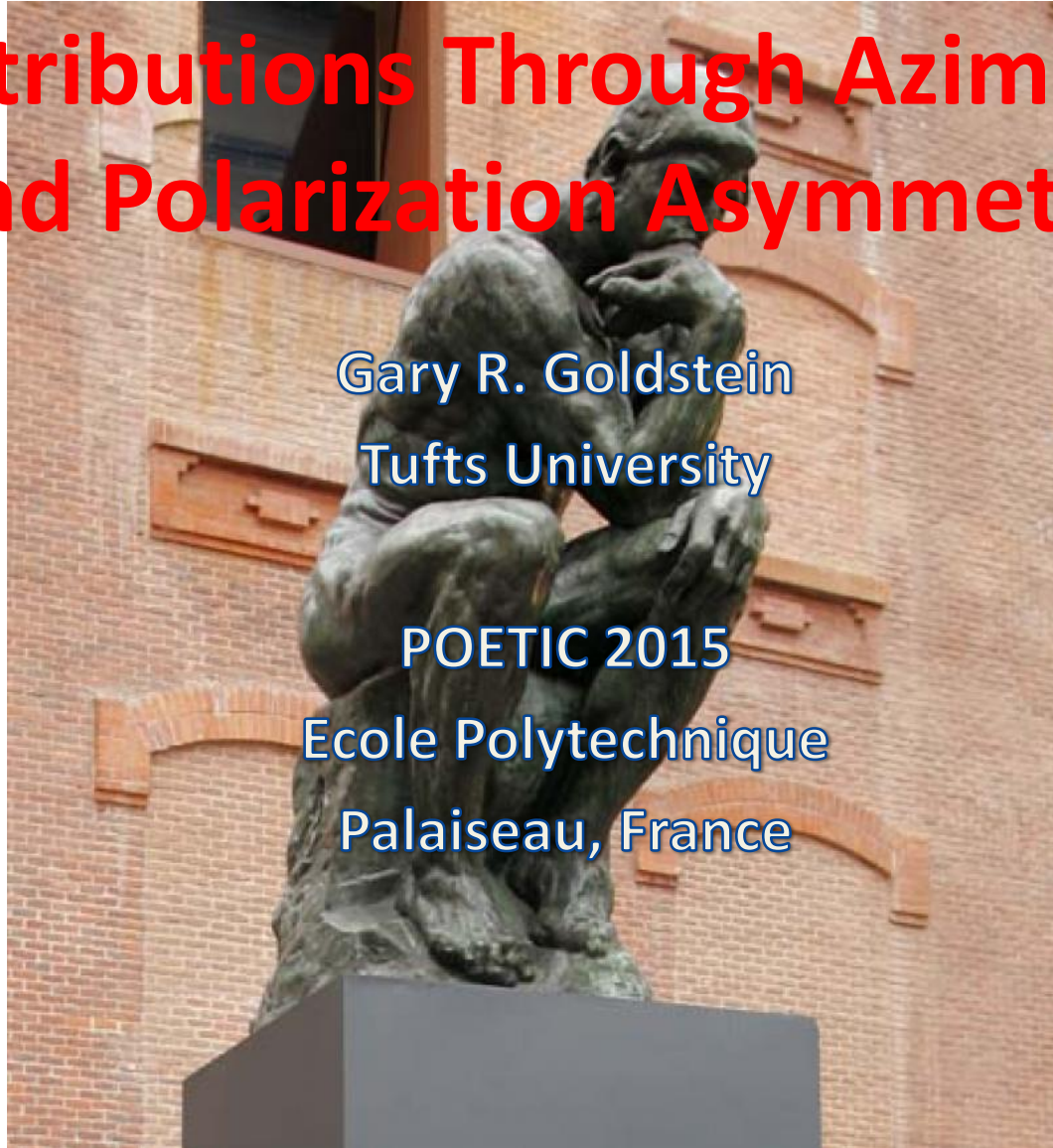


Probing Spin Dependent Quark Distributions Through Azimuthal and Polarization Asymmetries

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POETIC 2015
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Collaborators

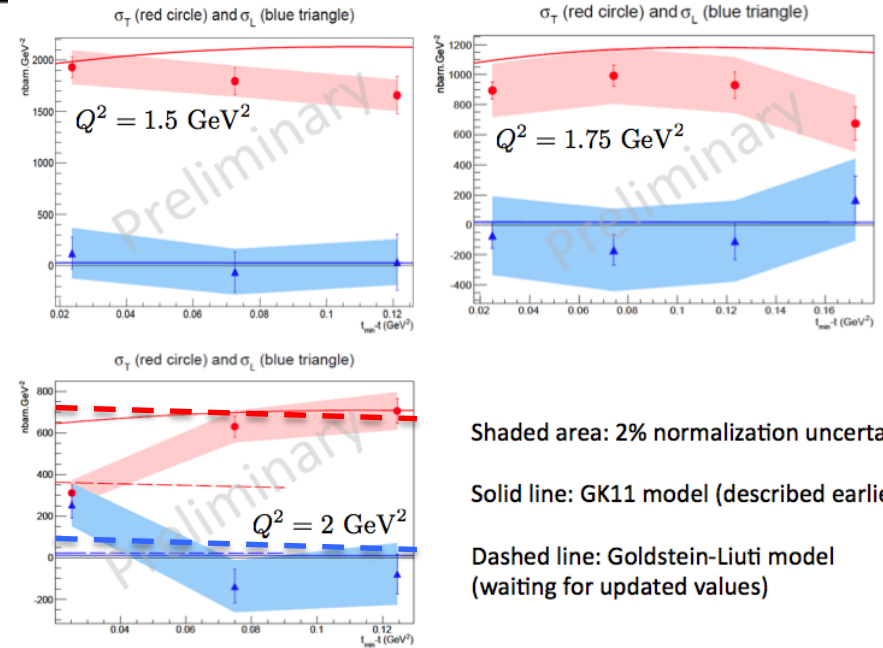
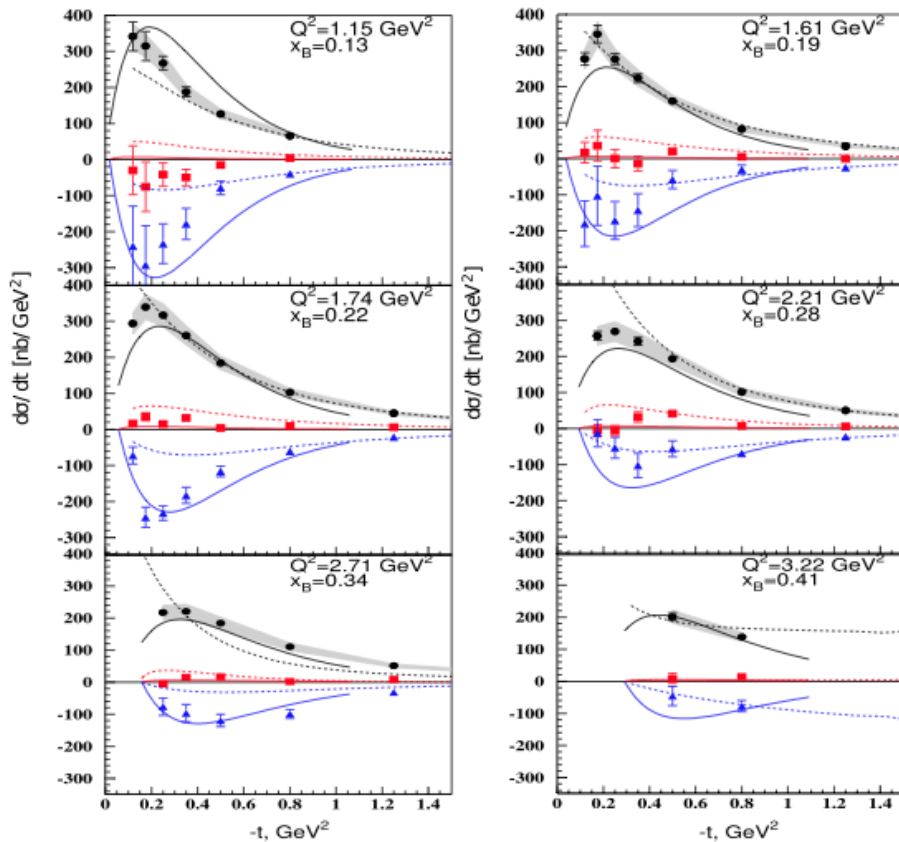
GPDs, Extension to Chiral Odd Sector

S. Liuti, O. Gonzalez Hernandez

- GRG, O. Gonzalez-Hernandez, S.Liuti, PRD91, 114013 (2015)
- GRG, O. Gonzalez-Hernandez, S.Liuti, arXiv:1401.0438
- Ahmad, GRG, Liuti, PRD79, 054014, (2009)
- Gonzalez, GRG, Liuti PRD84, 034007 (2011)
- GRG, Gonzalez Hernandez, Liuti, J. Phys. G: Nucl. Part. Phys. **39** 115001 (2012)
- Gonzalez Hernandez, Liuti, GRG, Kathuria, PRC88, 065206 (2013)

Why consider chiral-odd GPDs? Why go beyond leading twist?

π^0 electroproduction **data dictate** necessity of transverse photons
 CLAS; Hall A separated cross sections ; **Asymmetries** distinguish models



Shaded area: 2% normalization uncertainty

Solid line: GK11 model (described earlier)

Dashed line: Goldstein-Liuti model
 (waiting for updated values)

courtesy F. Sabatie, Hall A @ CIPANP

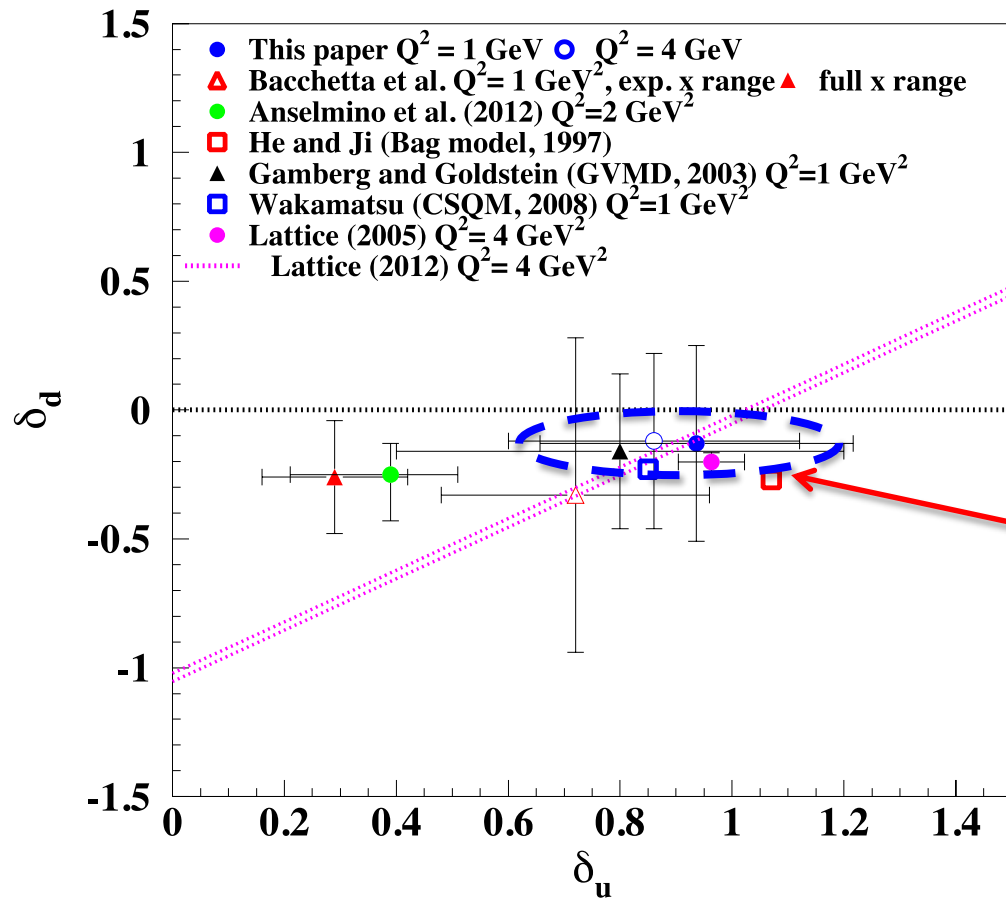
Dashed curve: GGL . . Solid curve: G&K
 CLAS: Bedlinskiy, et al., PRL 109, 112001 (2012)

G. R. Goldstein, J. O. Gonzalez Hernandez, and S. Liuti,
 Phys. Rev. D 84, 034007 (2011).
 S. V. Goloskokov and P. Kroll, Eur. Phys. J. A 47, 112 (2011).

Brief interlude

- Rudolf Haag postulated that the interaction picture does not exist in an interacting, relativistic quantum field theory, known as **Haag's Theorem**. 1955. (wikipedia)
- **The S matrix does not exist!**
- But strangeness, hyperons, partial wave analyses, vector mesons . . . Then what?

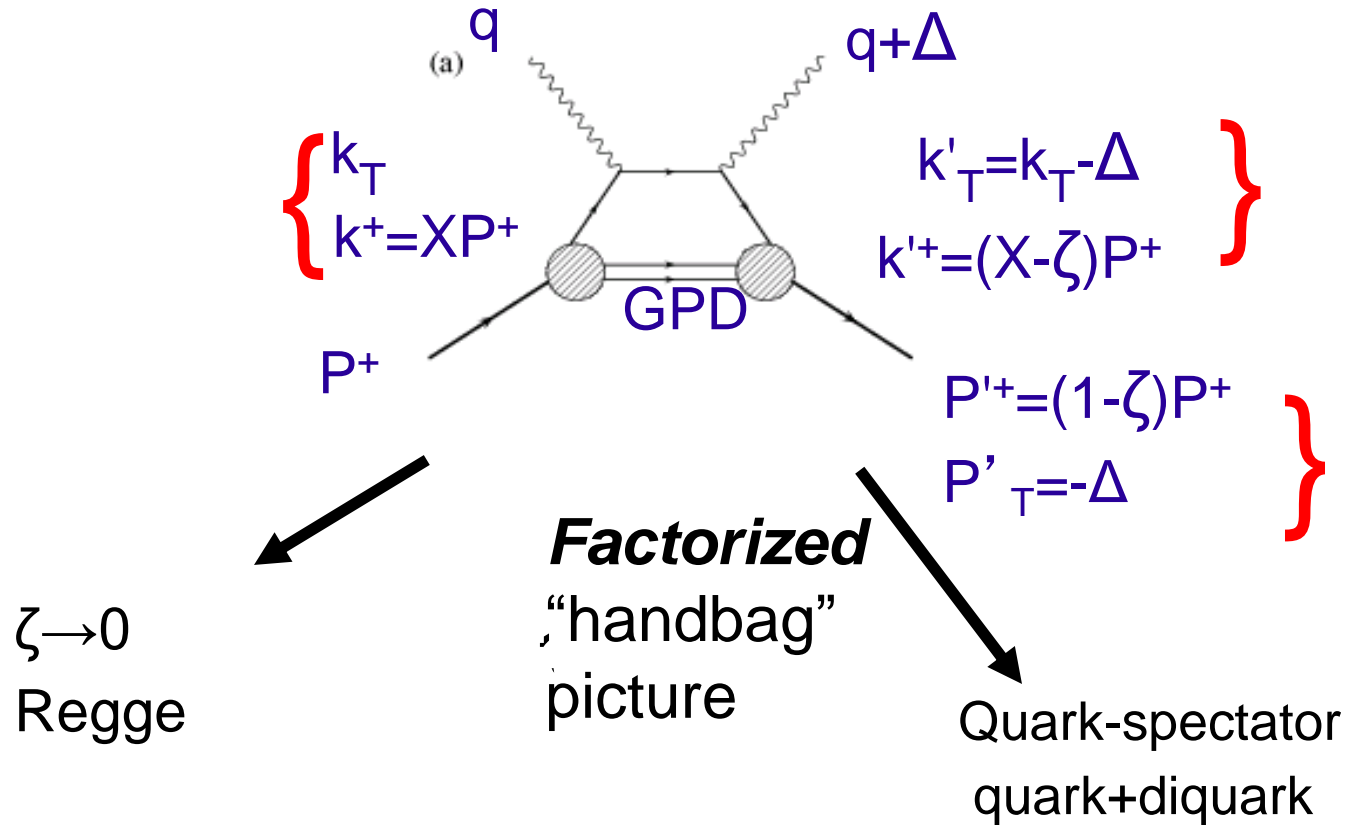
We look at Chiral odd GPDs - why? $\rightarrow H_T(x, \xi, t) \rightarrow$
 $h_1(x)$ **Transversity** \rightarrow tensor charges δ_q
to get complete picture of spin decomposition



GG, Gonzalez, Liuti,
PRD91, 114013 (2015)



DVCS & DVMP $\gamma^*(Q^2)+P \rightarrow (\gamma \text{ or meson})+P'$ partonic picture – leading twist



$X > \zeta$ DGLAP $\Delta_T \rightarrow b_T$ transverse spatial
 $X < \zeta$ ERBL $x = (X - \zeta/2)/(1 - \zeta/2)$; $\xi = \zeta/(2 - \zeta)$

see Ahmad, GG, Liuti, PRD79, 054014, (2009) for first chiral odd GPD
 parameterization Gonzalez, GG, Liuti PRD84, 034007 (2011); PRD91, 114013 (2015)



GPD definitions – 8 quark + 8 gluon (twist 2)

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \gamma^+ \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[H^q \gamma^+ + E^q \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} \right] u(p, \lambda),$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \gamma^+ \gamma_5 \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[\tilde{H}^q \gamma^+ \gamma_5 + \tilde{E}^q \frac{\gamma_5 \Delta^+}{2m} \right] u(p, \lambda),$$

Chiral even GPDs

-> Ji sum rule

talks by Mueller, Kumericki, Guidal.

Liuti, et al. → “flexible parameterization”

$$\langle J_q^x \rangle = \frac{1}{2} \int dx [H(x, 0, 0) + E(x, 0, 0)] x$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) i\sigma^{+i} \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[H_T^q i\sigma^{+i} + \tilde{H}_T^q \frac{P^+ \Delta^i - \Delta^+ P^i}{m^2} \right. \\ \left. + E_T^q \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2m} + \tilde{E}_T^q \frac{\gamma^+ P^i - P^+ \gamma^i}{m} \right] u(p, \lambda)$$

Chiral odd GPDs

-> transversity

How to measure

and/or

parameterize them?

Normalizing GPDs - Chiral even

Form factor,

Forward limit

$$\int_0^1 H_q(x, \lambda, t) dx = F_1^q(t), \quad H_q(x, 0, 0) = q(x) \quad \text{Integrates to charge}$$

$$\int_0^1 E_q(x, \lambda, t) dx = F_2^q(t) \quad \rightarrow \text{Anomalous magnetic moments}$$

$$\int_0^1 \tilde{H}_q(x, \lambda, t) dx = g_A^q(t), \quad \tilde{H}_q(x, 0, 0) = Dq(x) = q_{\rightarrow}(x) - q_{\leftarrow}(x)$$

Integrates to axial charge

$$\int_0^1 \tilde{E}_q(x, \lambda, t) dx = g_P^q(t)$$

One question is: how do we **normalize** chiral-odd GPDs?

The only Physical constraints on the various chiral-odd GPDs are

Forward limit

$$H_T(x, 0, 0) = q_{\uparrow\uparrow}^{\uparrow}(x) - q_{\uparrow\uparrow}^{\downarrow}(x) = h_1(x) \quad \text{Transversity}$$

Integrates to tensor charge δ_q

Form Factors

$$\int H_T^q(x, \xi, t) dx = \delta q(t)$$

$$\int \bar{E}_T^q(x, \xi, t) dx = \int \left(2\tilde{H}_T^q + E_T^q \right) dx = \kappa_T^q(t)$$

"transverse moment" κ_T^q

$$\int \tilde{E}_T(x, \xi, t) dx = 0$$

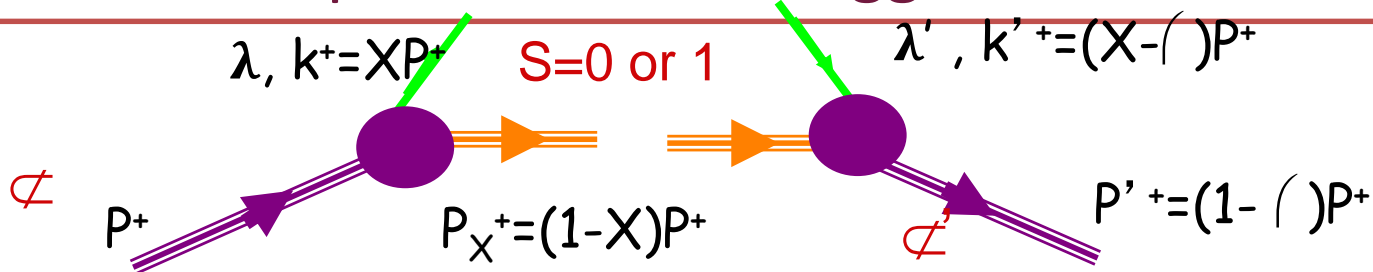
No direct interpretation of E_T .

The Model – Reggeized Diquarks

The Model – first for Chiral Even – Reggeized Diquarks

Procedure to construct Chiral Odd GPDs & observables

Spectator diquark model & Reggeization



Product of diquark l.c.w.f.'s $\rightarrow A_{\Lambda\lambda; \Lambda'\lambda'=\lambda}$

$A_{\Lambda\lambda; \Lambda'\lambda} \rightarrow$ chiral even GPDs

$g \otimes A \rightarrow$ exclusive process helicity amps

pdf's, FF's, $d\sigma/d\Omega$ & Asymmetries: parameters & predictions

vertex parity $\rightarrow A_{\Lambda\lambda; -\Lambda'\lambda'} \rightarrow$ chiral odd GPDs \rightarrow pdf's, ...

Recursive fit

GRG, Gonzalez Hernandez, Liuti, PRD84 (2011)

Advantage: control over the number of parameters to be fitted at different stages so that it can be optimized

Functional form:

From DIS

$$q(x, Q_o^2) = A_q x^{-a_q} (1-x)^{b_q} F(x, c_q, d_q, \dots)$$

to DVCS, DVMP

$$H_q(x, \chi, t; Q_o^2) = N_q x^{-[a_q + a'_q (1-x)^p t]} G^{a_1 a_2 a_3 \dots}(x, \chi, t)$$
$$a_1 = m_q, a_2 = M_X^q, a_3 = M_L^q, \dots$$

pdf's
Form Factors
 $d\sigma/d\Omega$
Asymmetries

"Flexible" parameterization based on the Reggeized quark-diquark model.

Sea quarks and gluon parametrization, work in progress

Details of Regge-diquark model

$$A_{\Lambda'\lambda',\Lambda\lambda}^{(0)} = \int d^2k_{\perp} \phi_{\Lambda'\lambda'}^*(k', P') \phi_{\Lambda\lambda}(k, P), \quad \text{Scalar diquark} \rightarrow \text{helicity amps}$$

$$\phi_{\Lambda,\lambda}(k, P) = \Gamma(k) \frac{\bar{u}(k, \lambda) U(P, \Lambda)}{k^2 - m^2} \quad \text{l.c.w.f. 's}$$

$$\phi_{\Lambda'\lambda'}^*(k', P') = \Gamma(k') \frac{\bar{U}(P', \Lambda') u(k', \lambda')}{k'^2 - m^2}, \quad \Gamma = g_s \frac{k^2 - m_q^2}{(k^2 - M_{\Lambda}^q)^2}$$

$$\phi_{++}^*(k, P) = \mathcal{A}(m + MX)$$

$$\phi_{+-}^*(k, P) = \mathcal{A}(k_1 + ik_2),$$

$$\phi_{--}(k, P) = \phi_{++}(k, P)$$

$$\phi_{-+}(k, P) = -\phi_{+-}^*(k, P).$$

Add axial vector diquark \rightarrow helicity amps

Chiral odd GPDs = helicity amps

$$\begin{aligned} \tau \left[2\tilde{H}_T(X, 0, t) + E_T(X, 0, t) \right] &= A_{++,+-} + A_{--,-} \\ &= A_{++,++}^{T_Y} - A_{+-,+}^{T_Y} + A_{-+,-}^{T_Y} - A_{--,--}^{T_Y} \quad \frac{c_{\perp}^2}{-X}. \\ H_T(X, 0, t) &= A_{++,--} + A_{--,+} \\ &= A_{++,++}^{T_X} - A_{+-,+}^{T_X} - A_{-+,-}^{T_X} + A_{--,--}^{T_X} \\ \tau^2 \tilde{H}_T(X, 0, t) &= -A_{--,+} \\ &= A_{++,++}^{T_Y} - A_{+-,+}^{T_Y} - A_{-+,-}^{T_X} + A_{--,--}^{T_X} \\ \tilde{E}_T(X, 0, t) &= A_{++,+-} - A_{--,-} = 0 \end{aligned}$$

Reggeizing diquark model

Diquark+quark & fixed masses (e.g. at $\xi=0$)

$$H_{M_X, m_q}^{M_\Lambda^q} = \mathcal{N}_q \int \frac{d^2 k_\perp}{1-x} \frac{[(m_q + Mx)(m_q + Mx) + \mathbf{k}_\perp \cdot \tilde{\mathbf{k}}_\perp]}{[\mathcal{M}_q^2(x) - k_\perp^2/(1-x)]^2 [\mathcal{M}_q^2(x) - \tilde{k}_\perp^2/(1-x)]^2},$$

$$E_{M_X, m_q}^{M_\Lambda^q} = \mathcal{N}_q \int \frac{d^2 k_\perp}{1-x} \frac{-2M/\Delta_\perp^2 [(m_q + Mx)\tilde{\mathbf{k}}_\perp \cdot \mathbf{\Delta}_\perp - (m_q + Mx)\mathbf{k}_\perp \cdot \mathbf{\Delta}_\perp]}{[\mathcal{M}_q^2(x) - k_\perp^2/(1-x)]^2 [\mathcal{M}_q^2(x) - \tilde{k}_\perp^2/(1-x)]^2},$$

Diquark mass “spectrum”
as in Brodsky, Close & Gunion
Phys. Rev. D8, 3678 (1973)

$$F_T^q(X, \zeta, t) = \mathcal{N}_q \int_0^\infty dM_X^2 \rho(M_X^2) F_T^{(m_q, M_\Lambda^q)}(X, \zeta, t; M_X).$$

$$\rho(M_X^2) \approx \begin{cases} (M_X^2)^\alpha & M_X^2 \rightarrow \infty \\ \delta(M_X^2 - \bar{M}_X^2) & M_X^2 \text{ few GeV}^2 \end{cases}$$

$$F_T^q(X, \zeta, t) \approx \mathcal{N}_q X^{-\alpha_q + \alpha'_q(X)t} F_T^{(m_q, M_\Lambda^q)}(X, \zeta, t; \bar{M}_X) = R_{p_q}^{\alpha_q, \alpha'_q}(X, \zeta, t) G_{M_X, m}^{M_\Lambda}(X, \zeta, t)$$

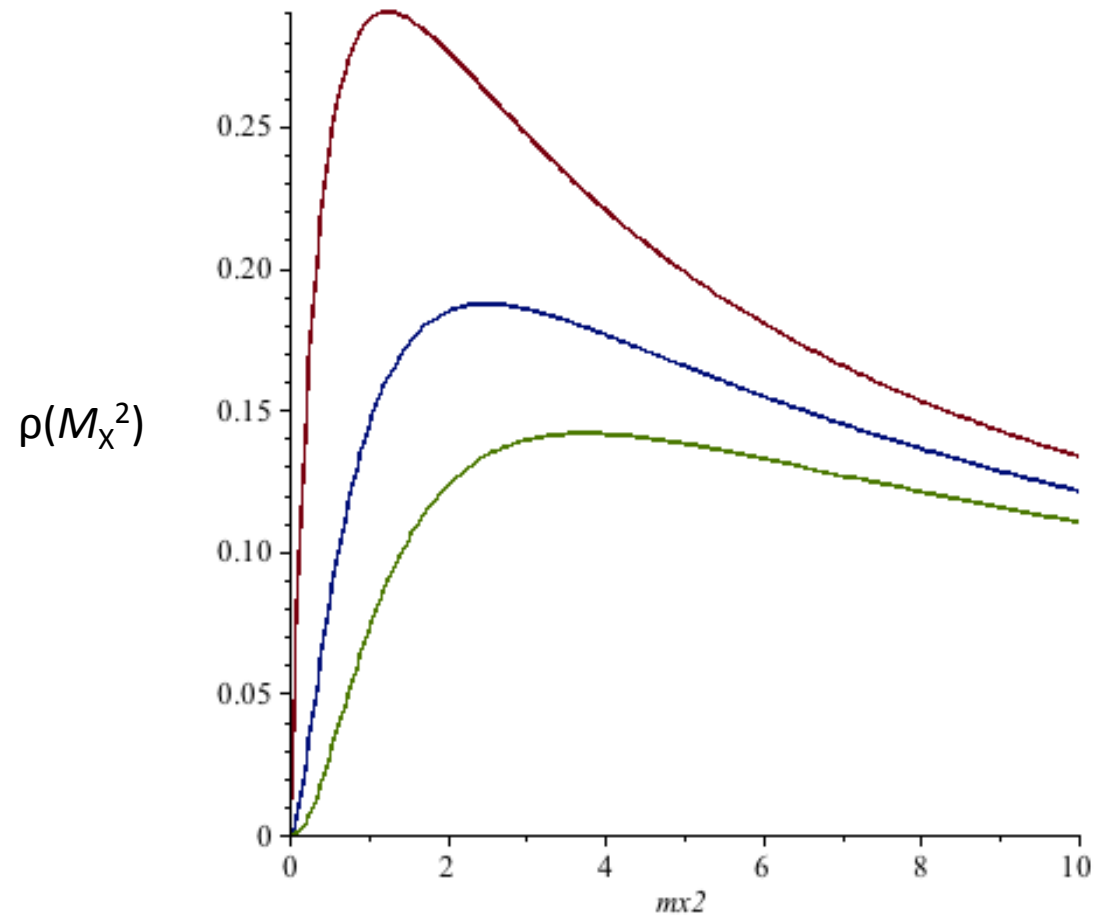
R × Dq

Spectral distribution

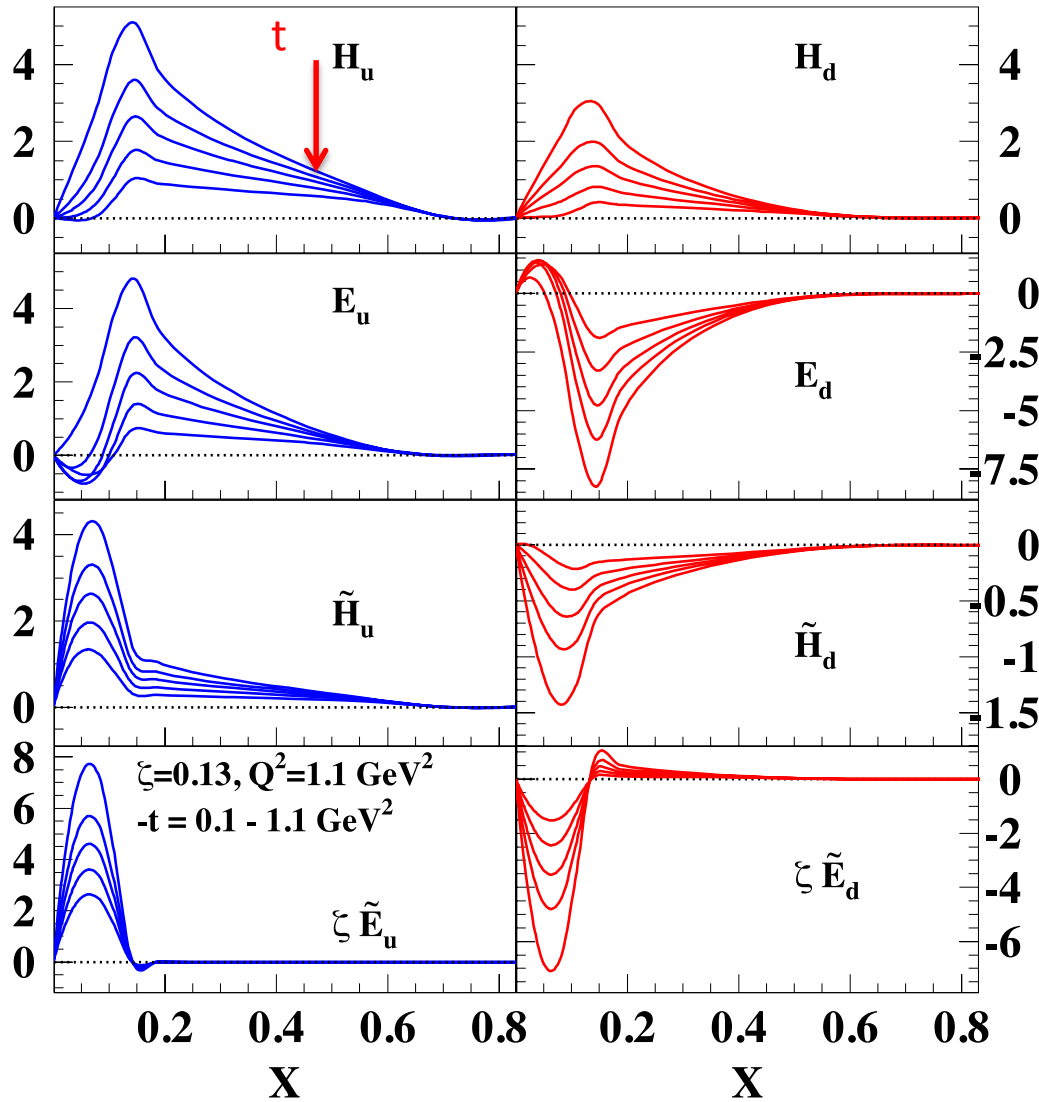
of form $\rho(M_x^2, k^2) \approx \rho(M_x^2) \beta(k^2)$

$$\rho(M_x^2) = (M_x^2/M_0^2)^\beta / (1 + M_x^2/M_0^2)^{\beta-\alpha+1}$$

$\beta(k^2)$ chosen to give large
 k_T^2 falloff behavior



Chiral even GPDs

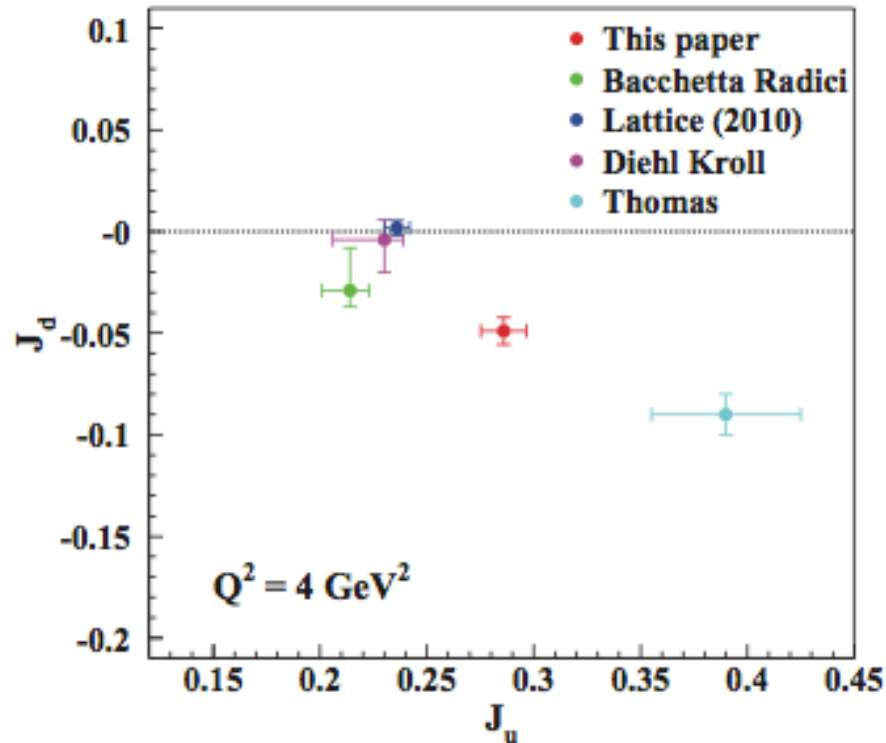


From GPDs
with evolution
to Compton
Form Factors
↓
CFFs to helicity
amps
↓
helicity amps to
observables
↓
<-> parameters

Valence quark angular momenta - from "flexible" chiral even model applied to EM form factors, pdf's & some cross section & asymmetry data

Gonzalez Hernandez, Liuti, GRG, Kathuria

PHYSICAL REVIEW C 88, 065206 (2013)

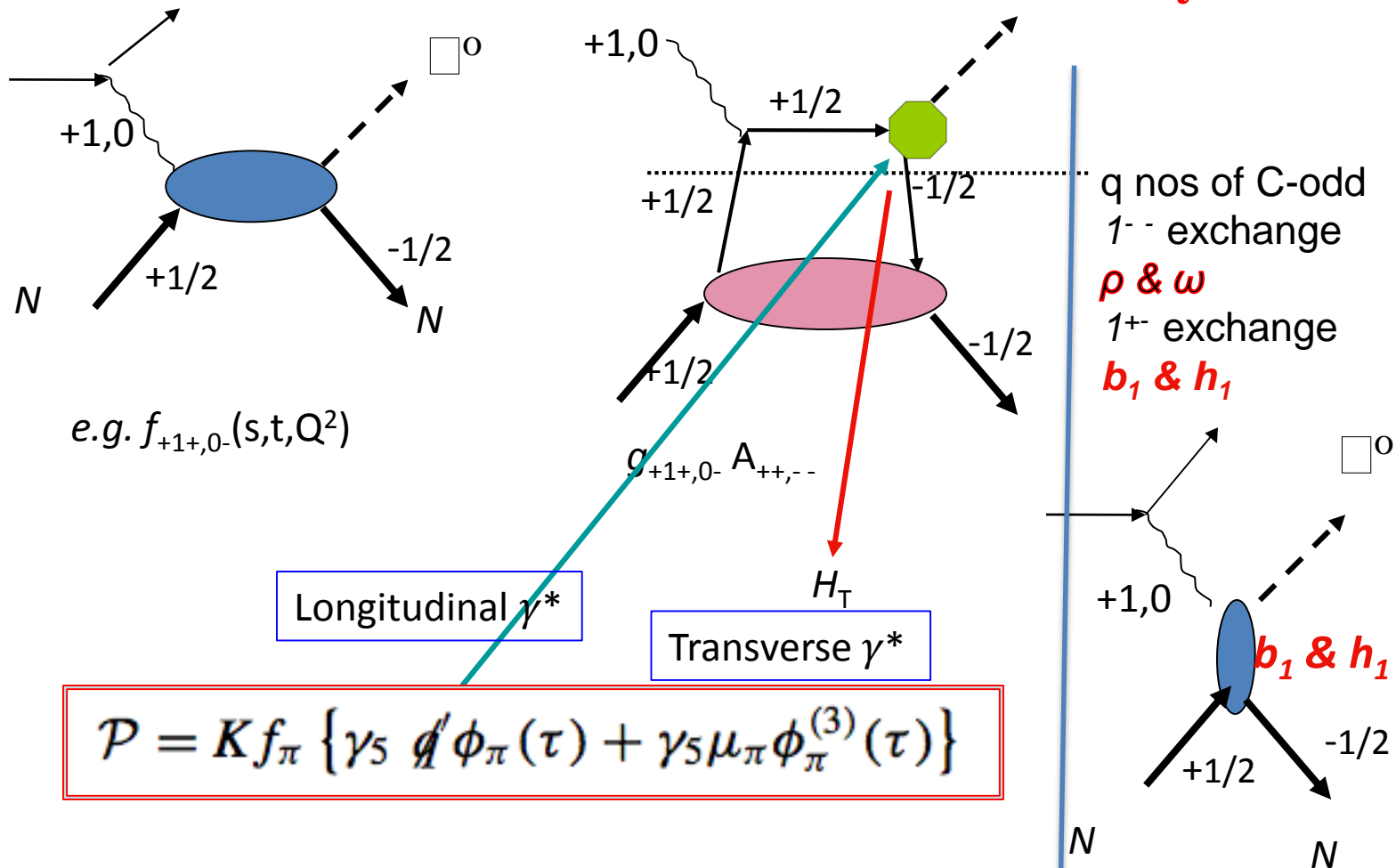


Improved precision based on EM Form Factor measurements
G. D. Cates, et al., Phys. Rev. Lett. **106**, 252003 (2011).



How to single out **chiral odd GPDs**?

Exclusive Lepto-production of \square^0 or η, η'
to measure **chiral odd GPDs & Transversity**



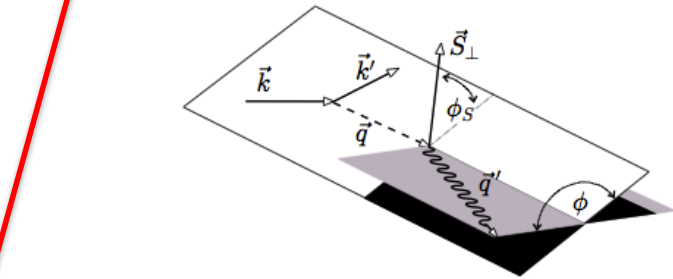
t-channel \mathcal{J}^{PC} quantum numbers enhance chiral odd

Cross section with φ modulations & beam/target polarized

$$\begin{aligned} \frac{d^4\sigma}{dx_{Bj}dyd\phi dt} = & \Gamma \left\{ \left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{2\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} + h \sqrt{2\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \right] \right. \\ & + S_{\parallel} \left[\sqrt{2\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + h \left(\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} \right) \right] \\ & + S_{\perp} \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) + \epsilon \left(\sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right) \right. \\ & + \left. \sqrt{2\epsilon(1+\epsilon)} \left(\sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right) \right] \\ & \left. + S_{\perp} h \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \left(\cos \phi_S F_{LT}^{\cos \phi_S} + \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \right\} \end{aligned}$$

$$A_{LU} = \sqrt{\epsilon(1-\epsilon)} \frac{F_{LU}^{\sin \phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

$$A_{UL} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$



$$\begin{aligned} F_{UU,T} &= \frac{d\sigma_T}{dt}, & F_{UU,L} &= \frac{d\sigma_L}{dt}, & F_{UU}^{\cos \phi} &= \frac{d\sigma_{LT}}{dt}, \\ F_{UU}^{\cos 2\phi} &= \frac{d\sigma_{TT}}{dt}, & F_{LU}^{\sin \phi} &= \frac{d\sigma_{LT'}}{dt} \end{aligned}$$

$$A_{LL} = \frac{N_{s_z=+}^{\rightarrow} - N_{s_z=-}^{\rightarrow} + N_{s_z=+}^{\leftarrow} - N_{s_z=-}^{\leftarrow}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{1-\epsilon^2} F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

from Diehl & Sapeta; Bacchetta, *et al.* SIDIS Review; ...

Observables expressed in bilinears of helicity amps –

6 amps for π^0

Compton Form Factors

$$f_1 \quad f_{10}^{++} = g_{\pi}^{V,odd}(Q) \frac{\sqrt{t_0-t}}{4M} \left[2\tilde{\mathcal{H}}_T + (1+\xi) (\mathcal{E}_T + \tilde{\mathcal{E}}_T) \right]$$

$$= g_{\pi}^{V,odd}(Q) \frac{\sqrt{t_0-t}}{2M} \left[\tilde{\mathcal{H}}_T + \frac{1}{2-\zeta} \mathcal{E}_T + \frac{1}{2-\zeta} \tilde{\mathcal{E}}_T \right],$$

Couplings $g_{\pi}^{V \text{ \&/or } A}(Q^2)$

$$f_2 \quad f_{10}^{+-} = \frac{g_{\pi}^{V,odd}(Q) + g_{\pi}^{A,odd}(Q)}{2} \sqrt{1-\xi^2} \left[\mathcal{H}_T + \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T + \frac{\xi^2}{1-\xi^2} \mathcal{E}_T + \frac{\xi}{1-\xi^2} \tilde{\mathcal{E}}_T \right]$$

$$= \frac{g_{\pi}^{V,odd}(Q) + g_{\pi}^{A,odd}(Q)}{2} \frac{\sqrt{1-\zeta}}{1-\zeta/2} \left[\mathcal{H}_T + \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T + \frac{\zeta^2/4}{1-\zeta} \mathcal{E}_T + \frac{\zeta/2}{1-\zeta} \tilde{\mathcal{E}}_T \right]$$

$$f_3 \quad f_{10}^{-+} = -\frac{g_{\pi}^{A,odd}(Q) - g_{\pi}^{V,odd}(Q)}{2} \sqrt{1-\xi^2} \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T$$

$$= -\frac{g_{\pi}^{A,odd}(Q) - g_{\pi}^{V,odd}(Q)}{2} \frac{\sqrt{1-\zeta}}{1-\zeta/2} \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T$$

$$f_4 \quad f_{10}^{--} = g_{\pi}^{V,odd}(Q) \frac{\sqrt{t_0-t}}{4M} \left[2\tilde{\mathcal{H}}_T + (1-\xi) (\mathcal{E}_T - \tilde{\mathcal{E}}_T) \right]$$

$$= g_{\pi}^{V,odd}(Q) \frac{\sqrt{t_0-t}}{2M} \left[\tilde{\mathcal{H}}_T + \frac{1-\zeta}{2-\zeta} \mathcal{E}_T + \frac{1-\zeta}{2-\zeta} \tilde{\mathcal{E}}_T \right]$$

$$f_5 \quad f_{00}^{+-} = g_{\pi}^{A,odd}(Q) \sqrt{1-\xi^2} \left[\mathcal{H}_T + \frac{\xi^2}{1-\xi^2} \mathcal{E}_T + \frac{\xi}{1-\xi^2} \tilde{\mathcal{E}}_T \right] \sqrt{t_0-t}$$

Also Chiral Even CFFs

$$f_6 \quad f_{00}^{++} = -g_{\pi}^{A,odd}(Q) \frac{\sqrt{t_0-t}}{2M} \left[\xi \mathcal{E}_T + \tilde{\mathcal{E}}_T \right] \sqrt{t_0-t}$$

Asymmetries & helicity amps

structure functions for the unpolarized beam and single transversely polarized target,

$$\begin{aligned}
 F_{UT,T}^{\sin(\phi-\phi_S)} &= \Im m F_{11}^{+-} = \Im m \sum_{\Lambda'} f_{10}^{+\Lambda'} * f_{10}^{-\Lambda'} = \Im m [f_{10}^{+++} f_{10}^{-+} + f_{10}^{+-*} f_{10}^{--}] \\
 F_{UT,L}^{\sin(\phi-\phi_S)} &= \Im m F_{00}^{+-} = \Im m \sum_{\Lambda'} f_{00}^{+\Lambda'} * f_{00}^{-\Lambda'} = \Im m [f_{00}^{+++} f_{00}^{-+} + f_{00}^{+-*} f_{00}^{--}] \\
 F_{UT}^{\sin(\phi+\phi_S)} &= \Im m F_{1-1}^{+-} = \Im m \sum_{\Lambda'} f_{10}^{+\Lambda'} * f_{-10}^{-\Lambda'} = \Im m [-f_{10}^{+++} f_{10}^{+-} + f_{10}^{+-*} f_{10}^{++}] \\
 F_{UT}^{\sin(3\phi+\phi_S)} &= \Im m F_{1-1}^{-+} = \Im m \sum_{\Lambda'} f_{10}^{-\Lambda'} * f_{-10}^{+\Lambda'} = \Im m [f_{10}^{-+*} f_{10}^{--} - f_{10}^{--*} f_{10}^{-+}] \\
 F_{UT}^{\sin\phi_S} &= \Im m F_{10}^{+-} = \Im m \sum_{\Lambda'} f_{10}^{+\Lambda'} * f_{00}^{-\Lambda'} = \Im m [f_{10}^{+++} f_{00}^{-+} + f_{10}^{+-*} f_{00}^{--}] \\
 F_{UT}^{\sin(2\phi-\phi_S)} &= \Im m F_{10}^{-+} = \Im m \sum_{\Lambda'} f_{10}^{-\Lambda'} * f_{00}^{+\Lambda'} = \Im m [f_{10}^{-+*} f_{00}^{++} + f_{10}^{--*} f_{00}^{+-}],
 \end{aligned}$$

and three for the longitudinally polarized lepton and transversely polarized target,

$$\begin{aligned}
 F_{LT}^{\cos(\phi-\phi_S)} &= \Re e F_{11}^{+-} = \Re e \sum_{\Lambda'} f_{10}^{+\Lambda'} * f_{10}^{-\Lambda'} = \Re e [f_{10}^{+++} f_{10}^{-+} + f_{10}^{+-*} f_{10}^{--}] \\
 F_{LT}^{\cos\phi_S} &= \Re e F_{10}^{+-} = \Re e \sum_{\Lambda'} f_{10}^{+\Lambda'} * f_{00}^{-\Lambda'} = \Re e [f_{10}^{+++} f_{00}^{-+} + f_{10}^{+-*} f_{00}^{--}] \\
 F_{LT}^{\cos(2\phi-\phi_S)} &= \Re e F_{10}^{-+} = \Re e \sum_{\Lambda'} f_{10}^{-\Lambda'} * f_{00}^{+\Lambda'} = \Re e [f_{10}^{-+*} f_{00}^{++} + f_{10}^{--*} f_{00}^{+-}].
 \end{aligned}$$

Selecting transversity

$$f_{10}^{++} \propto \Delta \left(2\tilde{\mathcal{H}}_T + (1 + \xi)\mathcal{E}_T - (1 + \xi)\tilde{\mathcal{E}}_T \right)$$

$$f_{10}^{+-} \propto \boxed{\mathcal{H}_T} + \frac{t_0 - t}{4M^2} \tilde{\mathcal{H}}_T \boxed{-} \frac{\xi^2}{1 - \xi^2} \mathcal{E}_T + \frac{\xi}{1 - \xi^2} \tilde{\mathcal{E}}_T$$

$$f_{10}^{-+} \propto \Delta^2 \tilde{\mathcal{H}}_T$$

$$f_{10}^{--} \propto \Delta \left(2\tilde{\mathcal{H}}_T + (1 \boxed{-} \xi)\mathcal{E}_T + (1 \boxed{-} \xi)\tilde{\mathcal{E}}_T \right), \leftarrow \boxed{2\tilde{\mathcal{H}}_T + \mathcal{E}_T \circ \tilde{\mathcal{E}}_T}$$

Compare also $f_{\text{long}}^{\text{odd}}$ & with chiral even $f_{\text{long}}^{\text{even}}$

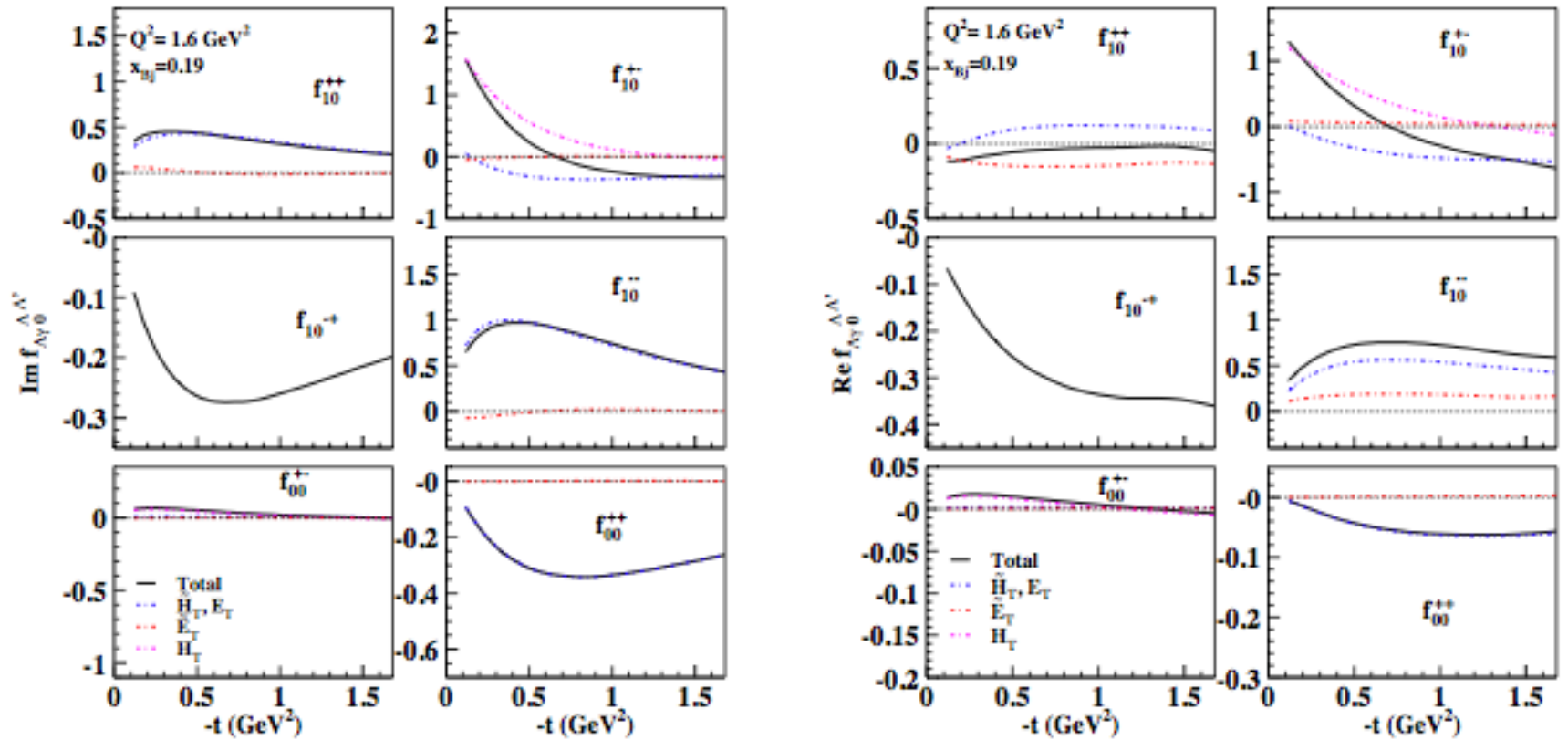
$$f_{00}^{+-} = g_{\pi}^{A, \text{odd}}(Q) \sqrt{1 - \xi^2} \left[\mathcal{H}_T + \frac{\xi^2}{1 - \xi^2} \mathcal{E}_T + \frac{\xi}{1 - \xi^2} \tilde{\mathcal{E}}_T \right] \frac{\sqrt{t_0 - t}}{2M}$$

$$f_{00}^{++} = -g_{\pi}^{A, \text{odd}}(Q) \left(\frac{\sqrt{t_0 - t}}{2M} \right)^{\frac{1}{2}} \left[\xi \mathcal{E}_T + \tilde{\mathcal{E}}_T \right].$$

$$f_{00}^{+-, \text{even}} = \frac{\zeta}{\sqrt{1 - \zeta}} \frac{1}{1 - \zeta/2} \frac{\sqrt{t_0 - t}}{2M} \tilde{\mathcal{E}},$$

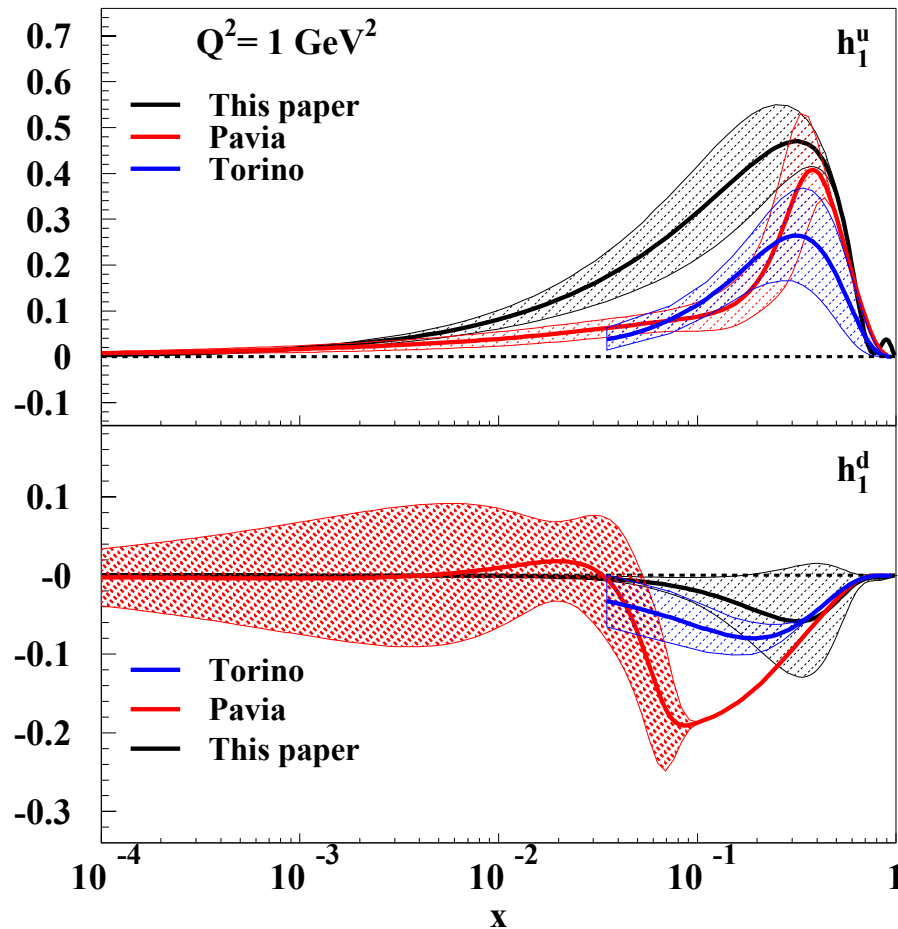
$$f_{00}^{++, \text{even}} = \frac{\sqrt{1 - \zeta}}{1 - \zeta/2} \tilde{\mathcal{H}} + \frac{-\zeta^2/4}{(1 - \zeta/2) \sqrt{1 - \zeta}} \tilde{\mathcal{E}},$$

6 helicity amps for π^0
after Compton Form Factors

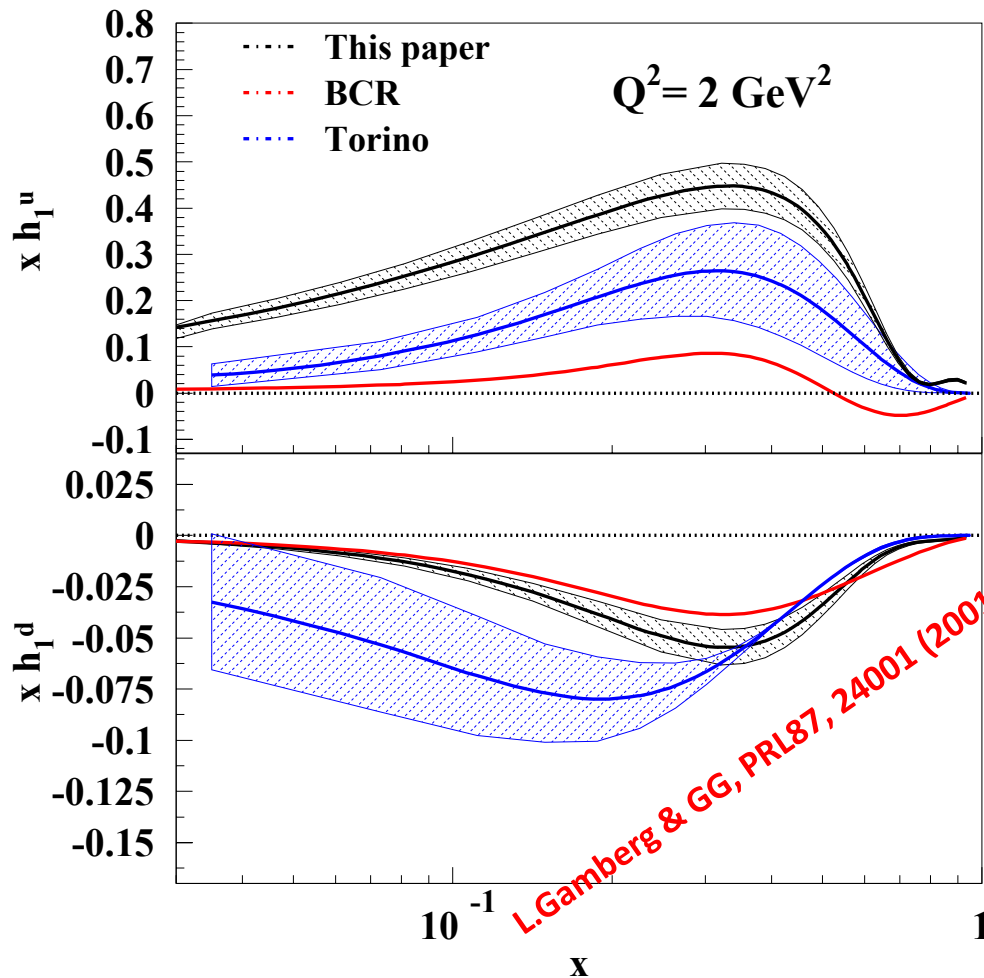


Chiral odd GPDs →

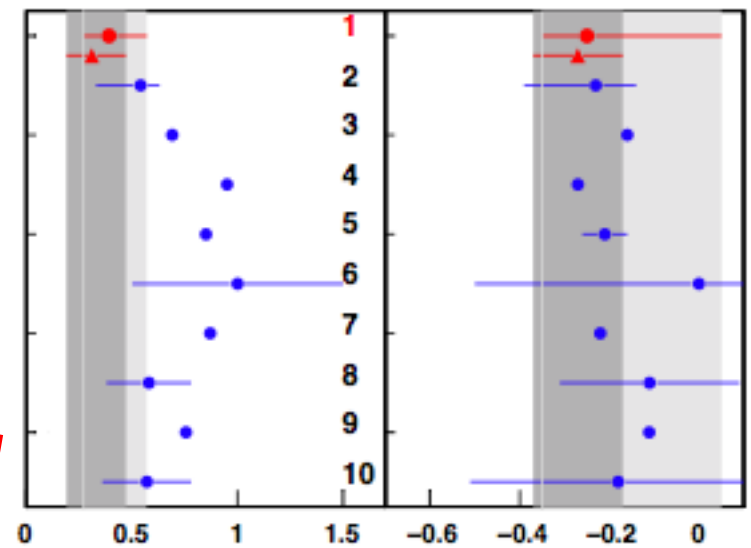
Transversity → pdf's: $h_1^q(x, Q^2)$



GG, Gonzalez, Liuti,
arXiv:1311.0483 [hep-ph]
1401.0438 PRD91, 114013 (2015)



$\bullet \delta u = 0.39^{+0.18}_{-0.12}$ $\bullet \delta d = -0.25^{+0.30}_{-0.10}$
 $\blacktriangle \delta u = 0.31^{+0.16}_{-0.12}$ $\blacktriangle \delta d = -0.27^{+0.10}_{-0.10}$

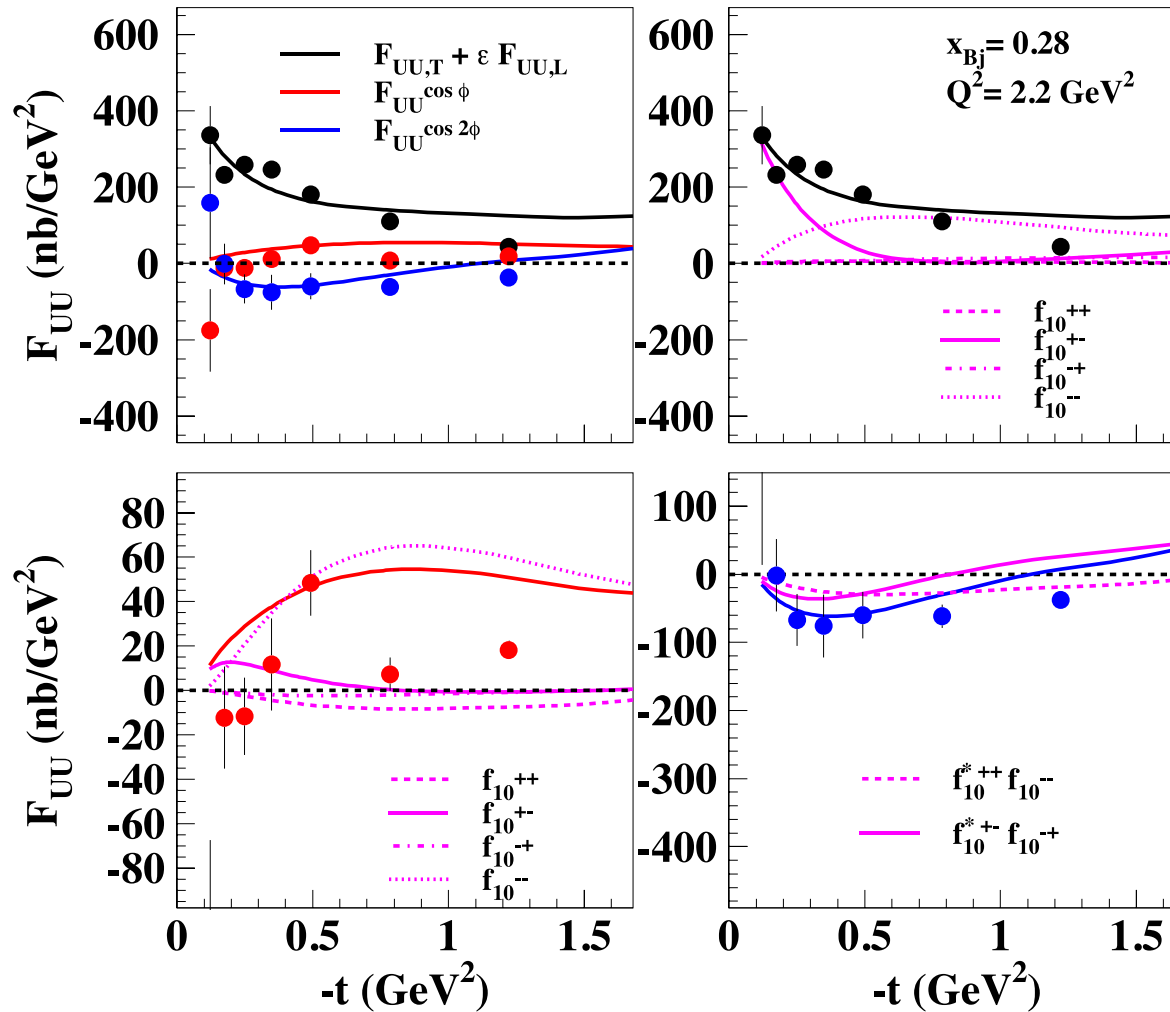


Anselmino, Boglione, et al.,
 Phys.Rev. D87 (2013) 094019
 $\delta u = 0.31^{+0.16}_{-0.12}$ $\delta d = -0.27^{+0.10}_{-0.10}$

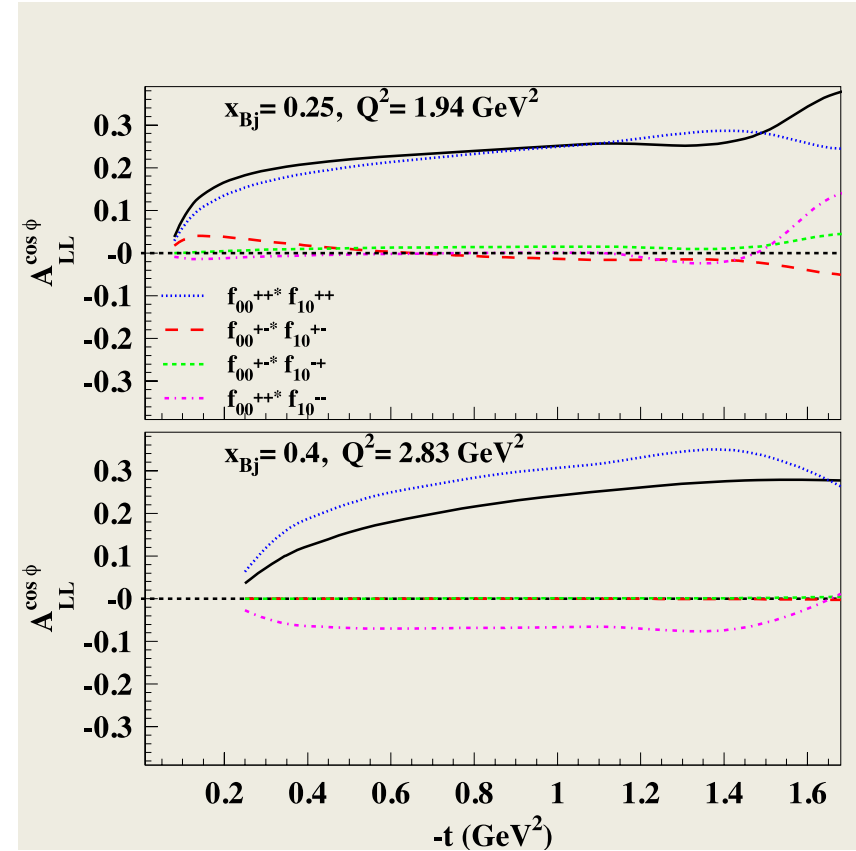
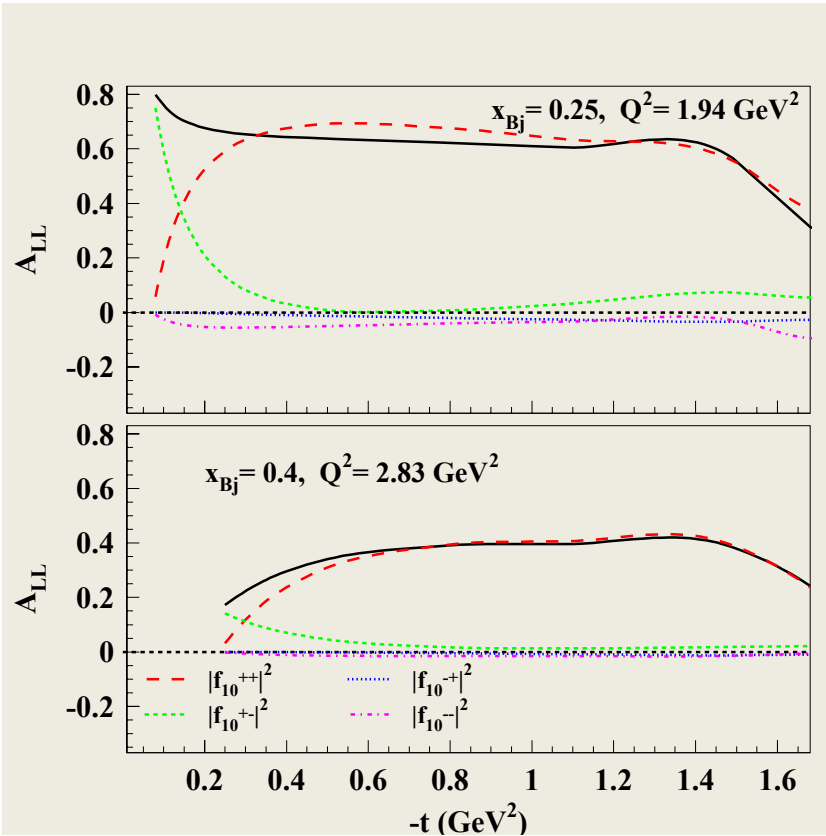
From our Reggeized form
 $\delta u \approx 1.2$ $\delta d \approx -0.08$
 Closer to QCD sum rule values

How well do the parameters fixed with DVCS data reproduce π^0 electroproduction data?

Hall B data, Bedlinskii, et al. PRL 109, 112001 (2012)



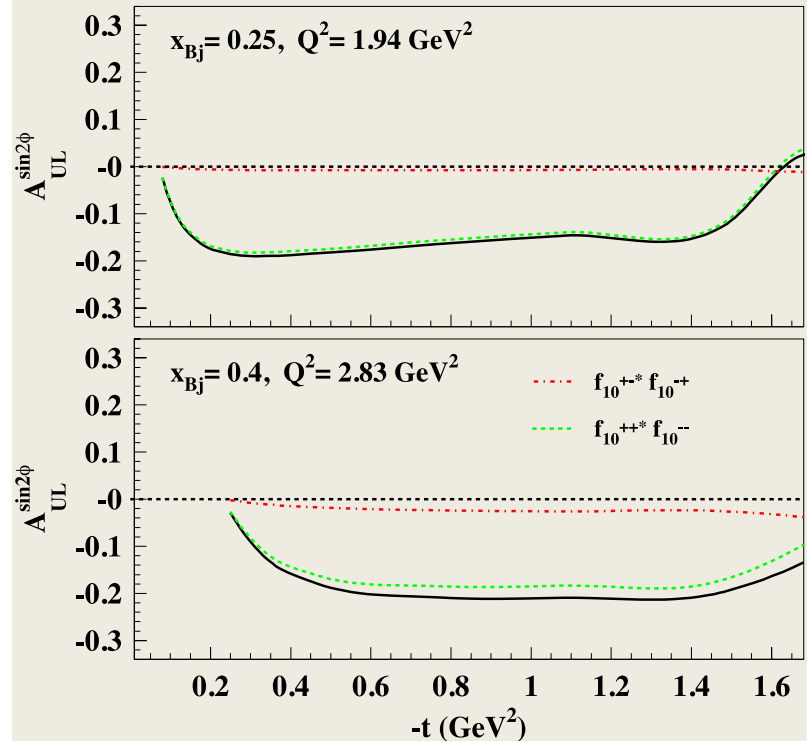
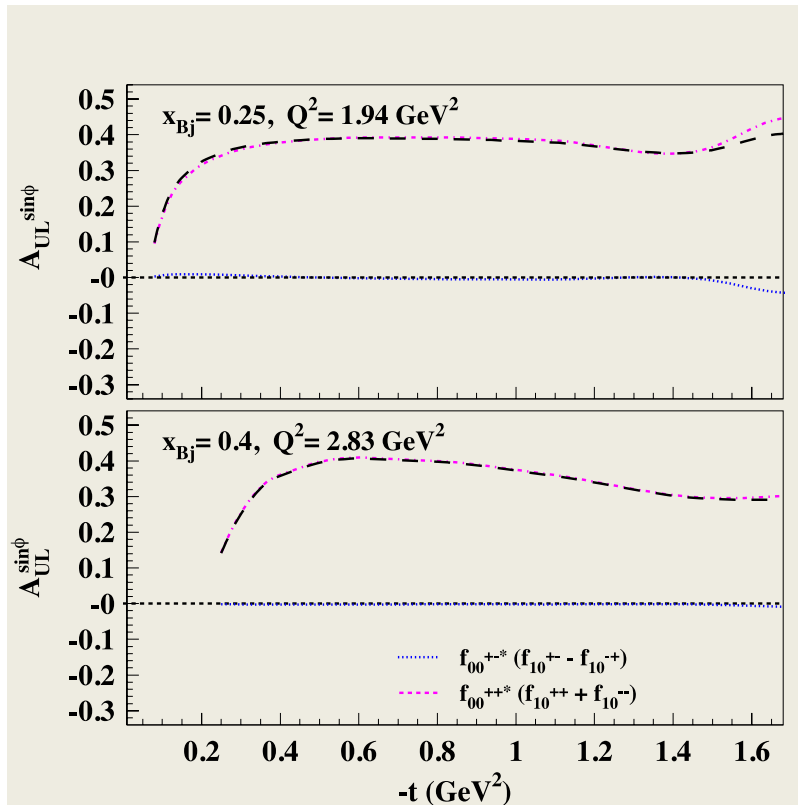
Longitudinally polarized beam and target



Look for tensor charge in f_{10}^{+}

Transverse dipole moment in f_{10}^{++}, f_{10}^{-}

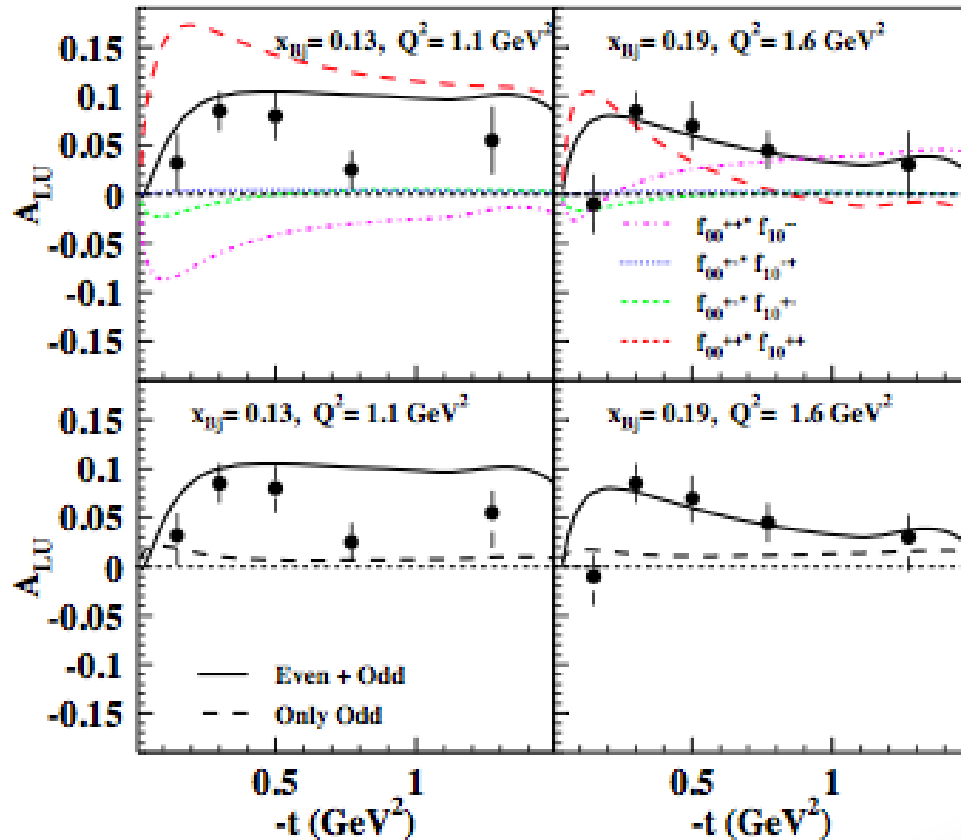
Longitudinally polarized target



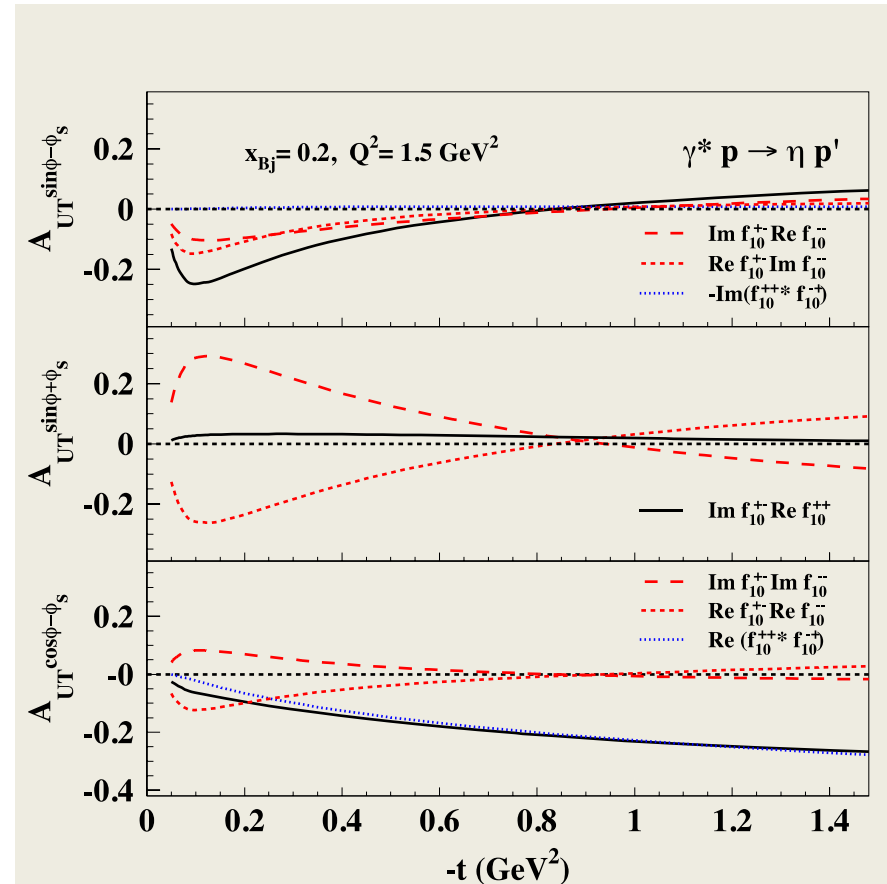
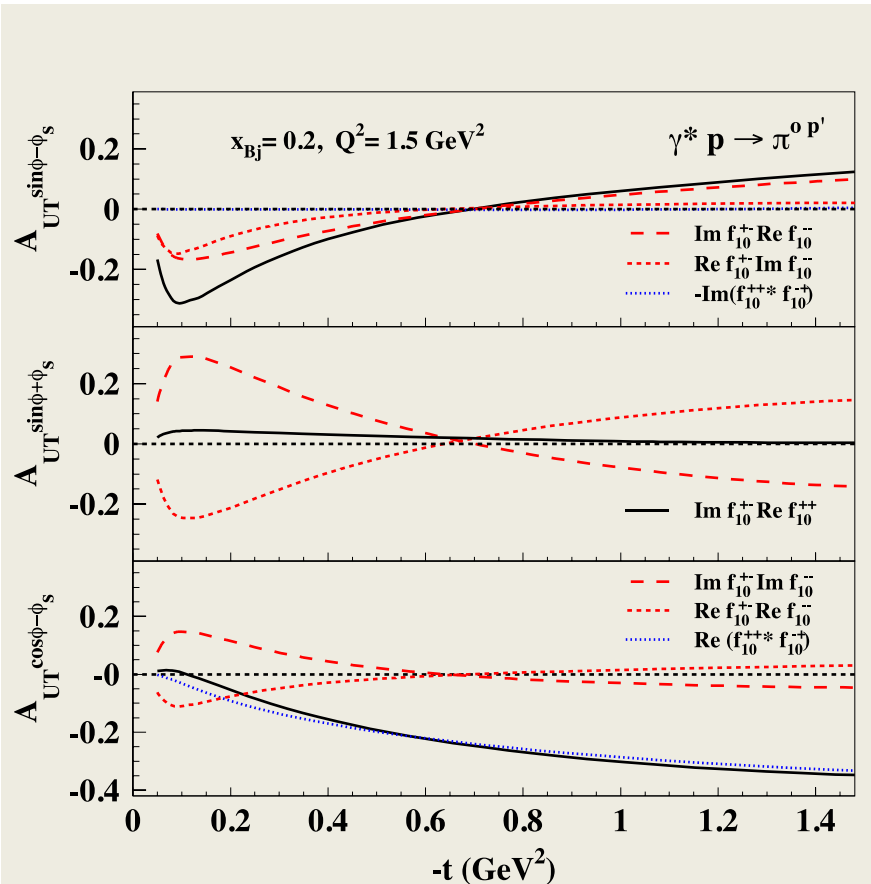
Look for tensor charge in f_{10}^{+-}

Transverse dipole moment in f_{10}^{++}, f_{10}^{--}

Beam spin asymmetry
 shows importance of H chiral even (CLAS data -DeMasi, et al.)



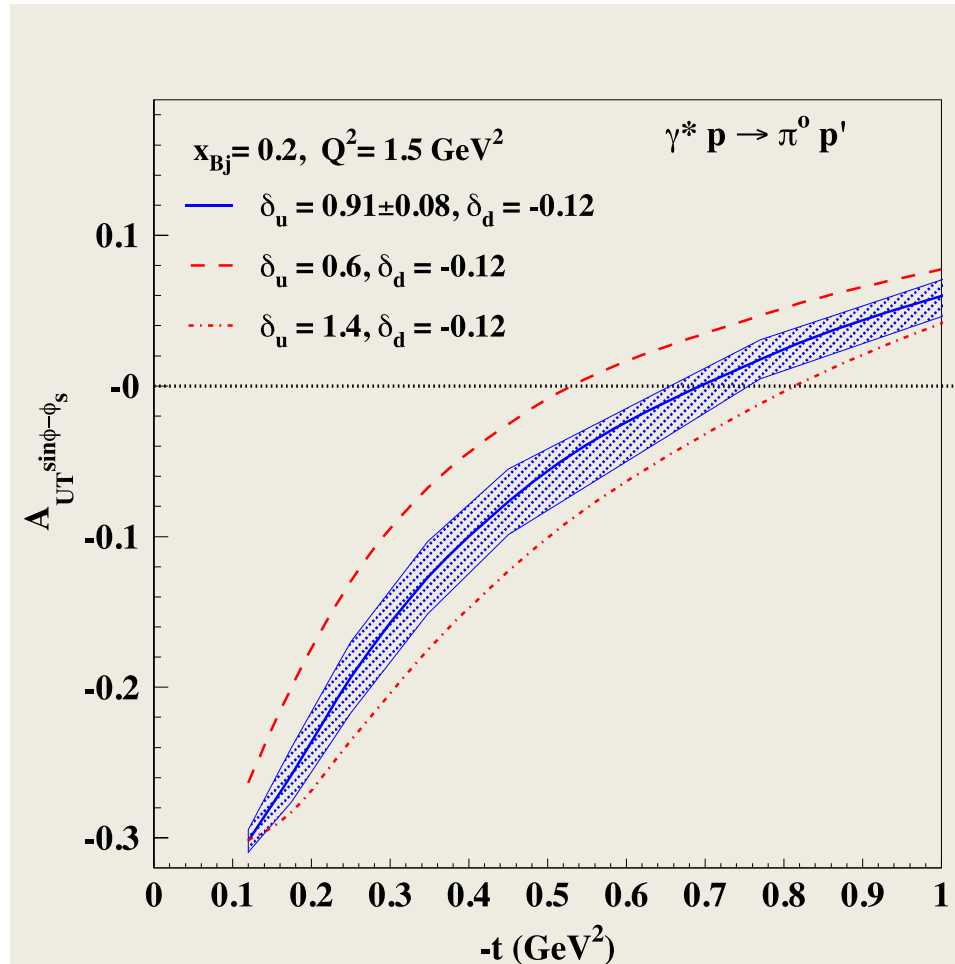
Transverse target



Look for tensor charge in f_{10}^{++}

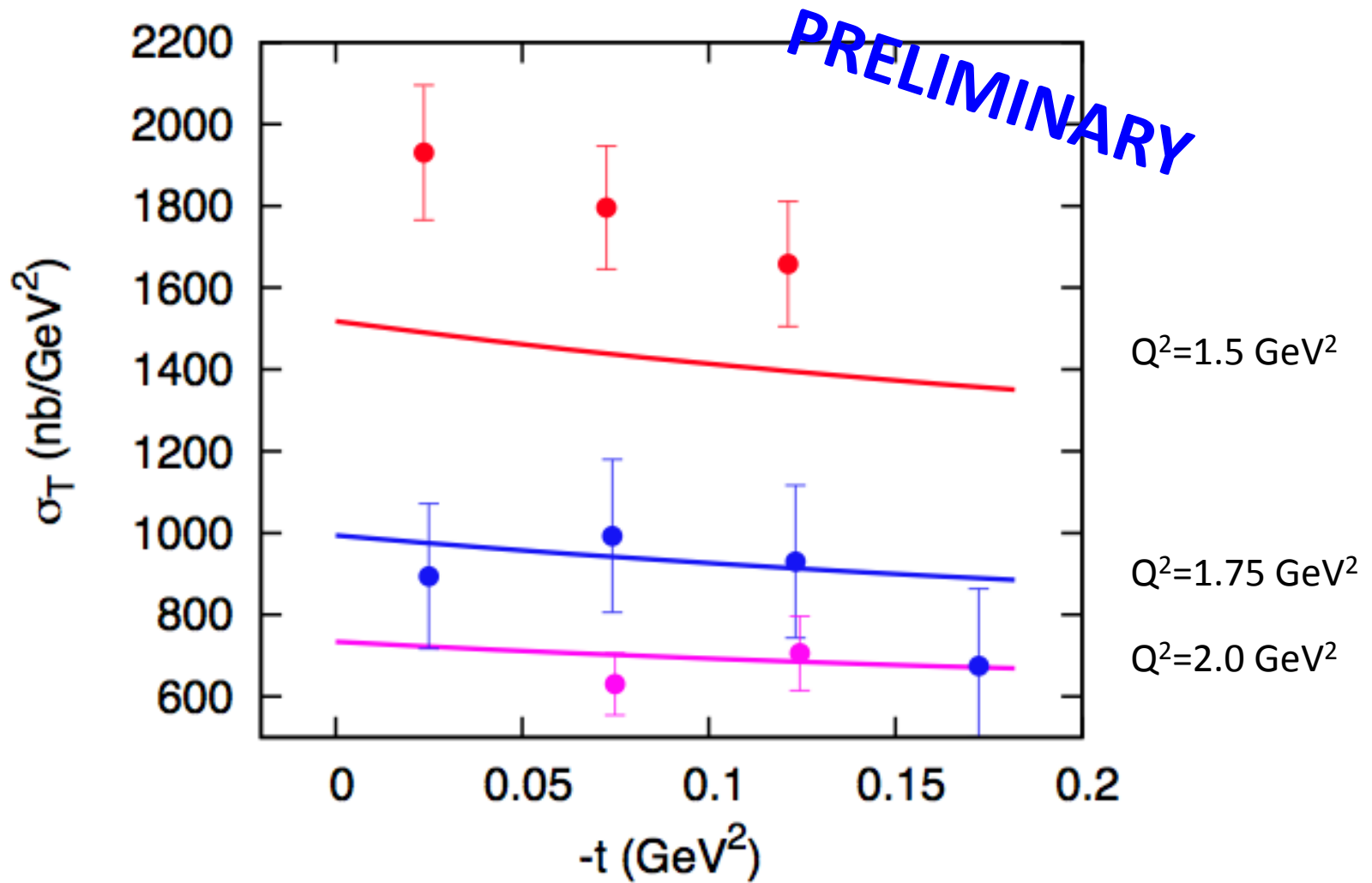
Transverse dipole moment in f_{10}^{++}, f_{10}^{--}

Asymmetry sensitive to tensor charge



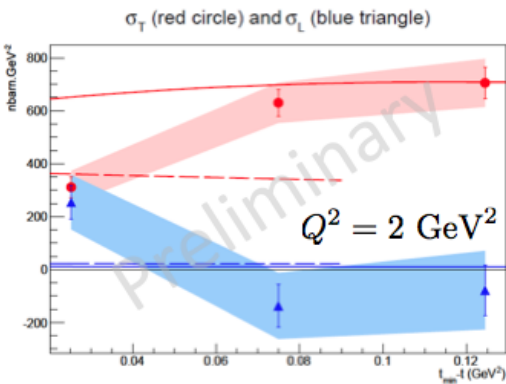
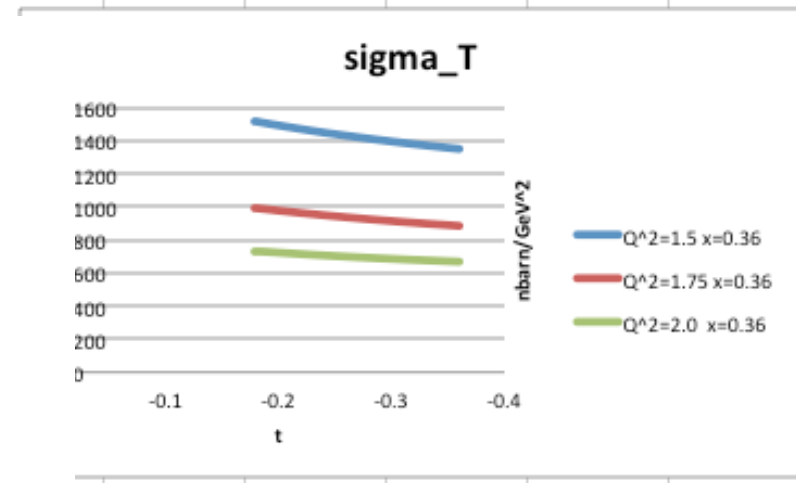
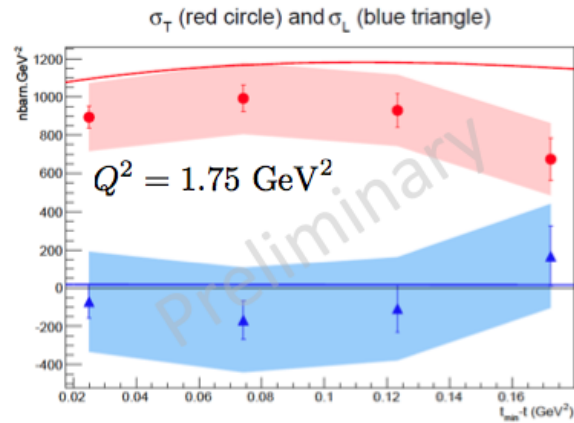
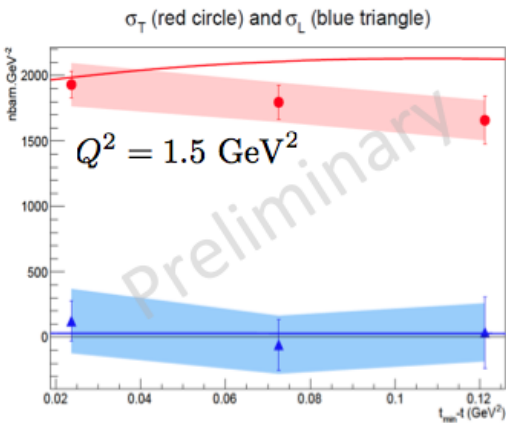
$$A_{UT}^{\sin(\phi+\phi_s)} = -\frac{\epsilon}{2} \frac{F_{UT}^{\sin(\phi+\phi_s)}}{F_{UU,T} + \epsilon F_{UU,L}} = -\epsilon \frac{\Re f_{10}^{+-} \Im m f_{10}^{++} - \Re f_{10}^{++} \Im m f_{10}^{+-}}{d\sigma/dt}$$

Hall A data $x_B=0.36$
courtesy F. Sabatie & M. Defurne



Comparing with new data from Hall A

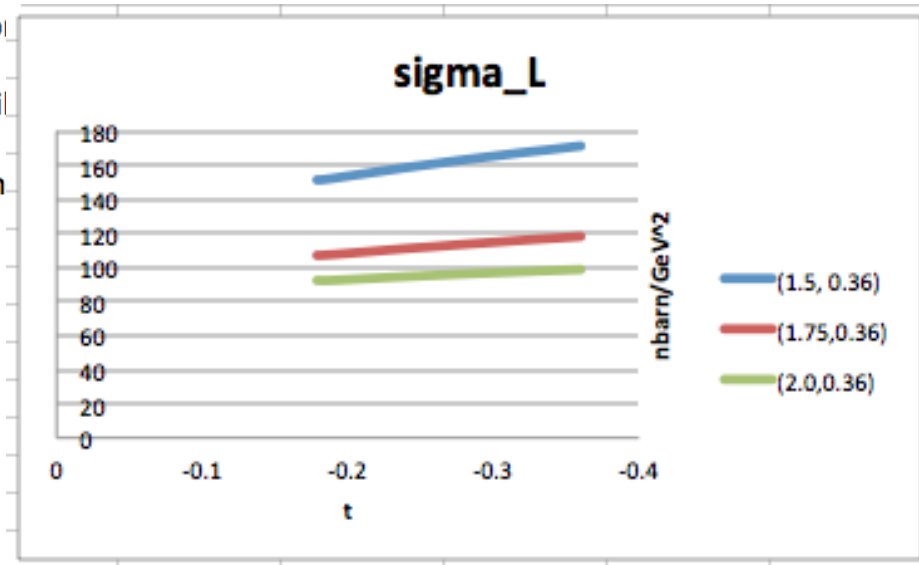
courtesy F. Sabatie, CIPANP



Shaded area: 2% normalization

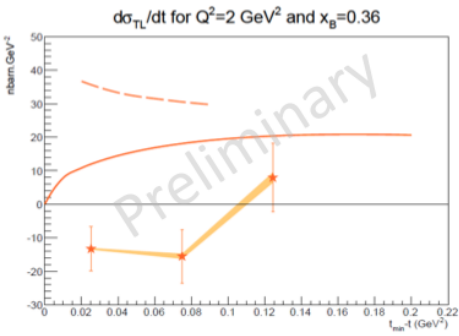
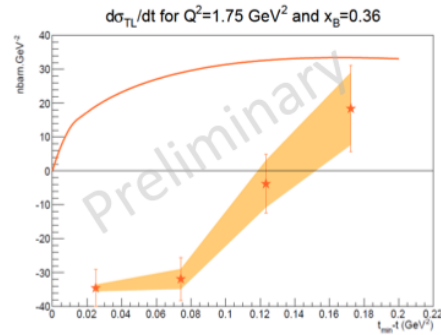
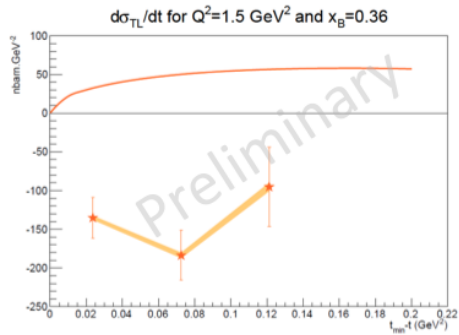
Solid line: GK11 model (descri

Dashed line: Goldstein-Liuti m
(waiting for updated values)



Comparing with new data from Hall A

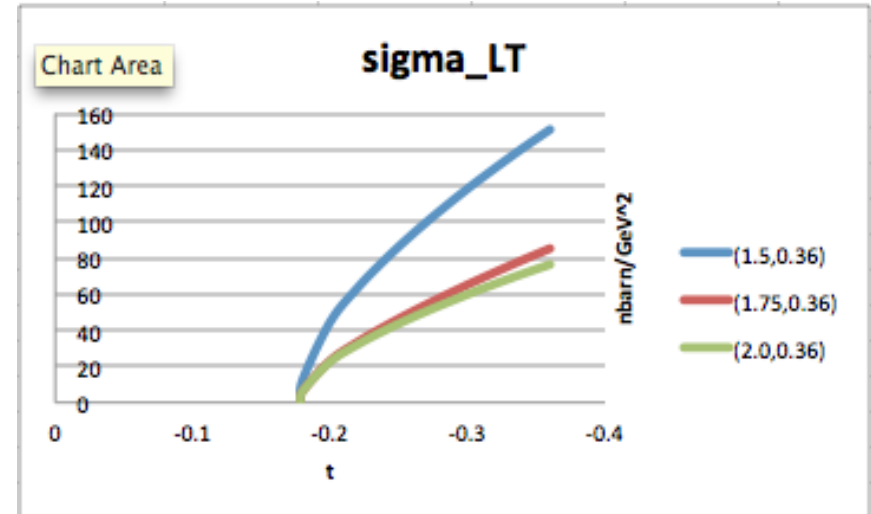
courtesy F. Sabatie, CIPANP



Shaded area: 2% normalization uncertainty

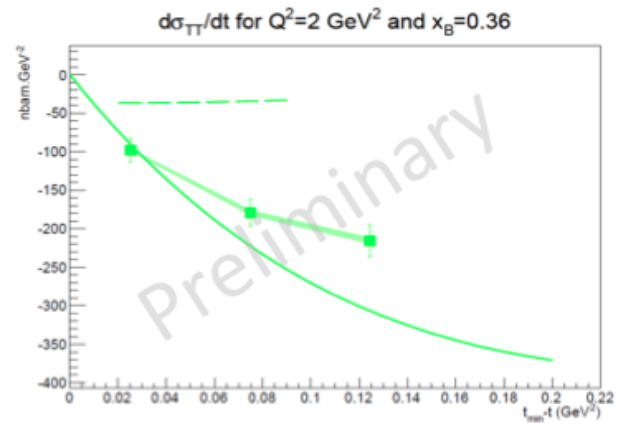
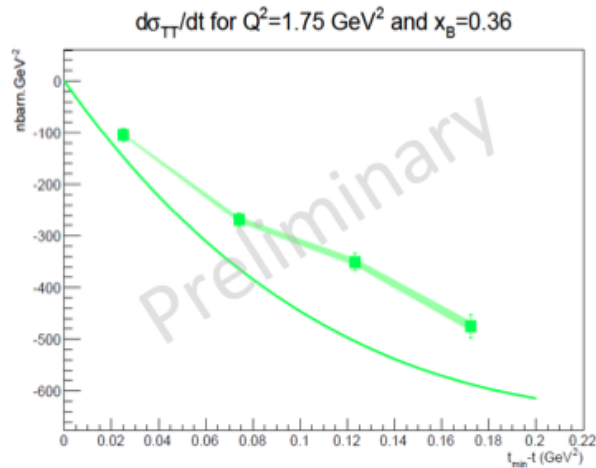
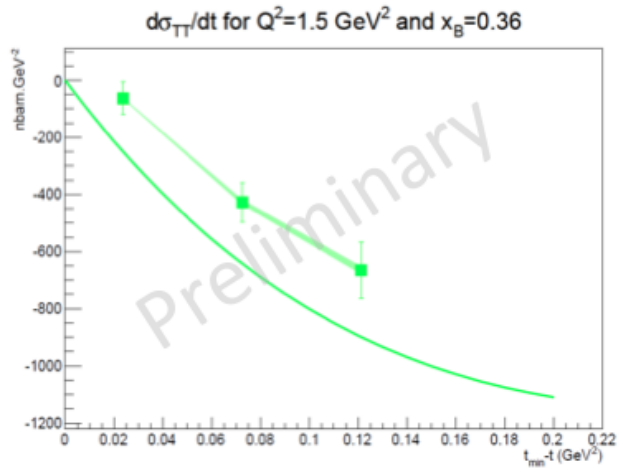
Solid line: GK11 model (described earlier)

Dashed line: Goldstein-Liuti model
(waiting for updated values)



Comparing with new data from Hall A

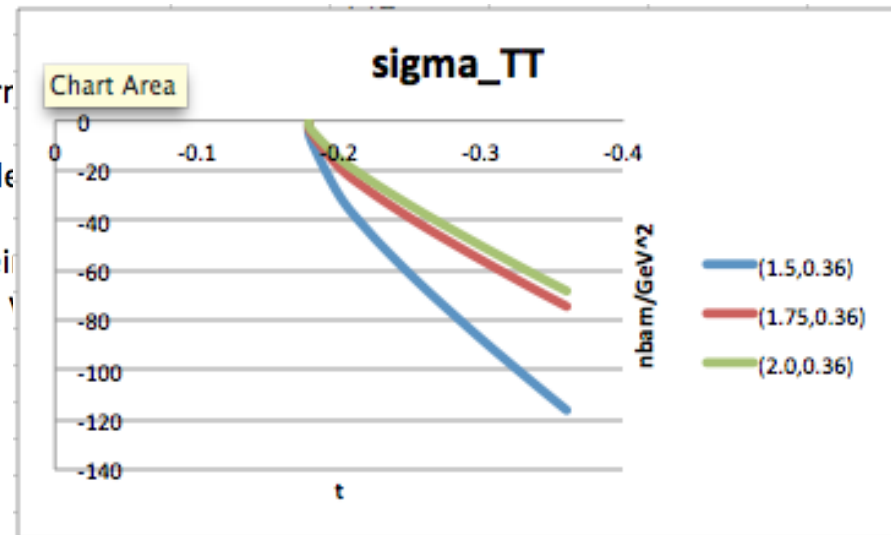
courtesy F. Sabatie, CIPANP



Shaded area: 2% norm

Solid line: GK11 model

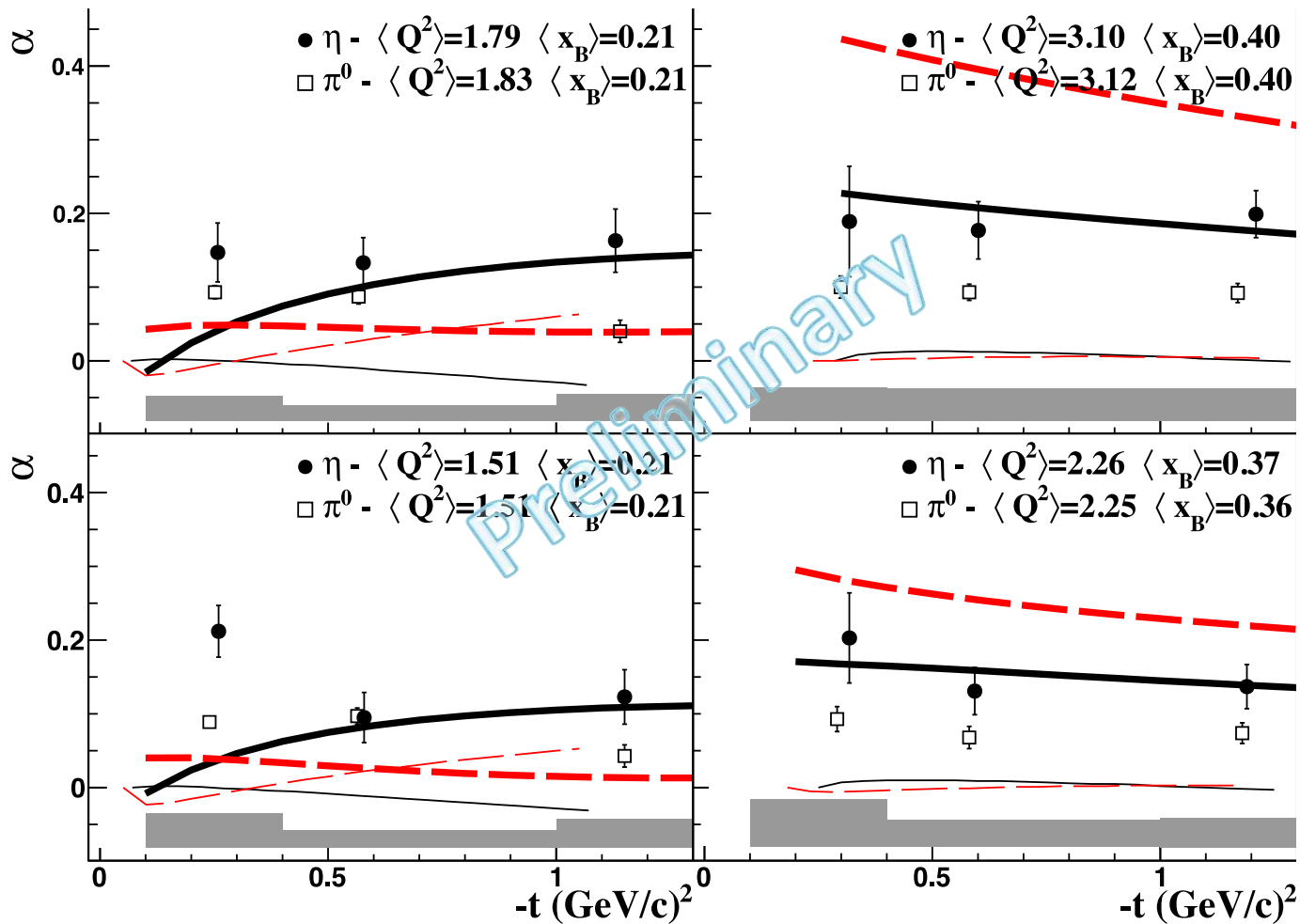
Dashed line: Goldstein
(waiting for updated)



Comparing to other models

- The $t \rightarrow 0$ feature for us is that f_{10}^{+-} dominates & it is driven by H_T .
But f_{10}^{++} & f_{10}^{--} also contribute as $\sim \sqrt{t_0 - t}$, however weaker.
- f_{10}^{++} & f_{10}^{--} are not equal in magnitude, especially vs. ζ or ξ . $\rightarrow E_T^{\sim}$ is significant.
- $\ln A_{LL} \sim |f_{10}^{++}|^2 + |f_{10}^{+-}|^2 - |f_{10}^{-+}|^2 - |f_{10}^{--}|^2$ sensitive to differences
- Our normalization is set by Chiral-odd \leftrightarrow even
- c.f. Goloskokov & Kroll – different dominant amps.

Ratio of unpolarized η / π^0 for flavor decomposition
 (CLAS data in progress, Andrey Kim, Harut Avakian)



t- dependence from EM form factor results

$$R_p^{\alpha, \alpha'} = X^{-[\alpha + \alpha'(X)t + \beta(\zeta)t]}, \quad \longrightarrow$$

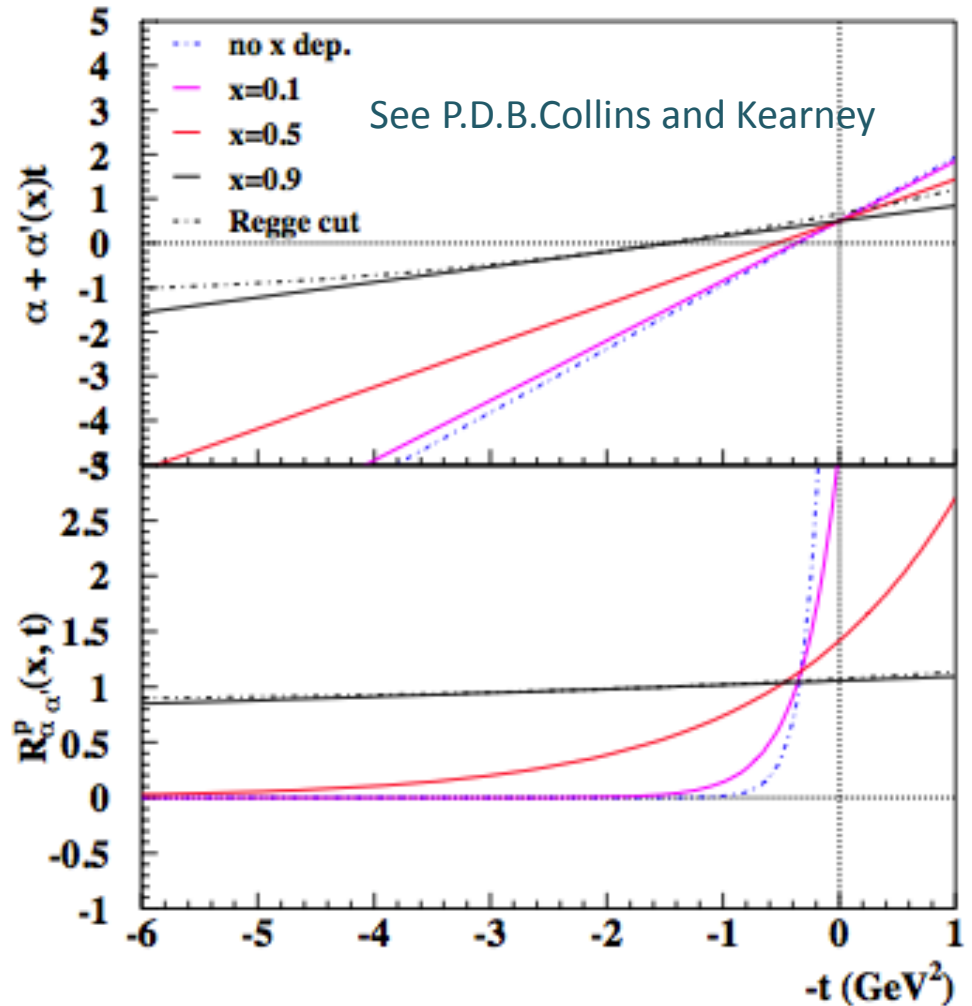
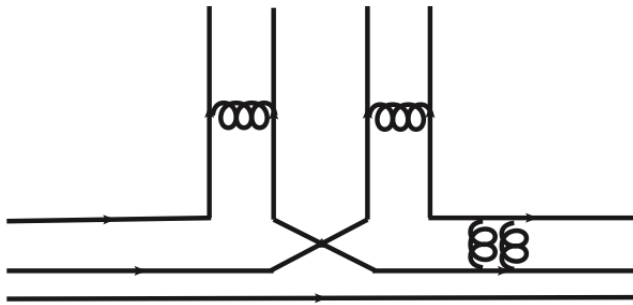
to account for coordinate space behavior (Initially introduced by Radyushkin, Burkardt, ...)

$$\alpha'(X) \equiv \alpha'(1 - X)^p \quad \beta = 0.$$



Now see as effectively taking into account Regge cuts

O. Gonzalez Hernandez, GG, S. Liuti, K. Kathuria
PRC88, 065206 (2013)

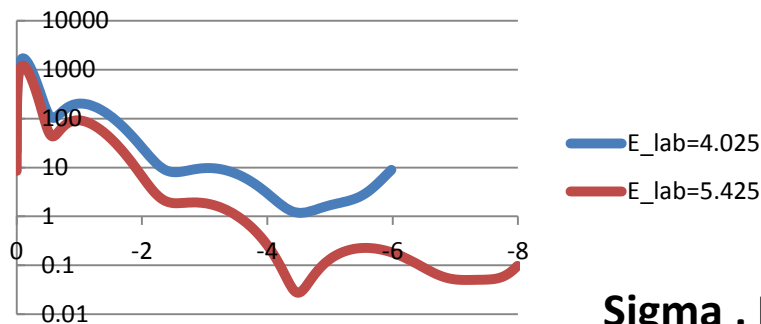


Want can be done with that $\alpha(t)$?

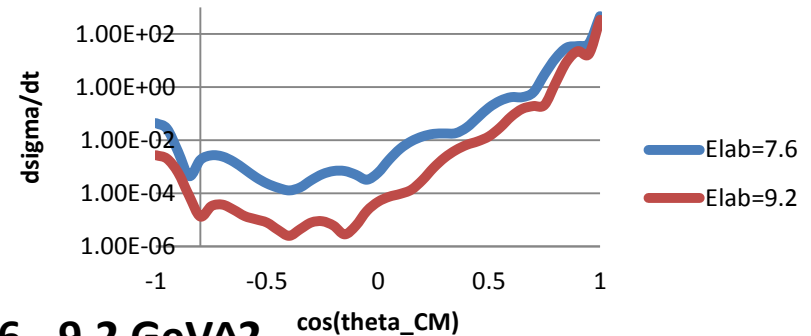
π^0 “photoproduction”

- Regge poles & cuts generate diffractive type minima in $d\sigma/dt$

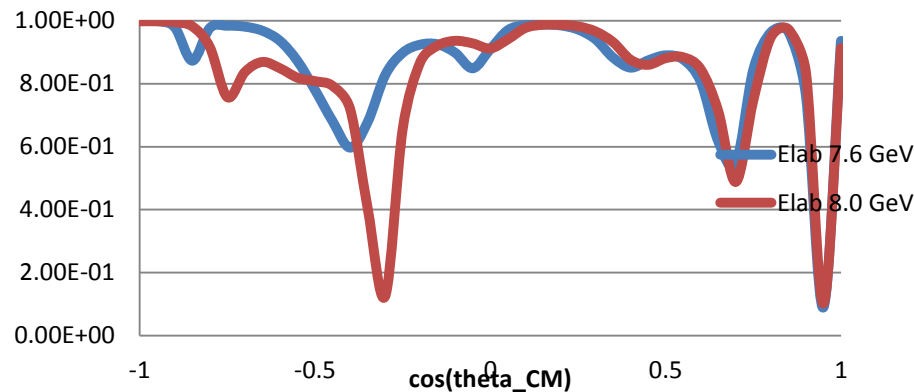
Smaller Regge cuts (3.16)



Elab=7.6-9.2 GeV



Sigma , Elab=7.6 - 9.2 GeV²



Summary

- Flexible parameterization for chiral even from form factors, pdfs & DVCS **R ✖ Dq**
- Extended **R ✖ Dq** to chiral odd sector
- DVMP – π^0 many $d\sigma$'s & Asymmetries measure ***Transversity***
- **Compared to new Hall A data – showed agreement within error bands.**

Backup Slides

Different notations

- $$f_{\Lambda\gamma, \Lambda^N; 0, \Lambda^{N'}} = M_{0, \Lambda^{N'}; \Lambda\gamma, \Lambda^N} = f_{\Lambda\gamma, 0}^{\Lambda_N, \Lambda_{N'}}$$

$$= -(-1)^{\Lambda_\gamma - \Lambda_N + \Lambda_{N'}} f_{-\Lambda\gamma, -\Lambda^N; 0, -\Lambda^{N'}}$$

$$f_1 = f_{1,+1/2; 0,+1/2} = f_{10}^{++} = M_{0,+1/2; +1,+1/2} \text{ or } M_{0+, ++} : \text{single helicity flip}$$

$$f_2 = f_{1,+1/2; 0,-1/2} = f_{10}^{+-} = M_{0-, ++} : \text{non-flip}$$

$$f_3 = f_{1,-1/2; 0,+1/2} = f_{10}^{-+} = M_{0+, +-} : \text{double flip}$$

$$f_4 = f_{1,-1/2; 0,-1/2} = f_{10}^{--} = M_{0-, +-} : \text{single flip}$$

$$f_5 = f_{0,+1/2; 0,-1/2} = f_{00}^{+-} = M_{0-, 0+} : \text{single flip}$$

$$f_6 = f_{0,+1/2; 0,+1/2} = f_{00}^{++} = M_{0+, 0+} : \text{non-flip}$$

n helicity flips get factor $[\sqrt{(t_0 - t) / 2M}]^n = \Delta^n / (2M)^n$

Polynomiality!

GRG, O. Gonzalez Hernandez,
S. Liuti, PRD84, 034007 (2010).

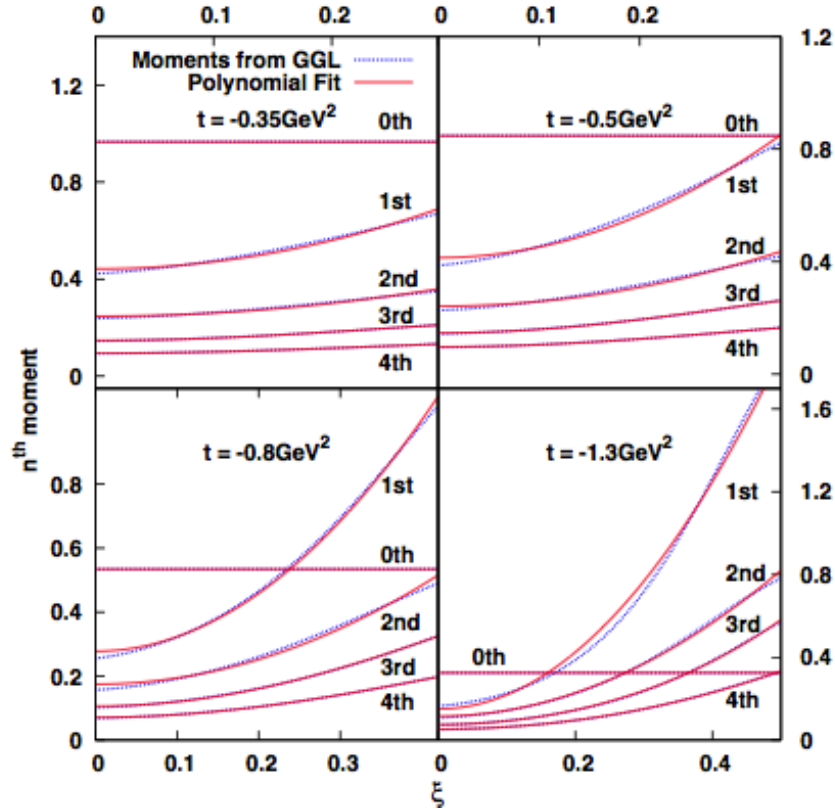
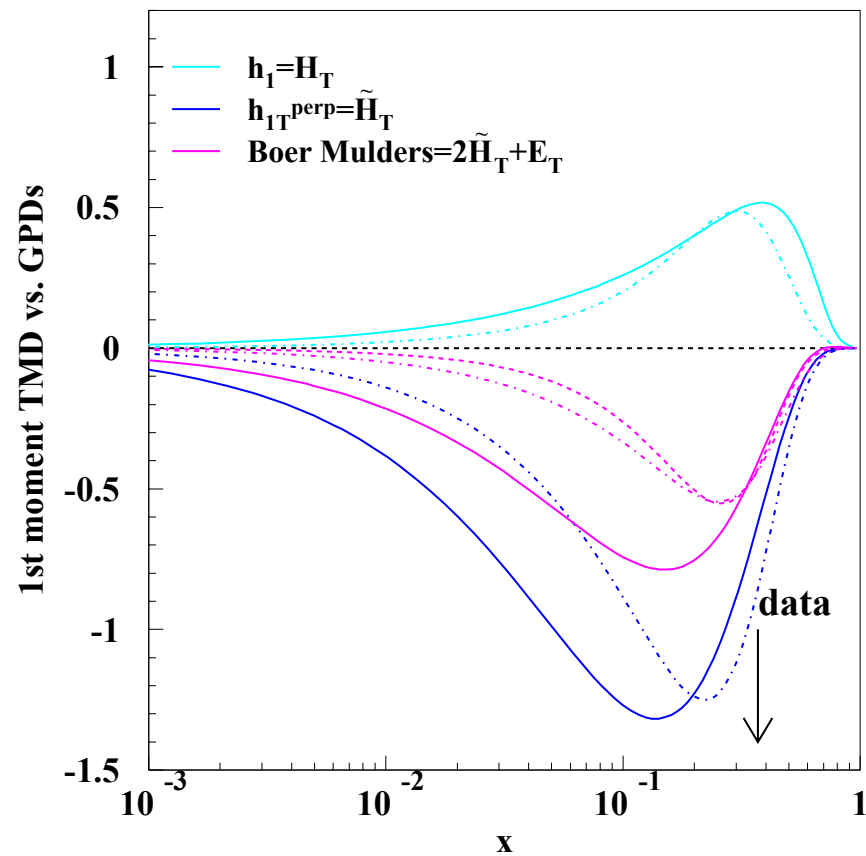
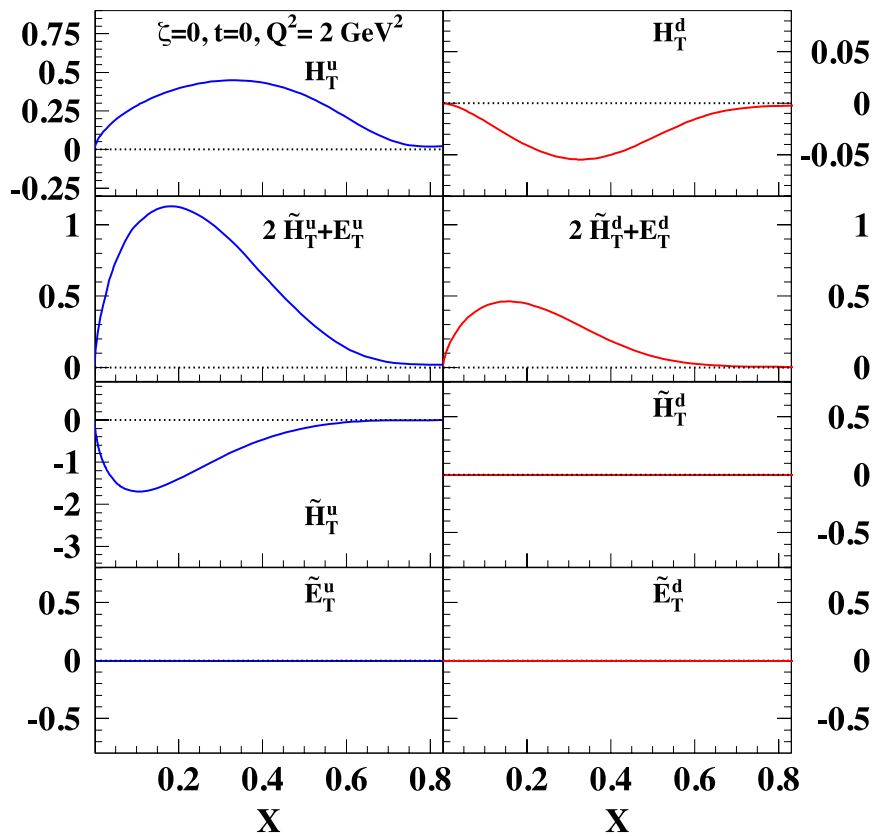


FIG. 8 (color online). The polynomiality property in our parametrization. The dashed lines are the theoretical results using the parametrization from this paper; the solid lines correspond to a polynomial fit in §2 to the theoretical curves. The two sets of curves display an excellent agreement with each other, thus demonstrating that polynomiality is satisfied in our model.

the minimal number of parameters
 necessary to fit X and t ?" Results of Recursive Fit

Parameters	H	E	\tilde{H}	\tilde{E}
m_u (GeV)	0.420	0.420	2.624	2.624
M_X^u (GeV)	0.604	0.604	0.474	0.474
M_Λ^u (GeV)	1.018	1.018	0.971	0.971
α_u	0.210	0.210	0.219	0.219
α'_u	2.448 ± 0.0885	2.811 ± 0.765	1.543 ± 0.296	5.130 ± 0.101
p_u	0.620 ± 0.0725	0.863 ± 0.482	0.346 ± 0.248	3.507 ± 0.054
\mathcal{N}_u	2.043	1.803	0.0504	1.074
χ^2	0.773	0.664	0.116	1.98
m_d (GeV)	0.275	0.275	2.603	2.603
M_X^d (GeV)	0.913	0.913	0.704	0.704
M_Λ^d (GeV)	0.860	0.860	0.878	0.878
α_d	0.0317	0.0317	0.0348	0.0348
α'_d	2.209 ± 0.156	1.362 ± 0.585	1.298 ± 0.245	3.385 ± 0.145
p_d	0.658 ± 0.257	1.115 ± 1.150	0.974 ± 0.358	2.326 ± 0.137
\mathcal{N}_d	1.570	-2.800	-0.0262	-0.966
χ^2	0.822	0.688	0.110	1.00



Observables

Chiral Even

$$A_{L'\pm, L\pm} \Leftrightarrow H, E, \tilde{H}, \tilde{E}$$

Chiral Odd

$$A_{\Lambda'\pm, \Lambda\mp} \Leftrightarrow H_T, E_T, \tilde{H}_T, \tilde{E}_T$$

Flexible parameterization for chiral even from form factors, pdfs & DVCS **R** \times **Dq**

Compton Form Factors

$$\mathcal{H}(\xi, t; Q^2) = \int dx \left[\frac{1}{x - \xi - i\epsilon} \mp \frac{1}{x + \xi - i\epsilon} \right] H(x, \xi, t; Q^2)$$

$$\rightarrow \left(P.V. \int dx \frac{H(x, \xi, t; Q^2)}{x - \xi} + i\pi H(\xi, \xi, t; Q^2) \right) \mp (\text{symm. term})$$

Re H

Im H