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The 3D structure of QCD
the roots of the standard model

Piet Mulders

mulders@few.vu.nl

European Research Council

Introduction

- High-energy (short-distance) QCD shows strong/successful ‘collinear’ features
  - Jets, PDFs, CSS formalism, SCET, …
- This makes it challenging to look at transverse structure
  - TMDs, azimuthal (spin) asymmetries & their QCD operator structure
  - Need to understand TMD factorization and TMD evolution
- Double role: TMDs encode novel QCD-structure (like GPDs and DPDs) and provides possible tools

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- Double role: TMDs encode novel QCD-structure (like GPDs and DPDs) and provides possible tools

- Several TMDs still have interpretations as densities, but they no longer fully decouple from interactions
  - Gauge links and process dependence
  - Sign changes generalized into gluonic pole factors
  - Multiple functions for quark Pretzelocity and linear gluon polarization

- See proceedings DIS 2015 or QCD evolution 2015

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  - TMDs, azimuthal (spin) asymmetries & their QCD operator structure
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- Is there something underlying 1D to 3D transition in QCD ?
- Maybe!

Speculative & Preliminary

Excitations in the world

- D = 2 is more natural than D = 4
  - Basic symmetry IO(1,1): H (hamiltonian), P (momentum), K (boost)
  - In D = 2 one has dim[ϕ] = 0 and dim[ψ] = ½
- Real bosonic or fermionic excitations: ϕ_R, ϕ_L, and ξ_R, ξ_R (decoupled right/left)

$$M = 0 \;\Rightarrow\; \phi_R(x) = \exp(ik^+x^-) \;\Rightarrow\; i\partial_+\phi_R = [P^-, \phi_R] = 0$$

- Supersymmetric starting point is very natural in D = 2:

$$[Q_{R/L}, \phi_{R/L}] = \xi_{R/L} \qquad \{Q_{R/L}, Q^\dagger_{R/L}\} = 2\,P^\pm$$
$$\{Q_{R/L}, \xi_{R/L}\} = [P^\pm, \phi_{R/L}]$$

- CP symmetry
- Interactions: M (couples right-left)

$$[P^-, \xi_R] = -iM\xi_L \qquad [P^+, \xi_L] = iM\xi_R$$

- Boosts: $[K, P^\pm] = \pm iP^\pm \qquad [K, Q_{R/L}] = \pm\frac{1}{2}iQ_{R/L}$

A basic supersymmetric starting point

- Wess-Zumino (1974), but let us look at D=2 (superrenormalizable):
  Interactions: M (couples right-left), g (Yukawa coupling)
  dim[M] = dim[g] = 1

$$L = \frac{1}{2}\partial_-\phi_R\,\partial_+\phi_R + \frac{1}{2}\partial_+\phi_L\,\partial_-\phi_L + \frac{i}{2}\,\xi_R\partial_+\xi_R + \frac{i}{2}\xi_L\partial_-\xi_L - V$$
$$= \frac{1}{2}\partial^\mu\phi_S\,\partial_\mu\phi_S + \frac{1}{2}\partial^\mu\phi_P\,\partial_\mu\phi_P + \overline{\psi}\,i\,\gamma^\mu\partial_\mu\,\psi - V$$

$$\psi = \frac{1}{\sqrt{2}}\begin{pmatrix}\xi_R \\ -i\xi_L\end{pmatrix} \qquad \phi_{S/P} = (\phi_R \pm \phi_L)/\sqrt{2}$$

$$V = \frac{1}{2}(M + g\phi_S)^2(\phi_S^2 + \phi_P^2) + \frac{1}{2}g^2\phi_P^2(\phi_S^2 + \phi_P^2)$$
$$+ \overline{\psi}(M + g\phi_S + g\phi_P\gamma^1)\psi$$

- Constraint: $(\phi_R\sqrt{2} + v)(\phi_L\sqrt{2} + v) - v^2 = (\phi_S + v)^2 - \phi_P^2 - v^2 = 0$

$$v = M/2g$$

Bosonic excitations

- Constraint using $v = M/2g$

$$(\phi_R\sqrt{2} + v)(\phi_L\sqrt{2} + v) - v^2 = (\phi_S + v)^2 - \phi_P^2 - v^2 = 0$$

- Scalar and pseudoscalar fields + constraint

$$v\Phi_S = \phi_S + v \quad \text{with} \quad \langle \Phi_S \rangle = 1$$
$$v\Phi_P = \phi_P \quad \text{with} \quad \langle \Phi_P \rangle = 0$$

$$\Phi_S = \cosh\eta \qquad \Phi_P = \sinh\eta$$

or

$$\Phi_S = \cos\theta \qquad \Phi_P = i\,\sin\theta$$

- In d=2 pseudoscalar → vector field $\phi_P = A^+ = -A^-$

$$\tfrac{1}{2}\,\partial^\mu\phi_P\,\partial_\mu\phi_P = \tfrac{1}{2}(\partial_+\phi_P)(\partial_-\phi_P) = -\tfrac{1}{4}\,F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}(\partial_\mu A^\mu)^2$$

- Left and right fields with opposite phases (in Hilbert space)

$$\Phi_L^* = \Phi_R \text{ with } \langle \Phi_R \rangle = \langle \Phi_L \rangle = 1/\sqrt{2}$$

A world with three (real) boson and fermion fields

- Extend to three real fields with SO(3) field symmetry (N = 3)
  - Requires M to be proportional to unit matrix and one coupling g
  - Including complex phases, look at SU(3) as full symmetry
- Symmetry of lagrangian and ground state: SO(3), but also P and T
  - IO(1,1) x SO(3) = IO(1,3): D = 4 Poincaré symmetry
  - Real SO(3) symmetric fluctuations identify space-time
- Asymptotic fields living in E(1,4)

!

- Internal symmetries
  - (Oriented) embedding of SO(3), say λ_2, $-\lambda_5$, λ_7 in SU(3) →
  - Identify SU(2) x U(1) generators $T_1 = \lambda_1/2$, $T_2 = \lambda_2/2$ $T_3 = \lambda_3/2$ (isospin plane) and $Y \sim \lambda_8$ (hypercharge). It also fixes the charge operator $Q = T_3 + Y/2$.
  - decoupling required for internal symmetries [Mandula-Coleman 1967]
  - SO(3) embedding is not unique: Z_3 or A_4 symmetry
  - identify A_4 singlets with families [Ma-Rajasekaran 2001, Altarelli-Feruglio 2006]

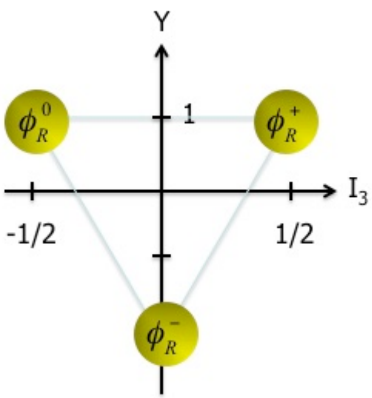
$$SU(3) \supset SO(3) \times A_4 \times [SU(2) \otimes U(1)] \rightarrow SO(3) \otimes [SU(2)_I \otimes U(1)_Y]$$

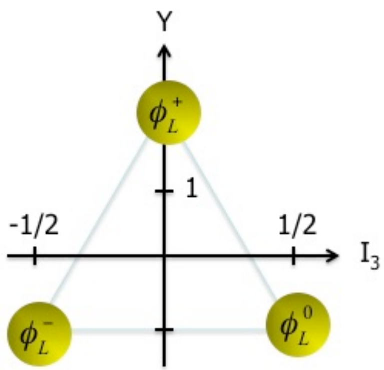
Bosonic excitations

$$IO(1,1) \otimes SU(3) \;\supset\; \underbrace{IO(1,1) \times SO(3)}_{IO(1,3)} \otimes \underbrace{SU(2)_I \otimes U(1)_Y}_{\to\, U(1)_Q} .$$

$$E(1,1): \quad iD_\mu \Phi^i = i\partial_\mu \Phi^i + g \sum_{a=1,\ldots 8} A^a_\mu (T_a)^i_j \Phi^j$$

$$E(1,3): \quad iD_\mu \Phi^i = i\partial_\mu \Phi^i + g \sum_{a=1,2,3,8} A^a_\mu (T_a)^i_j \Phi^j$$





$$\Phi_L = \frac{1}{\sqrt{2}} \exp\Big(-\tfrac{i}{2} \sum_{a=1,2,3} \theta^a \lambda_a\Big) \begin{pmatrix} 0 \\ 1+H \\ 0 \end{pmatrix}$$

$$\Phi_R = \frac{1}{\sqrt{2}} \exp\Big(+\tfrac{i}{2} \sum_{a=1,2,3} \theta^a \lambda_a\Big) \begin{pmatrix} 1+H \\ 0 \\ 0 \end{pmatrix}$$

Bosonic excitations

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$$iD_\mu \Phi = i\partial_\mu \Phi + \frac{g}{2} \left(\sum_{i=1}^{3} W^i_\mu \lambda_i + B_\mu \lambda_8 \right) \Phi$$

$$= i\partial_\mu \Phi + \frac{g}{\sqrt{2}} (W^+_\mu I_- + W^-_\mu I_+) \Phi + g \left(W^0_\mu I_3 + \frac{1}{2\sqrt{3}} B_\mu Y \right) \Phi$$

$\sin \theta_W = 1/2$ [Weinberg, 1972]

$$M = M_{\text{top}} \longrightarrow M_W = M_{\text{top}}/2, M_Z = M_{\text{top}}/\sqrt{3}, M_H = M_{\text{top}}/\sqrt{2}$$

$$M/2g = v = 1 \longrightarrow e/M = 1/4 \text{ (note } \sqrt{4\pi\alpha} \approx 0.3)$$

Fermionic excitations (leptons)

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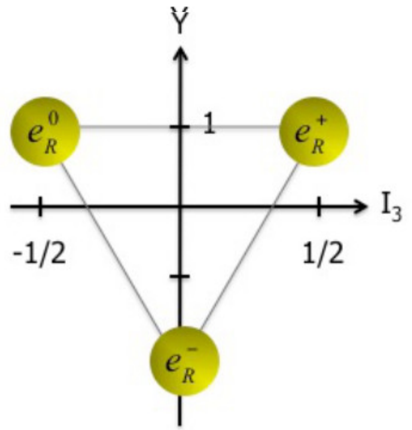

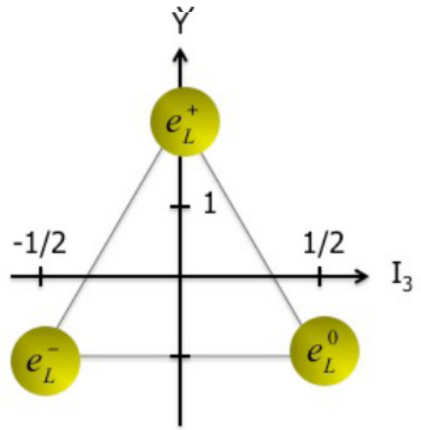

- leptons: 3 majorana's
  → charged Dirac + neutral majorana
- lefthanded doublet + righthanded singlet
  lefthanded singlet + righthanded doublet
- custodial symmetry
- Electroweak charges (in 3D space) can be free!

Fermionic excitations (quarks)

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- Lagrangian in D = 2 includes massless scalar [XQCD, D.B. Kaplan]
- Confinement automatic (color charges in 1D space)
- Electroweak quantum numbers (valence picture)
  - Frozen color (valence) scheme
  - Requirement that (I_3, Y) are SU(3) roots
- Structure as in Rishon Model [Harari & Seiberg 1982]
- Doublets of lefthanded quarks and righthanded antiquarks
- Two singlets of lefthanded and righthanded singlets

Fermionic excitations (quarks)

- Quarks live in E(1,1), coming in 3 families and 3 colors. Only instantaneous interaction. Confinement automatic
- Electroweak interactions
  - In d = 1: ξ^0, ξ^+, or ξ^- charge/momentum eigenstates
  - In d = 2: $(\xi^0\,\xi^0)$, $(\xi^+\,\xi^+)$ and $(\xi^-\,\xi^-)$ charge/helicity eigenstates
  - In d = 3: $\xi_L^0\,(\xi_L^0\,\xi_L^0)$ is acceptable SU(3) root [I_3 quantum numbers]

    $\xi_L^0\,(\xi_L^+\,\xi_L^+)$ and $\xi_L^0\,(\xi_L^-\,\xi_L^-)$ are not acceptable!
- E(1,3): gluons dynamical and 'electroweak properties' of quarks (= QCD)

  Freeze color, e.g. R = red (in triplet 3), L = anti-red (in anti-triplet 3*)
  - For ξ_L^0 only $\xi_L^0\,(\xi_R^+\,\xi_R^+)$ and $\xi_L^0\,(\xi_L^-\,\xi_R^-)$ are acceptable giving the quarks u_L (red) and u_L^* (anti-red), the latter being an iso-singlet
  - $\xi_L^0\,(\xi_R^+\,\xi_R^+)$ and $\xi_L^-\,(\xi_R^0\,\xi_R^0)$ form a red iso-doublet u_L and d_L
- Structure similar as in Rishon Model [Harari & Seiberg 1982]

Fermionic content of standard model

particle	space			isospin		hypercharge	charge	color
	L	T_1	T_2	I	I_3	Y	Q	$\underline{c}$
ν_L	ξ_L^0	ξ_L^0	ξ_L^0	$1/2$	$+1/2$	-1	0	$\underline{1}$
e_L^-	ξ_L^-	ξ_L^-	ξ_L^-	$1/2$	$-1/2$	-1	-1	$\underline{1}$
e_L^+	ξ_L^+	ξ_L^+	ξ_L^+	0	0	$+2$	$+1$	$\underline{1}$
ν_R	ξ_R^0	ξ_R^0	ξ_R^0	$1/2$	$-1/2$	$+1$	0	$\underline{1}$
e_R^+	ξ_R^+	ξ_R^+	ξ_R^+	$1/2$	$+1/2$	$+1$	$+1$	$\underline{1}$
e_R^-	ξ_R^-	ξ_R^-	ξ_R^-	0	0	-2	-1	$\underline{1}$
u_L	ξ_L^0	$(\xi_R^+$	$\xi_R^+)$	$1/2$	$+1/2$	$+1/3$	$+2/3$	$\underline{3}$
d_L	ξ_L^-	$(\xi_R^0$	$\xi_R^0)$	$1/2$	$-1/2$	$+1/3$	$-1/3$	$\underline{3}$
$\overline{u}_L$	ξ_L^0	$(\xi_L^-$	$\xi_R^-)$	0	0	$-4/3$	$-2/3$	$\underline{3}^*$
$\overline{d}_L$	ξ_L^+	$(\xi_L^0$	$\xi_R^0)$	0	0	$+2/3$	$+1/3$	$\underline{3}^*$
$\overline{u}_R$	ξ_R^0	$(\xi_L^-$	$\xi_L^-)$	$1/2$	$-1/2$	$-1/3$	$-2/3$	$\underline{3}^*$
$\overline{d}_R$	ξ_R^+	$(\xi_L^0$	$\xi_L^0)$	$1/2$	$+1/2$	$-1/3$	$+1/3$	$\underline{3}^*$
u_R	ξ_R^0	$(\xi_L^+$	$\xi_R^+)$	0	0	$+4/3$	$+2/3$	$\underline{3}$
d_R	ξ_R^-	$(\xi_L^0$	$\xi_R^0)$	0	0	$-2/3$	$-1/3$	$\underline{3}$

Fermionic content of standard model

particle	space			isospin		hypercharge	charge	color
	L	T_1	T_2	I	I_3	Y	Q	$\underline{c}$
ν_L	ξ_L^0	ξ_L^0	ξ_L^0	$1/2$	$+1/2$	-1	0	$\underline{1}$
e_L^-	ξ_L^-	ξ_L^-	ξ_L^-	$1/2$	$-1/2$	-1	-1	$\underline{1}$
e_L^+	ξ_L^+	ξ_L^+	ξ_L^+	0	0	$+2$	$+1$	$\underline{1}$
ν_R	ξ_R^0	ξ_R^0	ξ_R^0	$1/2$	$-1/2$	$+1$	0	$\underline{1}$
e_R^+	ξ_R^+	ξ_R^+	ξ_R^+	$1/2$	$+1/2$	$+1$	$+1$	$\underline{1}$
e_R^-	ξ_R^-	ξ_R^-	ξ_R^-	0	0	-2	-1	$\underline{1}$
u_L	ξ_L^0	$(\xi_R^+$	$\xi_R^+)$	$1/2$	$+1/2$	$+1/3$	$+2/3$	$\underline{3}$
d_L	ξ_L^-	$(\xi_R^0$	$\xi_R^0)$	$1/2$	$-1/2$	$+1/3$	$-1/3$	$\underline{3}$
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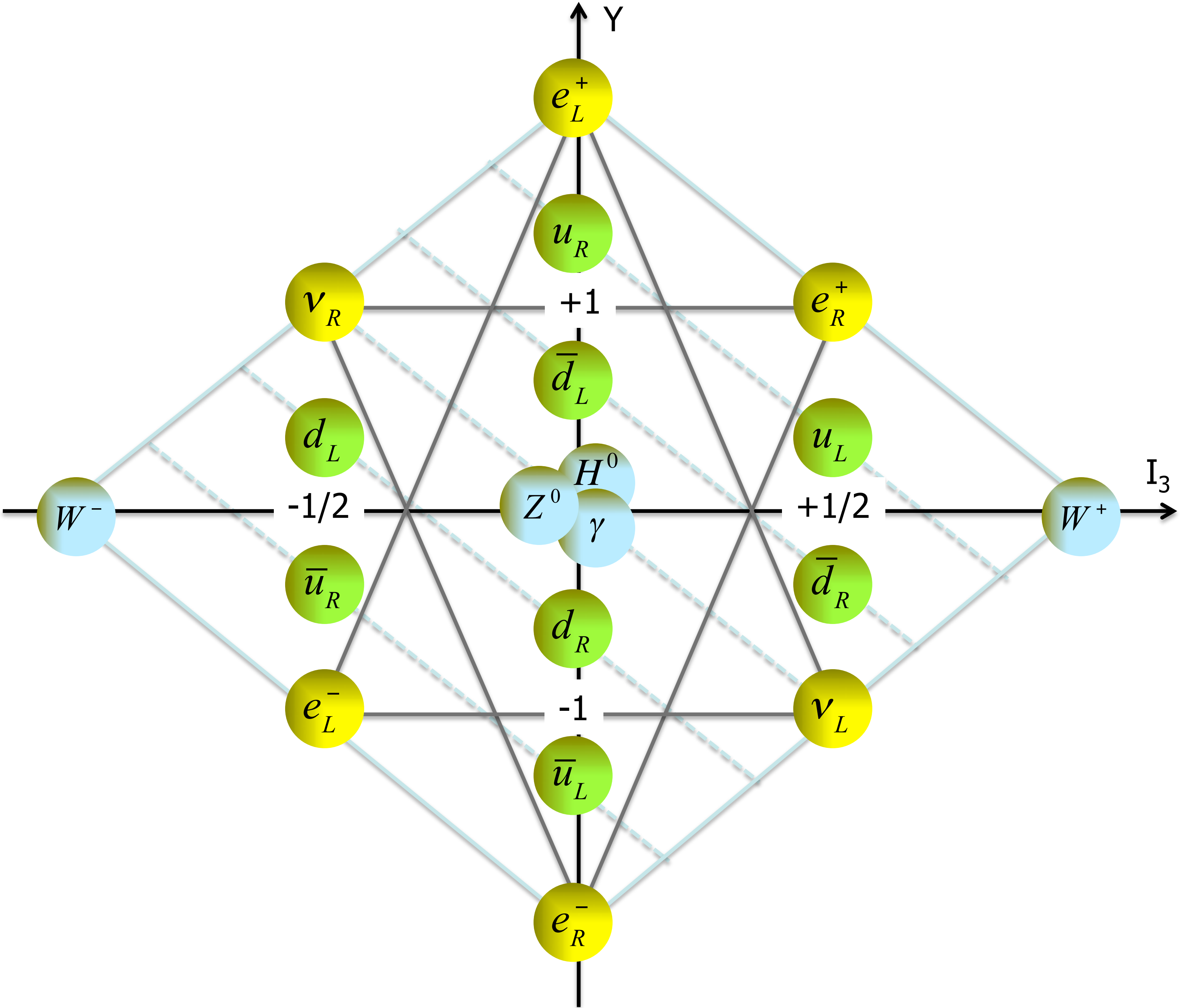
$\overline{u}_R$	ξ_R^0	$(\xi_L^-$	$\xi_L^-)$	$1/2$	$-1/2$	$-1/3$	$-2/3$	$\underline{3}^*$
$\overline{d}_R$	ξ_R^+	$(\xi_L^0$	$\xi_L^0)$	$1/2$	$+1/2$	$-1/3$	$+1/3$	$\underline{3}^*$
u_R	ξ_R^0	$(\xi_L^+$	$\xi_R^+)$	0	0	$+4/3$	$+2/3$	$\underline{3}$
d_R	ξ_R^-	$(\xi_L^0$	$\xi_R^0)$	0	0	$-2/3$	$-1/3$	$\underline{3}$


Y
I_3
e_L^+
u_R
ν_R
+1
e_R^+
$\bar{d}_L$
d_L
u_L
H^0
W^-
-1/2
Z^0
γ
+1/2
W^+
$\bar{u}_R$
$\bar{d}_R$
d_R
e_L^-
-1
ν_L
$\bar{u}_L$
e_R^-

Fermionic excitations (quarks)

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- Two singlets of lefthanded and righthanded singlets
- In zeroth order one family takes all mass: top-quark, t ~ ξ^0 (ξ^+ ξ^+)

Concluding remarks

$$IO(1,1) \otimes SU(3) \supset \underbrace{IO(1,1) \times SO(3)}_{IO(1,3)} \otimes \underbrace{SU(2)_I \otimes U(1)_Y}_{\rightarrow U(1)_Q} .$$

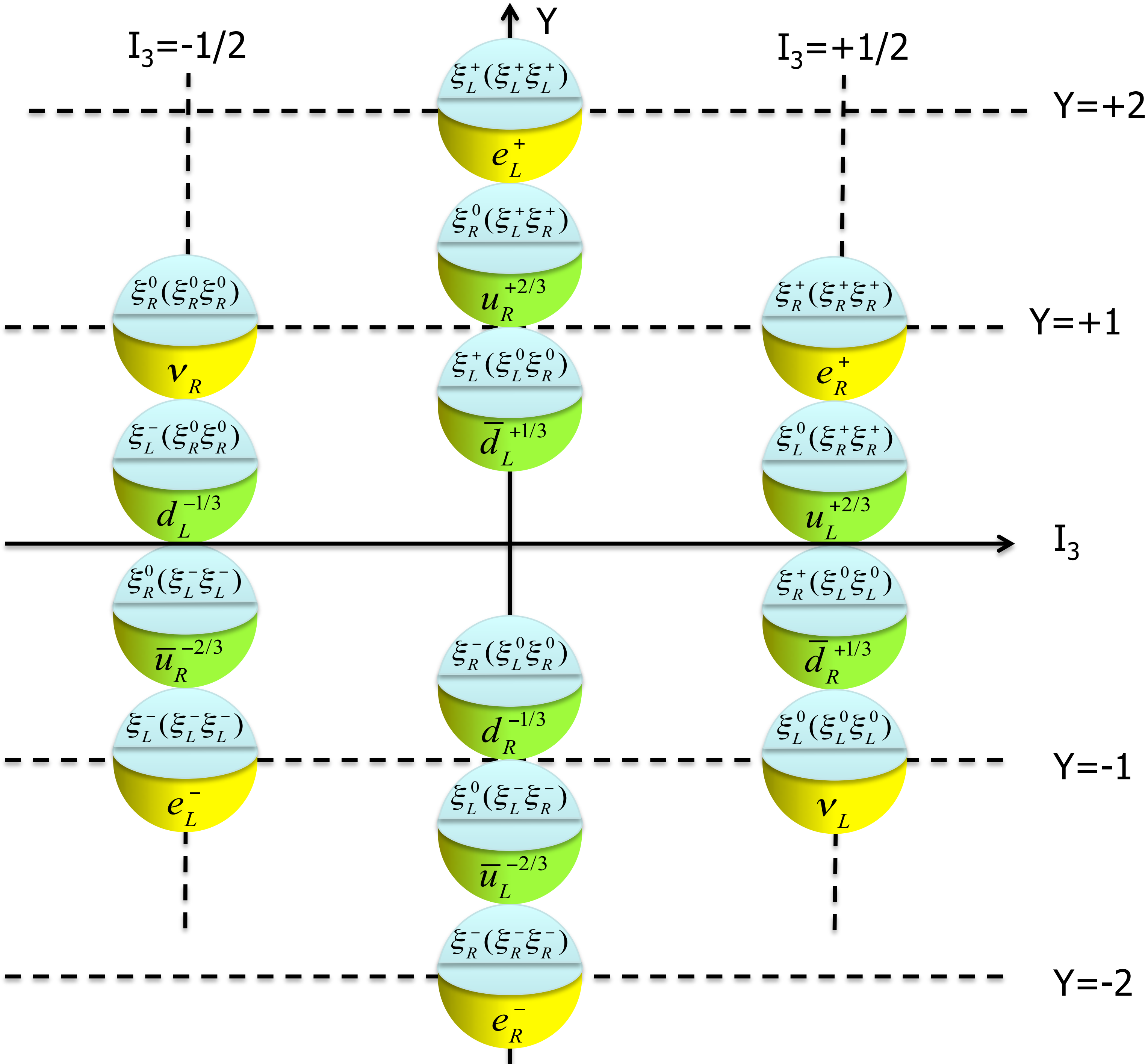
- Basic supersymmetric starting point, solves hierarchy and naturalness problems
- Links # space dimensions, # colors, # families
- Provides spectrum of bosons and fermions in standard model
- Allows for family mixing (M and g can be complex symmetric), role for A_4
- Left-right symmetric starting point and custodial symmetry
- B-L symmetry
- D = 2 and D = 4 worlds meet at QCD scale !!!
- Provides a new view for many phenomena in QCD (Confinement, importance of SCET for PDFs, PFFs including TMDs, multitude of effective models for QCD)
- Electroweak charges are interesting region: $1 - 4 \sin^2\theta_W$
- Hydrogen involves all excitations in lowest family: $(\xi^+\xi^-\xi^0)^4$

  Family-breaking effects when different embeddings meet: proton radius puzzle

- There are still many open ends !

Frozen color scheme

Disney Frozen Color Changers Elsa Anna Royal Changing Dolls


Y
I3=-1/2
I3=+1/2
Y=+2
Y=+1
I3
Y=-1
Y=-2
ξ+L(ξ+Lξ+L)
e+L
ξ0R(ξ+Lξ+R)
u+2/3R
ξ0R(ξ0Rξ0R)
νR
ξ+R(ξ+Rξ+R)
e+R
ξ+L(ξ0Lξ0R)
d̄+1/3L
ξ−L(ξ0Rξ0R)
d−1/3L
ξ0L(ξ+Rξ+R)
u+2/3L
ξ0R(ξ−Lξ−L)
ū−2/3R
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