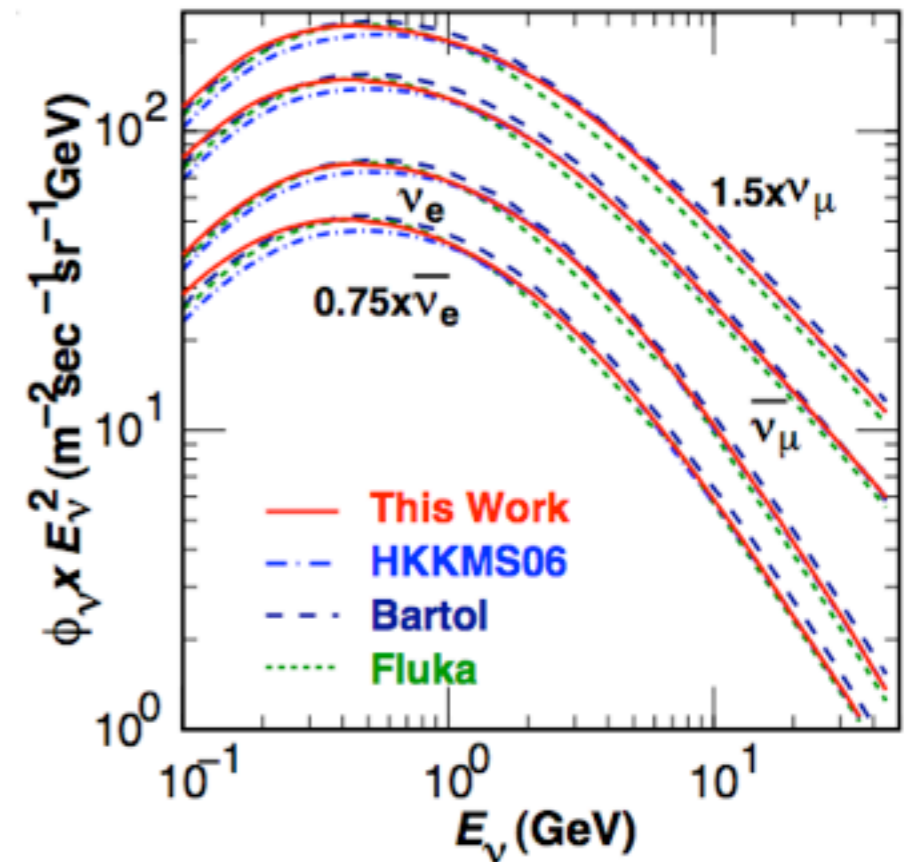
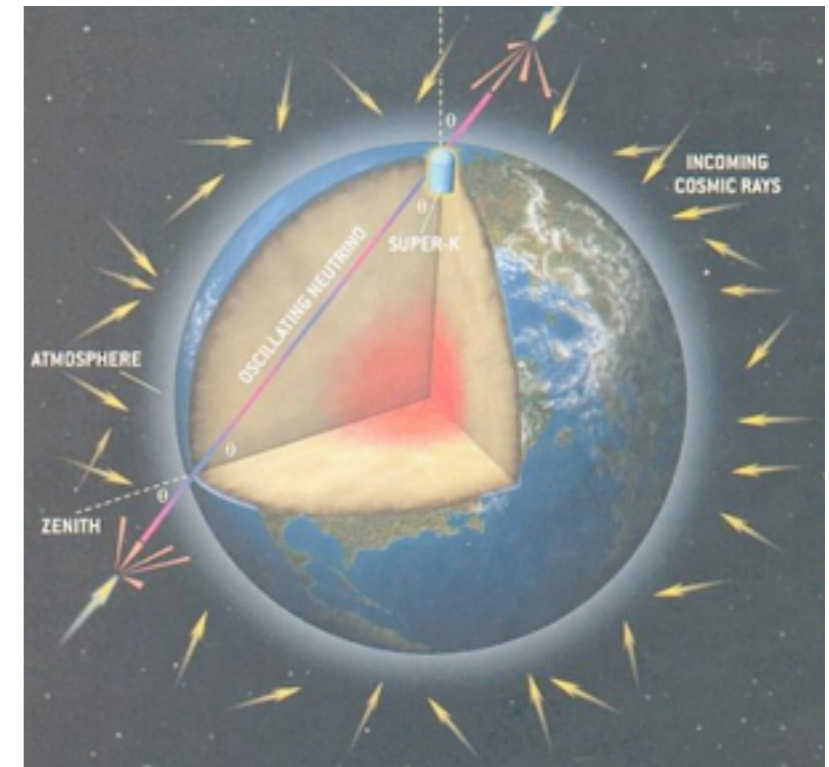


Over-constrain the PMNS Unitarity with HyperK

**Akira Konaka (TRIUMF)
@HK-EU meeting, April 2015**

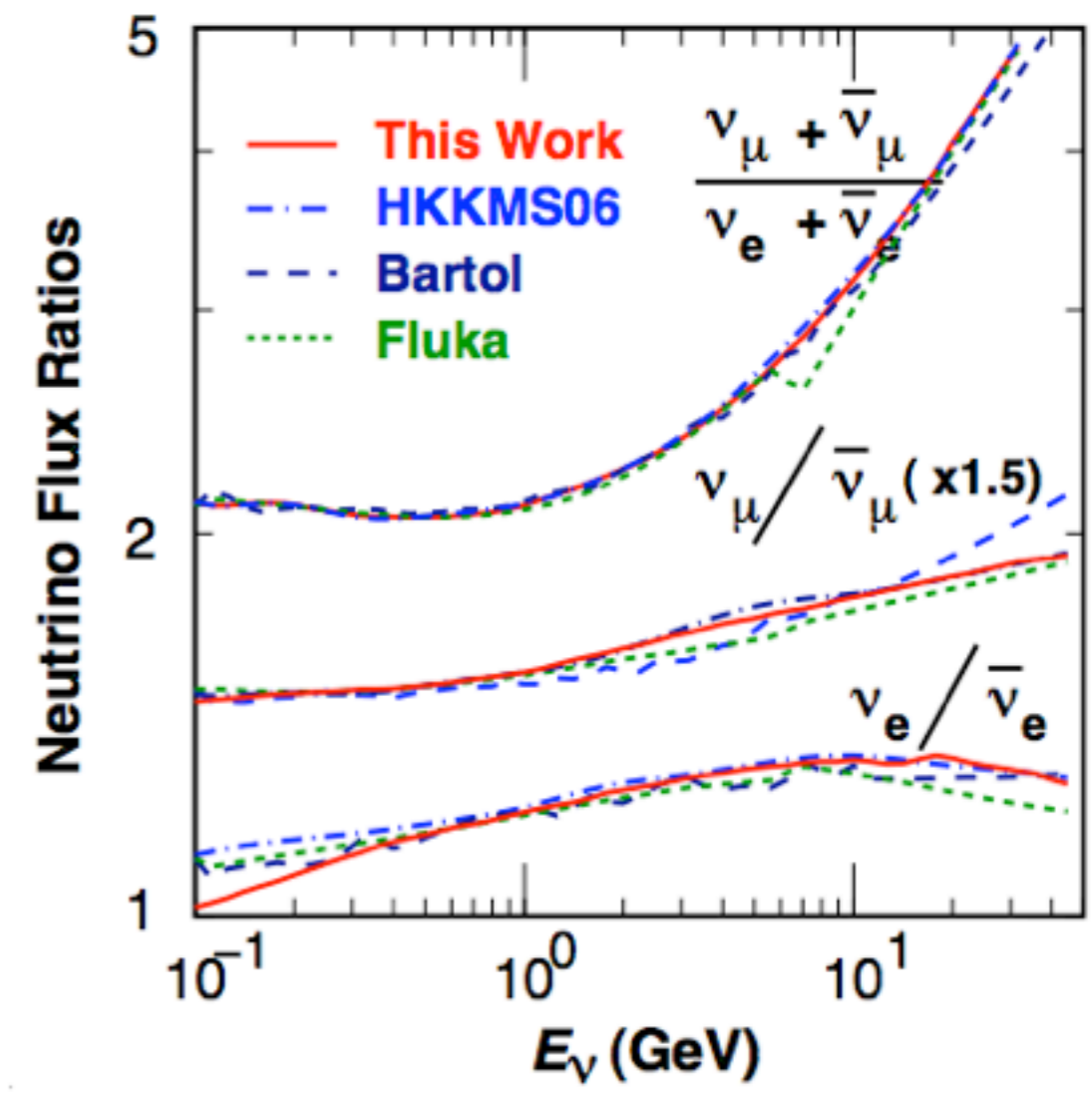
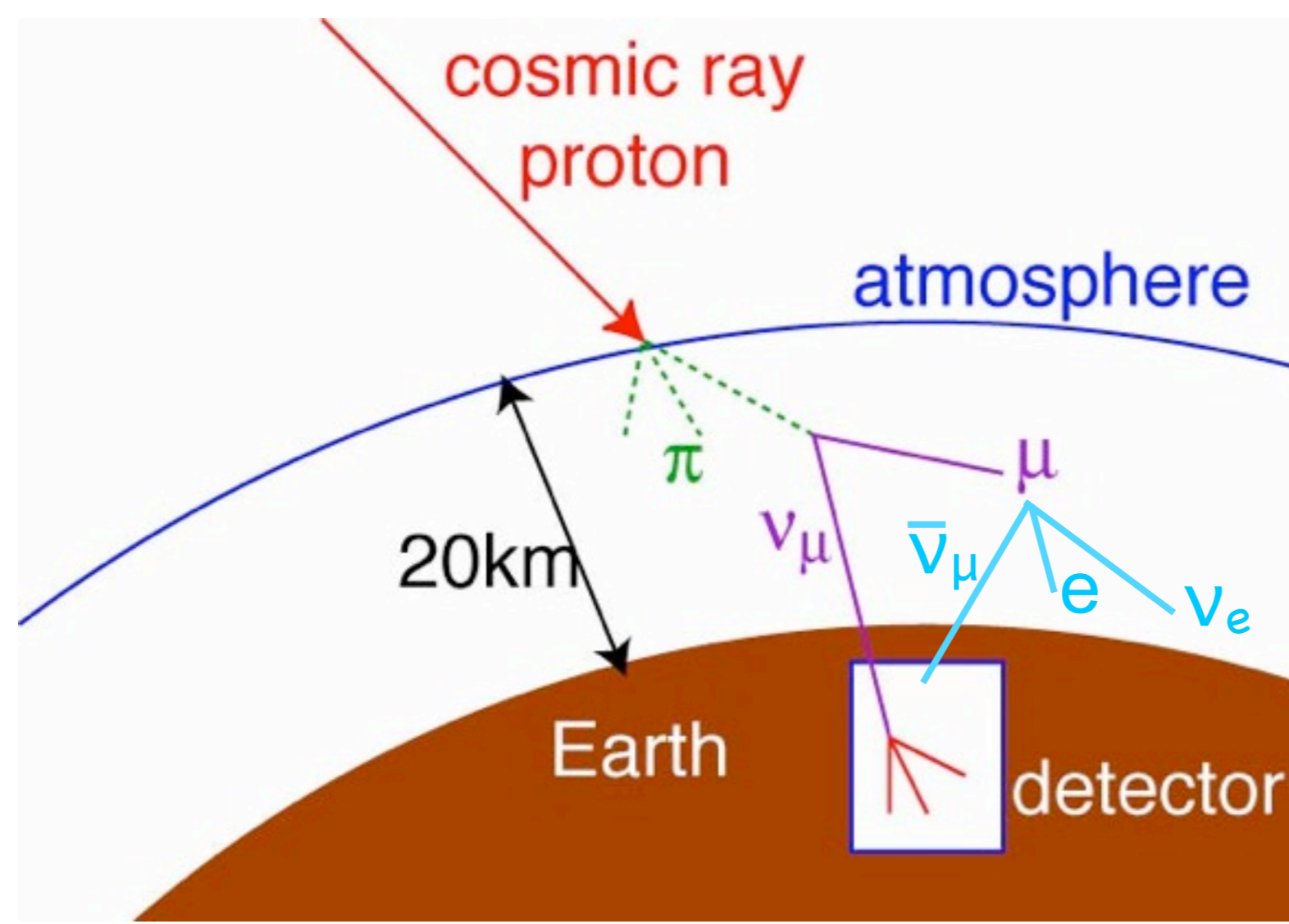
- Atmospheric ν covers very wide range of oscillation parameter space:
 - Baseline length up to 13000km tagged by $\cos\theta_{\text{zenith}}$
 - Different types of neutrinos: ν_{μ} , ν_e and their anti-particles
 - Energy from subGeV to 10GeV up
- Significant statistics:
 - SK 250kton-years:
 - SubGeV: 10k 1Re evts, 10k 1R μ evts
 - Multi-GeV: 2.5k 1Re evts, 2.5k 1R μ evts
- Challenge: Systematics uncertainty
 - flux and cross section



Atmospheric neutrinos

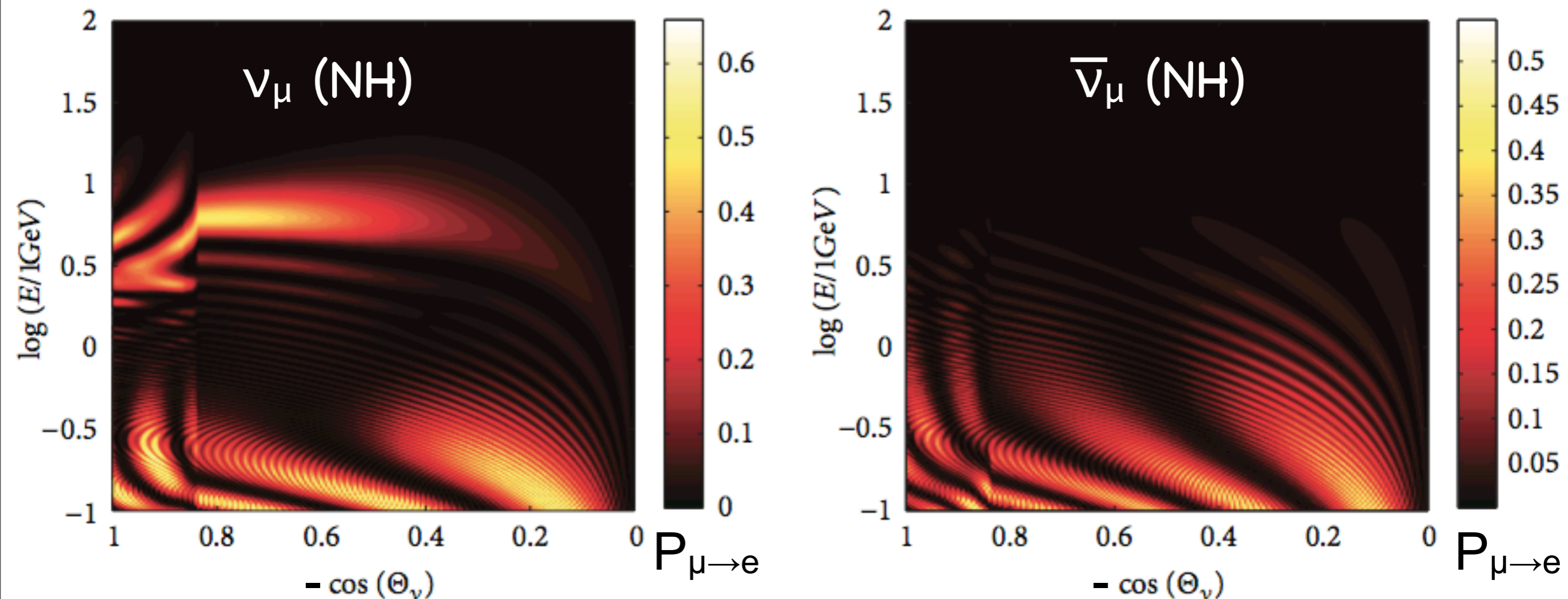
$$r = \Phi(\nu_\mu) / \Phi(\nu_e) \sim 2 \text{ for } E_\nu < 3 \text{ GeV}$$

Produced by cosmic ray proton interacting in earth atmosphere:

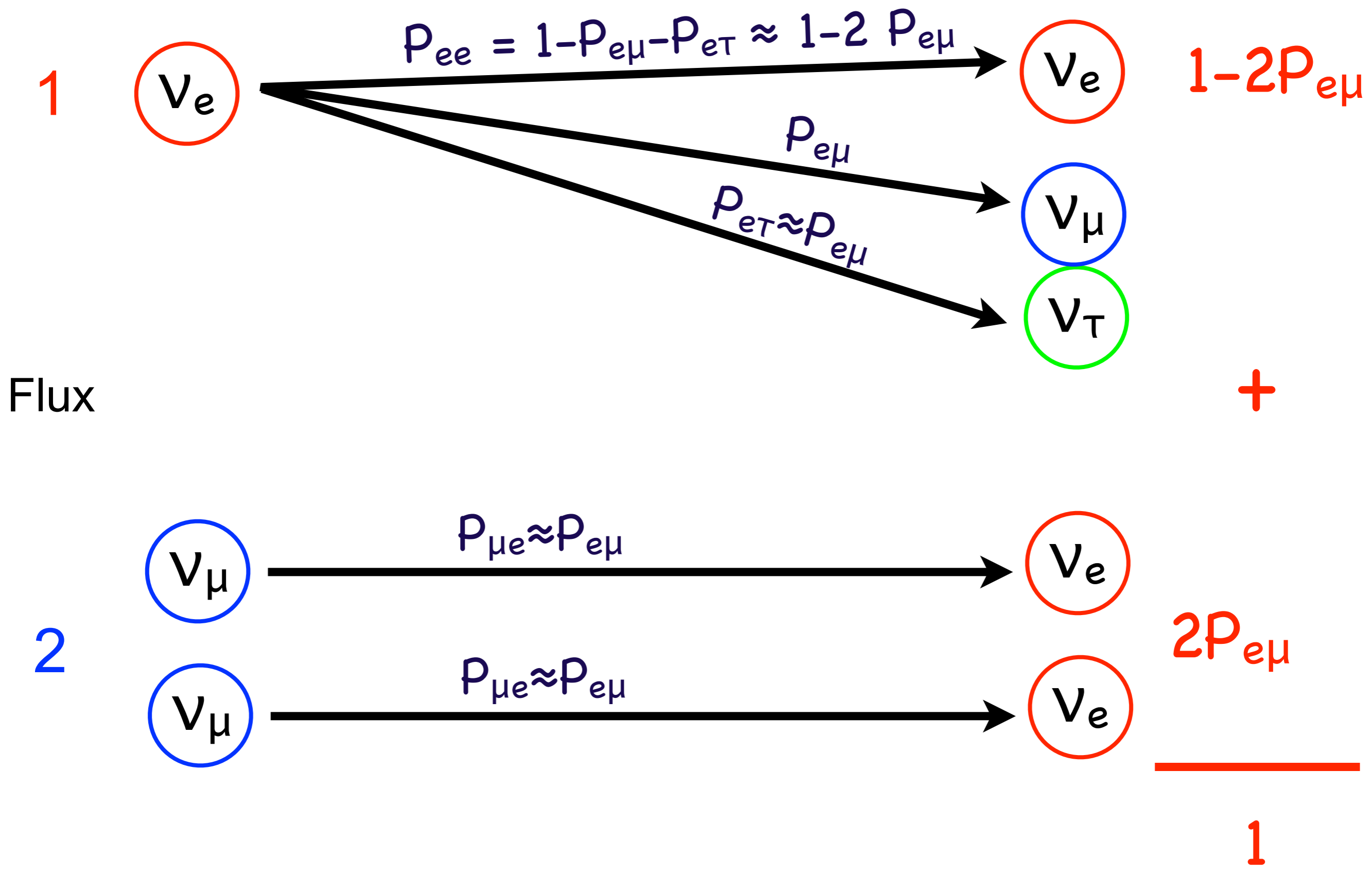


arXiv:1102.2688

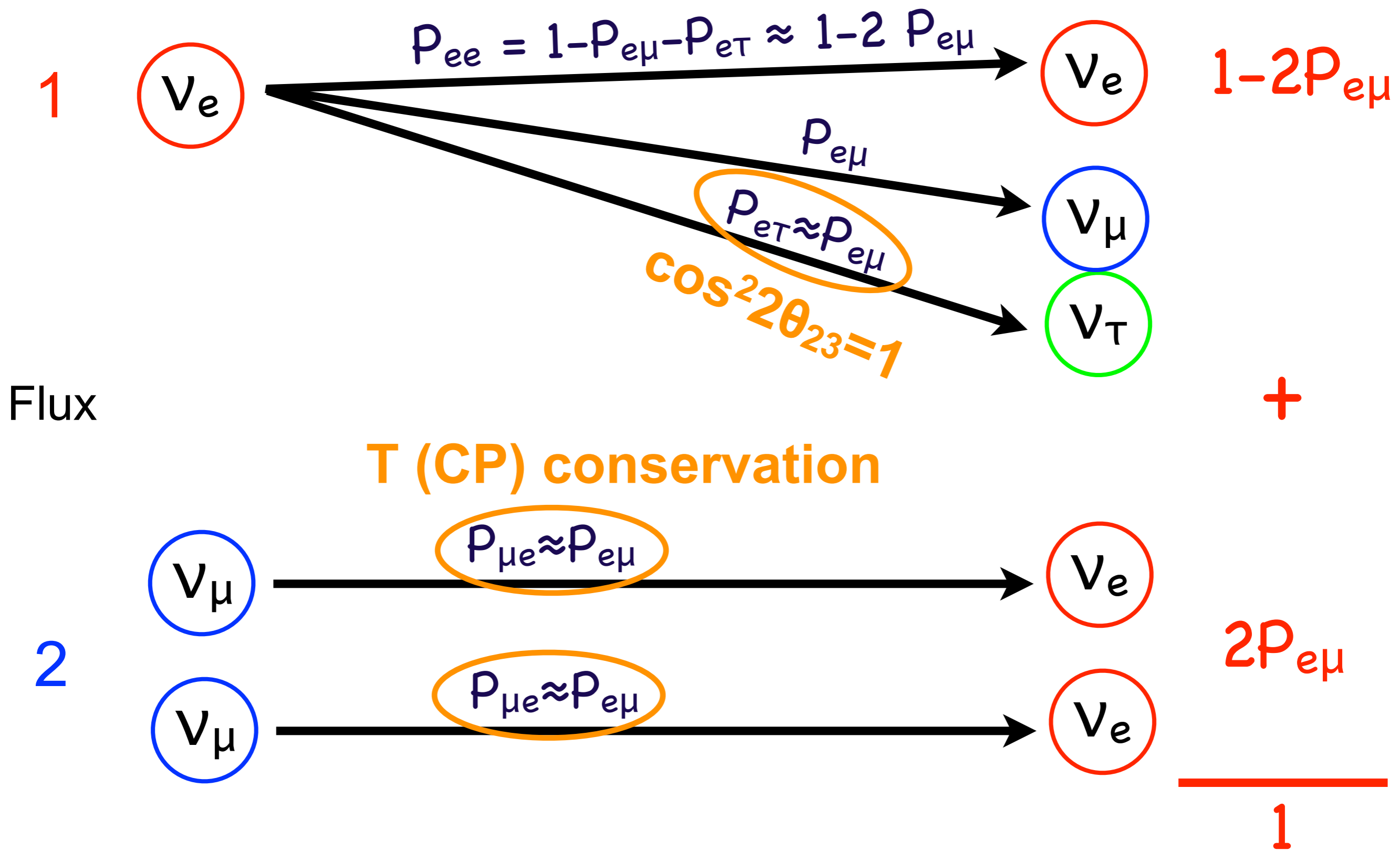
- $\nu_\mu \rightarrow \nu_e$ at several GeV: matter oscillation resonance
 - Resonance in ν_μ for Normal hierarchy: hierarchy determination
- Δ_{12} oscillation in the sub-GeV region: large θ_{12} effect
 - Crossing the earth core at $\cos\theta_\nu \approx -0.85$
 - phase difference between ν and $\bar{\nu}$ due to matter effect



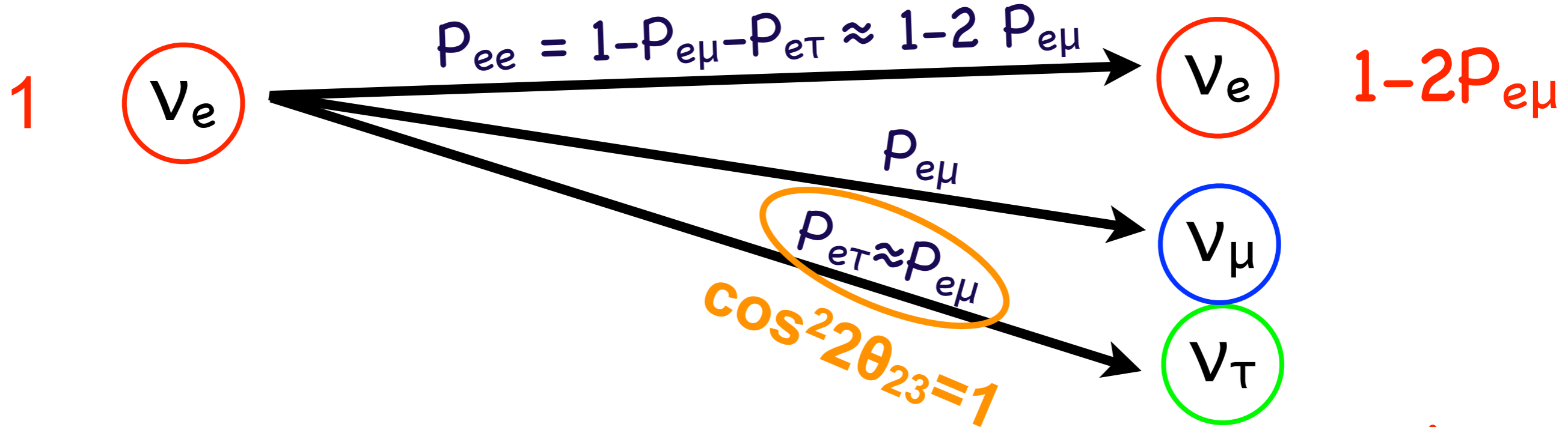
Cancellation of atm. ν_e appearance



atm. ν_e app. sensitive to T(CP) viol.

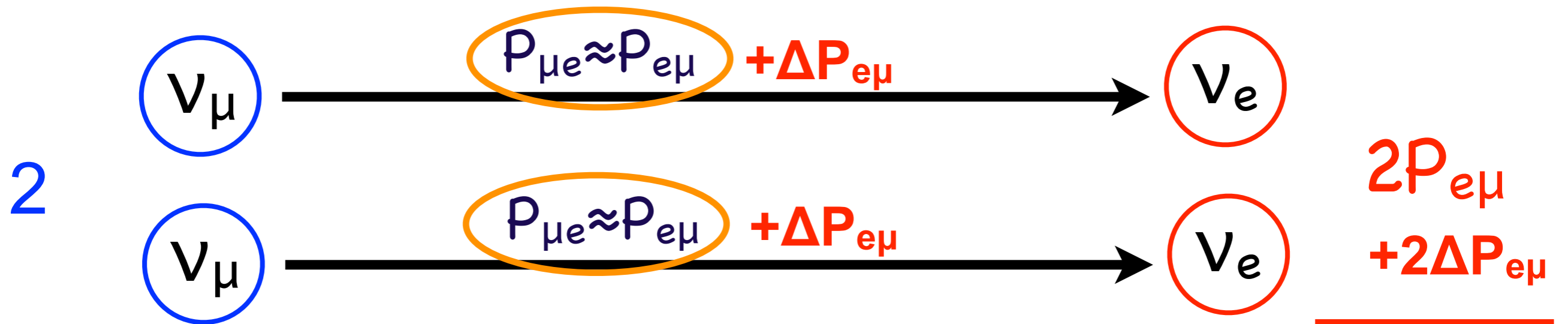


atm. ν_e app. sensitive to T(CP) viol.



Flux

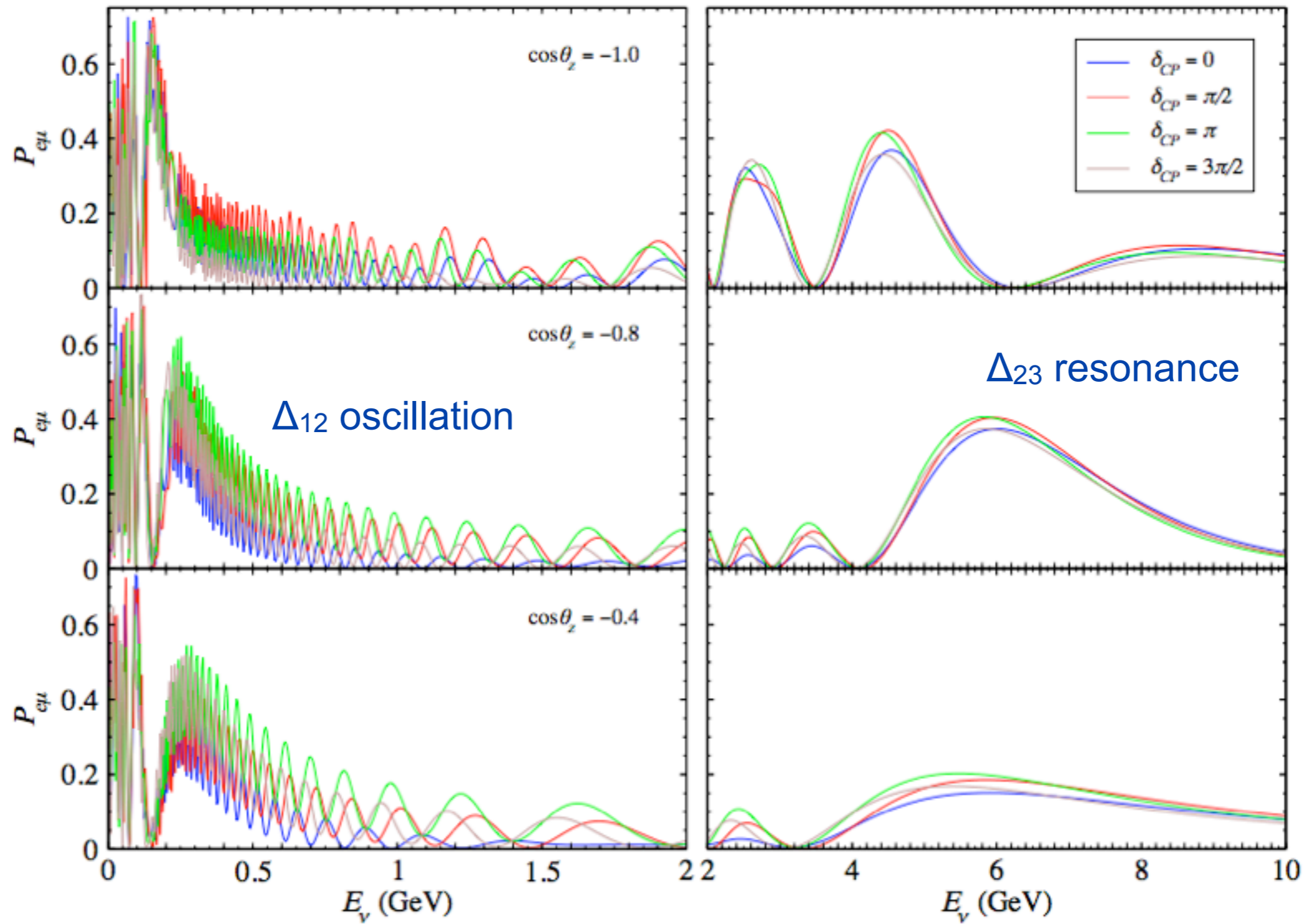
T (CP) conservation



$\Delta P_{e\mu} = P_{\mu e} - P_{e\mu}$

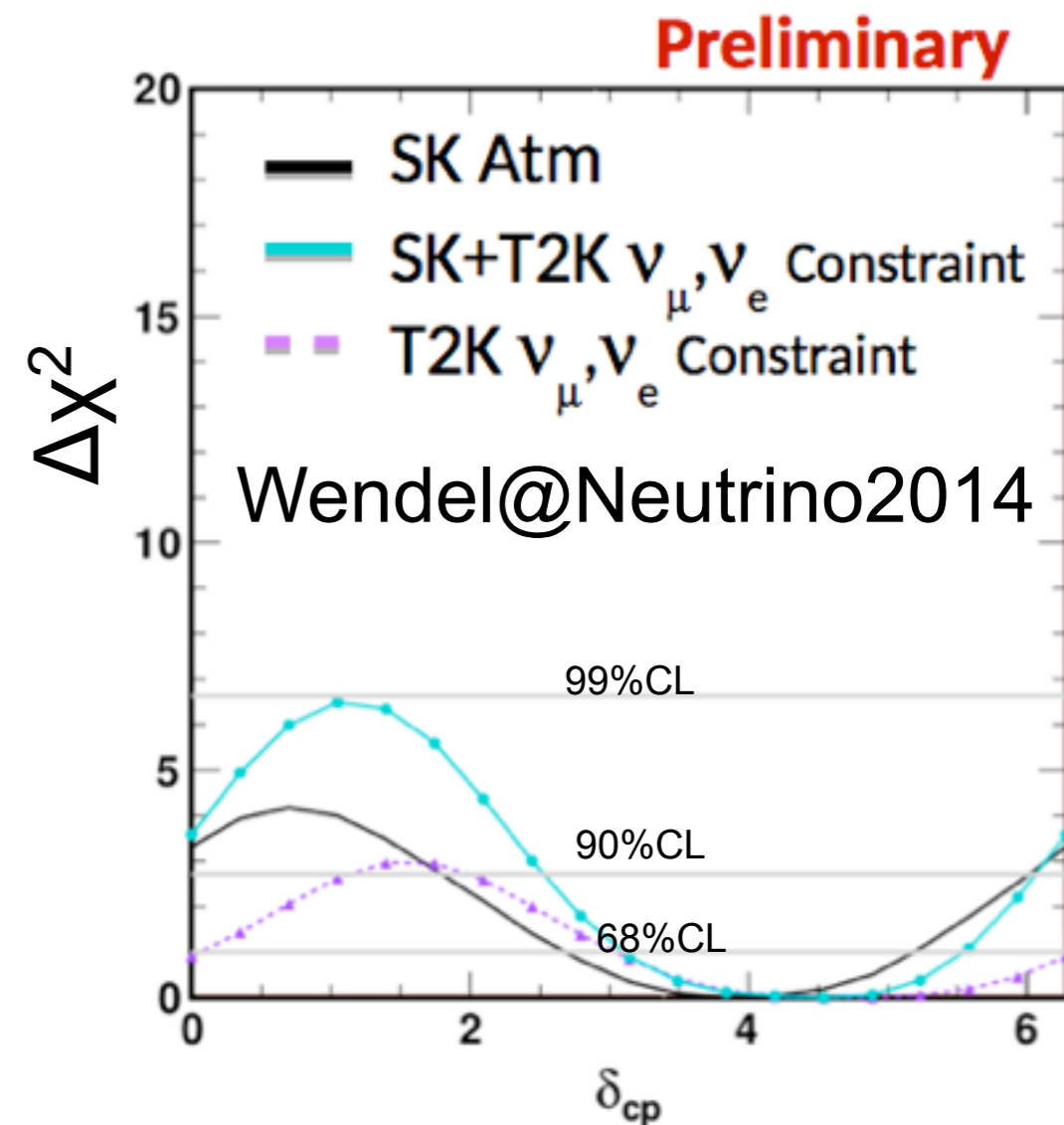
1

 $+ 2\Delta P_{e\mu}$



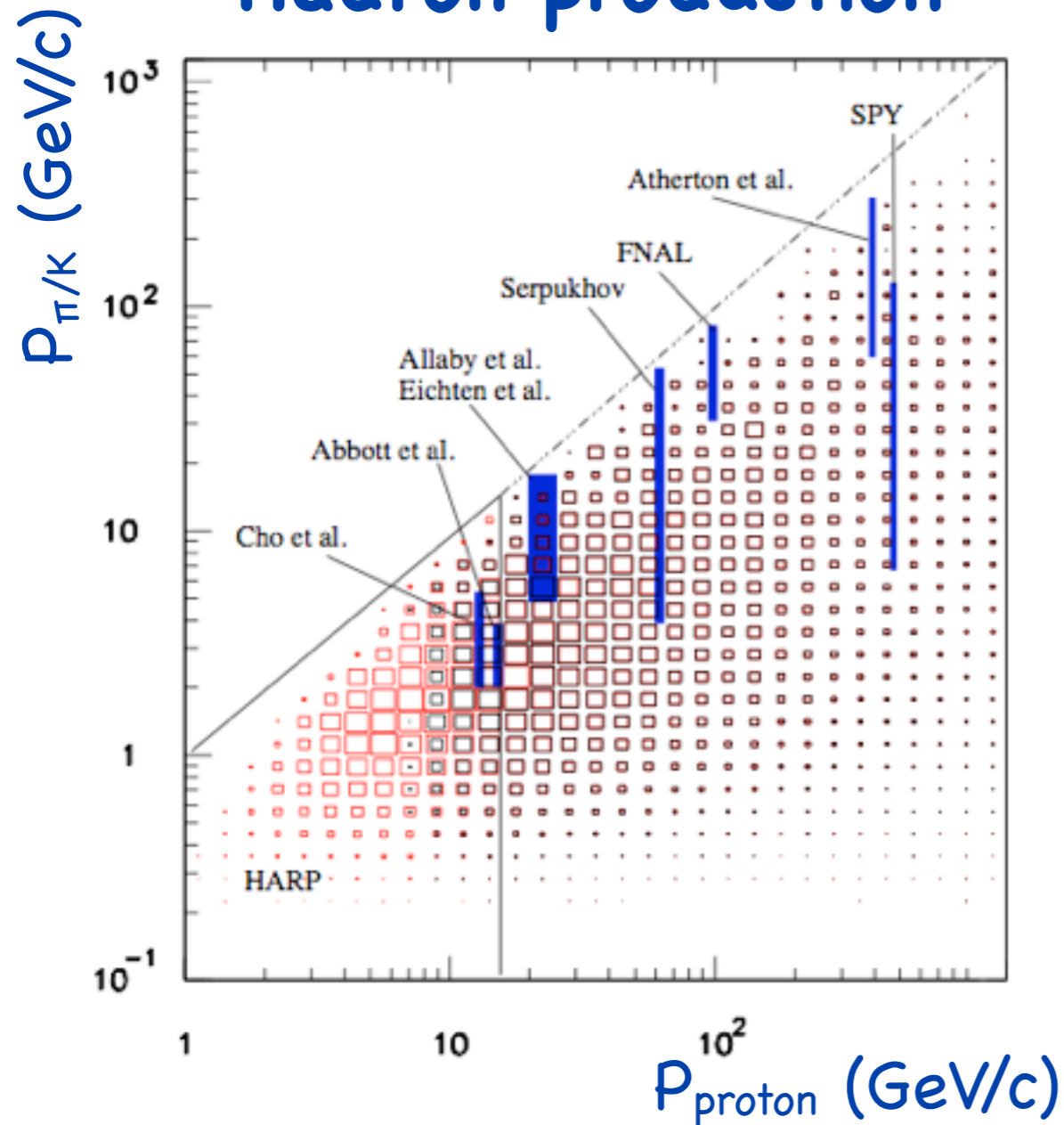
CP violation effect is large! $P_{\mu e} \sim$ up to 20%

- Current SK CP sensitivity (~ 250 kton-years):
 - CP signal size: up to several %
 - Stat. error = 1% [10k subGeV atm. ν_e events @SK]
 - $\Delta\chi^2=3.5$ at $\delta_{cp}=0$ (rejects at 90%CL)
 - Large systematic uncertainties:
- Systematics to be improved:
 - Flux:
 - Cross section:
 - Detector response:

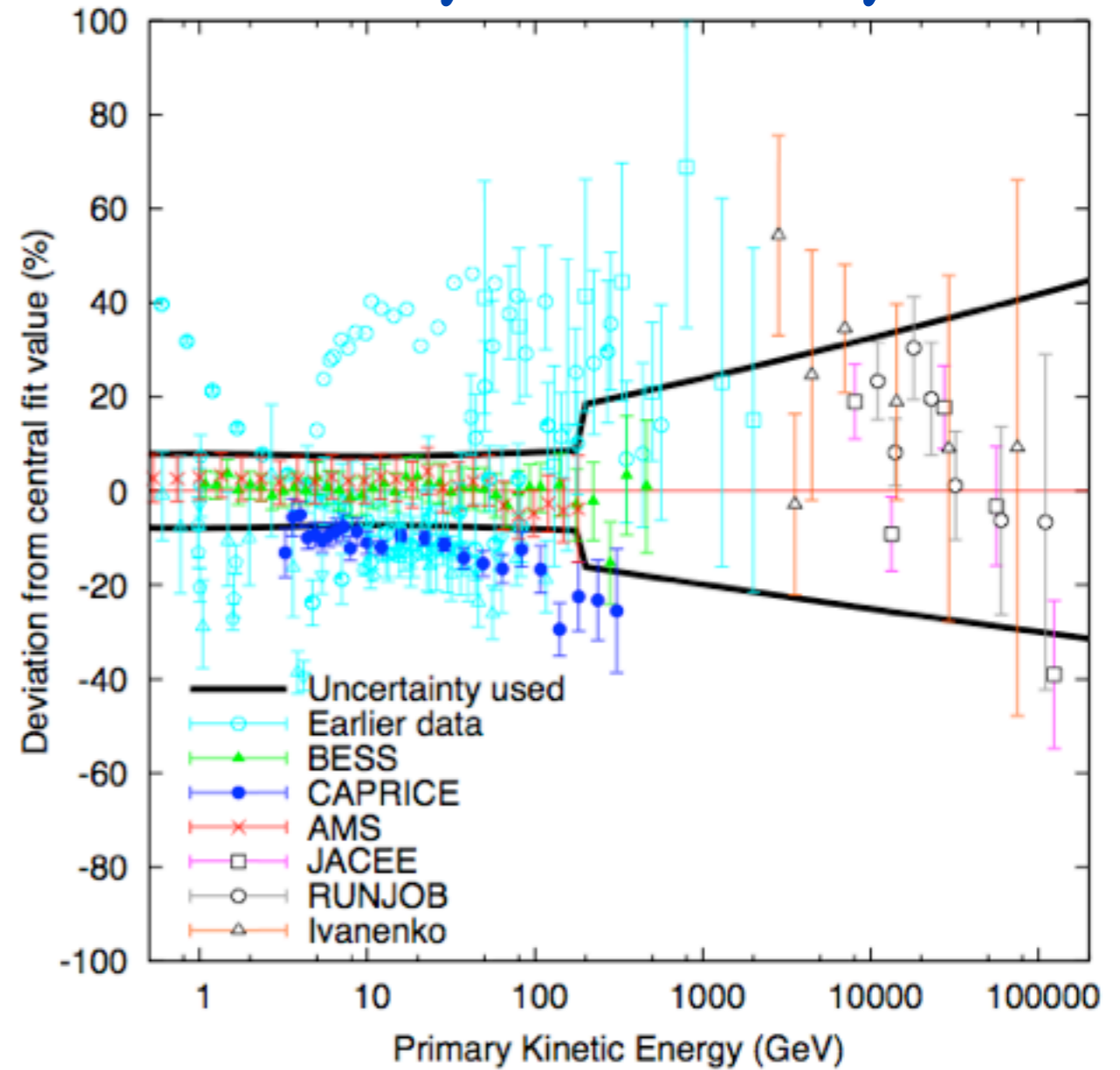


- **Atm. ν offers wide range of L/E in oscillation studies**
 - However, taken by the community with "grain of salt"
 - Good statistics, limited by "unknown" systematics
 - Accelerator ν may also hit such limit (e.g. nuclear effect)
- **Exciting opportunity for precision atmospheric ν :
Study of systematic errors on well controlled data samples**
 - Neutrino flux
 - AMS data provides precise primary cosmic ray flux
 - Limited by hadron production: **NA61** with $E < 15 \text{ GeV}$ capability?
 - Neutrino cross section
 - Model independent cross section study by **nuPRISM**
 - SK detector efficiency
 - **Calibration** to estimate errors based on PMT and water parameters

Hadron production



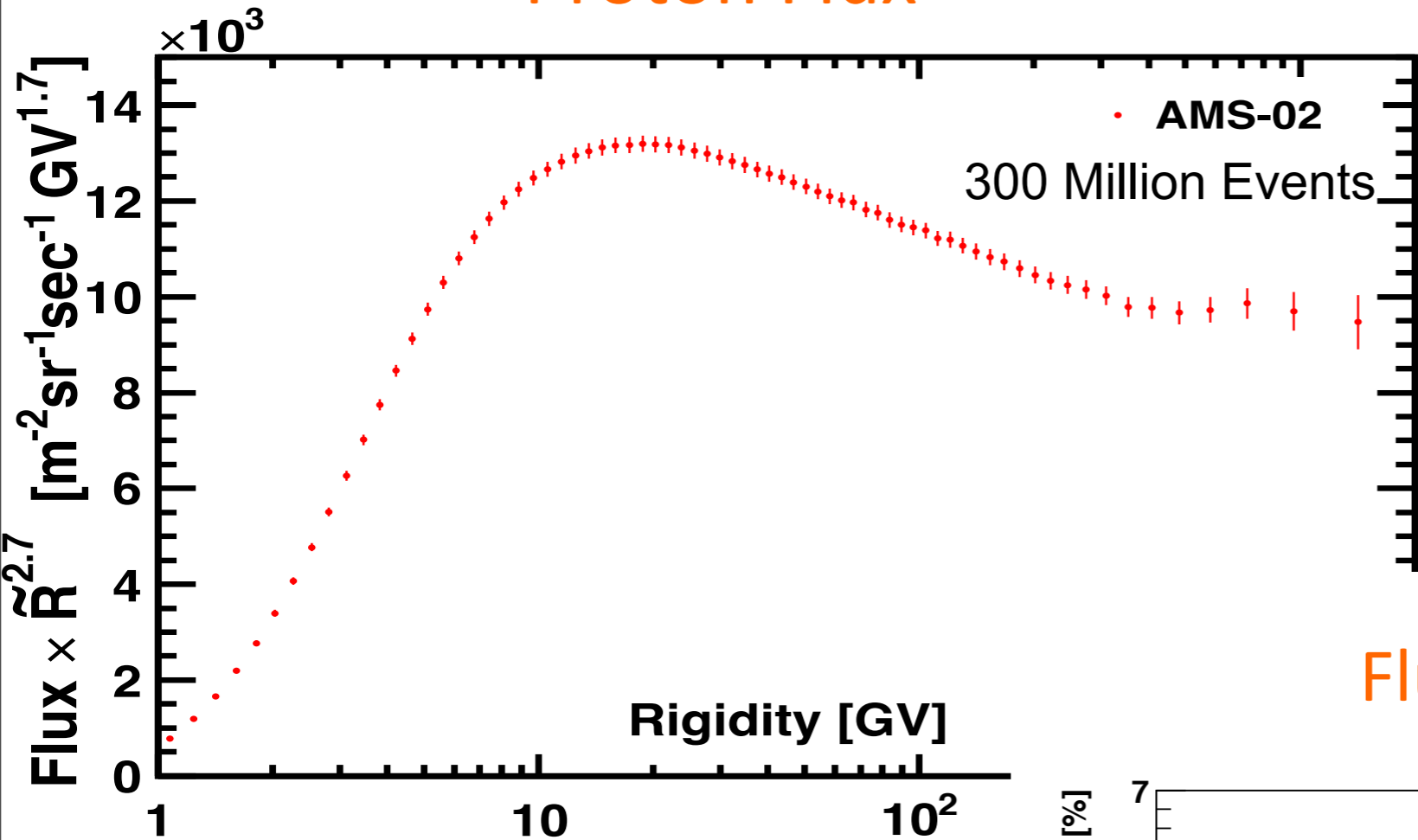
Primary cosmic ray flux



E_i (GeV)	Pions			Kaons		
	0	0.5	1	0	0.5	1
<8	10%	30%		40%		
8-15	30%	10%	30%	40%		
15-30	30	10	5%	30	20	10%
30-500	30	15%		40	30%	
>500	30	15%+Energy dep.		40	30%+Energy dep.	

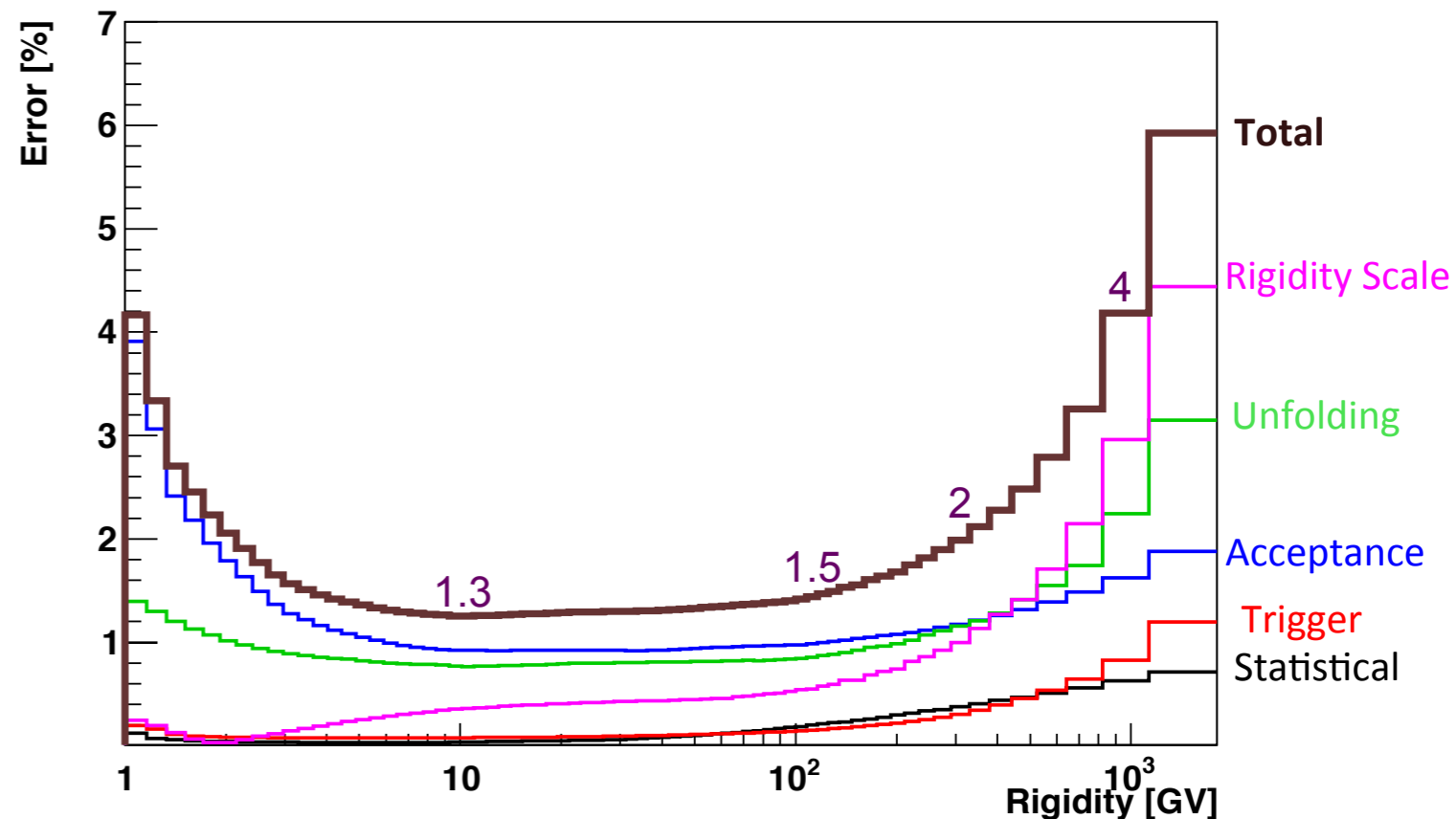
Parameter	Proton fluxes	Nuclear fluxes
a (normalization)	1.49 ± 0.10	0.060 ± 0.004
b	2.15 ± 0.025	1.25 ± 0.03
c	-2.21 ± 0.02	-0.14 ± 0.02
d (index) < 200GeV/n	2.74 ± 0.01	2.64 ± 0.02
> 200GeV/n	2.74 ± 0.03	2.64 ± 0.04

Proton Flux



AMS: April 2015

Flux Errors Breakdown



- Primary cosmic ray flux is now measured at 1-2% level.
- Open the door for very precise atm.v studies

- T2K/NOvA (with reactor θ_{13}) are getting close to observing the leptonic CP violation
 - Could be discovered before HyperK or in the 1st year of HyperK, if δ_{cp} is at the current favored value.
 - Staged (quicker) version of HyperK?
- Multiple experiments are in preparation to determine the neutrino mass hierarchy
 - T2K/NOvA, JUNO, PINGU, INO, ...
- Can we go beyond at HyperK/DUNE (2025-)?
 - Testing the unitarity of PMNS matrix
 - Leptonic version of B-factory
 - Establish unitarity and CP violation from PMNS phase
Or discover new physics (sterile ν , new interaction,...)

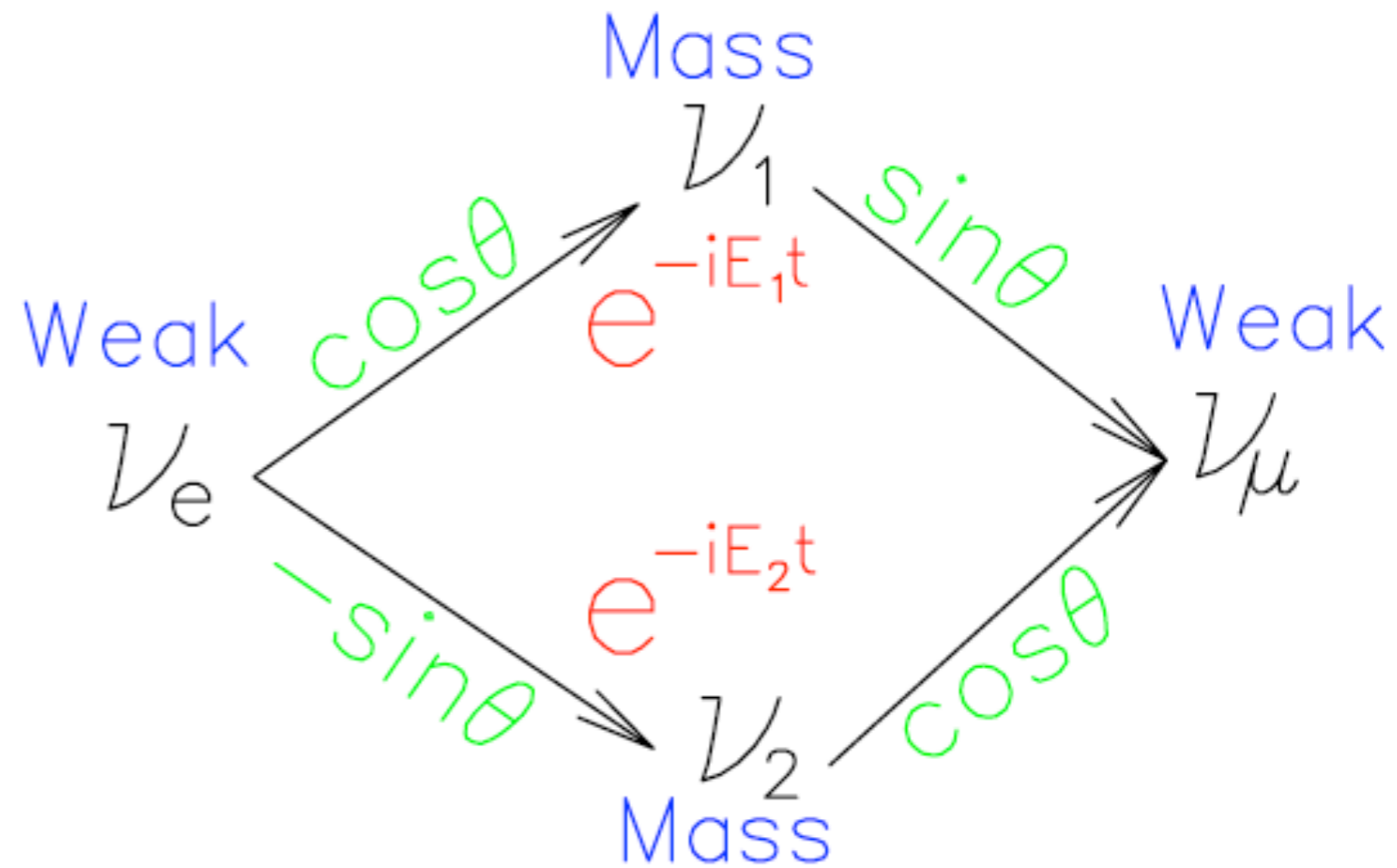
$\langle \nu_l | \nu_i \rangle = U_{li}$: *PMNS matrix* (Unitary matrix)

$l = e, \mu, \tau$: *weak eigenstates*

$i = 1, 2, 3$: *mass eigenstates*

Neutrinos are produced and detected as weak eigenstates and travels as mass eigenstates:

Interference



$$P_{l \rightarrow l'} = |\langle \nu_{l'}(t) | \nu_l(t) \rangle|^2$$

$$= \left| \sum_i \langle \nu_{l'} | \nu_i \rangle e^{-iE_i t} \langle \nu_i | \nu_l \rangle \right|^2$$

$$= \left| \sum_i U_{li}^* U_{l'i} e^{-im_i^2 L/2E} \right|^2 = \sum_{i,j} U_{li}^* U_{l'i} (U_{lj}^* U_{l'j})^* e^{-i(m_i^2 - m_j^2)L/2E}$$

- Unitarity condition:

$$U^\dagger U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Normalization

$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1$$

$$|U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 = 1$$

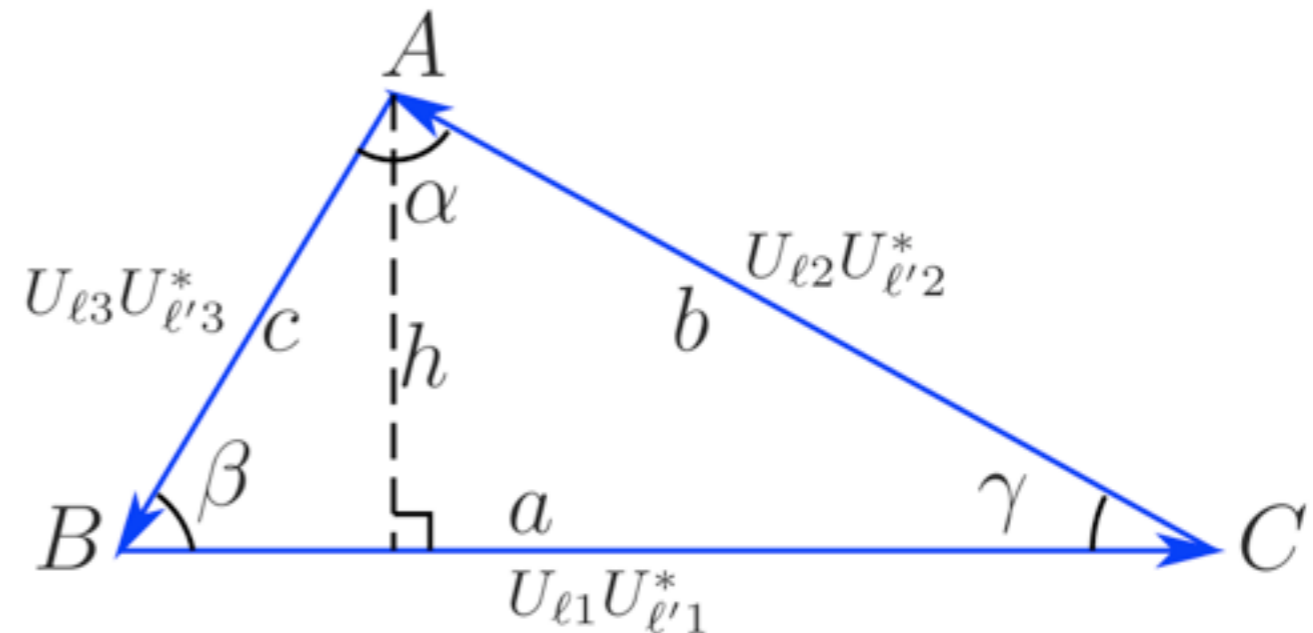
$$|U_{\tau 1}|^2 + |U_{\tau 2}|^2 + |U_{\tau 3}|^2 = 1$$

- Unitarity triangle

$$U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$$

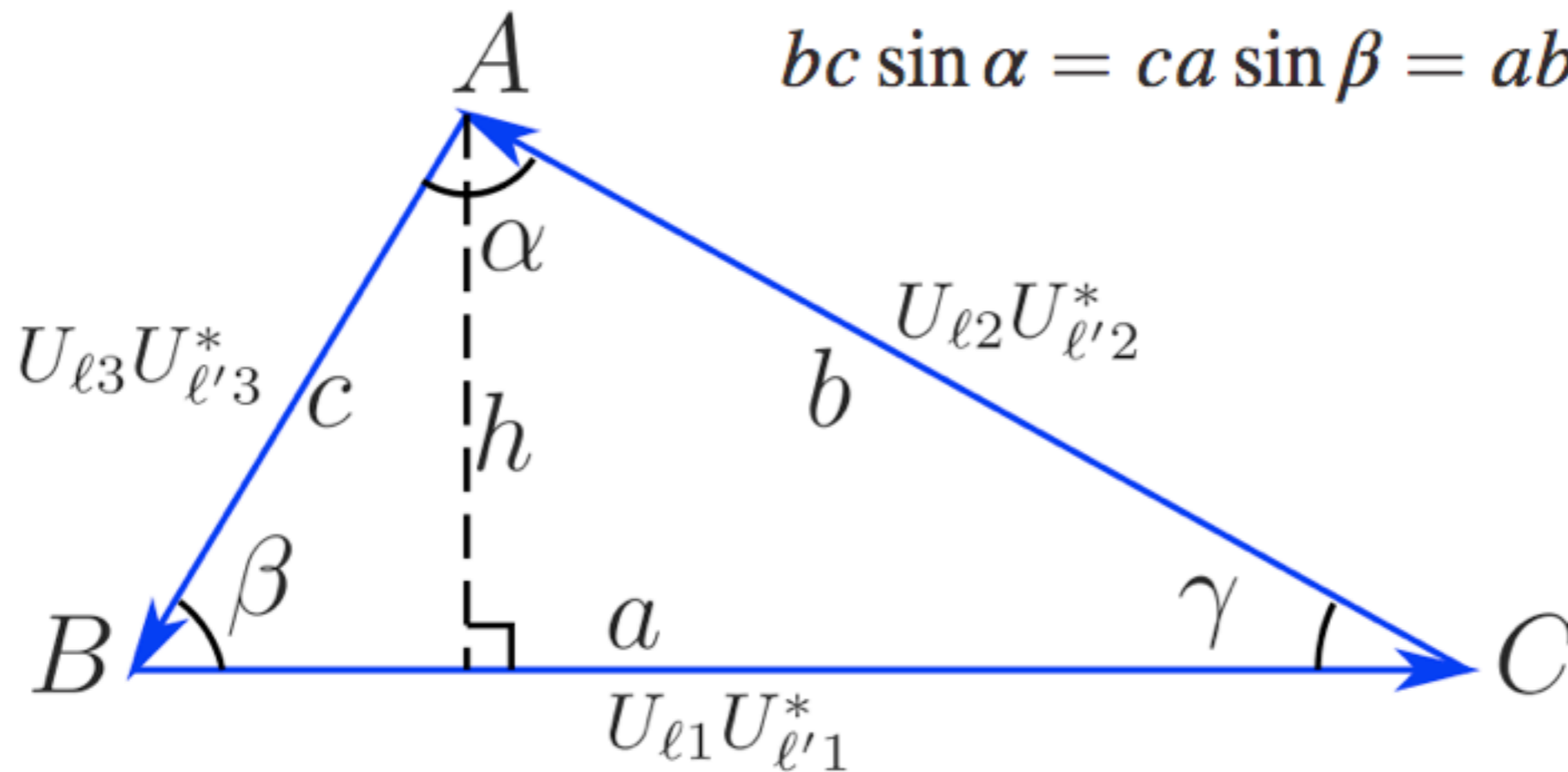
$$U_{\mu 1}U_{\tau 1}^* + U_{\mu 2}U_{\tau 2}^* + U_{\mu 3}U_{\tau 3}^* = 0$$

$$U_{\tau 1}U_{e1}^* + U_{\tau 2}U_{e2}^* + U_{\tau 3}U_{e3}^* = 0$$



- New physics can break the Unitarity of PMNS

- sterile neutrino or new interactions



$$a = |U_{\ell 1}| |U_{\ell' 1}|$$

$$b = |U_{\ell 2}| |U_{\ell' 2}|$$

$$c = |U_{\ell 3}| |U_{\ell' 3}|$$

$$\Delta_{12} = \Delta m_{12}^2 L / 4E$$

$$\Delta_{23} = \Delta m_{23}^2 L / 4E$$

$$\Delta_{31} = \Delta m_{31}^2 L / 4E$$

Appearance

$$\begin{aligned}
 P_{\ell \rightarrow \ell'} = & 4ab \sin(\Delta_{12} \pm \gamma) \sin \Delta_{12} \\
 & + 4bc \sin(\Delta_{23} \pm \alpha) \sin \Delta_{23} \\
 & + 4ac \sin(\Delta_{31} \pm \beta) \sin \Delta_{31}
 \end{aligned}$$

Each of the CP angles (α, β, γ) correspond to each L/E oscillation length; $\Delta_{23}, \Delta_{31}, \Delta_{12}$

- Disappearance tests normalization part of unitarity and the sides of the unitarity triangle:

$$\begin{aligned}
 P_{\text{disapp}} &= 1 - P_{\ell \rightarrow \ell} \\
 &= 4|U_{\ell 1}|^2 |U_{\ell 2}|^2 \sin^2 \Delta_{12} \\
 &\quad + 4|U_{\ell 2}|^2 |U_{\ell 3}|^2 \sin^2 \Delta_{23} \\
 &\quad + 4|U_{\ell 3}|^2 |U_{\ell 1}|^2 \sin^2 \Delta_{31}
 \end{aligned}$$

$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1$$

$$|U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 = 1$$

$$|U_{\tau 1}|^2 + |U_{\tau 2}|^2 + |U_{\tau 3}|^2 = 1$$

$$a = |U_{\ell 1}| |U_{\ell' 1}|$$

$$b = |U_{\ell 2}| |U_{\ell' 2}|$$

$$c = |U_{\ell 3}| |U_{\ell' 3}|$$

- Different L/E scale to untangle each contributions
 - solar (Δ_{12}) and atmospheric (Δ_{23}, Δ_{31}) scales
 - At 1st oscillation max. $\Delta_{23} \sim \Delta_{13} \sim 0.03 \Delta_{12}$:
 - Higher osc. max and matter effect provide additional handles

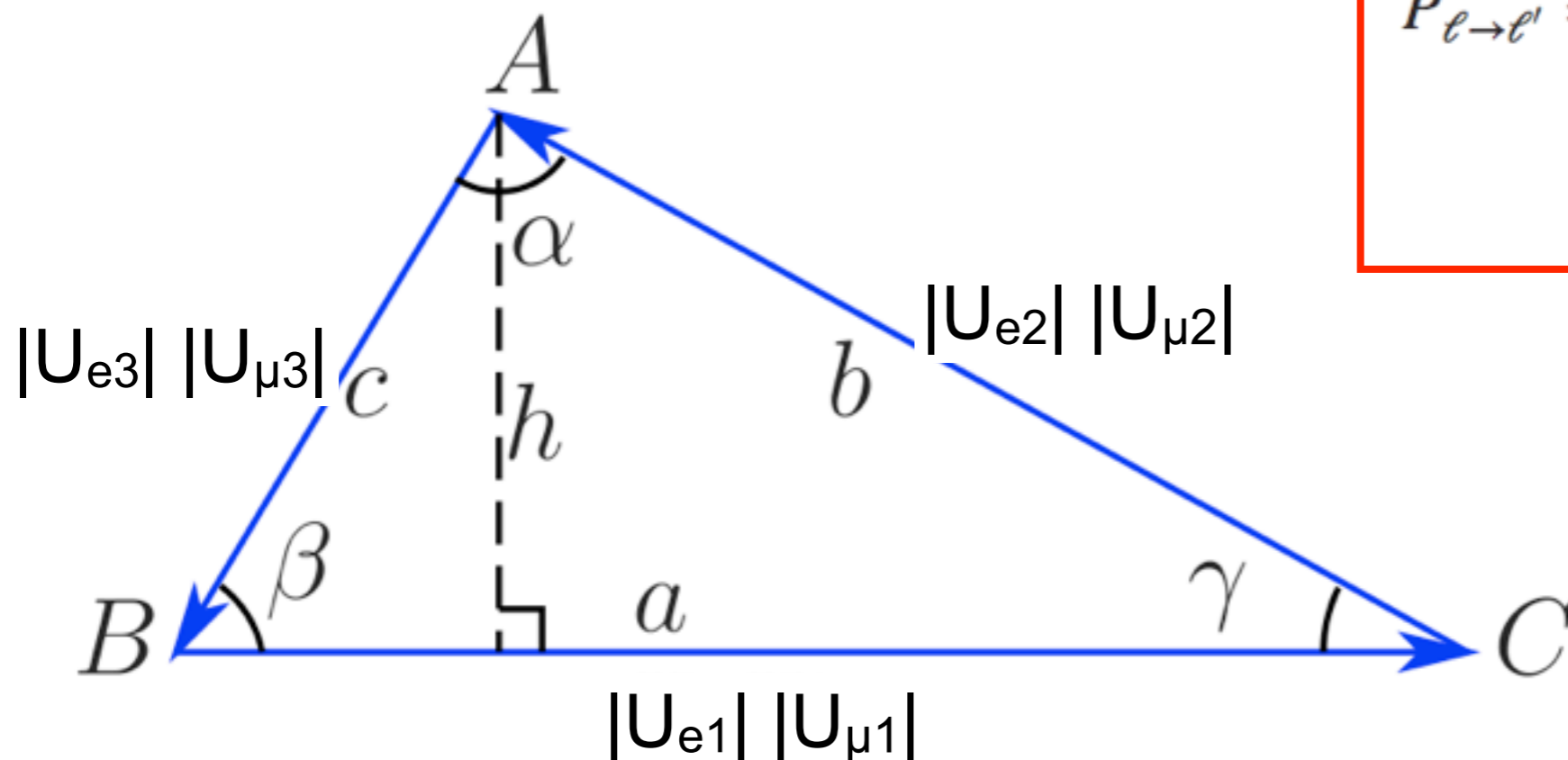
- e- μ part of unitarity can be studied best:

$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1$$

$$|U_{\mu1}|^2 + |U_{\mu2}|^2 + |U_{\mu3}|^2 = 1$$

Normalization terms and sides of the triangle can be determined by disappearance

$$U_{e1}U_{\mu1}^* + U_{e2}U_{\mu2}^* + U_{e3}U_{\mu3}^* = 0$$



$$P_{e \rightarrow e'} = 4ab \sin(\Delta_{12} \pm \gamma) \sin \Delta_{12} + 4bc \sin(\Delta_{23} \pm \alpha) \sin \Delta_{23} + 4ac \sin(\Delta_{31} \pm \beta) \sin \Delta_{31}$$

$$\Delta_{13} : \beta \sim \delta_{cp}$$

$$\Delta_{12} : \gamma \sim \delta_{\text{subGeV-atm}}$$

$$\Delta_{23} : \alpha \sim \delta_{\text{nth-max}}$$

$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1$$

- Solar neutrino: $|U_{e2}|^2$
 - MSW conversion in the sun: $\nu_e \rightarrow \nu_2$ and detect ν_e
- KamLand: $|U_{e1}|^2 |U_{e2}|^2$
 - Reactor ν_e disappearance at Δ_{12} scale of L/E
- Reactor θ_{13} : $|U_{e3}|^2 (|U_{e1}|^2 + |U_{e2}|^2)$
 - Reactor ν_e disappearance at Δ_{13} scale of L/E
- All the parameters are measured:
 - Better precision for $|U_{e1}|^2$ and $|U_{e2}|^2$ expected by JUNO, untangling Δ_{23} and Δ_{31} contributions

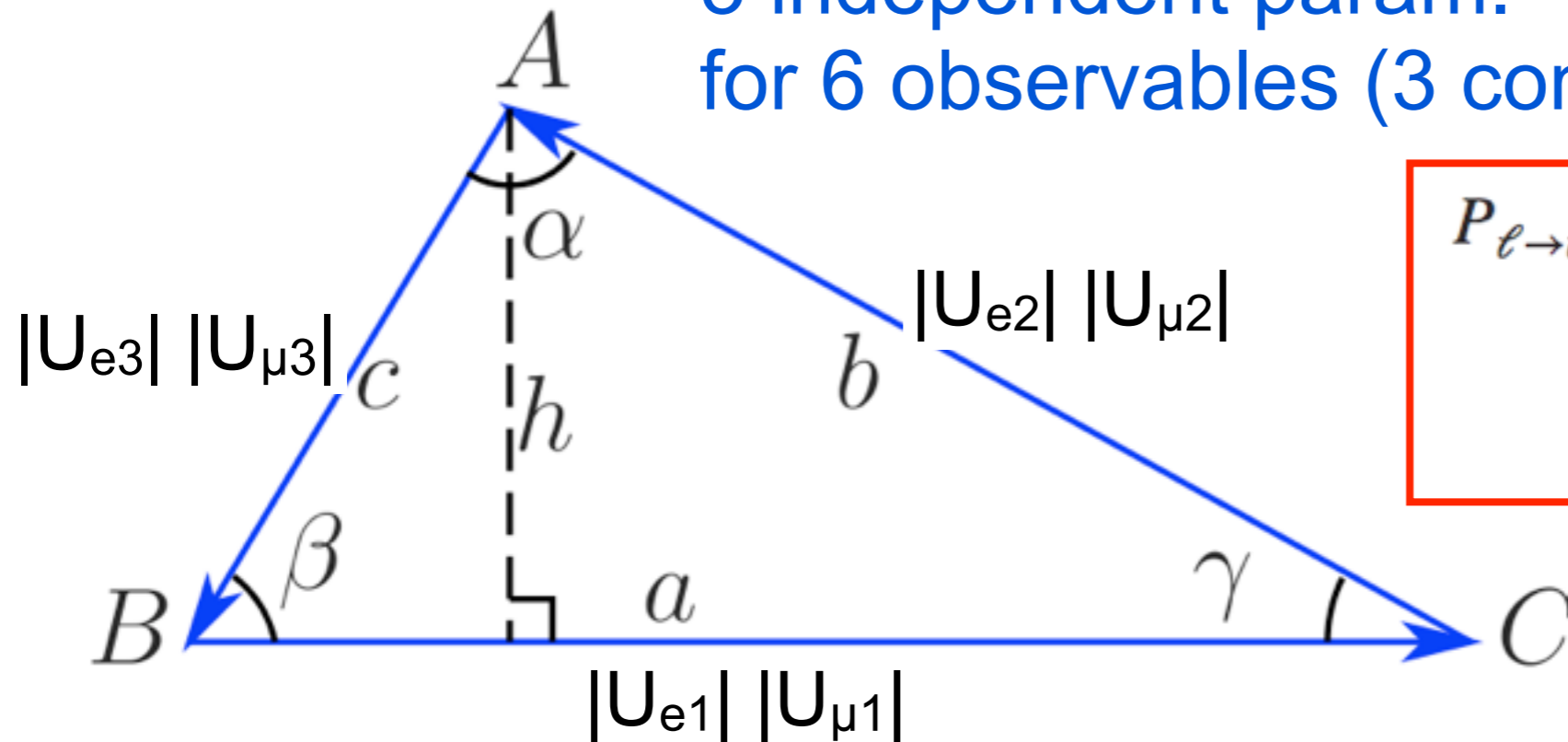
$$|U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 = 1$$

- LBL ν_{μ} disappearance: $|U_{\mu 3}|^2 (|U_{\mu 1}|^2 + |U_{\mu 2}|^2)$
 - T2K/MINOS/NOvA measure this.
- $\nu_{\mu} \rightarrow \nu_e$ atm. matter resonance (3–8 GeV): $(r|U_{\mu 3}|^2 - 1)$
 - Amplitude of the atm. ν mass hierarchy study [HK]
- Solar scale atm. ν_{μ} disapp. : $|U_{\mu 1}|^2 |U_{\mu 2}|^2$
 - 0.4–0.8 GeV up-going atm. ν_{μ} disapp. [HK]
 - Precise zenith angle needed: $\bar{\nu}_{\mu}$ (with neutron tag)
- 1–3 GeV atm. ν_{μ} disapp.: $|U_{\mu 1}|^2 / |U_{\mu 2}|^2$
 - 4–6th oscl. max: 20–30% phase shift btw Δ_{13} & Δ_{23} [HK]
- Good potential to untangle all three parameters:
HyperK will play a major role

$$U_{e1}U_{\mu1}^* + U_{e2}U_{\mu2}^* + U_{e3}U_{\mu3}^* = 0$$

3 independent param.

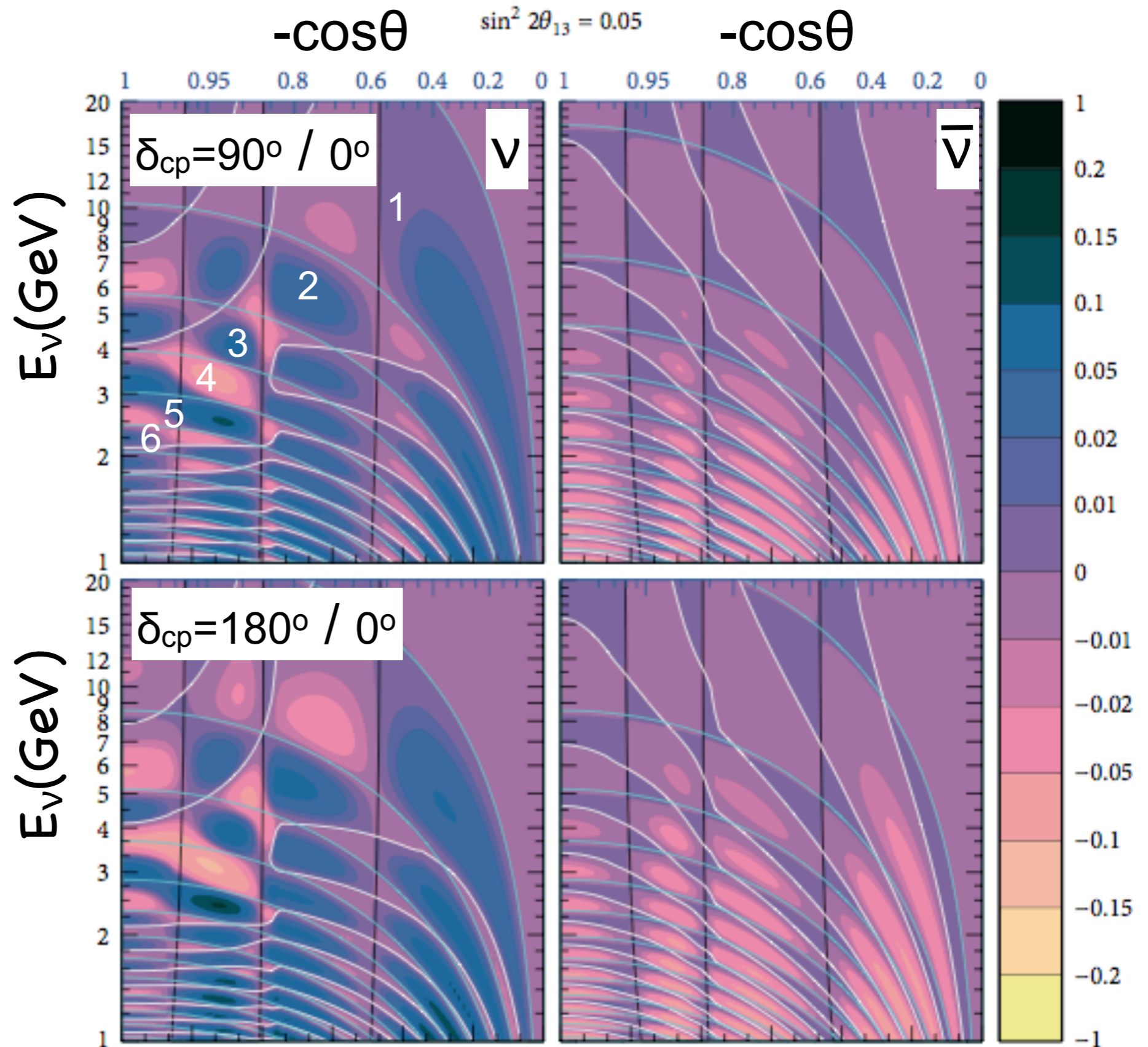
for 6 observables (3 constraints)



$$P_{\ell \rightarrow \ell'} = 4ab \sin(\Delta_{12} \pm \gamma) \sin \Delta_{12} \\ + 4bc \sin(\Delta_{23} \pm \alpha) \sin \Delta_{23} \\ + 4ac \sin(\Delta_{31} \pm \beta) \sin \Delta_{31}$$

- a, b, c can be determined by the disappearance
 - amplitude of $\nu_{\mu} \rightarrow \nu_e$ appearance also constrains $a+b$
- α, β from LBL $\nu_{\mu} \rightarrow \nu_e$ appearance, γ from atm. $\nu_{\mu} \rightarrow \nu_e$
 - T2K/T2HK/DUNE will determine combination of α and β [HK]
 - γ can be determined by the 0.5–0.8 GeV atm. $\nu_{\mu} \rightarrow \nu_e$ [HK]
 - Atm. $\nu_{\mu} \rightarrow \nu_e$ (2nd to 10th max.) untangles $\beta(\Delta_{31})$ and $\alpha(\Delta_{23})$ [HK]

CP effect:
 $E \sim 2-5 \text{ GeV}$
 $\cos\theta \sim 0.9$
 (1st-10th
 Δ_{23} max)



- **Atm. ν covers wide range of oscillation parameters**
 - Realistic scenario emerges to control systematic uncertainties:
 - Precise AMS cosmic ray measurement
 - Precision hadron production study established by NA61: need to be extended also below 15GeV by beamline modification
 - nuPRISM proposal for model independent ν cross section study
- **Precision test of e - μ PMNS Unitarity**
 - 9 parameters with 5 constraints to be tested
 - $|U_{e1}|^2, |U_{e2}|^2, |U_{e3}|^2, |U_{\mu1}|^2, |U_{\mu2}|^2, |U_{\mu3}|^2, \alpha, \beta, \gamma$
 - Unitarity triangle (3 constraints), normalization (2)
 - All 9 parameters can be measured:
 - Typical expected accuracy would be 10-20%
 - HyperK will lead the new measurements: $|U_{\mu i}|^2, \alpha, \beta, \gamma$ (JUNO will improve $|U_{e i}|^2$)
 - HyperK will play leading role in establishing PMNS paradigm, similar to B-factory in CKM

Unitarity triangle

$$\begin{aligned}
 P_{l \rightarrow l'} &= |\langle \nu_{l'}(t) | \nu_l(t) \rangle|^2 \\
 &= \left| \sum_i \langle \nu_{l'} | \nu_i \rangle e^{-iE_i t} \langle \nu_i | \nu_l \rangle \right|^2 \\
 &= \left| \sum_i U_{li}^* U_{l'i} e^{-im_i^2 L/2E} \right|^2 = \sum_{i,j} U_{li}^* U_{l'i} (U_{lj}^* U_{l'j})^* e^{-i(m_i^2 - m_j^2)L/2E}
 \end{aligned}$$

Calculation is simpler without Unitarity

$$\begin{aligned}
 &= |U_{l1}^* U_{l'1}|^2 + |U_{l2}^* U_{l'2}|^2 + |U_{l3}^* U_{l'3}|^2 \\
 &+ 2\text{Re}[U_{l1}^* U_{l'1} [U_{l2}^* U_{l'2}]^* e^{-i(m_1^2 - m_2^2)L/2E}] \\
 &+ 2\text{Re}[U_{l2}^* U_{l'2} [U_{l3}^* U_{l'3}]^* e^{-i(m_2^2 - m_3^2)L/2E}] \\
 &+ 2\text{Re}[U_{l3}^* U_{l'3} [U_{l1}^* U_{l'1}]^* e^{-i(m_3^2 - m_1^2)L/2E}] \\
 &= a^2 + b^2 + c^2
 \end{aligned}$$

$$\Delta_{12} = \Delta m_{12}^2 L/4E$$

$$\Delta_{23} = \Delta m_{23}^2 L/4E$$

$$\Delta_{31} = \Delta m_{31}^2 L/4E$$

$$\begin{aligned}
 &-2ab \cos(2\Delta_{12} + \gamma) \\
 &-2bc \cos(2\Delta_{12} + \alpha) \\
 &-2ca \cos(2\Delta_{12} + \beta) \\
 &= 4ab \sin(\Delta_{12} + \gamma) \sin \Delta_{12} \\
 &+ 4ab \sin(\Delta_{23} + \alpha) \sin \Delta_{23} \\
 &+ 4ab \sin(\Delta_{31} + \beta) \sin \Delta_{31}
 \end{aligned}$$

