Forward dijet production and improved TMD factorization in dilute-dense hadronic collisions

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in collaboration with:
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Phys.Rev. D89 (2014) 9, 094014 and work in progress

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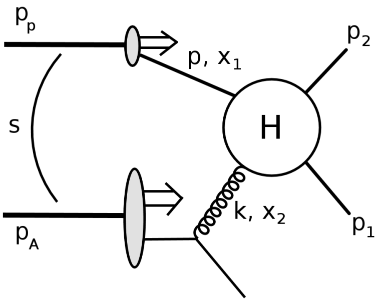
Forward dijets in *dilute-dense* hadronic collisions



$$\hat{s} = (p+k)^2$$
$$\hat{t} = (p_2 - p)^2$$
$$\hat{u} = (p_1 - p)^2$$

Incoming partons' energy fractions:

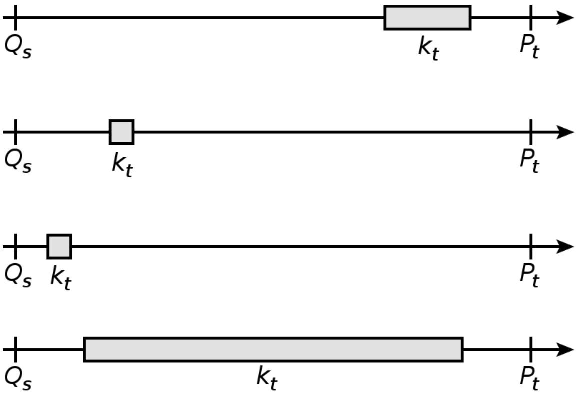
$$x_1 = \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \qquad x_2 = \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \quad \xrightarrow{y_1, y_2 \gg 0} \quad x_1 \sim 1, \qquad x_2 \ll 1$$

Gluon's transverse momentum (p_{1t}, p_{2t} imbalance):

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}| \cos \Delta\phi$$

Scales and regimes

P_t – average jet transverse momentum
k_t – target gluon's transverse momentum (dijets imbalance)
Q_s – saturation scale



High Energy Factorization

Generalized Transverse Momentum Dependent Factorization

Collinear factorization

Improved Transverse Momentum Dependent Factorization (this talk)

HEF and generalized TMD factorization

High Energy Factorization *[Catani, Ciafaloni, Hautmann 1991]*

$$\frac{d\sigma^{pA\to\mathrm{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} \propto \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}_{g/A}(x_2,k_t)$$

- $x_1 f_{a/p}(x_1,\mu^2)$ – collinear PDF in p, suitable for $x_1 \sim 1$
- $|\overline{\mathcal{M}_{ag^*\to cd}}|^2$ – matrix element with off-shell incoming gluon
- $\mathcal{F}_{g/A}(x_2,k_t)$ – unintegrated gluon PDF in A, suitable for $x_2 \ll 1$

Generalized TMD factorization *[Dominguez, Marquet, Xiao, Yuan 2011]*

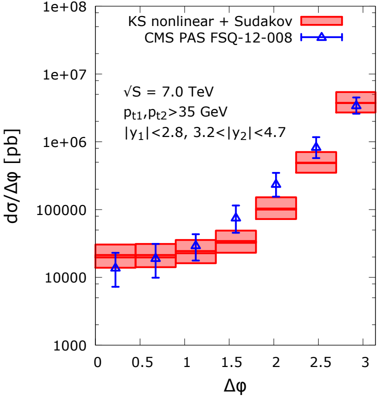
$$\frac{d\sigma^{pA\to\mathrm{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} \propto \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t)$$

- $H^{(i)}_{ag\to cd}$ – hard factor of i-th type, with on-shell incoming gluon
- $\mathcal{F}^{(i)}_{ag}(x_2,k_t)$ – unintegrated gluon distribution of i-th type in A

Successes, strengths and limitations of HEF (1/2)

Forward-central dijet production: proton-proton

[van Hameren, Kotko, Kutak and SS 2014]



HEF approach with:

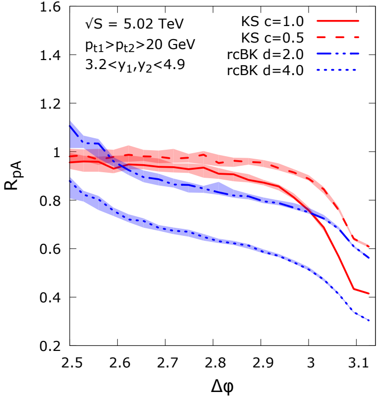
- KS gluon: BK with kinematic constraint, non-singular DGLAP pieces, running coupling – fitted to combined F_2 HERA data
- Sudakov resummation: ensures no emission between k_t and μ (turns out to be relevant for moderate $\Delta\phi$)

Successful description of azimuthal decorrelations!

Successes, strengths and limitations of HEF (2/2)

Forward-forward dijet production: proton-proton vs proton-lead

[van Hameren, Kotko, Kutak, Marquet and SS 2014]



HEF approach with:

- Nuclear modification factor

$$R_{\text{pA}} = \frac{d\sigma^{p+A}}{A\, d\sigma^{p+p}}$$

- KS gluon: parameter c controls strength of the non-linear term
- rcBK gluon: BK equation with running coupling, parameter d modifies saturation scale in the initial condition

Sizable effects of gluon saturation at $\Delta\phi = \pi$ $\rightarrow$ Predictions qualitatively consistent for all gluons and model parameters.

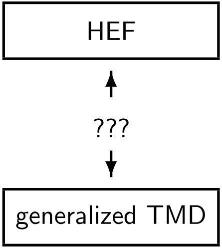
Observations:

- HEF does a good job in describing data and provides interesting predictions. Reliable when $k_t \sim P_t$. Not applicable in the strict back-to-back limit with $\Delta\phi \simeq \pi$, which corresponds to $k_t \ll P_t$.

- In the strict back-to-back limit, one needs to turn into generalized TMD factorization – reliable when $k_t \ll P_t$. But that approach misses important ingredient of HEF namely k_t dependence in the matrix element. It is therefore not applicable when $k_t \sim P_t$.

Would there be a way to combine virtues of the two approaches?

Relating HEF and generalized TMD factorization



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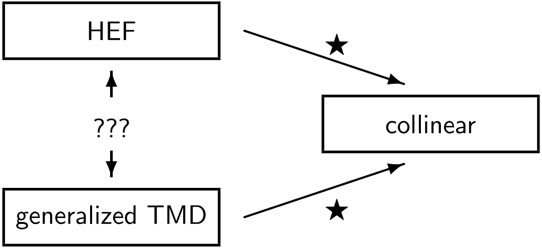
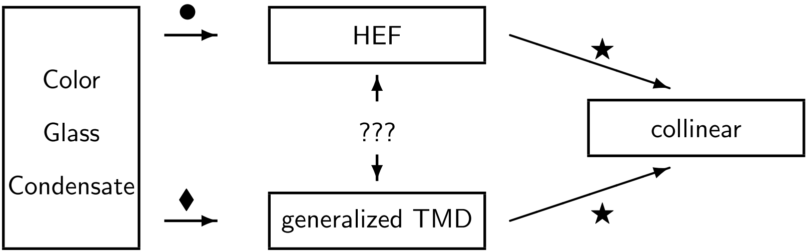


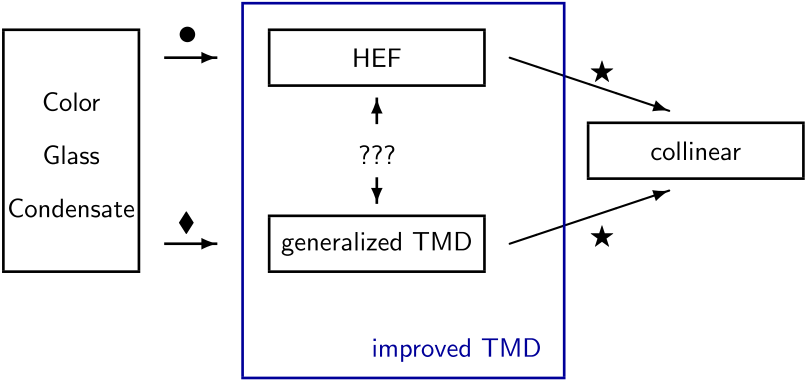
★ easy

Relating HEF and generalized TMD factorization



- ★ easy
- ● *[Kotko, Kutak, Marquet, Petreska, SS and van Hameren 2015]* (see Elena's talk)
- ♦ *[Dominguez, Marquet, Xiao, Yuan 2011]*

Relating HEF and generalized TMD factorization



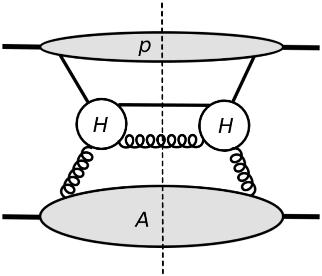
★ easy

● *[Kotko, Kutak, Marquet, Petreska, SS and van Hameren 2015]* (see Elena's talk)

♦ *[Dominguez, Marquet, Xiao, Yuan 2011]*

??? this talk

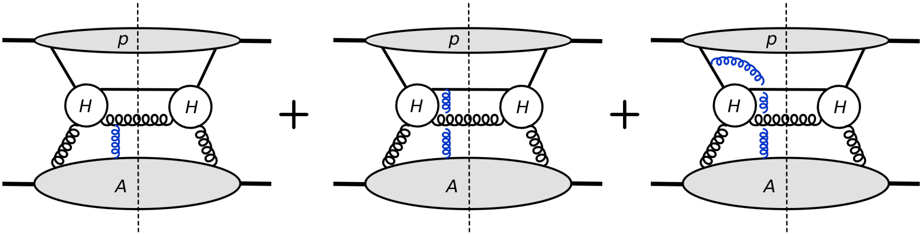
TMD gluon distribution (first try)



$$\mathcal{F}_{g/A}(x_2, k_t) \overset{\text{naive}}{=} 2 \int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \boldsymbol{\xi}_t} \left\langle A \right| \mathrm{Tr} \left[F^{i-}\left(\xi^+, \boldsymbol{\xi}_t\right) F^{i-}(0) \right] \left| A \right\rangle$$

This definition is gauge dependent!

TMD gluon distributions (proper definition)



+ similar diagrams with $2, 3, \ldots$ gluon exchanges

They all contribute at leading power and need to be resummed.

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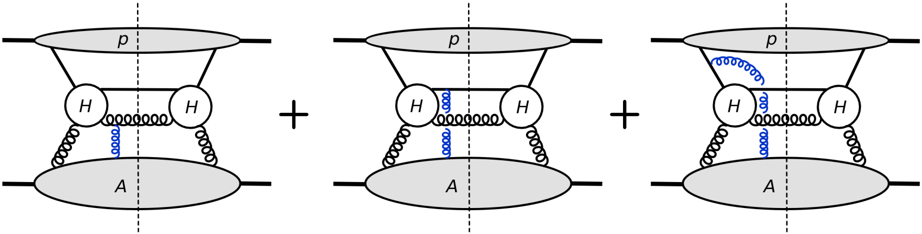

$+$ similar diagrams with $2, 3, \ldots$ gluon exchanges

They all contribute at leading power and need to be resummed.

That is done by gauge links $U_{[\alpha,\beta]}$

$$\mathcal{F}_{g/A}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \boldsymbol{\xi}_t} \langle A | \mathrm{Tr}\left[F^{i-}\left(\xi^+, \boldsymbol{\xi}_t\right) U_{[\xi,0]} F^{i-}(0) \right] | A \rangle$$

- $U_{[\alpha,\beta]}$ renders gluon distribution gauge invariant

Gauge links

Wilson lines along the path from α to β

$$W_{[\alpha,\beta]} = \mathcal{P}\exp\left[-ig\int_{\alpha}^{\beta} d\eta^{\mu} A^{a}(\eta)T^{a}\right]$$

The path $[\alpha,\beta]$ depends on the hard process.

- Gluon TMD, $\mathcal{F}$, is in general process-dependent.

Gauge links

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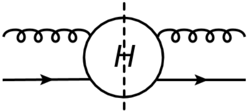
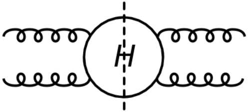
Cross section for dijet production in hadron-hadron collisions cannot be written down with just a single gluon! *[Bomhof, Mulders, Pijlman 2006]*



$\mathcal{F}_{qg}^{(1)}, \mathcal{F}_{qg}^{(2)}$



$\mathcal{F}_{gg}^{(1)}, \mathcal{F}_{gg}^{(2)}, \mathcal{F}_{gg}^{(3)}, \mathcal{F}_{gg}^{(4)}, \mathcal{F}_{gg}^{(5)}, \mathcal{F}_{gg}^{(6)}$

Generalized TMD factorization

[Dominguez, Marquet, Xiao, Yuan 2011]

$$\frac{d\sigma^{pA\to cdX}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\, x_1 f_{q/p}(x_1,\mu^2) \sum_{i=1}^{n} \mathcal{F}_{ag}^{(i)} H_{ag\to cd}^{(i)} \frac{1}{1+\delta_{cd}}$$

- $qg \to qg$: n $= 2$
  $gg \to q\bar{q}$: n $= 3$
  $gg \to gg$: n $= 6$
- $H_{ag\to cd}^{(i)}$ obtained in large N_c limit
- $H_{ag\to cd}^{(i)}$ obtained in the approximation $k_t = 0$

Towards improved TMD factorization (see Elena's talk)

[Kotko, Kutak, Marquet, Petreska, SS and van Hameren 2015]

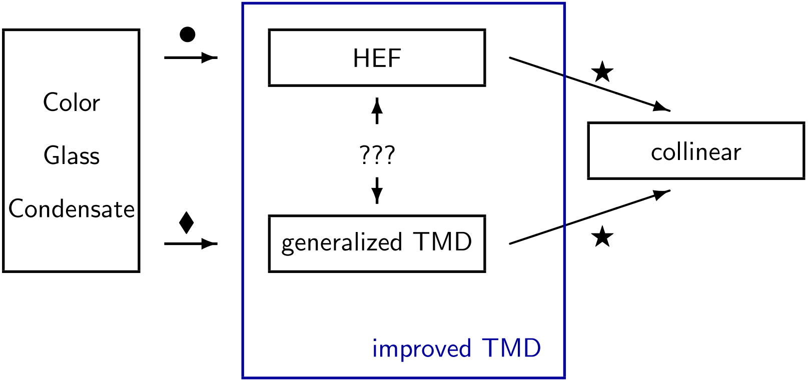
- Reduce to n = 2 for all channels

$$\frac{d\sigma^{pA\to\mathrm{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag\to cd}\Phi^{(i)}_{ag\to cd}\,\frac{1}{1+\delta_{cd}}$$

- Restore all the finite-N_c terms in the hard factors

	$K^{(1)}_{ag\to cd}$	$K^{(2)}_{ag\to cd}$
$qg\to qg$	$-\frac{\hat{s}^2+\hat{u}^2}{2\hat{t}^2\hat{s}\hat{u}}\left[\hat{u}^2+\frac{\hat{s}^2-\hat{t}^2}{N_c^2}\right]$	$-\frac{C_F}{N_c}\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{\hat{t}^2\hat{u}}$
$gg\to q\bar{q}$	$\frac{1}{2N_c}\frac{(\hat{t}^2+\hat{u}^2)^2}{\hat{s}^2\hat{t}\hat{u}}$	$-\frac{1}{2C_FN_c^2}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$
$gg\to gg$	$\frac{2N_c}{C_F}\frac{(\hat{s}^2-\hat{t}\hat{u})^2(\hat{t}^2+\hat{u}^2)}{\hat{t}^2\hat{u}^2\hat{s}^2}$	$\frac{2N_c}{C_F}\frac{(\hat{s}^2-\hat{t}\hat{u})^2}{\hat{t}\hat{u}\hat{s}^2}$

Relating HEF and generalized TMD factorization



★ easy

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⧫ *[Dominguez, Marquet, Xiao, Yuan 2011]*

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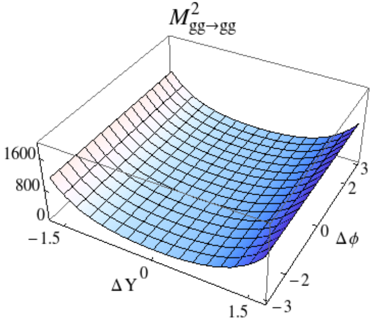
Off-shell matrix elements



- On-shell matrix element is a very bad approximation for dijet configurations with the two jets close in azimuthal angle $\Delta\phi \simeq 0$.
- These configurations arise when the gluon transverse momentum $k_t \gg 0$.

Off-shell matrix elements

- We cannot just use the usual prescriptions for calculating on-shell matrix elements and keep the gluon off-shell. Such result would not be gauge-invariant!

The process $ag^* \to cd$ must be embedded in an on-shell process.

Off-shell matrix elements

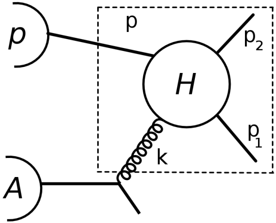
- We cannot just use the usual prescriptions for calculating on-shell matrix elements and keep the gluon off-shell. Such result would not be gauge-invariant!

The process $ag^* \to cd$ must be embedded in an on-shell process.

Effective procedure: *[Catani, Ciafaloni, Hautmann 1991]*

- Standard Feynman rules with the gauge vector $n = p_A$.
- Off-shell gluon's longitudinal polarization vector:

$$\epsilon^0_\mu = \frac{i\sqrt{2}\,x_2}{|k_t|} p_{A\,\mu}\,.$$



This procedure is just enough to render the result gauge-invariant.

Generalizations to arbitrary gauge and higher multiplicities exist. *[Lipatov 1995; Antonov, Lipatov, Kuraev, Chrednikov 2005] [van Hameren, Kotko, Kutak 2013]*

Colour ordered amplitudes

N-gluon amplitude:

$$\mathcal{M}^{a_1\ldots a_N}\left(\varepsilon_1^{\lambda_1},\ldots,\varepsilon_N^{\lambda_N}\right) = \sum_{\sigma\in S_{N-2}} \left(F^{a_{\sigma_2}}\ldots F^{a_{\sigma_{N-1}}}\right)_{a_1 a_N} \mathcal{M}\left(1^{\lambda_1},\sigma_2^{\lambda_{\sigma_2}},\ldots,\sigma_{N-1}^{\lambda_{\sigma_{N-1}}},N^{\lambda_N}\right),$$

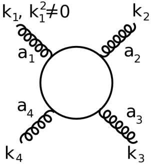
where $(F^a)_{bc} = f_{abc}$ and S_{N-2} are $(N-2)!$ non-cyclic permutations.

- colour ordered amplitudes $\mathcal{M}\left(1^{\lambda_1},\sigma_2^{\lambda_{\sigma_2}},\ldots,\sigma_{N-1}^{\lambda_{\sigma_{N-1}}},N^{\lambda_N}\right)$ are gauge invariant
- only $(N-2)!$ of them needed for amplitude with N external legs

TMDs from colour ordered amplitudes

In the $gg^* \to gg$ channel:

$$\mathcal{M}^{a_1 a_2 a_3 a_4}_{gg^* \to gg}\left(n_1, \varepsilon_2^{\lambda_2}, \varepsilon_3^{\lambda_3}, \varepsilon_4^{\lambda_4}\right) = f_{a_1 a_2 c} f_{c a_3 a_4}\, \mathcal{M}_{gg^* \to gg}\left(1^*, 2, 3, 4\right) + f_{a_1 a_3 c} f_{c a_2 a_4}\, \mathcal{M}_{gg^* \to gg}\left(1^*, 3, 2, 4\right),$$



where $k_1 = n_1 + k_T$.

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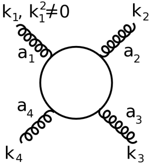


where $k_1 = n_1 + k_T$.

Only two unique colour structures multiplying the gauge links corresponding to $\mathcal{F}^{(i)}_{gg}$ $\rightarrow$ two independent TMDs.

colour-ordered amplitude squared	gluon TMD
$\left\lvert\mathcal{M}_{gg^* \to gg}(1^*, 2, 3, 4)\right\rvert^2$ $\left\lvert\mathcal{M}_{gg^* \to gg}(1^*, 3, 2, 4)\right\rvert^2$	$\Phi^{(1)}_{gg \to gg} = \frac{1}{2N_c^2}\left(N_c^2 \mathcal{F}^{(1)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2 \mathcal{F}^{(6)}_{gg}\right)$
$\mathcal{M}_{gg^* \to gg}(1^*, 2, 3, 4)\, \mathcal{M}^*_{gg^* \to gg}(1^*, 3, 2, 4)$ $\mathcal{M}^*_{gg^* \to gg}(1^*, 2, 3, 4)\, \mathcal{M}_{gg^* \to gg}(1^*, 3, 2, 4)$	$\Phi^{(2)}_{gg \to gg} = \frac{1}{N_c^2}\left(N_c^2 \mathcal{F}^{(2)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2 \mathcal{F}^{(6)}_{gg}\right)$

Similar story for the other two channels $qg \to qg$ and $gg \to q\bar{q}$.

Improved TMD factorization

The final formula

$$\frac{d\sigma^{pA\to\mathrm{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}\Phi^{(i)}_{ag\to cd}\,\frac{1}{1+\delta_{cd}}$$

i	1	2
$K^{(i)}_{gg^*\to gg}$	$\frac{N_c}{C_F}\,\frac{(\bar{s}^4+\bar{t}^4+\bar{u}^4)(\bar{u}\hat{u}+\bar{t}\hat{t})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$	$-\frac{N_c}{2C_F}\,\frac{(\bar{s}^4+\bar{t}^4+\bar{u}^4)(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$
$K^{(i)}_{gg^*\to q\bar{q}}$	$\frac{1}{2N_c}\,\frac{(\bar{t}^2+\bar{u}^2)(\bar{u}\hat{u}+\bar{t}\hat{t})}{\bar{s}\hat{s}\hat{t}\hat{u}}$	$\frac{1}{4N_c^2C_F}\,\frac{(\bar{t}^2+\bar{u}^2)(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s})}{\bar{s}\hat{s}\hat{t}\hat{u}}$
$K^{(i)}_{qg^*\to qg}$	$-\frac{\bar{u}\,(\bar{s}^2+\bar{u}^2)}{2\bar{t}\hat{t}\hat{s}}\left(1+\frac{\bar{s}\hat{s}-\bar{t}\hat{t}}{N_c^2\;\bar{u}\hat{u}}\right)$	$-\frac{C_F}{N_c}\,\frac{\bar{s}(\bar{s}^2+\bar{u}^2)}{\bar{t}\hat{t}\hat{u}}$

Modified Mandelstam variables:

- $\bar{s}=(x_2p_A+p)^2,\qquad \bar{t}=(x_2p_A-p_1)^2,\qquad \bar{u}=(x_2p_A-p_2)^2$

Recovery of the on-shell limit:

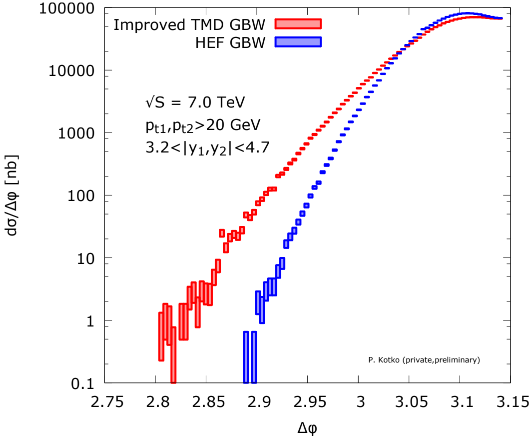
- $\lim_{|k_t|\to 0}(\bar{s}-\hat{s})=0,\quad \lim_{|k_t|\to 0}(\bar{t}-\hat{t})=0,\quad \lim_{|k_t|\to 0}(\bar{u}-\hat{u})=0$

First numerical results: improved TMD vs HEF

- GBW model used to compute TMDs
- off-shell hard factors
- large N_c limit

Improved TMD factorization points towards less correlations.

Azimuthal correlations in forward dijets production



Conclusions

- We have developed the improved TMD factorization approach for forward dijet production in dilute-dense hadronic collisions.
- Our framework unifies HEF (off-shell ME but single gluon TMD) and generalized TMD factorization (multiple TMDs but on-shell ME).
- The improved TMD factorization formula is valid in the limit $|p_{1t}|, |p_{2t}| \gg Q_s$ with an arbitrary value of $|k_t|$.
- Our result provides a robust framework for studies of saturation domain with hard objects.
- We have just begun numerical studies: first results with GBW model point towards slightly smaller azimuthal correlations in forward dijet production.

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Future

- Phenomenology of improved TMD factorization with gluons from a range of models.
- Exploration of all-order validity of the improved TMD factorization in for hadronic dijet production in dilute-dense collisions.