation, time evolution) are **trivial and exact**.
- **Number of equations reduced by two.**

$$\begin{aligned}
\frac{dF_\nu(E)}{dx} &= -i[\mathrm{H},\mathrm{F}_\nu(\mathrm{E})] - \sum_\alpha \frac{1}{2\lambda^\alpha(E)}\{\Pi_\alpha, F_\nu(E)\} \\
&+ \int_{E'} \frac{1}{\lambda_{\mathrm{NC}}(E')}\sum_\alpha \{\Pi_\alpha, F_\nu(E')\} NC(E',E) \\
&+ \int_{E'} \frac{1}{\lambda^\tau(E')} F_\tau(E') CC_\tau(E',E)\Pi_\tau \\
&+ \mathrm{Br}_\mu \int_{E'} \frac{1}{\lambda^\tau(E')} \bar{F}_\tau(E') CC_\tau(E',E)\Pi_\mu\,, \\
&+ \mathrm{Br}_e \int_{E'} \frac{1}{\lambda^\tau(E')} \bar{F}_\tau(E') CC_{\bar{\tau}}(E',E)\Pi_e \\
\frac{dF_\tau(E)}{dx} &= -\frac{1}{\lambda^\tau(E)} F_\tau(E) \\
&+ \int_{E'} \frac{1}{\lambda^\tau_{\mathrm{CC}}(E')} \mathrm{Tr}[\Pi_\tau\, F_\nu(E')] CC_\tau(E_\nu, E_\tau)\,.
\end{aligned}$$

RED: 3 x 3 Complex matrices → **18** ODE

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We implemented this method in a software called SQuIDS

SQuIDS
Simple Quantum Integro Differential Solver

We can **expand** the *F* as a function of the **SU(4) generators + Identity** (λ_i):

$$F_\nu(E,x) = \sum_i f_i(E,x)\lambda_i$$

Advantages

- **QM operations** (commutation, anticommutation, time evolution) are **trivial and exact**.
- **Number of equations reduced by two.**

$$\begin{aligned}
\frac{dF_\nu(E)}{dx} &= -i[\mathrm{H}, \mathrm{F}_\nu(\mathrm{E})] - \sum_\alpha \frac{1}{2\lambda^\alpha(E)}\{\Pi_\alpha, F_\nu(E)\} \\
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SQuIDS/nu-SQuIDS

Argüelles. et al. [arXiv:**1412.3832, CPC 2015.06.022.**]
Argüelles. et al. [*In preparation*]

What is it?

Is a software framework written in C++ that **evolves quantum mechanical ensembles**. nu-SQuIDS **calculates neutrino propagation** (oscillation+interactions).

What can it do?

- ❑ Calculate neutrino oscillation probabilities in 3 generations (can configure mixing angles, CP phases, and mass splittings).
- ❑ Ready to use in: short baseline, long baseline, atmospheric, and solar neutrino oscillation experiments.
- ❑ Incorporates neutrinos' non-coherent interactions (includes tau regeneration).
- ❑ Can handle collective neutrino interactions (e.g. super nova), as well as neutrino-antineutrino interactions.
- ❑ Easily extendable to BSM physics scenarios. **Sterile neutrinos,** NSI, and LV already implemented!

Get it here:

https://github.com/jsalvado/SQuIDS

https://github.com/arguelles/nuSQuIDS

SQuIDS
Simple Quantum Integro Differential Solver

Take home message

- ★ **Starting** the MC in the **detector vicinity** gives **greater flexibility**.(change neutrino cross section, new neutrino physics, Earth models, etc.)
- ★ **Neutrino propagation** in the Earth can be done through the ***Master* equation**. (Fast+accurate +oscillations included+easy to add new physics)
- ★ ***Master* equation** readily solved with **SQuIDS/nu-SQuIDS**.

SQuIDS
Simple Quantum Integro Differential Solver