Towards a Direct Measurement of the Quark Orbital Angular Momentum Distribution

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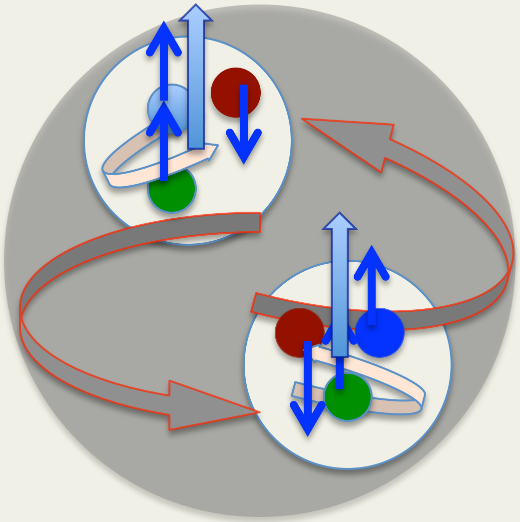

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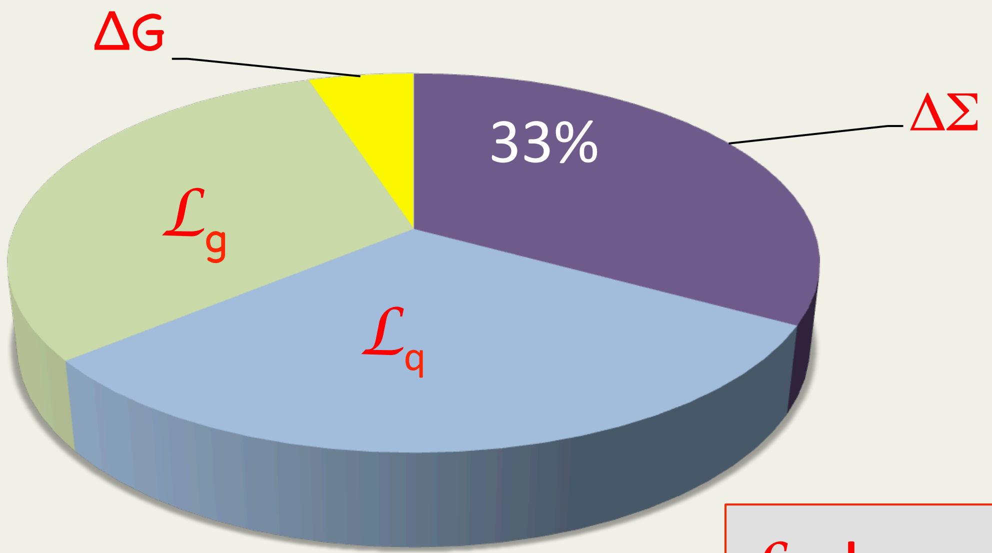
In collaboration with: Aurore Courtoy, Michael Engelhardt, Abha Rajan

The spin crisis in a cartoon

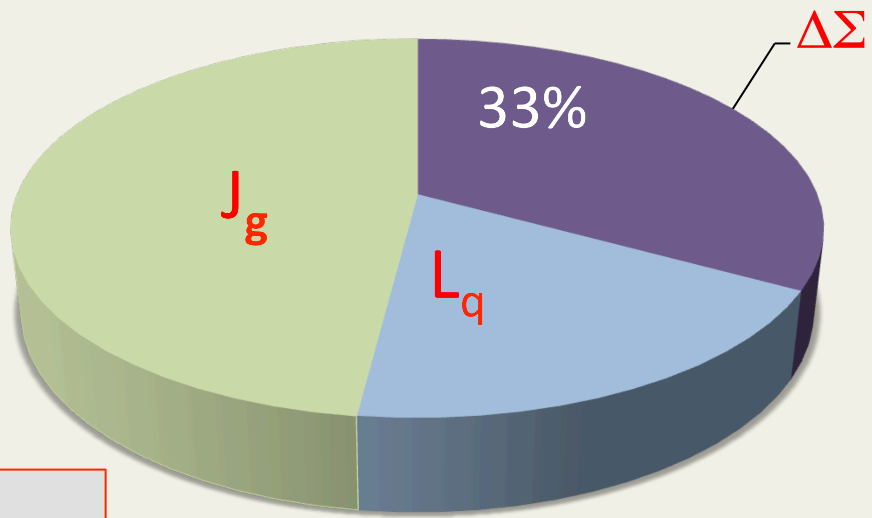
Jaffe Manohar

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$



Ji

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

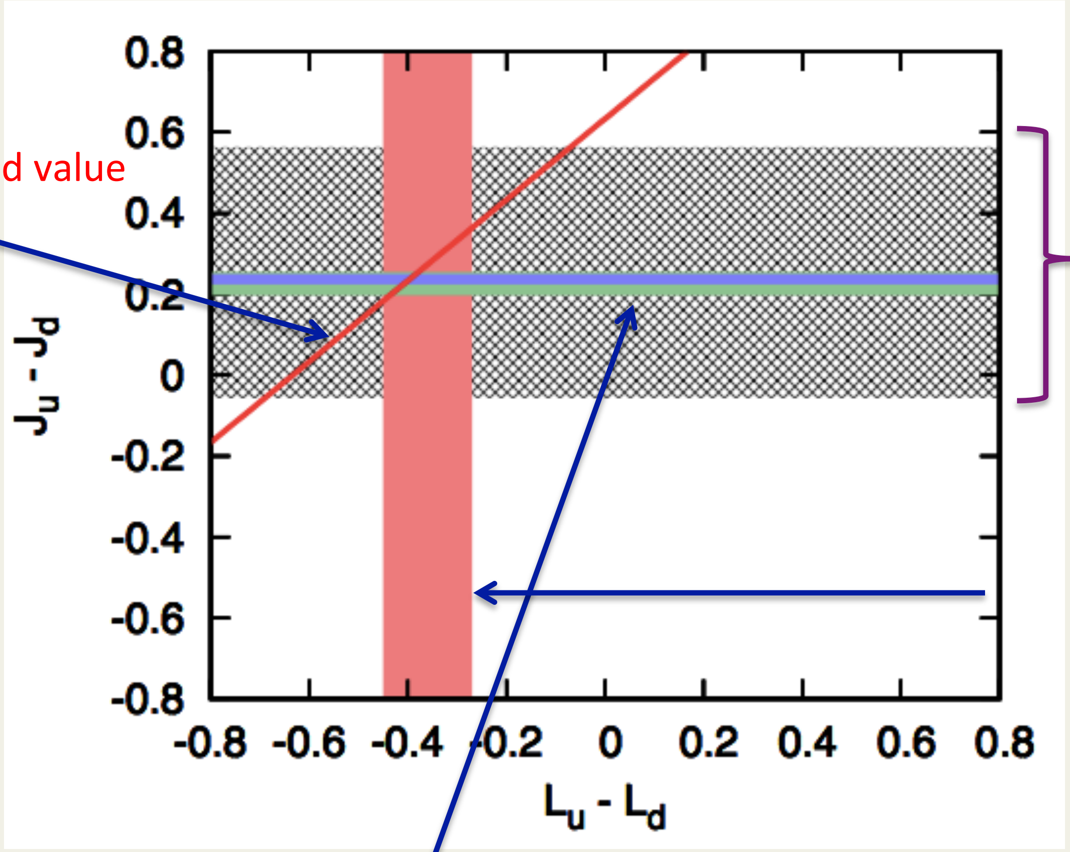


$\mathcal{L}_q \neq L_q$

$J_g \neq \mathcal{L}_g + \Delta G$

Ji's Sum Rule: $J_q = L_q + \frac{1}{2}\Delta\Sigma_q$ ➔ three independently measured quantities

Using the measured value of $\Delta\Sigma_{u-d}$



Mazous et al. PRL (2007)

DVCS + VGG

M. Engelhardt, this meeting

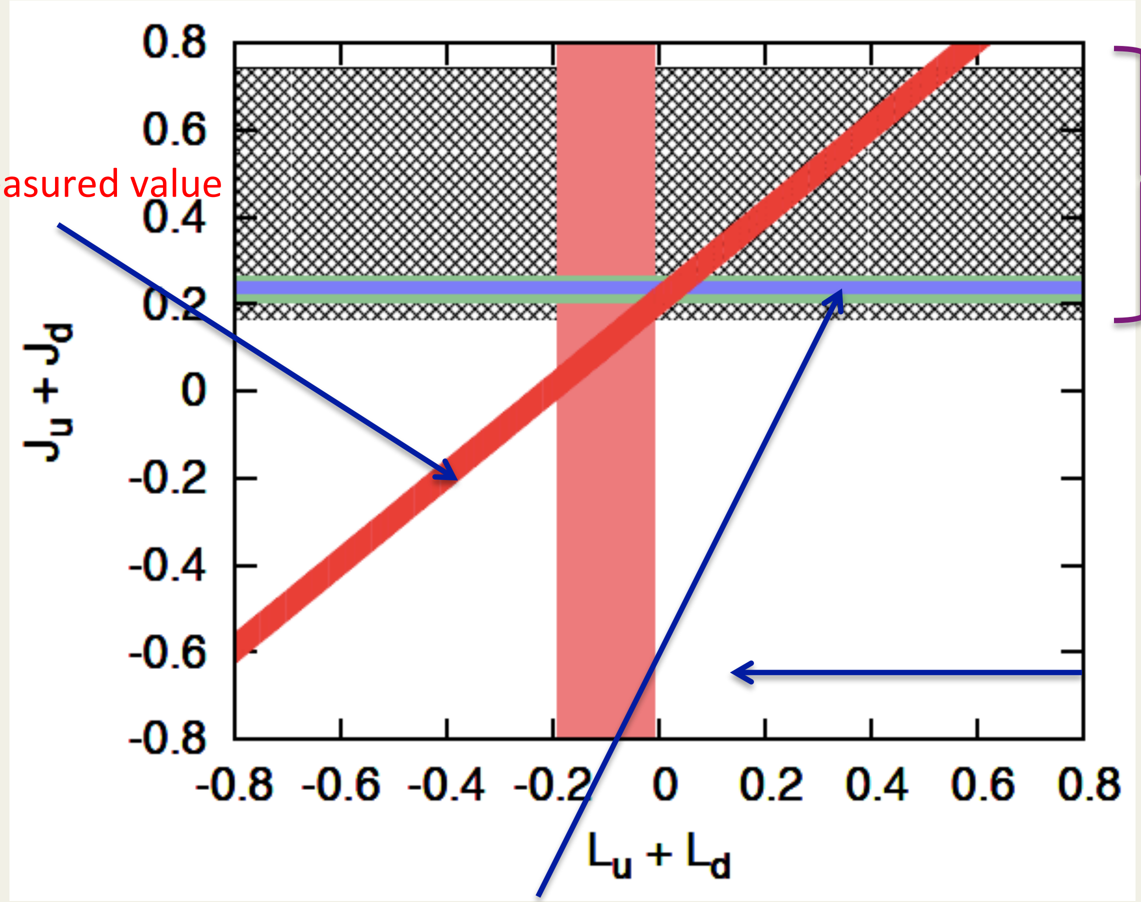
Lattice QCD evaluation of GTMD F_{14}

- O. Gonzalez Hernandez et al., Phys. Rev. C88; arXiv:1206.1876
- M. Diehl and P. Kroll, Eur. Phys. J. C73; arXiv:1302.4604

Using flavor separated nucleon form factors from Jlab

Ji's Sum Rule Singlet: $J_q = \frac{1}{2} - J_g^{Ji} = L_q^{Ji} + \frac{1}{2}\Delta\Sigma_q$

Using the measured value of $\Delta\Sigma_{u+d}$



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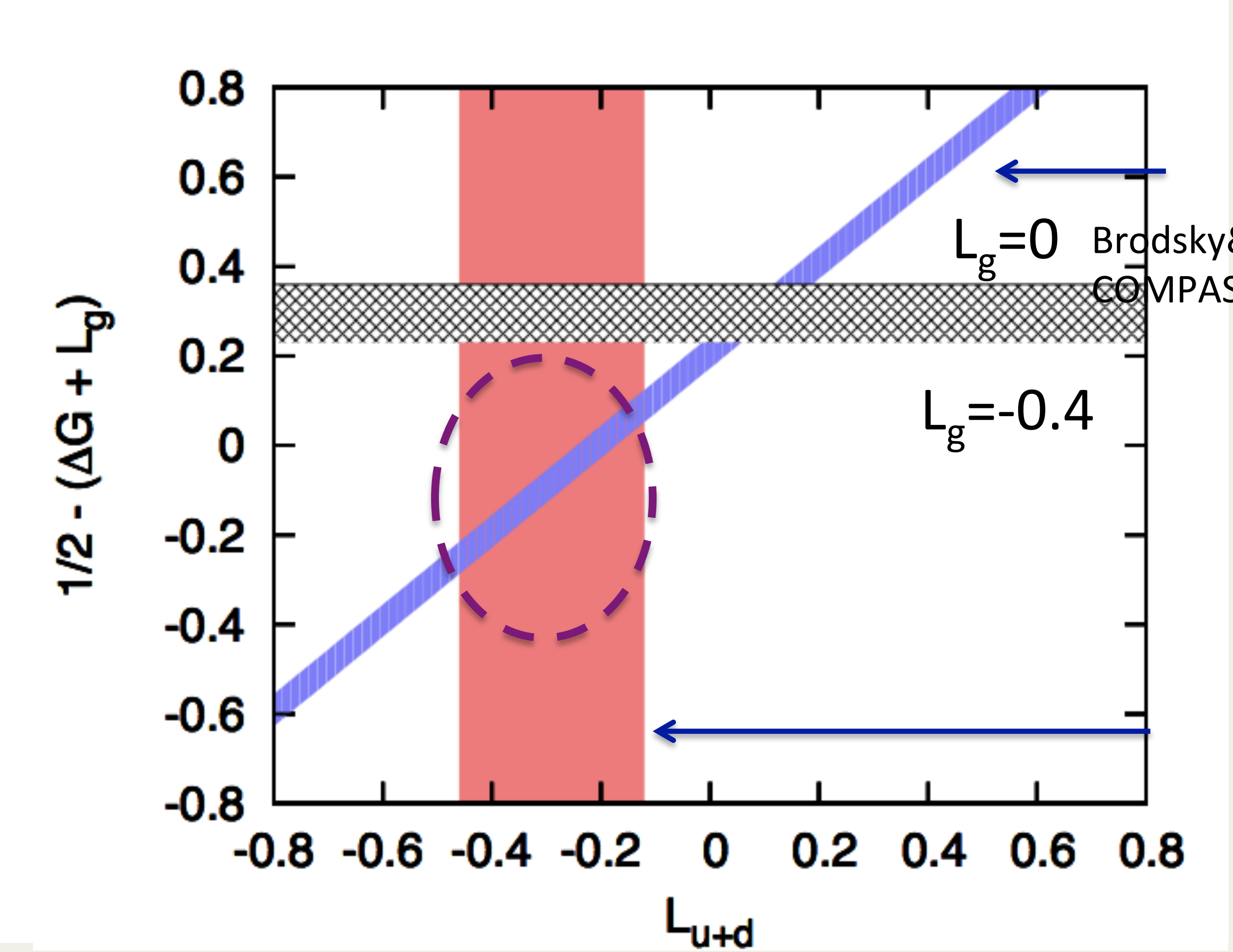
Using flavor separated nucleon form factors from Jlab

Jaffe Manohar's Sum Rule: $$\frac{1}{2} - (\Delta G + L_g^{JM}) = L_q^{JM} + \frac{1}{2}\Delta\Sigma_q$$

➔ four independently measured quantities



Using the measured value of $\Delta\Sigma_{u+d}$

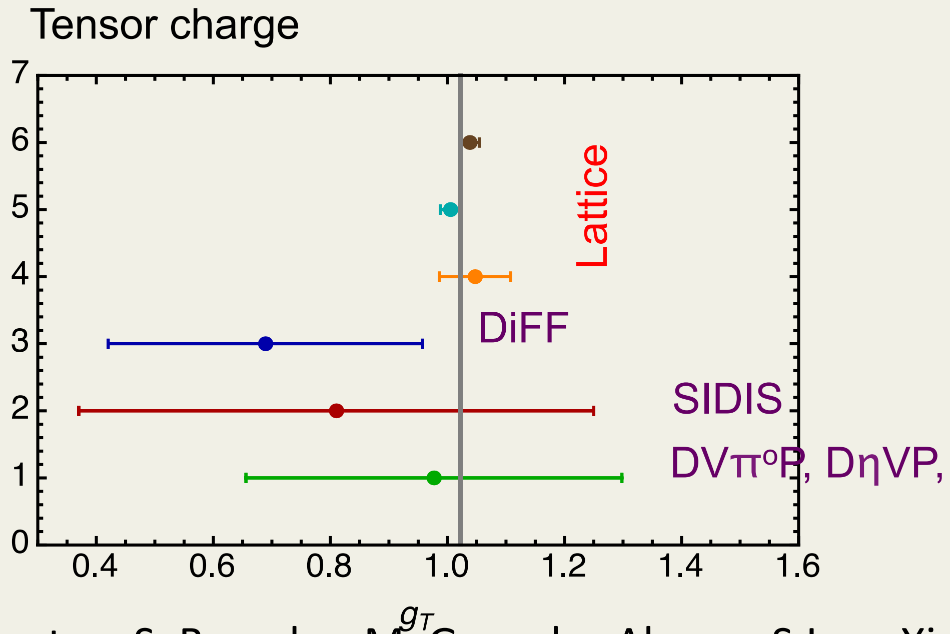
Brodsky&Gardner using COMPASS data PLB 2011

Using the measured value of ΔG

M. Engelhardt, this meeting

Lattice QCD evaluation of GTMD F_{14}+ gauge link

The Electron Ion Collider will allow us to access partonic Orbital Angular Momentum experimentally



A. Courtoy, S. Baessler, M. Gonzalez Alonso, S.L., arXiv:1503.06814 & PRL 2015

Angular Momentum Sum Rules

Express the various components, $\Delta\Sigma$, ΔG, L_q, J_q, J_g, $\mathcal{L}_q$, $\mathcal{L}_g$ in terms of non-local light-cone operators of twist 2 and twist 3.

Jaffe Manohar: in LC gauge rewrite AM using Dirac eqn. to isolate spin terms

$$M^{+12} = \psi^\dagger \sigma^{12}\psi + \psi^\dagger\left[\vec{x}\times(-i\partial)\right]^3\psi + Tr(\varepsilon^{+-ij}F^{+j}A^j) + 2iTrF^{+j}(\vec{x}\times\partial)A^j$$

$\Delta\Sigma$ $\mathcal{L}_q$ ΔG $\mathcal{L}_g$

Ji:

$$M^{+12} = \psi^\dagger\ \sigma^{12}\ \psi + \psi^\dagger\left[\vec{x}\times\left(-i\vec{D}\right)\right]^3\psi + \left[\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right]^3$$

$\Delta\Sigma$ L_q J_g

1. **The two sum rules give equivalently valid representations**

2. **Different mechanisms for generating OAM in the proton can coexist**

3. **The ones that will make physical sense are the ones that can be measured**

Generalized Transverse Momentum Distribution F_{14} Description

OAM represents the correlation between the position and momentum of the quarks and gluons

$$\mathcal{L}_q, L_q = \int dx\, d^2b\, d^2k_T \left(\vec{b} \times \vec{k}_T\right)_3 \mathcal{W}\left(x, \vec{b}, \vec{k}_T\right)$$

Hatta (2011)
Lorce, Pasquini (2011)

GTMD

OAM Wigner Distribution

$$\mathcal{W}\left(x, \vec{b}, \vec{k}_T\right) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\Delta_T \cdot b} \int \frac{d^2 z_T d^2 z^-}{(2\pi)^3} e^{i(xP^+ z^- - k_T \cdot z_T)} \langle P', \Lambda' | \overline{\psi}(0)\gamma^+ U(0,\infty \mid n)\psi(z) | P, \Lambda \rangle \Big|_{z^+=0}$$

Hatta (2011)
Burkardt(2013)

Wilson-line gauge link, "n" defines the path along which we evaluate the vector potential

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Simonetta Liuti
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The Wigner distribution defining OAM through a ($b \times k_T$) term is the Fourier transform of a GTMD called F_{14} with coefficient ($\Delta_T \times k_T$) (Lorce, Pasquini, 2011; Meissner, Metz, Schlegel, 2009)

$$
\begin{aligned}
W^{\gamma^+}_{\Lambda\Lambda'} &= \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\gamma^+ F_{11} + \frac{i\sigma^{i+}\Delta_T^i}{2M}(2F_{13}-F_{11}) + \frac{i\sigma^{i+}\bar{k}_T^i}{2M}(2F_{12}) + \frac{i\sigma^{ij}\bar{k}_T^i\Delta_T^j}{M^2}F_{14}\right]U(p,\Lambda) \\
&= \delta_{\Lambda,\Lambda'}F_{11} + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\Delta_1 - i\Delta_2}{2M}(2F_{13}-F_{11}) + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\bar{k}_1 - i\bar{k}_2}{2M}(2F_{12}) + \delta_{\Lambda,\Lambda'}\, i\Lambda\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14}
\end{aligned}
$$

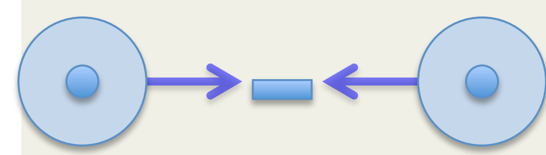
vector

A_{UL} correlation

helicity non-flip

A.Courtoy, G.Goldstein, O.Gonzalez Hernandez, S.Liuti and A.Rajan, PLB 731(2014)

$$
\begin{aligned}
i\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14} &= A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--} \\
-i\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}G_{11} &= A_{++,++} - A_{+-,+-} + A_{-+,-+} - A_{--,--}
\end{aligned}
$$



$$L_q = -\int_{-1}^{1} dx \int d^2 k_T \, \frac{k_T^2}{M^2} F_{14}(x, 0, k_T, 0, 0) = -F_{14}^{(1)}$$

Lorce&Pasquini, Hatta, Metz, Schlegel

Twist three Generalized Parton Distribution -- Polyakov's relation

In the hypothesis that a genuine twist three term integrates to 0 (generalized Burkhardt Cottingham SR) Ji's OAM distribution is identified with a twist 3 GPD

$$L_q(x) = -xG_2(x)$$

- ✓ Define twist three GPDs

$$W^{\gamma^i}_{\Lambda'\Lambda} = \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\frac{\Delta^i_T}{M}G_1 + \frac{i\sigma^{ji}\Delta_j}{M}G_2 + \frac{Mi\sigma^{i+}}{P^+}G_4 + \frac{\Delta^i_T}{P^+}\gamma^+ G_3\right]U(p,\Lambda),$$

Polyakov et al. (2000), Hatta&Yoshida (2012)

- ✓ Derive Wandzura Wilczeck relation using OPE tw 2 and tw 3 operators.
- ✓ Take off-forward matrix elements

Similarly to the forward case,

$$G_2(x,0,t,Q^2) = \left[\int_x^1 \frac{dy}{y}(H+E) - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \bar{G}_2$$

genuine twist three

Generalized Wandzura Wilczek relation

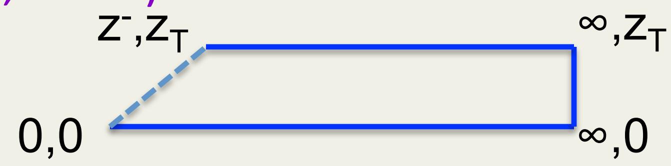
$$\underbrace{xG_2(x)}_{t=3} = \underbrace{-x\int_x^1 \frac{dy}{y}\left[H(x,0,0)+E(x,0,0)\right] + x\int_x^1 \frac{dy}{y^2}\,\tilde{H}(x,0,0)}_{G_2^{WW}\;\rightarrow\;\tau=2} + \underbrace{\left[\bar{G}_2^{tw3} - \int_x^1 \frac{dy}{y}\,\bar{G}_2^{tw3}\right]}_{\tau=3}$$

What do the two descriptions have in common?

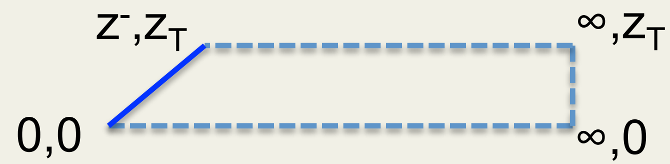
F_{14} can describe both Ji and Jaffe-Manohar OAM

Evaluate gauge links (Hatta Yoshida, 2012; Burkardt, 2013)

$\mathcal{L}_q^{JM}$ ➔ "staple" gauge link,



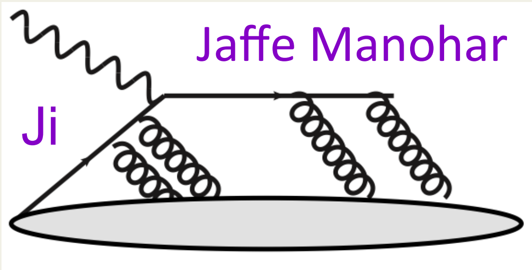
L_q^{Ji} ➔ "straight" gauge link



The difference of the two gives a torque,

$$\mathcal{L}_q^{JM} - L_q^{Ji} = \int \frac{d^2 z_T \, dz^-}{(2\pi)^3} \langle P',\Lambda' | \bar{\psi}(z)\gamma^+(-g)\int_{z^-}^{\infty} dy^- U\left[z_1 G^{+1}(y^-) - z_2 G^{+2}(y^-)\right] U\psi(z) | P,\Lambda \rangle \Big|_{z^+=0}$$

$\vec{\tau} = \vec{r} \times \vec{F}$ ➔ "Chromodynamic torque"*



*a Qiu-Sterman term type term analogous to f_{1T}^{perp}

$$G_2 \equiv \tilde{E}_{2T}$$

Meissner, Metz and Schlegel, JHEP(2009)

There exists a connection between F_{14} and G_2 that uncovers different types of quark-gluon interactions behind the Jaffe Manohar and Ji mechanisms for generating OAM

(A. Courtoy, M. Engelhardt, S. L., A. Rajan, 2015)

→ a unique probe of quark-gluon interactions

Using directly the unintegrated quark-quark correlator defining GTMDs,

$$W_{\Lambda\Lambda'}^{\sigma^{i+}\gamma_5} = \int \frac{d^2 z_T d^2 z^-}{(2\pi)^3} e^{ixP^+ z^- - i\vec{k}_T \cdot \vec{z}_T} \langle P', \Lambda' | \overline{\psi}(0) i\sigma^{i+}\gamma_5 \psi(z) | P, \Lambda \rangle \Big|_{z^+=0}$$

… and the Equations of Motion,

$$\int \frac{d^2 z_T d^2 z^-}{(2\pi)^3} e^{ixP^+ z^- - i\vec{k}_T \cdot \vec{z}_T} \langle P', \Lambda' | \overline{\psi}(0) i\sigma^{i+} [i\not{D}(0) - m] \psi(z) | P, \Lambda \rangle \Big|_{z^+=0} = 0$$

$$iD_\mu = i\partial_\mu + gA_\mu$$

$$i\not{D} = i\partial\_\gamma^+ - \partial_T \cdot \gamma_T + g\not{A}$$

Sum rules relating GTMD and twist 3 GPD descriptions

Lorentz Invariant Relation (LIR)

$$\frac{d}{dx}F_{14}^{(1)} = \tilde{E}_{2T} \quad \Rightarrow \quad F_{14}^{(1)} = -\int_x^1 dy\, \tilde{E}_{2T}(y) \quad \Rightarrow \int_{-1}^{1} dx F_{14}^{(1)} = \int_{-1}^{1} dx x \tilde{E}_{2T}$$

Equations of Motion (EoM) relation

$$\int_{-1}^{1} dx x \tilde{E}_{2T} = -\int_{-1}^{1} dx x (H+E) + \int_{-1}^{1} dx \tilde{H} + \int_{-1}^{1} dx G^{tw3}$$

In Ji's picture F_{14} and $\bar{E}_{2T}$ give us similar information on OAM!

Compare with gauge-links derivation

$$\int d^2k_T \frac{k_T^2}{M^2} F_{14}^{JM}\left(x,0,\vec{k}_T\right) = \int d^2k_T \frac{k_T^2}{M^2} F_{14}^{Ji}\left(x,0,\vec{k}_T\right) + \text{"Qiu - Sterman"}$$

$$\Updownarrow$$

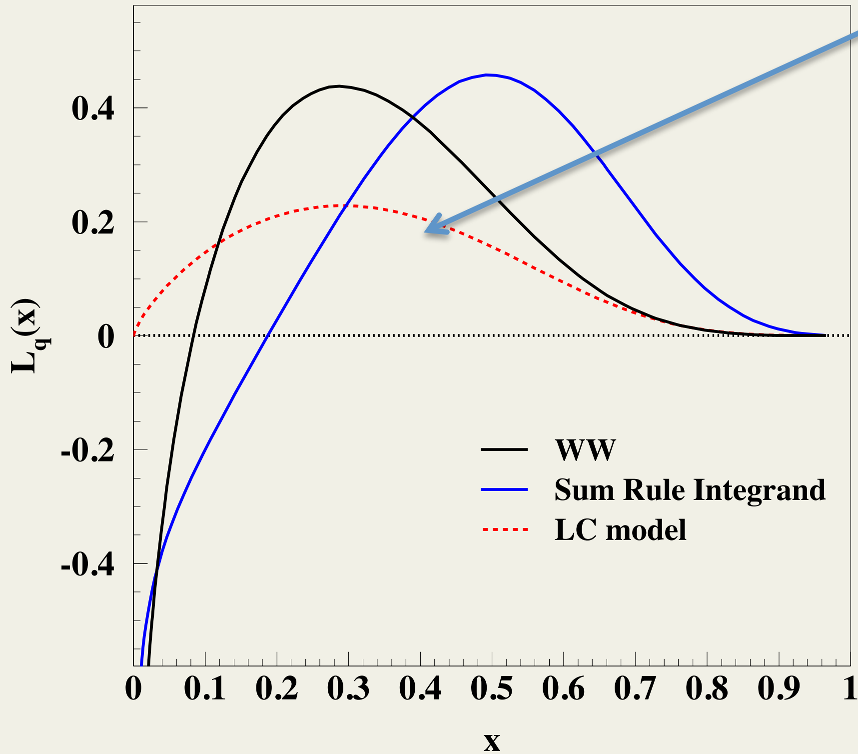
$$\int d^2k_T \frac{k_T^2}{M^2} F_{14}^{JM}\left(x,0,\vec{k}_T\right) = \int d^2k_T \frac{k_T^2}{M^2} F_{14}^{Ji}\left(x,0,\vec{k}_T\right) + \text{"}tw3^{Ji}\text{"} - \text{"}tw3^{JM}\text{"}$$

We are investigating further this connection

x dependence

$$L_q(x,0,0) = x\int_x^1 \frac{dy}{y}(H_q(y,0,0)+E_q(y,0,0)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y,0,0),$$

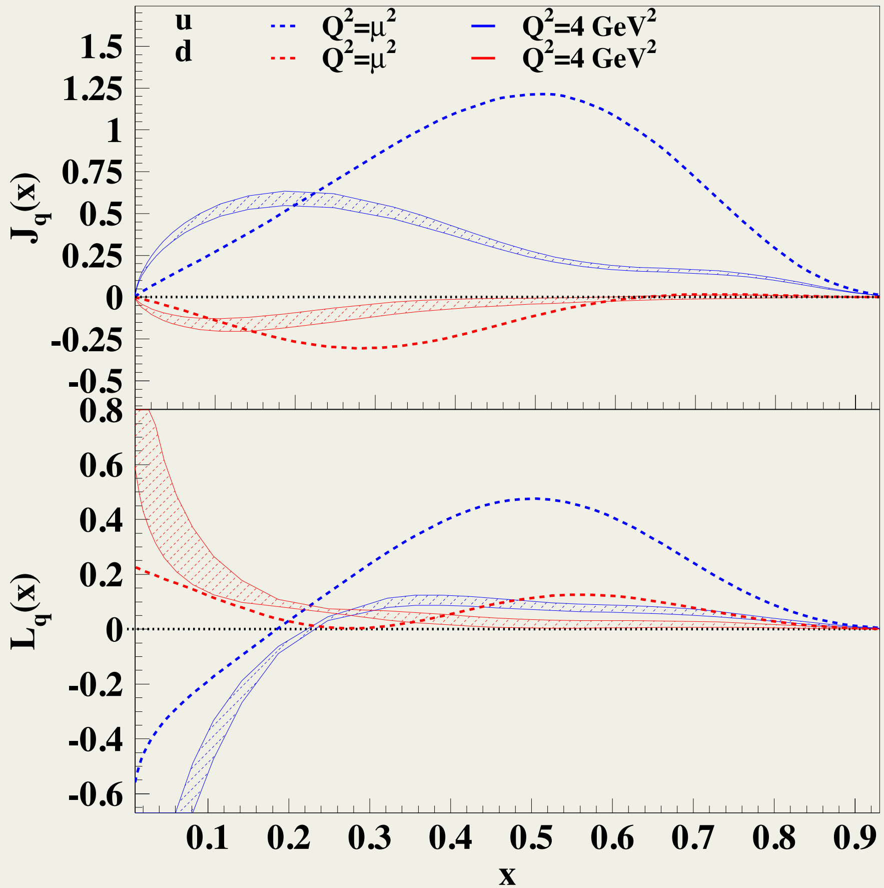
≠F_{14}!



u quark

GPDs calculated in
Reggeized diquark model
GGL PRD (2010),
O. Gonzalez et al, PRC(2013)

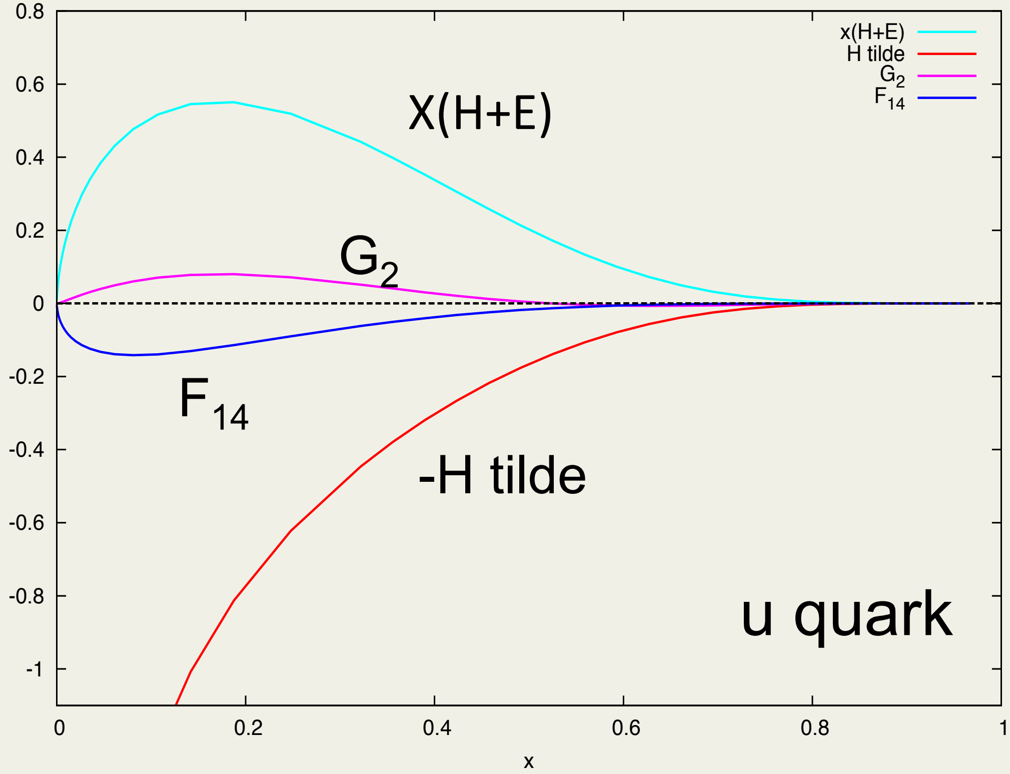
Effect of evolution



GPDs calculated in Reggeized diquark model

GGL PRD (2010), O. Gonzalez et al, PRC (2013)

Preliminary results in reggeized diquark model (With Abha Rajan)



Q^2=2 GeV^2

J_q	$\Delta\Sigma_q$	L_q	Int $-xG_2$	Int F_{14}
0.24	0.46	-0.22	-0.025	-0.045

Diquark Model calculation of Torque (Abha Rajan)

$$F_{14}^{"straight"} = \mathcal{N} \frac{M^2}{x\left[(k-\Delta)^2 - m_\Lambda^2\right]^2 \left[k^2 - m_\Lambda^2\right]^2} \Rightarrow L_q^{Ji}$$

$$F_{14}^{"staple"} = \mathcal{N} \frac{M^2(1-x)}{\left[(k-\Delta)^2 - m_\Lambda^2\right]^2} \int \frac{d^2 l_T}{(2\pi)^2} \frac{\left(1 + \frac{l_T \cdot k_T}{l_T^2}\right)}{\left[\left[(l_T - k_T)^2 - \mathcal{M}^2(x) - 2x l_T^2\right]\right]^2} \Rightarrow \mathcal{L}_q^{JM}$$

Torque is given by the difference

Normalization is not known

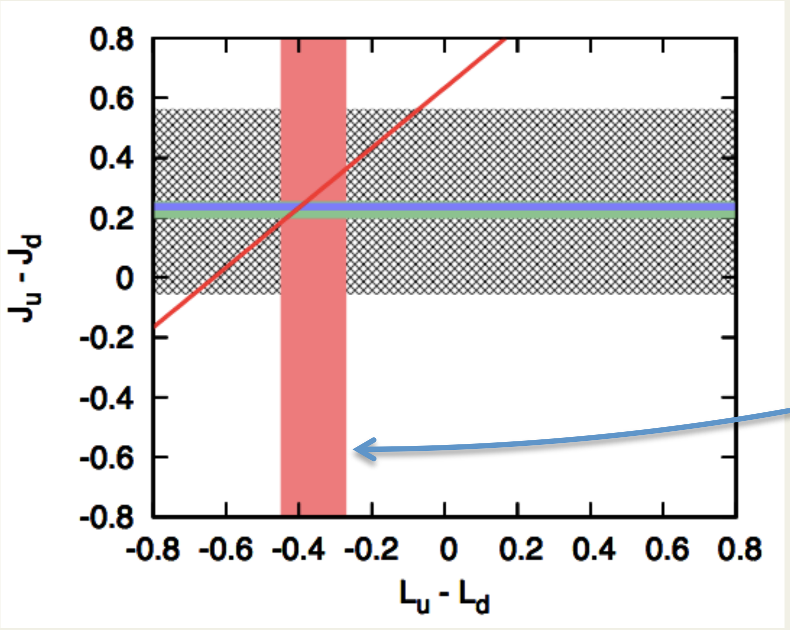
Connection with g_2, d_2

$$\underbrace{g_2(x)}_{t=3} = \underbrace{-g_1(x) + \int_x^1 \frac{dy}{y} g_1(x)}_{g_2^{WW} \to \tau=2} + \underbrace{\left[\bar{g}_2^{tw3} - \int_x^1 \frac{dy}{y} \bar{g}_2^{tw3} \right]}_{\tau=3}$$

$$d_2 = 2 \int dx\, x^2 g_1(x) + 3 \int dx\, x^2 g_2(x)$$

$$d_2 = 2 \int dx\, x^2 \left(H(x) + E(x)\right) + 3 \int dx\, x^2 \tilde{E}_{2T}(x)$$

d_2 measurements provide an independent normalization



A few observations

- ✓ The new expression relates a GTMD (F_{14}) with a twist 3 GPD (G_2), intrinsic k_T enters even if integrated over ➔ it establishes a connection between transverse spatial and momentum dependences

- ✓ Similar to the Sivers effect but here the function vanishes for a straight link, only staple links are probed

- ✓ A unique setup to study/test transverse momentum dependence and related effects, factorization issues, renormalization issues…

- ✓ A unique handle on quark-gluon interactions through the explicit appearance of the quark-gluon-quark correlator in the sum rule

- ✓ The role of partonic k_T and off-shellness, k^2 is manifest.

- ✓ Similarities with g_2

Observability: Cross section and asymmetries

$$\frac{d^4\sigma}{dx_{Bj}dyd\phi dt} = \Gamma\Big\{\Big[F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{2\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi} + h\sqrt{2\epsilon(1-\epsilon)}\,\sin\phi F_{LU}^{\sin\phi}\Big]$$
$$+\ S_{||}\Big[\sqrt{2\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi} + h\Big(\sqrt{1-\epsilon^2}\,F_{LL} + \sqrt{2\epsilon(1-\epsilon)}\,\cos\phi\, F_{LL}^{\cos\phi}\Big)\Big]$$
$$+\ S_{\perp}\Big[\sin(\phi-\phi_S)\Big(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\Big) + \epsilon\Big(\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}\Big)$$
$$+\ \sqrt{2\epsilon(1+\epsilon)}\Big(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\Big)\Big]$$
$$+\ S_{\perp}h\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\Big(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big)\Big]\Big\}$$

$$A_{LU} = \sqrt{\epsilon(1-\epsilon)}\,\frac{F_{LU}^{\sin\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

$$A_{UL} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}}{F_{UU,T} + \epsilon\, F_{UU,L}} + \frac{\epsilon\sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T} + \epsilon\, F_{UU,L}}$$

$$A_{LL} = \frac{N^{\rightarrow}_{s_z=+} - N^{\rightarrow}_{s_z=-} + N^{\leftarrow}_{s_z=+} - N^{\leftarrow}_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{1-\epsilon^2}\ F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)}\ \cos\phi\, F_{LL}^{\cos\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

We identified the quark-proton helicity amplitudes combinations

In terms of quark-proton helicity amplitudes,

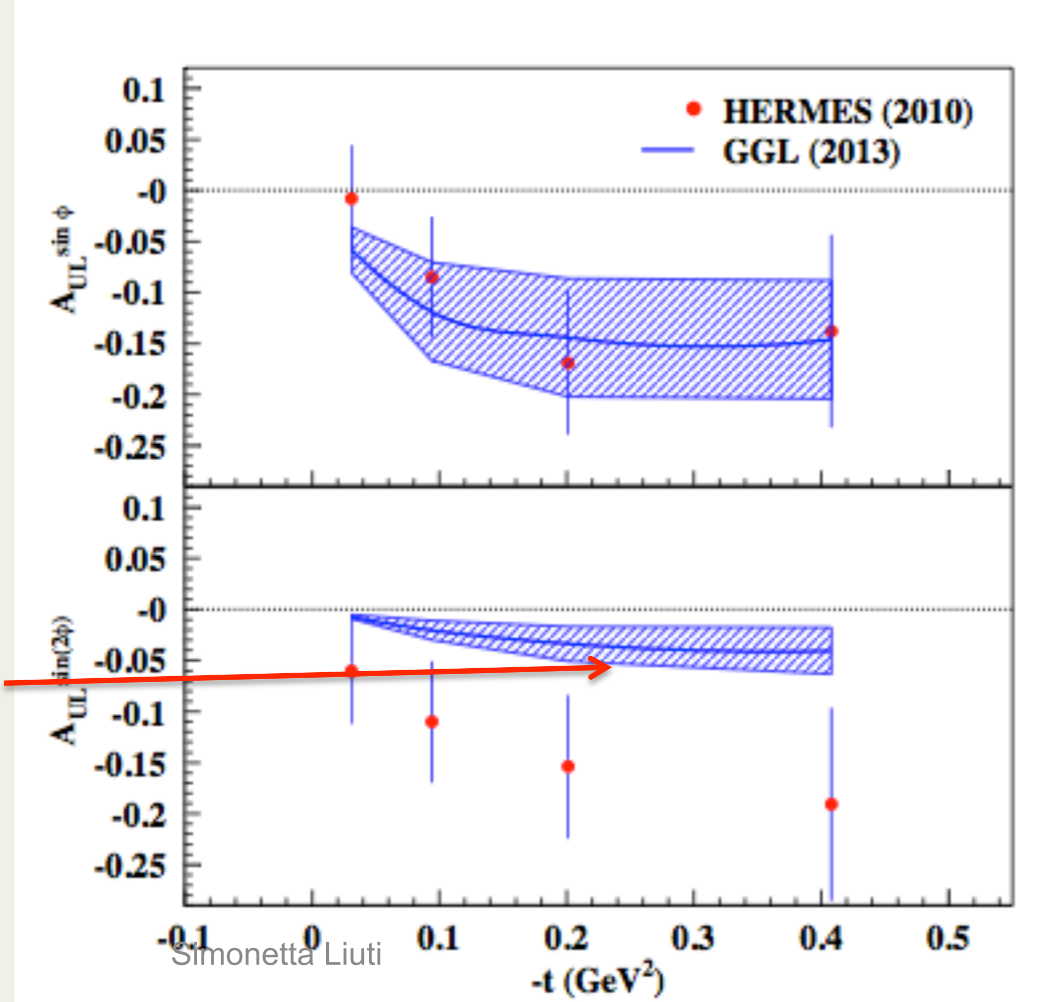
$$i(\mathbf{k} \times \mathbf{\Delta})_3 F_{14} = A^{tw2}_{++,++} + A^{tw2}_{+-,+-} - A^{tw2}_{-+,-+} - A^{tw2}_{--,--}$$

G_2

$$(k_1 - ik_2)F_{27} + (\Delta_1 - i\Delta_2)F_{28} = A^{tw3}_{+-^*,++} - A^{tw3}_{+-,++^*} - A^{tw3}_{--^*,-+} + A^{tw3}_{--,-+^*}$$

Direct measurement of OAM via G_2: DVCS on a longitudinally polarized target

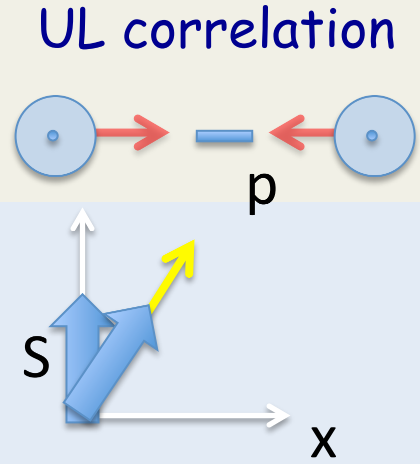
$$A_{UL,L} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{2\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$



WW, small ξ

Hall B analysis in progress!
Avakian, Pisano

Direct measurement of (k_T moment of) F_{14}



F_{14} is Parity even: its matrix element transforms like:

$$S_z = p_z \Lambda$$

it can therefore in principle represent OAM!

However, because F_{14} transforms opposite to helicity, we do raise the issue of its observability in terms of helicity amplitudes!!

Parity predicts a lack of a UL correlation at twist 2

TMDs

	U	T_x	T_y	L
U	f_1	$-i\frac{k_y}{M}\, h_1^\perp$	$i\frac{k_x}{M}\, h_1^\perp$	✗
T_x	$-i\frac{k_y}{M}\, f_{1T}$	$h_1 + \frac{k_x^2-k_y^2}{2M^2}\, h_{1T}^\perp$	$\frac{k_x k_y}{M^2}\, h_{1T}^\perp$	$\frac{k_x}{M}\, g_{1T}$
T_y	$i\frac{k_x}{M}\, f_{1T}$	$\frac{k_x k_y}{M^2}\, h_{1T}^\perp$	$h_1 - \frac{k_x^2-k_y^2}{2M^2}\, h_{1T}^\perp$	$\frac{k_y}{M}\, g_{1T}$
L	✗	$\frac{k_x}{M}\, h_{1L}^\perp$	$\frac{k_y}{M}\, h_{1L}^\perp$	g_{1L}

GPDs

	U	T_x	T_y	L
U	H	$i\frac{\Delta_y}{2M}\,(2\tilde{H}_T + E_T)$	$-i\frac{\Delta_x}{2M}\,(2\tilde{H}_T + E_T)$	✗
T_x	$i\frac{\Delta_y}{2M}\, E$	$H_T - \frac{\Delta_x^2-\Delta_y^2}{4M^2}\,\tilde{H}_T$	$-\frac{\Delta_x\Delta_y}{2M^2}\,\tilde{H}_T$	
T_y	$-i\frac{\Delta_x}{2M}\, E$	$-\frac{\Delta_x\Delta_y}{2M^2}\,\tilde{H}_T$	$H_T + \frac{\Delta_x^2-\Delta_y^2}{4M^2}\,\tilde{H}_T$	
L	✗			$\tilde{H}$

✓The amps will cancel unless they are imaginary:

$$A_{++,++} = A^*_{--,--}\ ;\ A_{+-,+-} = A^*_{-+,-+}$$

✓But this cannot be when the scattering happens in one single hadronic plane. In this case there can be no relative phase between helicity amps (this is what we referred to as Parity Odd, not F_{14} itself!).

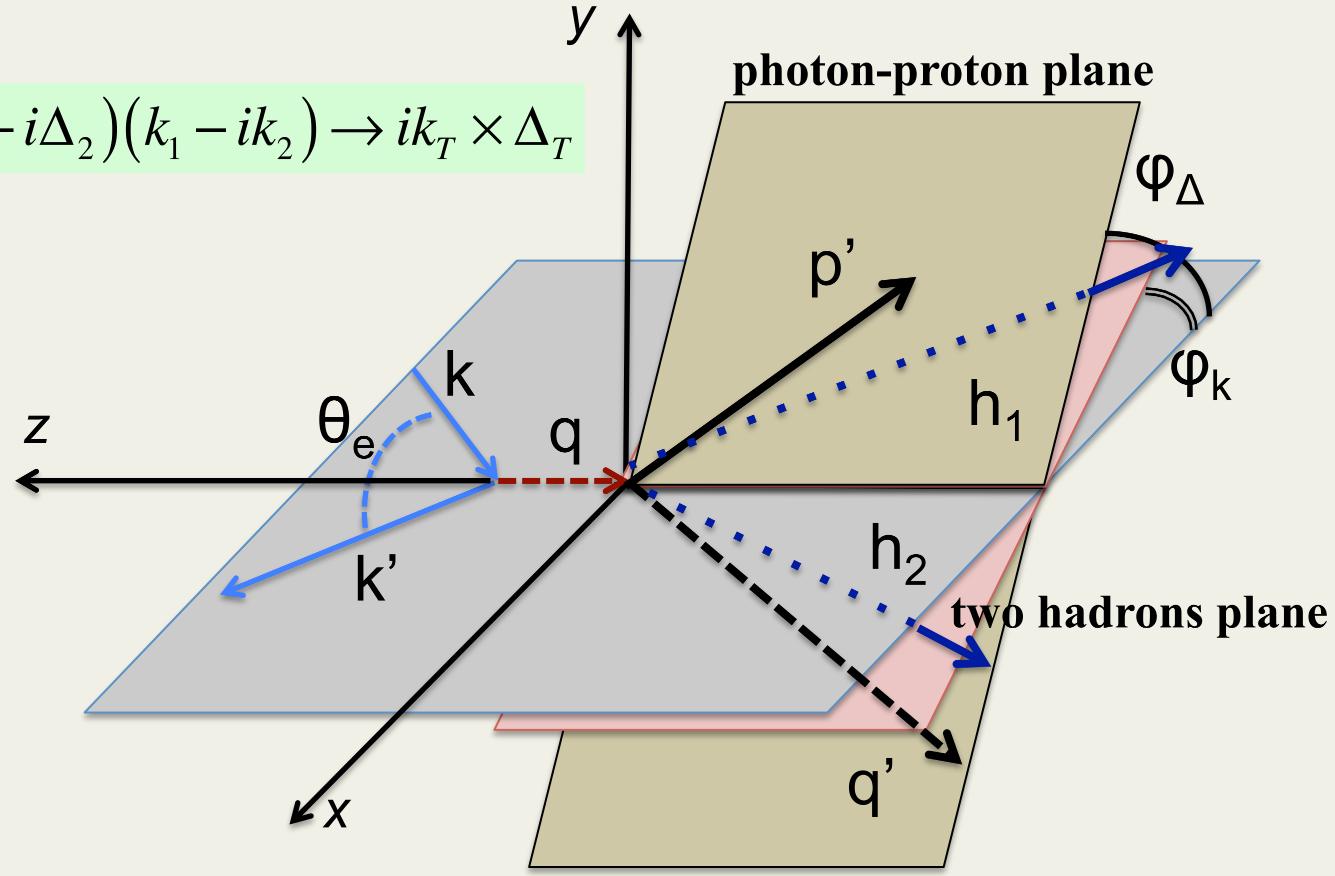
✓Different from GPD E where the amplitude's phase is a consequence of helicity flip and off-forward spinor rotation

✓The phase in F_{14} is obtained by introducing two scattering planes

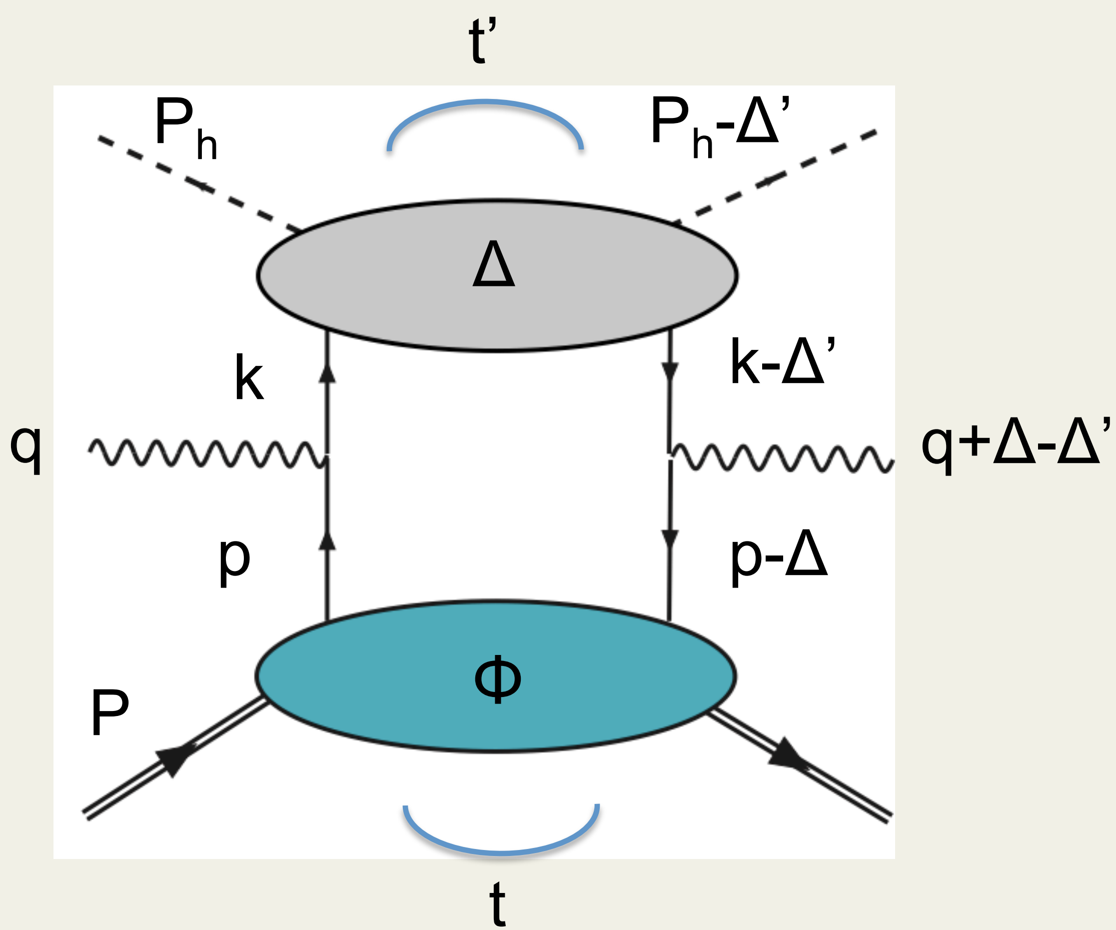
Off forward SIDIS

- To measure F_{14} one has to be in a frame where the reaction cannot be viewed as a two-body quark-proton scattering
- In the CoM the amplitudes are imaginary ➔UL connection goes to 0
- The way to accomplish this is to define two planes

$$|\Delta| e^{i\varphi_\Delta} |k_T| e^{i\varphi_k} (\Delta_1 - i\Delta_2)(k_1 - ik_2) \rightarrow ik_T \times \Delta_T$$



… "off-forward SIDIS" allows us to introduce additional degrees of freedom:



$t, t', P_h^2 < Q^2$

$$g_{\Lambda'_\gamma,\Lambda'_N,0;\Lambda_\gamma,\Lambda_N,0} = \sum_{\lambda,\lambda'} \tilde{g}^{\lambda'\lambda}_{\Lambda'_\gamma\Lambda_\gamma} \otimes A_{\Lambda'_N,\lambda',\Lambda_N,\lambda}(x,\xi,t) \otimes F^{\pi_1}_{\lambda 0}(z) F^{\pi_2}_{\lambda' 0}(v)$$

Φ Δ

Conclusions and Outlook

The connection we established through the new sum rules between (G)TMDs and GPDs, opens many interesting avenues:

- It allows us to study in detail the role of quark-gluon correlations, and of transverse momentum or off-shellness

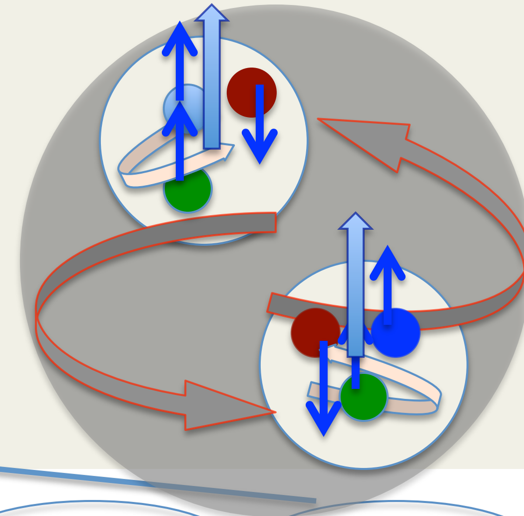
- OAM was obtained so far by subtraction (also in lattice). We can now both calculate OAM on the lattice (GTMD) and validate this through measurements (twist 3 GPD).

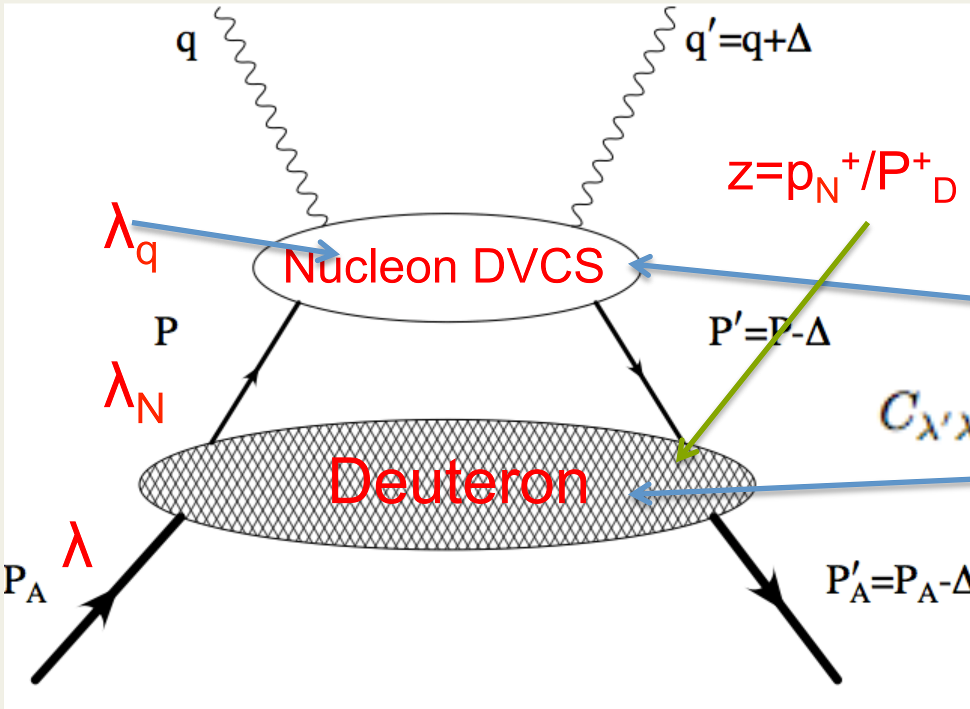
- Within this context we can study The difference between JM and Ji sheds light on the working of the quark-gluon correlations (twist analysis)

- Many more interesting new connections: with transverse spin (Sivers effect, transverse spin) and axial vector sector (g_2)

With observables in hand we can now state that OAM acquires different meanings depending on the way we probe it

ADVERTISEMENT: Sum Rules in Deuteron





$$C_{\Lambda'\lambda'_q,\Lambda\lambda_q} = \sum_{\lambda_N,\lambda'_N} B_{\Lambda'\lambda'_N,\Lambda\lambda_N} \otimes A_{\lambda'_N\lambda'_q,\lambda_N\lambda_q}$$

$$J_q = \frac{1}{2}\int dx\, x\,[H_q(x,0,0) + E_q(x,0,0)]\,,$$

Nucleon

$$\rightarrow J_q = \frac{1}{2}\int dx\, x\, H_2^q(x,0,0),$$

Deuteron

$\downarrow$ $F_1 + F_2 = G_M$

$\downarrow$ G_M

Back Up

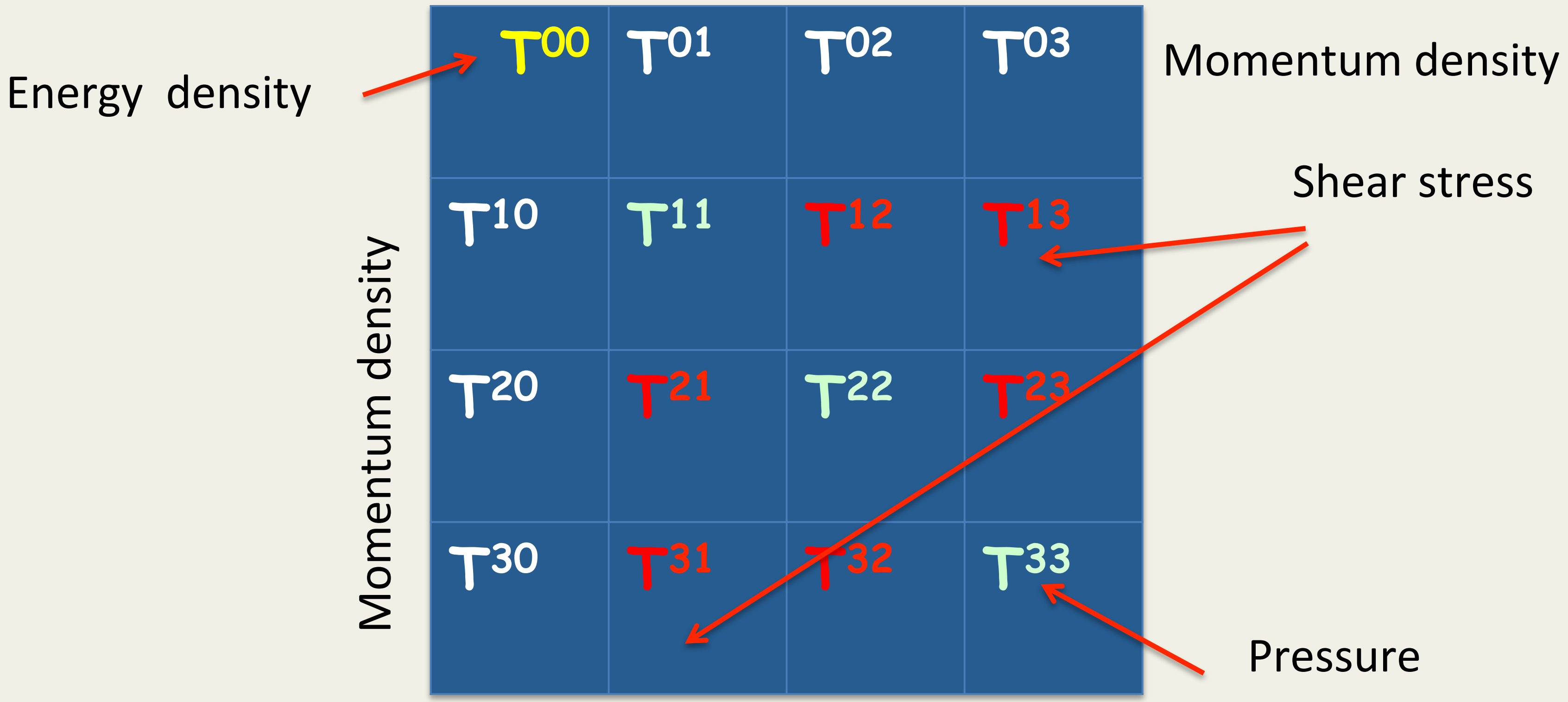
Twist 3 decomposition of hadronic tensor in various notations

Polyakov et al. [13]	$2G_1$	G_2	G_3	G_4
Meissner et al. [3]	$2\widetilde{H}_{2T}$	$\widetilde{E}_{2T}$	E_{2T}	H_{2T}
Belitsky et al. [16]	E^3_+	$\widetilde{H}^3_-$	$H^3_+ + E^3_+$	$\frac{1}{\xi}\widetilde{E}^3_-$

TABLE I: Comparison of notations for different twist 3 GPDs.

Courtoy et al, PLB (2014)arXiv:1310.5157

The QCD Energy Momentum Tensor



$$T^{\mu\nu} = \frac{1}{4} i q \bar{\psi} \left(\gamma^{\mu} \overleftrightarrow{D}^{\nu} + \gamma^{\nu} \overleftrightarrow{D}^{\mu} \right) \psi + Tr\left\{ F^{\mu\alpha} F^{\nu}_{\alpha} - \frac{1}{2} g^{\mu\nu} F^{2} \right\} \rightarrow M^{\mu\nu\lambda} = x^{\nu} T^{\mu\lambda} - x^{\lambda} T^{\mu\nu}$$

Angular Momentum density

Sum Rule: Part I

First define the angular momentum components

$$M^{\mu\nu\lambda} = x^{\nu} T^{\mu\lambda} - x^{\lambda} T^{\mu\nu} \quad \Longrightarrow \quad J_q^i = \varepsilon^{ijk} \int dz^- d^2 z \; M^{+jk}$$

then parametrize the EMT in terms of form factors A, B, C

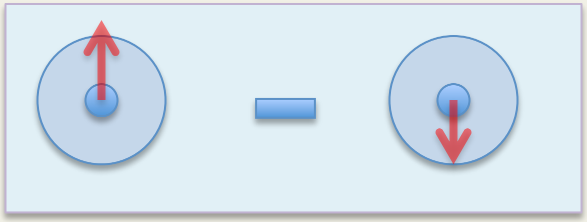
$$T^{\mu\nu} = \boxed{A}(\gamma^{\mu}\bar{P}^{\nu} + \gamma^{\nu}\bar{P}^{\mu}) + \boxed{B}\left(\frac{i\sigma^{\mu\alpha}\Delta_{\alpha}}{2M}\bar{P}^{\nu} + \frac{i\sigma^{\nu\alpha}\Delta_{\alpha}}{2M}\bar{P}^{\mu}\right) + \boxed{C}\frac{\Delta^{\mu}\Delta^{\nu} - \Delta^2 g^{\mu\nu}}{M}$$

Finally, connect the EMT matrix element with AM components

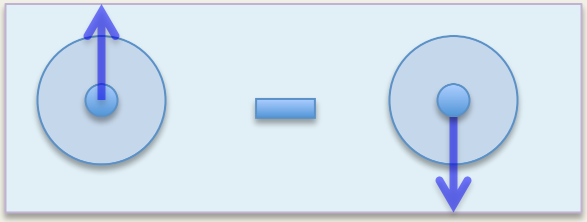
$$J_q = \frac{1}{2}\left(A_q + B_q\right) \Rightarrow \sum J_q + J_g = \frac{1}{2}$$

Jaffe Manohar (1990)
Ji (1997)

$h_1^\perp$



$f_{1T}^\perp$



Ah ha!
This is the same argument that allows us to observe the T-odd TMDs by understanding the role of the gauge links

$f_{1T}^\perp$

S.J. Brodsky et al. / Physics Letters B 530 (2002) 99–107

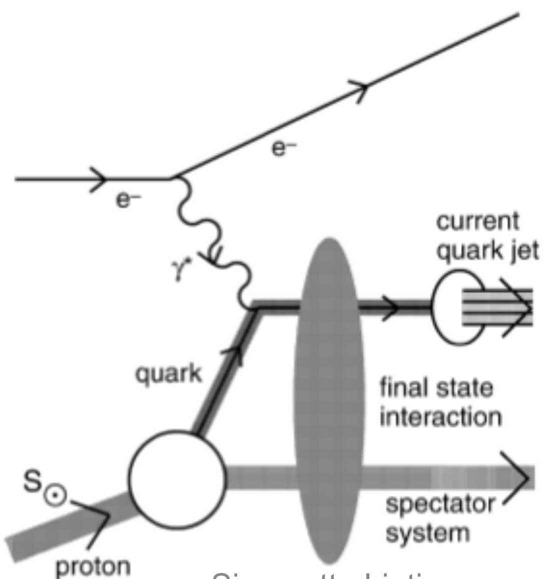


Fig. 1. The final-state interaction in the semi-inclusive deep inelastic lepton scattering $\ell p^\uparrow \to \ell' \pi X$.

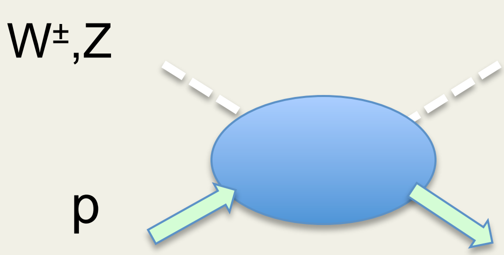
F_{14} appears in the unintegrated structure functions for deep inelastic scattering with electroweak currents

$$\tfrac{1}{4}\left(T_{1\,\frac12;1\,\frac12}+T_{1\,-\frac12;1\,-\frac12}+T_{-1\,\frac12;-1\,\frac12}+T_{-1\,-\frac12;-1\,-\frac12}\right)=T_1, \qquad F_1$$

$$\tfrac{1}{4}\left(T_{1\,\frac12;1\,\frac12}-T_{1\,-\frac12;1\,-\frac12}+T_{-1\,\frac12;-1\,\frac12}-T_{-1\,-\frac12;-1\,-\frac12}\right)=\frac{\nu}{M^2}\sqrt{1+\frac{M^2Q^2}{\nu^2}}\;A_1, \qquad A_1$$

$$\tfrac{1}{4}\left(T_{1\,\frac12;1\,\frac12}-T_{1\,-\frac12;1-\frac12}-T_{-1\,\frac12;-1\,\frac12}+T_{-1\,-\frac12;-1\,-\frac12}\right)=-\frac{\nu}{M^2}S_1+\frac{Q^2}{M^2}S_2+S_3, \qquad G_1$$

$$\tfrac{1}{4}\left(T_{1\,\frac12;1\,\frac12}+T_{1\,-\frac12;1\,-\frac12}-T_{-1\,\frac12;-1\,\frac12}-T_{-1\,-\frac12;-1\,-\frac12}\right)=\frac{\nu}{2M^2}\sqrt{1+\frac{Q^2M^2}{\nu^2}}\;T_3, \qquad F_3$$



X.Ji, NPB402 (1993)

$$G_1 \propto (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,+-})$$ g_1

$$+ (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,+-})$$ F_{14}

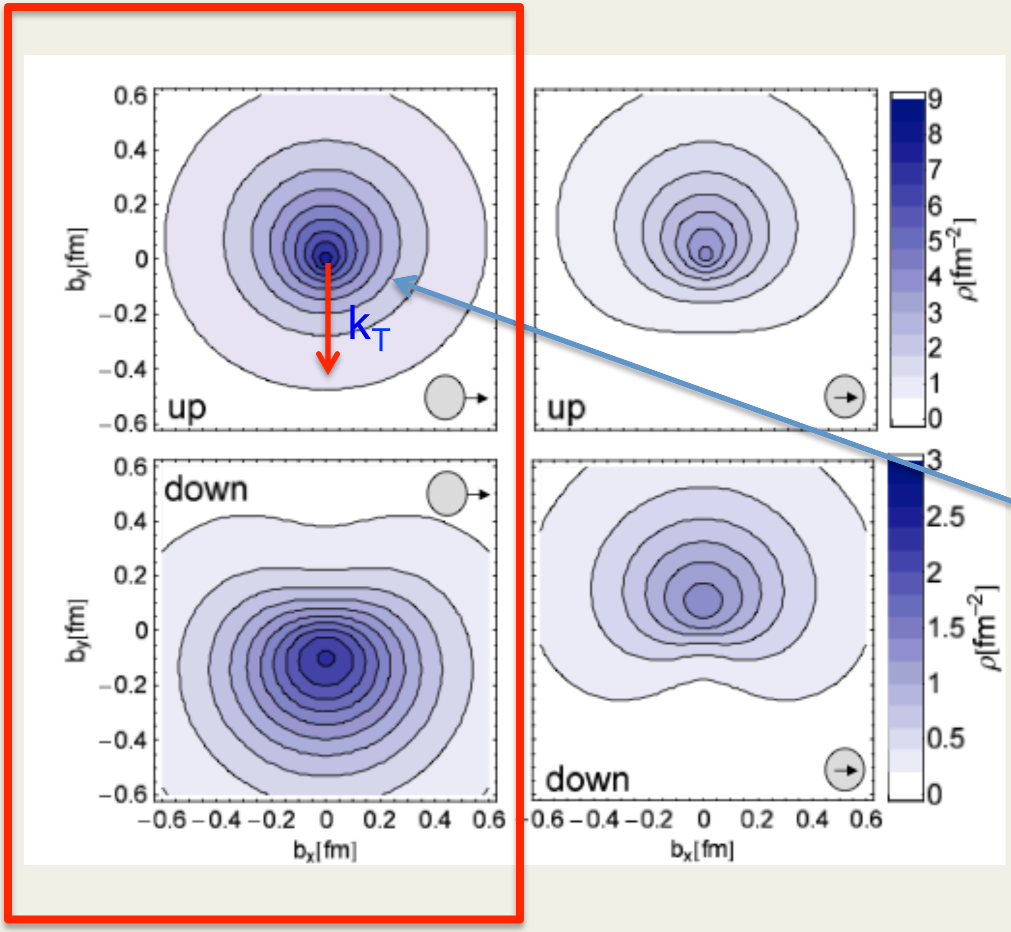
parity odd

$$A_1 \propto (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,+-})$$ F_{14}

$$+ (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,+-}),$$ g_1

F_{14} is the parity odd contribution to g_1 and the parity even contribution to A_1!

Analogous situation as for E wrt. transverse spin (M. Burkardt)



E $\rightarrow$ $\sigma^{+j}\Delta_j \Rightarrow \vec{S}_T \times \vec{\Delta}$

The net b corresponds to net k_T in the opposite direction (attractive color force due to FSI)

$$\left(A^X_{++,++} + A^X_{+-,+-} + A^X_{-+,-+} + A^X_{--,--}\right) + \left(A^X_{++,++} + A^X_{+-,+-} - A^X_{-+,-+} - A^X_{--,--}\right)$$

$$\approx H - i\Delta_2 E$$