Factorization theorem, gluon poles and new contributions in semi-inclusive processes

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based on

PLB690, 519 (2010); EPJC75, 184 (2015); arXiv:1508.00227
in collaboration with O.V. Teryaev

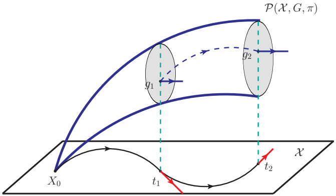
POETIC VI, Palaiseau, 2015

September 8, 2015

Main results

- In the Contour Gauge and the Collinear Factorization procedure applied for Drell-Yan and Direct Photon Production hadron tensor, we find new contributions to gluon poles.
- In the Feynman Gauge, we discuss the constraints for the gluon poles in the DY hadron tensor.

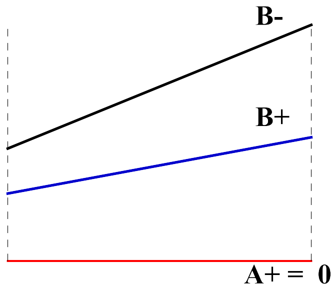
Geometrical interpretation of gluons



- Gluon field as a connection of $\mathcal{P}(\mathbb{R}^4, G, \pi)$: $\mathbb{R}^4$ – the base of the principal fiber bundle, G – the group and $\{\pi \,|\, \mathbb{R}^4 \to \mathcal{P}\}$.
- Each $\mathbf{g}(x)$ defines the gauge-transformed field and forms the orbit of the gauge-equivalent fields.
- The p.t.e. $\dot{x}_\alpha(v)\,\mathcal{D}_\alpha \mathbf{g}(x(v)) = 0$ has a solution $\mathbf{g}(x) = [x_0, x]$ (Hahn-Banach theorem).
- The contour gauge demands that $\mathbf{g}(x) = 1$ for $\forall x \in \mathbb{R}^4$ and

$$A^{c.g.}_\mu(x) = \int\limits_{\mathbb{P}(x_0,x)} dz_\alpha \frac{\partial z_\beta}{\partial x_\mu} G_{\alpha\beta}(z|A).$$

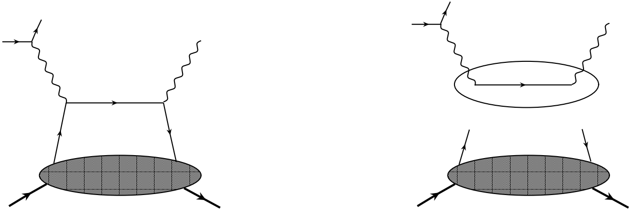
- The simple illustration of the use of the contour gauge conception, $[x_0, x] = 1$, which generates the usual axial-type gauges:



On the Contour Gauge, see S.Ivanov, G.Korchemsky, A.Radyushkin '85 - '90

Factorization theorem, in a nutshell

Schematically, F.T. (applied, for example, to DVCS) corresponds to



$$\text{Amplitude} = \{\text{Hard part (pQCD)}\} \otimes \{\text{Soft part (npQCD)}\},$$

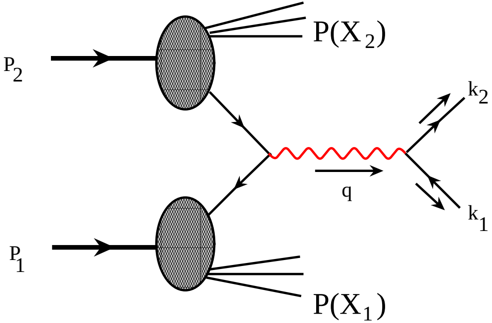
where both hard and soft parts are independent of each other, UV- and IR-renormalizable and, finally, parton distributions must possess the universality property.

Drell-Yan process

We study

$$N^{(\uparrow\downarrow)}(p_1) + N(p_2) \to \gamma^*(q) + X(P_X) \to \ell(l_1) + \bar{\ell}(l_2) + X(P_X),$$

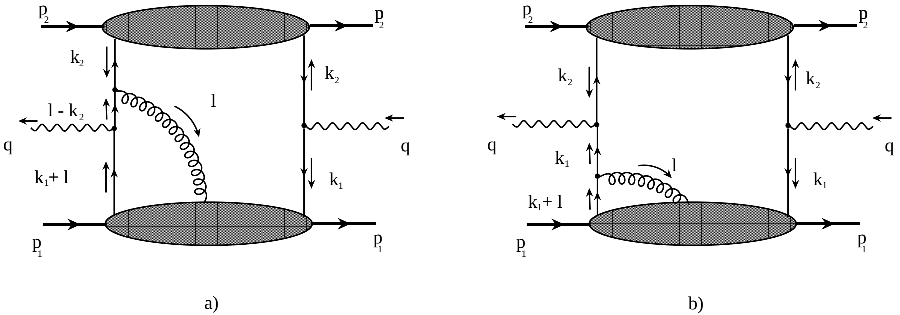
where $l_1 + l_2 = q$ has a large mass squared ($q^2 = Q^2$).



The cross-sections reads (kinematics: $p_1 \sim n^{*\,+}$, $p_2 \sim n^-$)

$$d\sigma = (dP.S.)^2 \; \mathcal{L}_{\mu\nu} \mathcal{W}^{GI}_{\mu\nu} \,,$$

where $\mathcal{L}_{\mu\nu}$ is a lepton tensor, and $\mathcal{W}^{GI}_{\mu\nu}$ – the QED gauge invariant hadron tensor.



- The standard diagram (a) and the non-standard diagram (b) differ by the hard parts. (Factorization links: IVA, O.V.Teryaev '09.)

Any SSA are defined as

$$\text{SSA} \sim d\sigma^{(\uparrow)} - d\sigma^{(\downarrow)} \sim \mathcal{L}_{\mu\nu} H_{\mu\nu} .$$

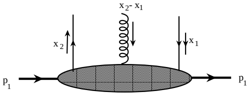
In our case, we deal with the unpolarized leptons, *i.e.* $\mathcal{L}_{\mu\nu} \in \Re\text{e}$. Therefore, the hadron tensor $H_{\mu\nu}$ should also be real one, *i.e.* $H_{\mu\nu} \in \Re\text{e}$, provided, at the same time, one of hadrons is transversely polarized. Usually, it is possible if

$$H^{(a)}_{\mu\nu} \sim \Im\text{m}\,[\text{Hard}] \otimes \left\{ \langle p_1, S_T | \mathcal{O}(\bar{\psi}, \psi, A) | S_T, p_1 \rangle \overset{\mathcal{F}}{\sim} i\varepsilon_{\alpha\beta S_T p_1} \Phi \right\} ,$$

$$H^{(b)}_{\mu\nu} \sim \text{Hard} \otimes \left\{ \langle p_1, S_T | \mathcal{O}(\bar{\psi}, \psi, A) | S_T, p_1 \rangle \overset{\mathcal{F}}{\sim} i\varepsilon_{\alpha\beta S_T p_1} \Im\text{m}\,[\Phi] \right\} .$$

Mulders, Boer et al. '94-96; Barone et al. '00; Boer, Qiu '02

However, if $B^V \in \Re\mathrm{e}$, which parametrizes



$$\langle p_1, S^T | \bar{\psi}(\lambda_1 \tilde{n}) \gamma^+ g A_T^\alpha(\lambda_2 \tilde{n}) \psi(0) | S^T, p_1 \rangle \stackrel{\mathcal{F}}{=} i\varepsilon^{\alpha + S^T -}(p_1 p_2) B^V(x_1, x_2)\,,$$

with

$$B^V(x_1, x_2) = \frac{\mathcal{P}}{x_1 - x_2} T(x_1, x_2),$$

$$T(x_1, x_2) \stackrel{\mathcal{F}}{\sim} \langle \bar{\psi}(\lambda_1 \tilde{n}) \gamma^+ \tilde{n}_\nu G_T^{\nu\alpha}(\lambda_2 \tilde{n}) \psi(0) \rangle,$$

the non-standard diagram (b) does NOT contribute to the SSA.

- As a result, we are faced to a problem with QED gauge invariance and, therefore, with the factorization breaking.

The inference on $B^V \in \Re e$ is based on the solution of the differential equation (within the gauge: $A^+ = 0$)

$$\partial^+ A^\alpha_T = G^{+\,\alpha}_T \,.$$

The solution has previously been assumed to have two equivalent representations:

$$\begin{aligned} A^\mu(z) &= \int\limits_{-\infty}^{\infty} d\omega^- \theta(z^- - \omega^-) G^{+\mu}(\omega^-) + A^\mu(-\infty) \\ &= -\int\limits_{-\infty}^{\infty} d\omega^- \theta(\omega^- - z^-) G^{+\mu}(\omega^-) + A^\mu(\infty) \,. \end{aligned}$$

Solution

I.V.A., O.V.Teryaev PLB690 (2010) 519

In fact, the two representations are NOT equivalent.

Using the contour gauge conception, one can easily check that

- the representation with $\theta(z^- - \omega^-)$ belongs to the gauge $[x, -\infty] = 1$;
- the representation with $\theta(\omega^- - z^-)$ belongs to the gauge $[+\infty, x] = 1$.

Therefore, there are no reasons to believe that two repres. are equivalent, *i.e.*

$$\left\{ \text{Rep}_{\theta(z^- - \omega^-)} \Rightarrow B_+^V(x_1, x_2) \right\} \neq \left\{ B_-^V(x_1, x_2) \Longleftarrow \text{Rep}_{\theta(\omega^- - z^-)} \right\}.$$

We get

$$B^V(x_1, x_2) = \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} + \delta(x_1 - x_2) B^V_{A(-\infty)}(x_1)\,,$$
$$B^V_{A(-\infty)}(x) = 0,$$

which leads to the non-zero contribution from the diagram (b).

Conclusions for DY:

$$\boxed{\textbf{ISI} \Rightarrow \frac{1}{\ell^+ - i\epsilon} \Rightarrow [z^-, -\infty^-] \Rightarrow \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} \Rightarrow \textbf{GI}}$$

and

$$\boxed{\mathcal{W}(\text{dia.B}) = \mathcal{W}(\text{dia.A})\,.}$$

DY hadron tensor in Feynman Gauge

The unintegrated tensor $\overline{\mathcal{W}}_{\mu\nu}$ for the factorized hadron tensor $\mathcal{W}_{\mu\nu}$ of the process reads

$$\mathcal{W}_{\mu\nu} = \int d^2\vec{\mathbf{q}}_T d\mathcal{W}_{\mu\nu} = \frac{2}{q^2} \int d^2\vec{\mathbf{q}}_T \, \delta^{(2)}(\vec{\mathbf{q}}_T) \times$$

$$i \int dx_1 \, dy \left[\delta(x_1/x_B - 1)\delta(y/y_B - 1)\right] \overline{\mathcal{W}}_{\mu\nu}.$$

The parametrizing functions are associated with the following correlators:

$$B^{(1)}(x_1, x_2) = \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} \stackrel{\mathcal{F}_2}{\Longleftarrow} \langle \bar{\psi}(\eta_1)\, \gamma^+\, A^T(z)\, \psi(0) \rangle\,,$$

$$B^{(2)}(x_1, x_2) \stackrel{\mathcal{F}_2}{\Longleftarrow} \langle \bar{\psi}(\eta_1)\, \gamma^\perp\, A^+(z)\, \psi(0) \rangle\,,$$

$$B^{(\perp)}(x_1, x_2) \stackrel{\mathcal{F}_2}{\Longleftarrow} \langle \bar{\psi}(\eta_1)\, \gamma^+\, \left(\partial^\perp\, A^+(z)\right)\, \psi(0) \rangle\,.$$

Let us now discuss the QED gauge invariance of the hadron tensor.

$$\overline{\mathcal{W}}_{\mu\nu} = \overline{\mathcal{W}}^{\text{(Stand.)}}_{\mu\nu} + \overline{\mathcal{W}}^{\text{(Stand.}, \partial_\perp)}_{\mu\nu} + \overline{\mathcal{W}}^{\text{(Non-stand.)}}_{\mu\nu} =$$

$$\bar{q}(y) \Bigg\{ \left[\frac{p_{2\,\mu}}{x_1} - \frac{p_{1\,\mu}}{y} \right] \varepsilon_{\nu S^T - p_2} \int dx_2 B^{(1)}(x_1, x_2) +$$

$$\frac{p_{2\,\mu}}{x_1} \varepsilon_{\nu S^T - p_2} \int dx_2 B^{(2)}(x_1, x_2) -$$

$$\left[\frac{p_{2\,\nu}}{x_1} \varepsilon_{\mu S^T - p_2} + \frac{p_{2\,\mu}}{x_1} \varepsilon_{\nu S^T - p_2} \right] x_1 \int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} +$$

$$\frac{p_{1\,\mu}}{y} \varepsilon_{\nu S^T - p_2} \int dx_2 \frac{B^{(\perp)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} \Bigg\},$$

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$$\bar{q}(y)\Bigg\{\left[\frac{p_{2\,\mu}}{x_1} - \frac{p_{1\,\mu}}{y}\right]\varepsilon_{\nu S^T - p_2}\int dx_2 B^{(1)}(x_1, x_2) +$$

$$\frac{p_{2\,\mu}}{x_1}\,\varepsilon_{\nu S^T - p_2}\int dx_2 B^{(2)}(x_1, x_2) -$$

$$\left[\frac{p_{2\,\nu}}{x_1}\varepsilon_{\mu S^T - p_2} + \frac{p_{2\,\mu}}{x_1}\varepsilon_{\nu S^T - p_2}\right] x_1 \int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} +$$

$$\frac{p_{1\,\mu}}{y}\,\varepsilon_{\nu S^T - p_2}\int dx_2 \frac{B^{(\perp)}(x_1, x_2)}{x_1 - x_2 + i\epsilon}\Bigg\},$$

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$$

$$
\frac{p_{2\,\mu}}{x_1} \varepsilon_{\nu S^T - p_2} \int dx_2 B^{(2)}(x_1, x_2) -
$$

$$
\left[\frac{p_{2\,\nu}}{x_1}\varepsilon_{\mu S^T - p_2} + \frac{p_{2\,\mu}}{x_1}\varepsilon_{\nu S^T - p_2}\right] x_1 \int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} +
$$

$$
\frac{p_{1\,\mu}}{y} \varepsilon_{\nu S^T - p_2} \int dx_2 \frac{B^{(\perp)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} \Bigg\},
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$$\bar{q}(y) \Big\{ \Big[\frac{p_{2\,\mu}}{x_1} - \frac{p_{1\,\mu}}{y} \Big] \varepsilon_{\nu S^T - p_2} \int dx_2 B^{(1)}(x_1, x_2) +$$

$$\frac{p_{2\,\mu}}{x_1} \varepsilon_{\nu S^T - p_2} \int dx_2 B^{(2)}(x_1, x_2) -$$

$$\Big[\frac{p_{2\,\nu}}{x_1} \varepsilon_{\mu S^T - p_2} + \frac{p_{2\,\mu}}{x_1} \varepsilon_{\nu S^T - p_2} \Big] x_1 \int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} +$$

$$\frac{p_{1\,\mu}}{y} \varepsilon_{\nu S^T - p_2} \int dx_2 \frac{B^{(\perp)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} \Big\},$$

$B^{(2)}(x_1, x_2)$ has no gluon poles

Consider the correlator:

$$\int (d\lambda_1 \, d\lambda_2) e^{-ix_1\lambda_1 - i(x_2 - x_1)\lambda_2} \times$$
$$\langle p_1, S^T | \bar{\psi}(\lambda_1 \tilde{n}) \, \gamma^{\perp}_{\beta} \, A^{+}_{\alpha}(\lambda_2 \tilde{n}) \, \psi(0) | S^T, p_1 \rangle$$

which can be parametrized with

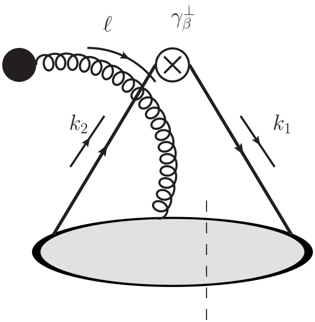
$$i\varepsilon_{\beta\alpha S^T -} \, (p_1 p_2) \, B^{(2)}(x_1, x_2) \, .$$

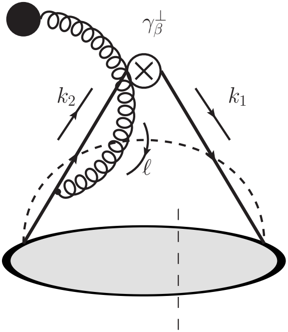
In the momentum representation, we have ($\ell = k_2 - k_1$)

$$\left[\bar{u}(k_1)\gamma^{\perp}_{\beta} u(k_2)\right] \times .... \times \frac{1}{\ell^2 + i\varepsilon},$$

where $k_1 = (x_1 p_1^+, k_1^-, \vec{\mathbf{k}}_{1\,\perp})$, $k_2 = (x_2 p_1^+, k_2^-, \vec{\mathbf{k}}_{2\,\perp})$.

To get the non-zero contribution we must have either $\vec{\mathbf{k}}_{1\,\perp} \neq 0$ or $\vec{\mathbf{k}}_{2\,\perp} \neq 0$.





One can conclude that, in the case with the substantial transverse component of the momentum, there are no sources for the gluon pole at $x_1 = x_2$.

As a result, for DY process,

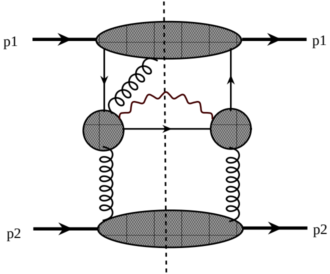
- the function $B^{(2)}(x_1, x_2)$ has no gluon poles (therefore, there is no $dT(x,x)/dx$)
- due to T-invariance ($B^{(2)}(x_1, x_2) = -B^{(2)}(x_2, x_1)$), the function obeys
$$B^{(2)}(x,x) = 0.$$
- The hadron tensor is gauge-independent.

Direct Photon Production in hadron collisions

We now dwell on the direct photon production in two hadron collisions:

$$N^{(\uparrow\downarrow)}(p_1) + N(p_2) \to \gamma(q) + X(P_X)\,.$$

where $x_F = 2q_3/\sqrt{S}$ is relatively large. The cross-section $d\sigma$ is defined by the hadron tensor as



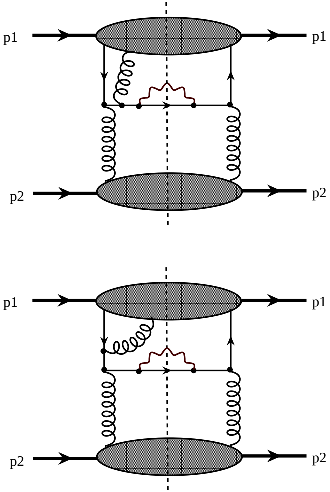
It is convenient to fix the dominant light-cone directions as

$$p_1 = \sqrt{\frac{S}{2}}\, n^* , \quad p_2 = \sqrt{\frac{S}{2}}\, n , \quad \text{with}$$
$$n^*_\mu = (1/\sqrt{2}, \mathbf{0}_T, 1/\sqrt{2}), \quad n_\mu = (1/\sqrt{2}, \mathbf{0}_T, -1/\sqrt{2}) .$$

The final on-shell photon and quark(anti-quark) momenta can be presented as

$$q = y_B \sqrt{\frac{S}{2}}\, n - \frac{q_\perp^2}{y_B\sqrt{2S}}\, n^* + q_\perp ,$$
$$k = x_B \sqrt{\frac{S}{2}}\, n^* - \frac{k_\perp^2}{x_B\sqrt{2S}}\, n + k_\perp .$$

To study the QCD gauge invariance, we consider the following diagrams:



In the process we consider, we have both ISI and FSI:

$$\textbf{ISI} \Rightarrow \frac{1}{\ell^+ - i\epsilon} \Rightarrow [z^-, -\infty^-] \Rightarrow \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon}$$

and

$$\textbf{FSI} \Rightarrow \frac{1}{\ell^+ + i\epsilon} \Rightarrow [+\infty^-, z^-] \Rightarrow \frac{T(x_1, x_2)}{x_1 - x_2 - i\epsilon}$$

QCD gauge invariance: final stage

$$\overline{W^{(1)}} \sim \mathbf{C}_2 \frac{1}{x_1} \int dx_2 \, \frac{x_2 - x_1}{x_2} \, \frac{T(x_1, x_2)}{x_1 - x_2 - i\epsilon} \, ,$$

$$\overline{W^{(2)}} \sim \mathbf{C}_2 \frac{1}{x_1} \int dx_2 \, \frac{1}{x_2} \, \frac{T(x_1, x_2)}{x_1 - x_2 - i\epsilon} \, ,$$

$$\overline{W^{(3)}} \sim \mathbf{C}_1 \frac{1}{x_1^2} \int dx_2 \, \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} \, ,$$

$$\overline{W^{(4)}} \sim \mathbf{C}_3 \frac{1}{x_1^2} \int dx_2 \, \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} \, ,$$

where $\mathbf{C}_i$ are corresponding colour factors. After calculation of imaginary parts, we get

$$\boxed{+\mathbf{C}_2 - \mathbf{C}_1 - \mathbf{C}_3 = -[t^a, t^b] \, t^b \, t^a - i f^{abc} \, t^c \, t^a \, t^b = 0}$$

The full expression for the hadron tensor can be split into two groups:

(i) the first type, before factorization,takes the following form

$$\mathcal{W}(\text{diag.H}) = \int \frac{d^3\vec{q}}{(2\pi)^3 2E} \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \, \text{C}_H \int (d^4 k_1)(d^4 k_2) \times$$

$$\delta^{(4)}(k_1 + k_2 - q - k) \, \Phi_g^{\alpha\beta}(k_2) \int (d^4\ell) \Phi_\perp^{[\gamma^+],\,\rho}(k_1,\ell) \, H^{\alpha\beta,\rho}(k_1, k_2, \ell) \, ,$$

(ii) the second type can be presented as

$$\mathcal{W}(\text{diag.D}) = \int \frac{d^3\vec{q}}{(2\pi)^3 2E} \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon}\, \mathrm{C}_D \int (d^4k_1)(d^4k_2) \times$$
$$\delta^{(4)}(k_1 + k_2 - q - k)\Phi_g^{\alpha\beta}(k_2)\mathrm{tr}_D\big[\Phi^{(1)}(k_1)\, D^{\alpha\beta}(k_1, k_2)\big]\,.$$

where the twist-3 quark distribution which is given by

$$\Phi^{(1)}(k_1) = \frac{\gamma^+\gamma_\perp^\rho\,\gamma^-}{2k_1^+ + i\epsilon} \int (d^4\eta_1)\, e^{ik_1\eta_1} \times$$
$$\langle p_1, S^T|\bar{\psi}(0)\gamma^+ A_\perp^\rho(0)\psi(\eta_1)|S^T, p_1\rangle\,,$$

Analysing the results for the diagrams H1, H7, D4 and H10, we can see that

$$d\mathcal{W}(\text{dia.H1}) + d\mathcal{W}(\text{dia.H7}) + d\mathcal{W}(\text{dia.D4}) = d\mathcal{W}(\text{dia.H10})\,.$$

In other words, as similar to the Drell-Yan process, the new (non-standard) contributions generated by the diagrams H1, H7 and D4 result again in the factor of 2 compared to the standard diagram H10 contribution to the corresponding hadron tensor.

This is our principle result.

P.S. Definitions:
Standard contributions – non-zero contributions for $B^V \in \Re\mathrm{e}$
Non-Standard contributions – zero contributions for $B^V \in \Re\mathrm{e}$

Summary

Drell-Yan process:

- It is mandatory to include a contribution of the extra diagram which naively does not have an imaginary part;
- This additional contribution emanates from the complex gluon pole prescription in the representation of the twist 3 correlator $B^V(x_1, x_2)$ owing to the corresponding contour gauge;
- In the Feynman gauge, the correlators with $\gamma^\perp A^+$ and $\gamma^+(\partial^\perp A^+)$ do not have the gluon poles and the gauge-invariant amplitude coincides with the amplitude derived within the axial-type gauge.

Direct Photon Production:

- In contact to DY, this process includes both **ISI** and **FSI** that leads to the different gluon pole prescriptions in the diagrams under our consideration; In turn, the different gluon pole prescriptions ensure the QCD gauge invariance.
- We find that the non-standard new terms, which exist in the case of the complex twist-3 B^V-function with the corresponding prescriptions, do contribute to the hadron tensor in the same way as the standard term known previously. This is another important result of our work. We also observe that this is exactly similar to the case of Drell-Yan process.
- We observed the universality breaking, which spoils the standard factorization. However, the factorization procedure we proposed can still be applied for calculations.

Inserting the above-mentioned presentations into the corresponding matrix elements, we thus obtain

$$\Phi^{\alpha}_{A}(x_1, x_2) = \delta(x_1 - x_2)\Phi^{\alpha}_{A(-\infty)}(x_1) + \frac{(-i)\,\Phi^{\alpha}_{G}(x_1, x_2)}{x_2 - x_1 - i\epsilon}\,,$$

and

$$\Phi^{\alpha}_{A}(x_1, x_2) = \delta(x_1 - x_2)\Phi^{\alpha}_{A(+\infty)}(x_1) + \frac{(-i)\,\Phi^{\alpha}_{G}(x_1, x_2)}{x_2 - x_1 + i\epsilon}\,.$$

Here, the corresponding prescriptions $\pm i\epsilon$ arise from the integral representation for the theta-function:

$$\theta(\pm x) = \frac{\pm i}{2\pi}\int\limits_{-\infty}^{+\infty} dk\,\frac{e^{-ikx}}{k \pm i\epsilon}\,.$$

Calculation the plus and minus combinations leads to

$$\Phi^{\alpha}_{A}(x_1,x_2) = \frac{1}{2}\Phi^{\alpha}_{A}(x_1,x_2) + \frac{1}{2}\Phi^{\alpha}_{A}(x_1,x_2) =$$
$$\frac{1}{2}\delta(x_1 - x_2)\Big\{\Phi^{\alpha}_{A(-\infty)}(x_1) + \Phi^{\alpha}_{A(+\infty)}(x_1)\Big\} +$$
$$\frac{\mathcal{P}}{x_2 - x_1}(-i)\Phi^{\alpha}_{G}(x_1,x_2)$$

and

$$0 = \Phi^{\alpha}_{A}(x_1,x_2) - \Phi^{\alpha}_{A}(x_1,x_2) =$$
$$\delta(x_1 - x_2)\Big\{\Phi^{\alpha}_{A(+\infty)}(x_1) - \Phi^{\alpha}_{A(-\infty)}(x_1)\Big\} -$$
$$2i\pi\,\delta(x_1 - x_2)(-i)\Phi^{\alpha}_{G}(x_1,x_2)\,.$$

Mulders, Boer et al. '94-96; Barone et al. '00; Boer, Qiu '02

So, this ambiguity ultimately gives us the standard representation:

$$B^V(x_1, x_2) = \frac{\mathcal{P}}{x_1 - x_2} T(x_1, x_2)\,,$$

$$T(x_1, x_2) \overset{\mathcal{F}}{\sim} \langle \bar{\psi}\, \gamma_\beta\, \tilde{n}_\nu G_{\nu\alpha}\, \psi \rangle \qquad T(x, x) \neq 0\,.$$

provided the asymmetric boundary condition for gluons:

$$B^V_{A(\infty)}(x) = -B^V_{A(-\infty)}(x)$$

Thus, for the considered DY, a pure real $B^V(x_1, x_2)$ will lead to the problem with QED gauge invariance which means factorization breaking.

After some algebra, we arrive at the following contributions for the unintegrated h. t. (which involves all relevant contributions except the mirror ones):

- the standard diagram gives

$$
\overline{\mathcal{W}}^{\text{(Stand.)}}_{\mu\nu} + \overline{\mathcal{W}}^{(\text{Stand.}, \partial_\perp)}_{\mu\nu} = \bar{q}(y) \Bigg\{
$$

$$
-\frac{p_{1\,\mu}}{y} \varepsilon_{\nu S^T - p_2} \int dx_2 \frac{x_1 - x_2}{x_1 - x_2 + i\epsilon} B^{(1)}(x_1, x_2)
$$

$$
-\left[\frac{p_{2\,\nu}}{x_1} \varepsilon_{\mu S^T - p_2} + \frac{p_{2\,\mu}}{x_1} \varepsilon_{\nu S^T - p_2}\right] x_1 \int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon}
$$

$$
+\frac{p_{1\,\mu}}{y} \varepsilon_{\nu S^T - p_2} \int dx_2 \frac{B^{(\perp)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} \Bigg\},
$$

- the non-standard diagram contributes as

$$\overline{\mathcal{W}}^{\text{(Non-stand.)}}_{\mu\nu} = \bar{q}(y)\frac{p_{2\,\mu}}{x_1}\varepsilon_{\nu S^T - p_2}\int dx_2 \Big\{ B^{(1)}(x_1, x_2) + B^{(2)}(x_1, x_2) \Big\}.$$

Summing up all contributions, we finally obtain the expression

$$
\begin{aligned}
&\overline{\mathcal{W}}_{\mu\nu} = \overline{\mathcal{W}}^{\text{(Stand.)}}_{\mu\nu} + \overline{\mathcal{W}}^{\text{(Stand.}, \partial_\perp)}_{\mu\nu} + \overline{\mathcal{W}}^{\text{(Non-stand.)}}_{\mu\nu} = \\
&\bar{q}(y) \Bigg\{ \left[\frac{p_{2\,\mu}}{x_1} - \frac{p_{1\,\mu}}{y} \right] \varepsilon_{\nu S^T - p_2} \int dx_2 B^{(1)}(x_1, x_2) + \\
&\frac{p_{2\,\mu}}{x_1} \varepsilon_{\nu S^T - p_2} \int dx_2 B^{(2)}(x_1, x_2) - \\
&\left[\frac{p_{2\,\nu}}{x_1} \varepsilon_{\mu S^T - p_2} + \frac{p_{2\,\mu}}{x_1} \varepsilon_{\nu S^T - p_2} \right] x_1 \int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} + \\
&\frac{p_{1\,\mu}}{y} \varepsilon_{\nu S^T - p_2} \int dx_2 \frac{B^{(\perp)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} \Bigg\},
\end{aligned}
$$

Notice that the first term coincides with the hadron tensor calculated within the light-cone (contour) gauge.

The Mandelstam variables for the process and subprocess are defined as

$$S = (p_1 + p_2)^2, \quad T = (p_1 - q)^2, \quad U = (q - p_2)^2,$$
$$\hat{s} = (x_1 p_1 + y p_2)^2 = x_1 y S,$$
$$\hat{t} = (x_1 p_1 - q)^2 = x_1 T, \quad \hat{u} = (q - y p_2)^2 = y U.$$

The quark-gluon correlator reads

$$\Phi^{\perp}_{\rho}(k_1,\ell) = -\int (d^4\eta_1\, d^4 z) e^{-ik_1\eta_1 - i\ell z} \langle p_1|\bar{\psi}(0)\gamma^+\psi(\eta_1)A^{\perp}_{\rho}(z)|p_1\rangle$$
$$= -\varepsilon^{\perp}_{\rho} \int (d^4\eta_1) e^{-ik_1\eta_1} \langle p_1|\bar{\psi}(0)\gamma^+\psi(\eta_1)a^+(\ell)|p_1\rangle \,.$$

Factorization procedure gives us

$$\Phi^{\perp}_{\rho}(x_1,x_2) = \int (d^4k_1\, d^4\ell)\delta(x_1 - k_1 n)\delta(x_{21} - \ell n)\Phi^{\perp}_{\rho}(k_1,\ell) =$$
$$-\varepsilon^{\perp}_{\rho} \int (d\lambda_1) e^{-ix_1\lambda_1} \langle p_1|\bar{\psi}(0)\gamma^+\psi(\lambda_1 n) \int (d^4\ell)\delta(x_{21} - \ell n) a^+(\ell)|p_1\rangle \,.$$

- For checking of the QCD gauge invariance, we make a replacement: $\hat{\varepsilon}^{\perp} \Rightarrow \hat{\ell}_L$ in the diagrams.

We now perform the factorization procedure, we obtain

$$d\mathcal{W}(\text{diag.H}) = \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \delta^{(2)}(\vec{\mathbf{k}}_\perp + \vec{\mathbf{q}}_\perp)\, \mathrm{C}_H \times$$
$$\int dx_1\, dy\, \delta(x_1 - x_B)\, \delta(y - y_B)\, \frac{2}{S} \mathcal{F}^g(y)\, g_\perp^{\alpha\beta} \times$$
$$\int dx_2\, \Phi_\perp^{[\gamma^+],\,\rho}(x_1, x_2)\, H^{\alpha\beta,\rho}(x_1, x_2)\,,$$

for the first type of contributions;

and

$$d\mathcal{W}(\text{diag.D}) = \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \delta^{(2)}(\vec{\mathbf{k}}_\perp + \vec{\mathbf{q}}_\perp)\, \mathrm{C}_D \, \times$$

$$\int dx_1\, dy\, \delta(x_1 - x_B)\, \delta(y - y_B)\, \frac{2}{S} \mathcal{F}^g(y)\, g_\perp^{\alpha\beta}\, \mathrm{tr}_D\big[\Phi^{(1)}(x_1)\, D^{\alpha\beta}(x_1)\big]\,,$$

for the second type of contributions.

To simplify our calculations without losing generality, we may impose the frame where $q_\perp^2 \ll S$. The Mandelstam variable defined for the subprocess, $\hat{u}$, is a small variable and can be neglected. It means that the Bjorken fraction y_B becomes independent of x_B, and $-x_F \approx y_B = -T/S$ (due to $\hat{s} + \hat{t} + \hat{u} = 0$).

After computing the corresponding traces and performing simple algebra within the frame we are choosing, it turns out that the only nonzero contributions to the hadron tensor come from the diagrams H1, H7, D4 and H10:

$$
d\mathcal{W}(\text{diag.H1}) = \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \delta^{(2)}(\vec{\mathbf{k}}_\perp + \vec{\mathbf{q}}_\perp)\, \mathrm{C}_2 \times
$$

$$
\int dx_1\, dy\, \delta(x_1 - x_B)\, \delta(y - y_B)\, \mathcal{F}^g(y) \times
$$

$$
\int dx_2 \frac{2S^2\, x_1\, y^2}{[x_2 yS + i\epsilon][x_1 yS + i\epsilon]^2} \frac{\varepsilon^{q_\perp + S_\perp -}}{p_1^+} B^V_{\underline{-}}(x_1, x_2)\,,
$$

$$d\mathcal{W}(\text{diag.H7}) = \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \delta^{(2)}(\vec{\mathbf{k}}_\perp + \vec{\mathbf{q}}_\perp)\, \mathrm{C}_1 \, \times$$

$$\int dx_1\, dy\, \delta(x_1 - x_B)\, \delta(y - y_B)\, \mathcal{F}^g(y) \;\times$$

$$\int dx_2 \, \frac{(-2) S\, T\, x_1\, (y - 3y_B)}{[x_2 T + i\epsilon][x_1 T + i\epsilon]^2} \, \frac{\varepsilon^{q_\perp + S_\perp -}}{p_1^+} \, B_+^V(x_1, x_2)\,,$$

$$
\begin{aligned}
d\mathcal{W}(\text{diag.D4}) &= \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \delta^{(1)}(\vec{\mathbf{k}}_\perp + \vec{\mathbf{q}}_\perp)\, \mathrm{C}_1 \times \\
&\int dx_1\, dy\, \delta(x_1 - x_B)\, \delta(y - y_B)\, \frac{2}{S} \mathcal{F}^g(y) \times \\
&\frac{2S^2\, x_1\, (y - 2y_B)}{[x_1 T + i\epsilon]^2} \, \frac{\varepsilon^{q_\perp + S_\perp -}}{2x_1 p_1^+ + i\epsilon} \int dx_2\, B_+^V(x_1, x_2)\,,
\end{aligned}
$$

$$d\mathcal{W}(\text{diag.H10}) = \frac{d^3\vec{q}}{(2\pi)^3 2E} \int \frac{d^3\vec{k}}{(2\pi)^3 2\varepsilon} \delta^{(2)}(\vec{\mathbf{k}}_\perp + \vec{\mathbf{q}}_\perp)\, \mathrm{C}_3 \times$$

$$\int dx_1 dy \delta(x_1 - x_B)\, \delta(y - y_B)\, \mathcal{F}^g(y) \times$$

$$\int dx_2 \frac{2T(x_1 - x_2)(2T + Sy)}{[x_1 T + i\epsilon][x_2 T + i\epsilon][(x_1 - x_2)yS + i\epsilon]} \frac{\varepsilon^{q_\perp + S_\perp -}}{p_1^+} B_+^V(x_1, x_2)\,.$$

Here, $\mathrm{C}_1 = C_F^2 N_c$, $\mathrm{C}_2 = -C_F/2$, $\mathrm{C}_3 = C_F\, N_c\, C_A/2$.

The other diagram contributions disappear owing to the following reasons:

- the γ-algebra gives $(\gamma^-)^2 = 0$;
- the common pre-factor $T + yS$ goes to zero,
- the diagrams H2 and H5 cancel each other.