Dispersion Representation of the D-term form factor in Deeply Virtual Compton Scattering

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Outline

- Dispersion Relations (DRs)
  for Deeply Virtual Compton Scattering (DVCS)

- D-Term Form Factor
  - ✓ subtraction function in s-channel DRs
  - ✓ predictions from DRs in the t-channel
  - ✓ physical content

talks of D. Mueller and K. Kumericki

parallel session Spin-3D : K. Semenov-Tian-Shansky

Nucleon Sructure Properties

em	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N' \vert J^\mu_{\text{em}} \vert N \rangle$	$\longrightarrow$	$Q,\ \mu$
weak	$\partial_\mu J^\mu_{\text{weak}} = 0$	$\langle N' \vert J^\mu_{\text{weak}} \vert N \rangle$	$\longrightarrow$	$g_A,\ g_p$
gravity	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N' \vert T^{\mu\nu}_{\text{grav}} \vert N \rangle$	$\longrightarrow$	$M_N,\ J,\ d_1$

$$Q_{\text{prot}} = 1.602176487(40) \times 10^{-19}\,\text{C}$$

$$\mu_{\text{prot}} = 2.792847356(23) \mu_N$$

$$M_{\text{prot}} = 938.272013(23)\,\text{MeV}$$

$$g_p = 8 - 12$$

$$g_A = 1.2694(28)$$

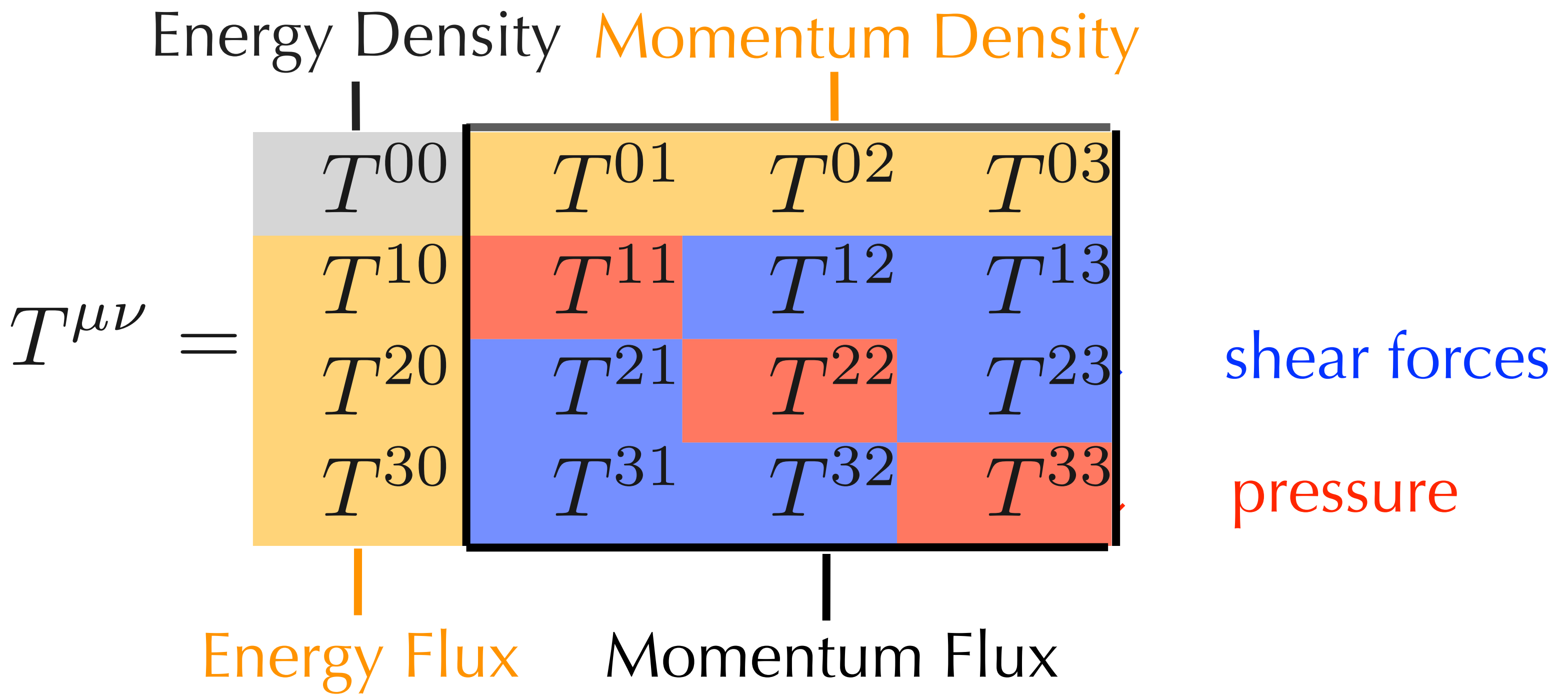
$$J = \frac{1}{2}$$

$$d_1 = ??$$

can be accessed from GPDs in hard exclusive reactions

Form Factors of Energy Momentum Tensor

talk of C. Lorcé and S. Liuti



Energy Density

Momentum Density

$$T^{\mu\nu} = \begin{array}{cccc} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{array}$$

shear forces

pressure

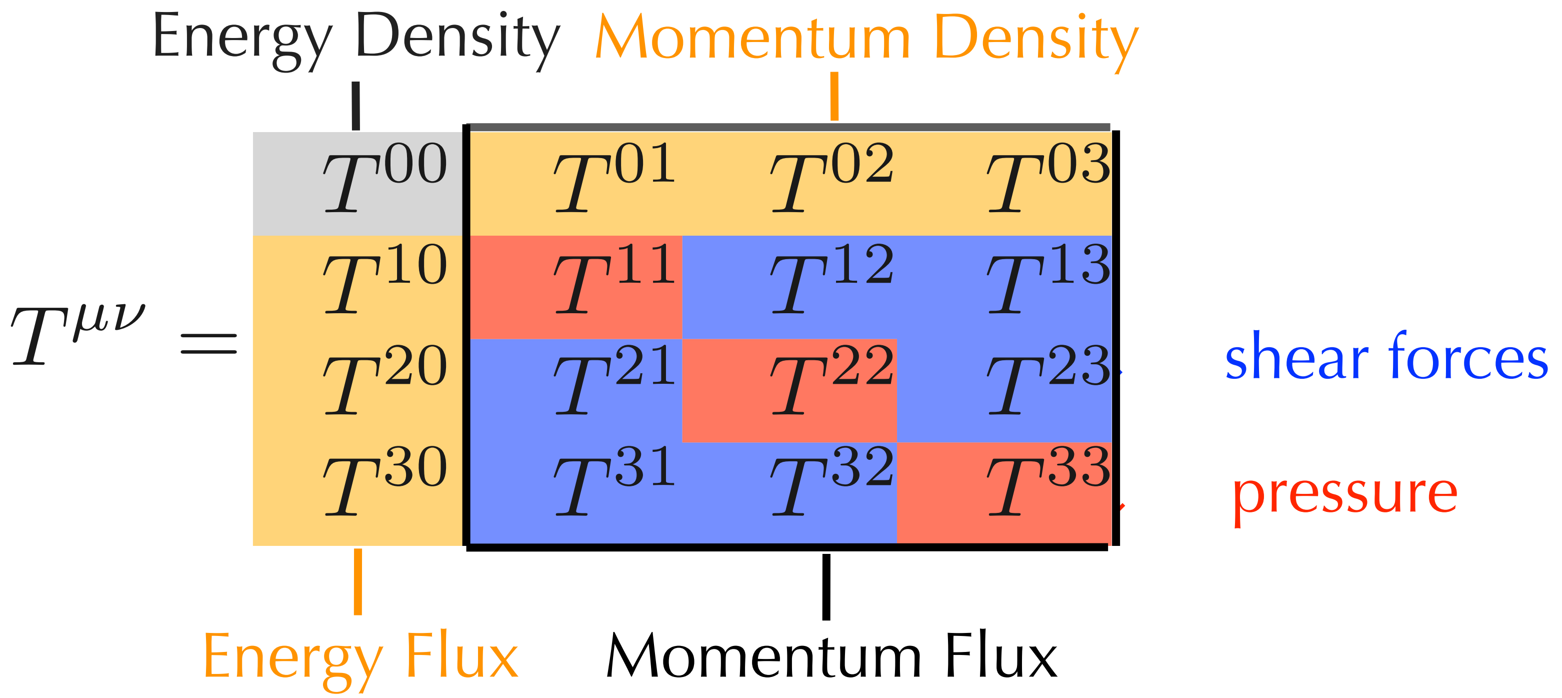
Energy Flux

Momentum Flux

$$\langle P'|T^{Q,G}_{\mu\nu}|P\rangle = \bar{u}(P')[M_2^{Q,G}(t)\,\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}]u(P)$$

Form Factors of Energy Momentum Tensor

talk of C. Lorcé and S. Liuti



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Relation with second-moments of GPDs:

“Charges” of the EM Tensor Form Factors at t=0

$$\sum_q \int \mathrm{d}x\, x\, H^q(x,\xi,t) = M_2^Q(t) + \frac{4}{5}\, d_1^Q(t)\xi^2$$

$$\sum_q \int \mathrm{d}x\, x\, E^q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5}\, d_1^Q(t)\xi^2$$

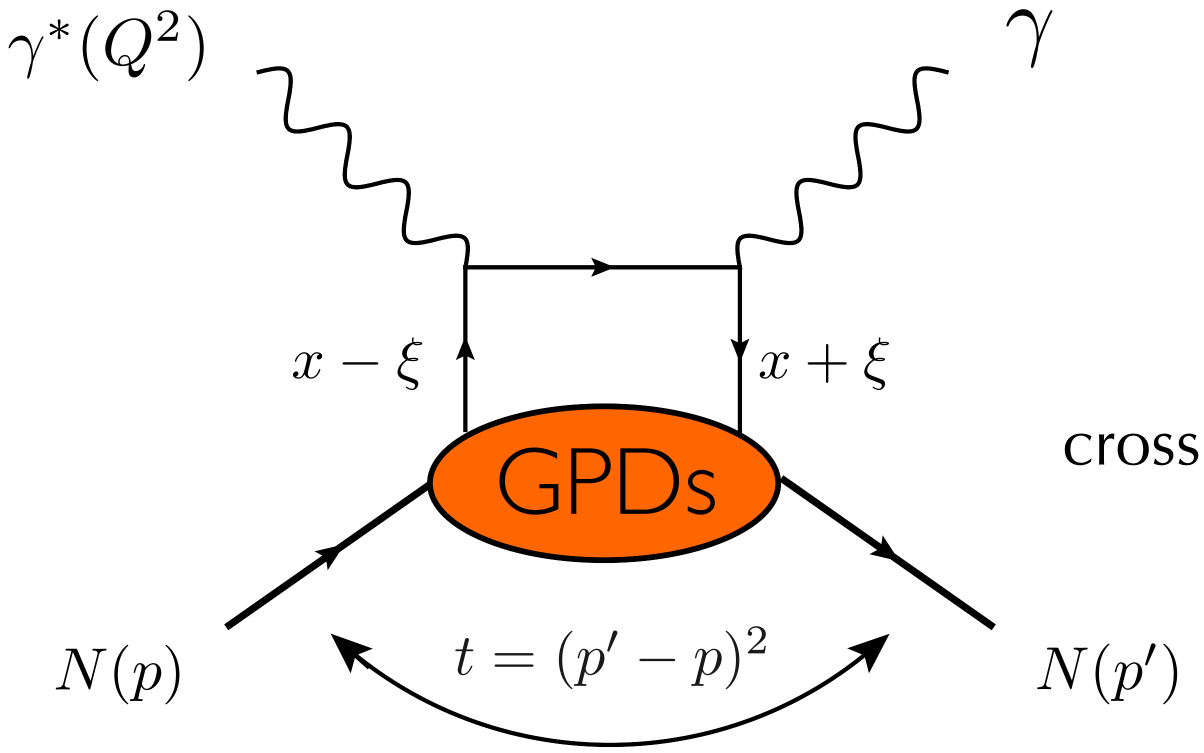
$M_2(0)$ nucleon momentum carried by partons

$J(0)$ angular momentum of partons

$d_1(0)$ D-term related to “stability” of the nucleon

DVCS at leading twist

talk of D. Müller



$t \ll \nu,\, Q^2$

$\downarrow$

crossing symmetric variable

$$\nu = \frac{s-u}{4M_N}$$

DVCS tensor at twist 2: $T^{\mu\nu} = \sum_{i=1}^{4} A_i(\nu, t, Q^2)\, O_i^{\mu\nu}$

unpolarized quark

$$A_1 = \mathcal{H} + \mathcal{E}$$

$$A_2 = \mathcal{E}$$

long. polarized quark

$$A_3 = \tilde{\mathcal{H}}$$

$$A_4 = \tilde{\mathcal{E}}$$

Compton form factors: $\mathcal{F} = \int_0^1 \mathrm{d}x\, \underset{\downarrow}{F^+}(x,\xi,t,Q^2) \left[\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon} \right]$ $\quad F = \left\{ H,\, E,\, \tilde{H},\, \tilde{E} \right\}$

singlet GPDs: $F^+(x,\xi,t) = F(x,\xi,t) - F(-x,\xi,t)$

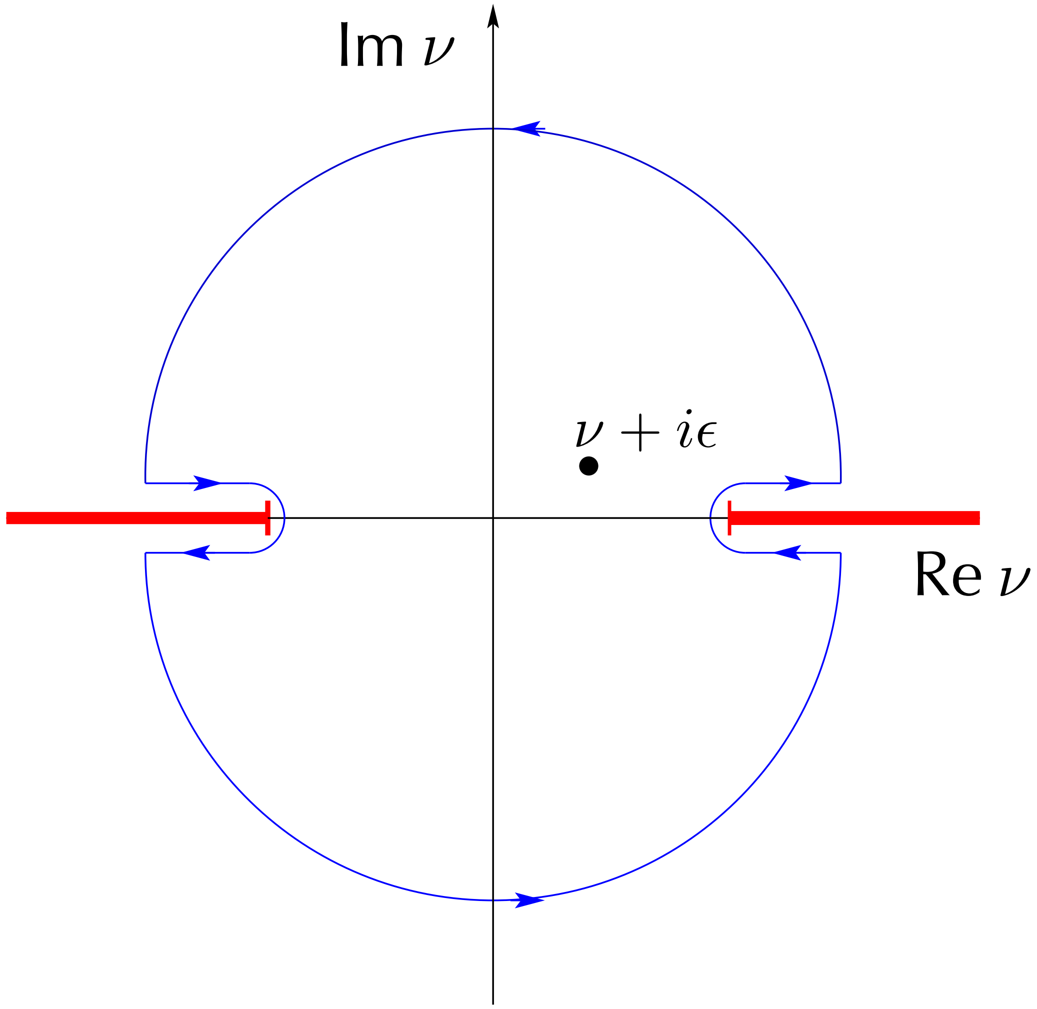
Dispersion Relations at fixed t and Q^2

$A_i(\nu, t, Q^2)$: analytical functions in the complex ν plane, with cuts on the real axis

Drechsel, Pasquini, Vanderhaeghen (2003)

Dispersion Relations at fixed t and Q^2

$A_i(\nu, t, Q^2)$: analytical functions in the complex ν plane, with cuts on the real axis



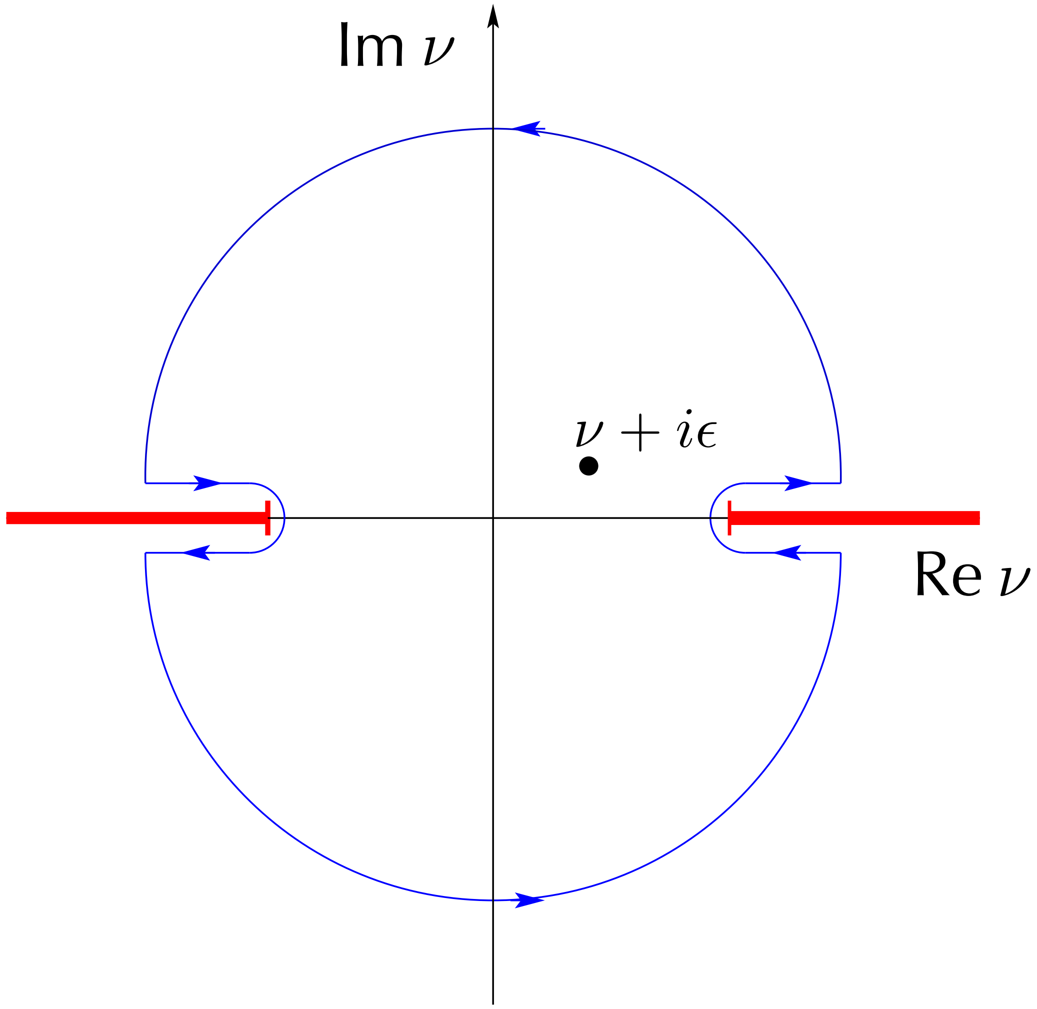
- Cauchy integral formula

$$A_i(\nu, t, Q^2) = \oint_C \mathrm{d}\nu' \frac{A_i(\nu', t, Q^2)}{\nu' - \nu}$$

Drechsel, Pasquini, Vanderhaeghen (2003)

Dispersion Relations at fixed t and Q²

$A_i(\nu, t, Q^2)$: analytical functions in the complex ν plane, with cuts on the real axis



- Cauchy integral formula

$$A_i(\nu, t, Q^2) = \oint_C \mathrm{d}\nu' \frac{A_i(\nu', t, Q^2)}{\nu' - \nu}$$

- Crossing symmetry and analyticity

$$A_i(\nu, t, Q^2) = A_i(-\nu, t, Q^2) \qquad A_i(\nu^*, t, Q^2) = A_i^*(\nu, t, Q^2)$$

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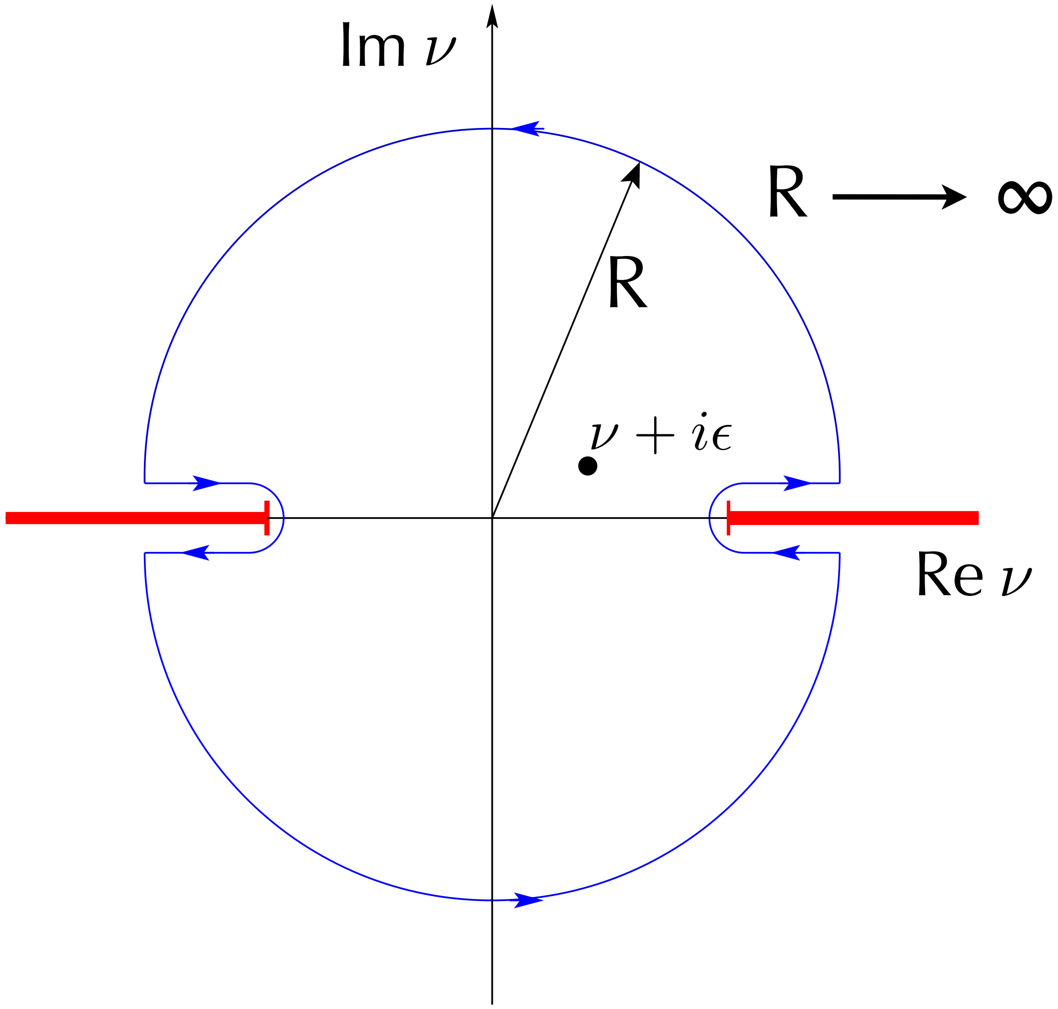

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Unsubtracted Dispersion Relations

$$\mathrm{Re}\, A_i(\nu, t, Q^2) = \frac{2}{\pi}\int_{\nu_0}^{\infty} \mathrm{Im}\, A_i(\nu', t, Q^2)\frac{\nu' \mathrm{d}\nu'}{\nu'^2 - \nu^2} \qquad (i = 1, \cdots, 4)$$

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non-convergent integrals

Subtracted Dispersion Relations

$$\mathrm{Re}\, A_2(\nu, t, Q^2) = A_2(0, t, Q^2) + \frac{2}{\pi} \nu^2 \mathcal{P} \int_{\nu_0}^{\infty} \mathrm{Im}\, A_2(\nu', t, Q^2) \frac{\mathrm{d}\nu'}{\nu'(\nu'^2 - \nu^2)}$$

subtraction at $\nu = 0$

Dispersion Relations in terms of GPDs

energy variables ⟶ $\nu = \frac{Q^2}{4M_N\xi}$ $\nu' = \frac{Q^2}{4M_N x}$

once subtracted fixed-t DRs in the variable x

$$\mathrm{Re}\,A_2(\nu,t,Q^2)=\Delta(t,Q^2)+\frac{2}{\pi}\mathcal{P}\int_0^1\frac{\mathrm{d}x}{x}\,\frac{\mathrm{Im}\,A_2(x,t,Q^2)}{(\xi^2/x^2-1)}$$

link with twist-2 GPDs: $\mathrm{Im}\,A_2(x,t,Q^2)=\pi E^+(x,\xi=x,t,Q^2)$

$$\mathrm{Re}\,A_2(\nu,t,Q^2)=\Delta(t,Q^2)+\mathcal{P}\int_0^1\mathrm{d}x\,E^+(x,x,t,Q^2)\left[\frac{1}{x-\xi}+\frac{1}{x+\xi}\right]$$

subtraction function

accessible through spin asymmetries

Anikin, Teryaev (2007); Kumericki-Passek, Mueller, Passek (2008); Diehl, Ivanov (2007);

Polyakov, Vanderhaeghen (2008); Goldstein, Liuti (2009); Mueller, Semenov (2015)

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$$\downarrow$$

$$\mathrm{Re}\,A_2(\nu,t,Q^2) = -\mathcal{P}\int_0^1 \mathrm{d}x\, E^+(x,\xi,t,Q^2)\left[\frac{1}{x-\xi}+\frac{1}{x+\xi}\right]$$

Anikin, Teryaev (2007); Kumericki-Passek, Mueller, Passek (2008); Diehl, Ivanov (2007);

Polyakov, Vanderhaeghen (2008); Goldstein, Liuti (2009); Mueller, Semenov (2015)

Subtraction Function

$$\Delta(t,Q^2)=\mathcal{P}\int_0^1 \mathrm{d}x\,[E^+(x,\xi,t,Q^2)-E^+(x,x,t,Q^2)]\left[\frac{1}{\xi-x}-\frac{1}{x-\xi}\right]$$

Make use of:

ξ-independence ⟶ take $\xi = 0$

Time-Reversal invariance ⟶ $E(x,x,t) = E(x,-x,t)$

Lorentz invariance ⟶ polinomiality of Mellin moments of GPDs

$$\int_{-1}^1 \mathrm{d}x\, x^n\, E(x,\xi,t) = e_0^{(n)}(t) + e_2^{(n)}(t)\xi^2 + \cdots + \underset{\downarrow}{e_{n+1}^{(n)}(t)}\xi^{n+1}$$

highest power generated by Polyakov-Weiss D-term $D(z,t)$

Anikin, Teryaev (2007); Radyushkin (2012)

talk of K. Semenov-Tian-Shansky

Subtraction Function

$$\Delta(t,Q^2)=\mathcal{P}\int_0^1 \mathrm{d}x\,[E^+(x,\xi,t,Q^2)-E^+(x,x,t,Q^2)]\left[\frac{1}{\xi-x}-\frac{1}{x-\xi}\right]$$

Make use of:

$\boxed{\xi\text{-independence}} \longrightarrow$ take $\xi = 0$

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$$\int_{-1}^{1} \mathrm{d}x\, x^n\, E(x,\xi,t) = e_0^{(n)}(t) + e_2^{(n)}(t)\xi^2 + \cdots + e_{n+1}^{(n)}(t)\xi^{n+1}$$

↓

highest power generated by
Polyakov-Weiss D-term $D(z,t)$

$$\Longrightarrow\ \boxed{\Delta(t,Q^2) = -\frac{4}{N_f}D(t,Q^2)}$$

with $D(t,Q^2) = \dfrac{1}{2}\displaystyle\int_{-1}^{1} \mathrm{d}z\, \frac{D(z,t,Q^2)}{1-z}$

Anikin, Teryaev (2007); Radyushkin (2012)

talk of K. Semenov-Tian-Shansky

Dispersion Relations for DVCS amplitudes

- s-channel DRs:

$$\mathrm{Re}\, A_2(\nu, t, Q^2) = \Delta(t, Q^2) + \frac{2}{\pi}\nu^2 \mathcal{P} \int_{\nu_0}^{\infty} \mathrm{Im}\, A_2(\nu', t, Q^2) \frac{\mathrm{d}\nu'}{\nu'(\nu'^2 - \nu^2)}$$

Dispersion Relations for DVCS amplitudes

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- t-channel DRs for subtraction function

$$\Delta(t, Q^2) = -\frac{4}{N_f} D(t, Q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \mathrm{d}t' \, \frac{\mathrm{Im}_t\, A_2(0, t', Q^2)}{t' - t} + \frac{1}{\pi} \int_{-\infty}^{-a} \mathrm{d}t' \, \frac{\mathrm{Im}_t\, A_2(0, t', Q^2)}{t' - t}$$

$$\big\downarrow$$

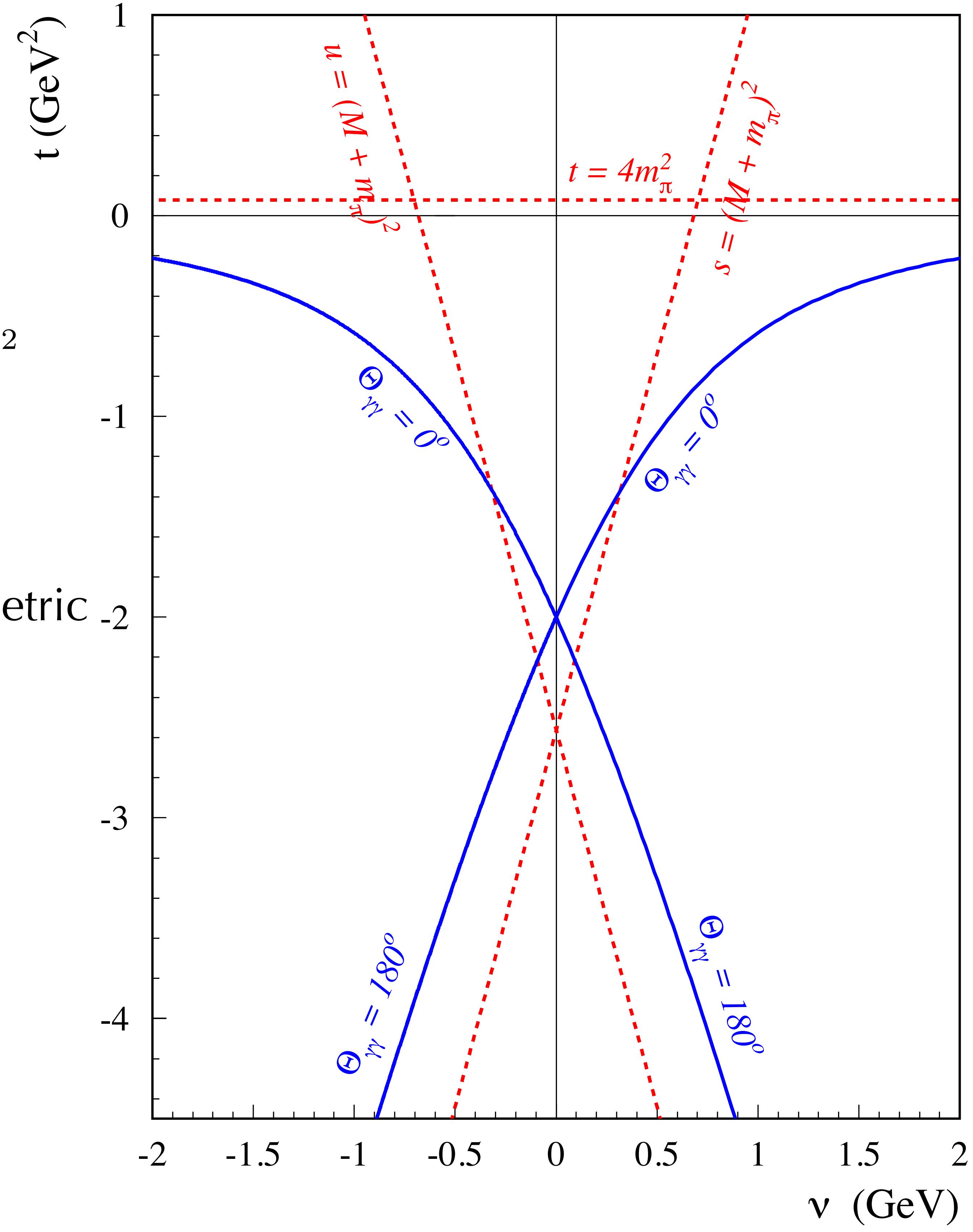
$$-a = -2(m_\pi^2 + 2M_N m_\pi) - Q^2$$

Fixed

$Q^2 = -2$ GeV^2

$\nu = \frac{s-u}{4M_N}$

crossing symmetric variable



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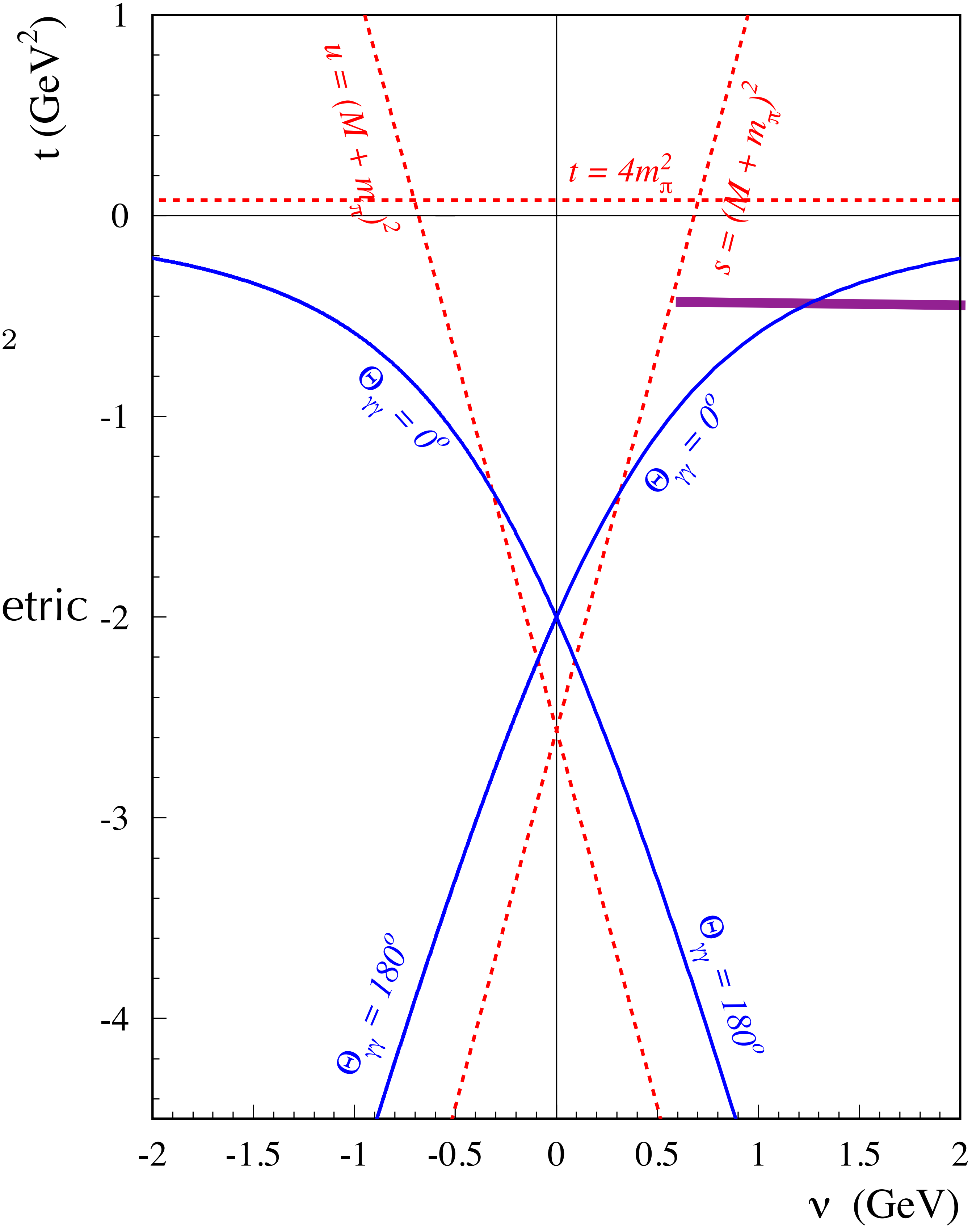


DRs in the s-channel at fixed t

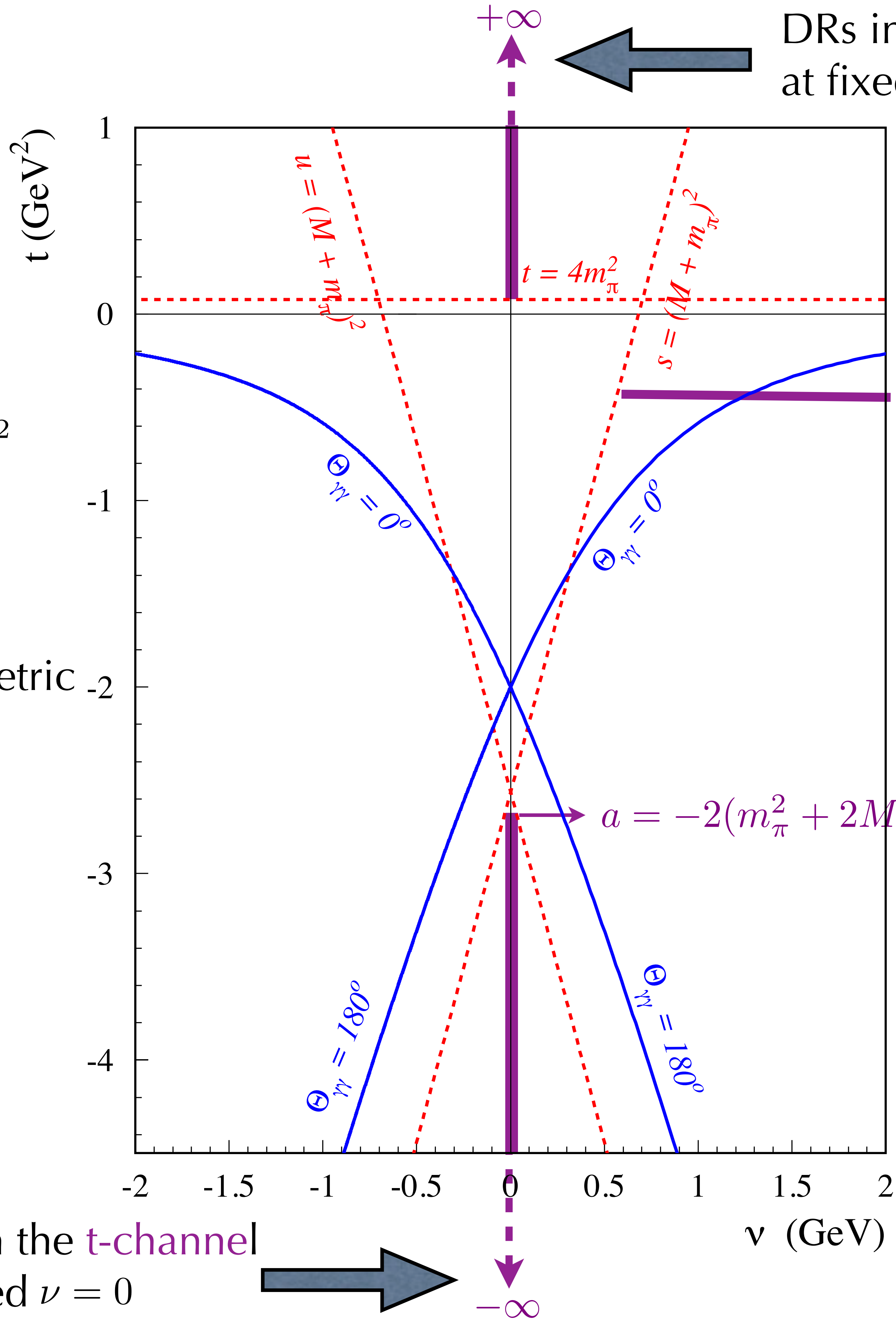
DRs in the t-channel
at fixed $\nu = 0$

Fixed

$Q^2 = -2\ \mathrm{GeV}^2$

$\nu = \frac{s-u}{4M_N}$

crossing symmetric
variable



DRs in the s-channel
at fixed t

DRs in the t-channel
at fixed $\nu = 0$

Dispersion Relations in the t-channel

$$\Delta(t,Q^2) = \mathrm{Re}\, A_2(0,t,Q^2) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}t' \, \frac{\mathrm{Im}_t\, A_2(0,t',Q^2)}{t'-t} + \frac{1}{\pi}\int_{-\infty}^{-a} \mathrm{d}t' \, \xcancel{\frac{\mathrm{Im}_t\, A_2(0,t',Q^2)}{t'-t}}$$

$$\downarrow$$

$$-a = -2(m_\pi^2 + 2M_N m_\pi) - Q^2 \to -\infty$$

Dispersion Relations in the t-channel

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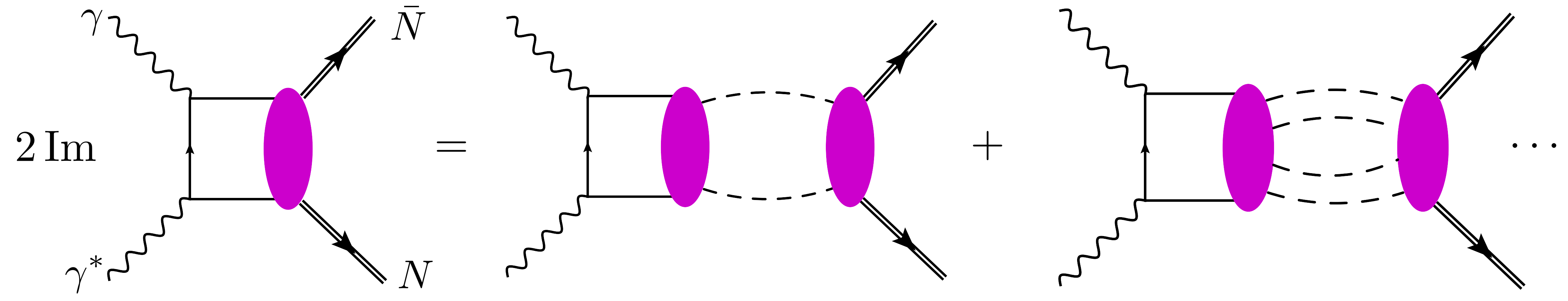
$$\downarrow$$

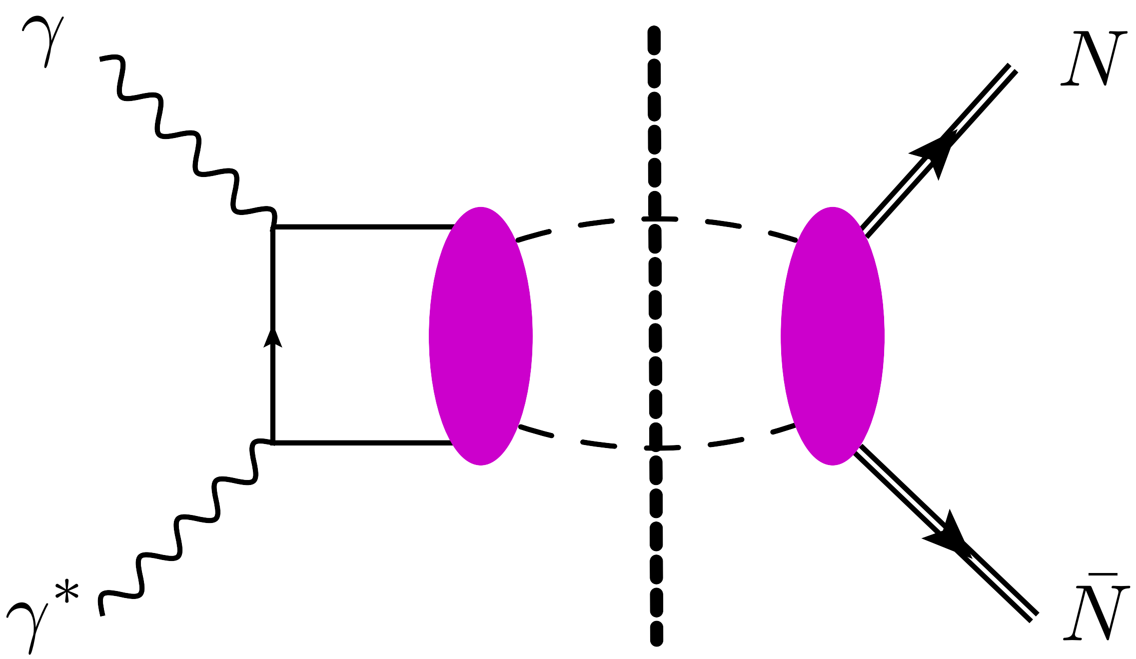
$$-a = -2(m_\pi^2 + 2M_N m_\pi) - Q^2 \to -\infty$$

Unitarity relation in t-channel

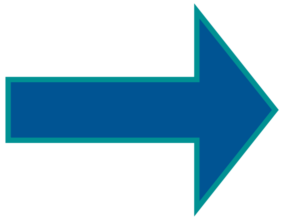


$t \geq (2m_\pi)^2$ $\qquad$ $t \geq (4m_\pi)^2$

Unitarity Relations in the t-channel

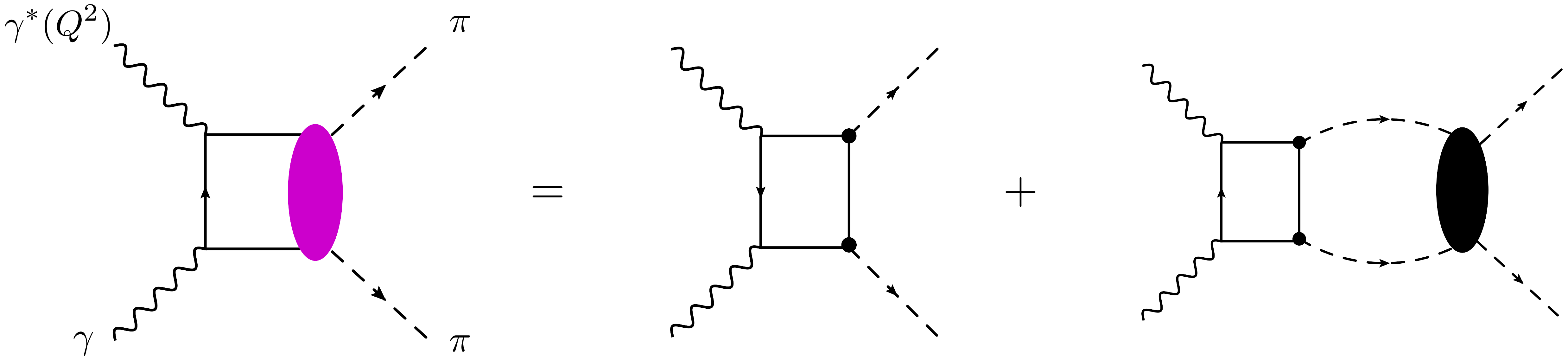


- Charge conjugation
- Partial wave expansion with $\nu = 0 \rightarrow \theta_t = 90^o$



two-pion intermediate state with
$I = 0 \qquad J = 0, 2, \cdots$

$\gamma^*\gamma \to \pi\pi$: two-pion GDAs



$$\Phi_q^{\pi\pi} = 6\,z(1-z) \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \sum_{\substack{l=0\\ \text{even}}}^{n+1} \tilde{B}^q_{nl}(t)\, C_n^{(3/2)}(2z-1)\, P_l(\cos\theta_{\pi\pi})$$

unitarized S- and D- waves: dispersive (Omnès) representation

S wave

$$\tilde{B}_{10}(t) = -B_{12}(0)\frac{3C-\beta^2}{2} f_0(t)$$

$$B_{12}(0) = \frac{10}{9}\int \mathrm{d}x\, x \frac{1}{N_f}\sum_f [q^f_\pi(x) + \bar{q}^f_\pi(x)]$$

↓ pion PDFs

D wave

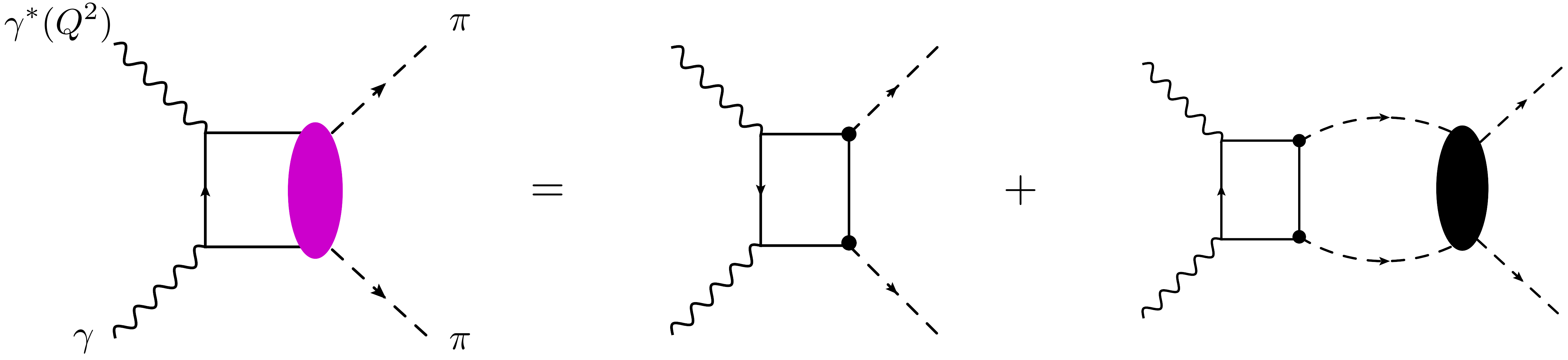
$$\tilde{B}_{12}(t) = \beta^2 B_{12}(0) f_2(t)$$

$$f_l(t) = \exp\left[\frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}t' \frac{\delta_l^0(t')}{t'(t'-t-i\epsilon)}\right]$$

↓ $\pi\pi$ phase shifts

M. Polyakov, 1999

$\gamma^*\gamma \to \pi\pi$: two-pion GDAs

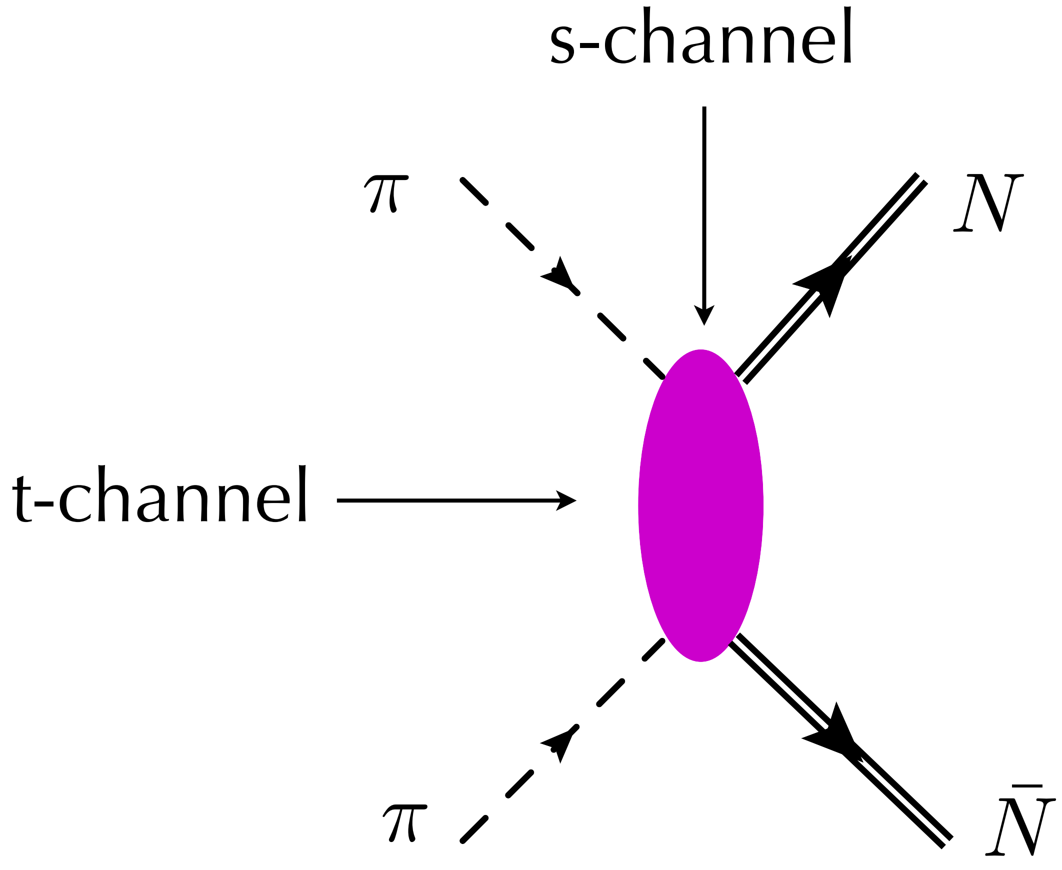


unitarized S- and D- waves: dispersive (Omnès) representation

input ⇒ $\pi\pi$ phase shifts
pion PDFs

M. Polyakov, 1999

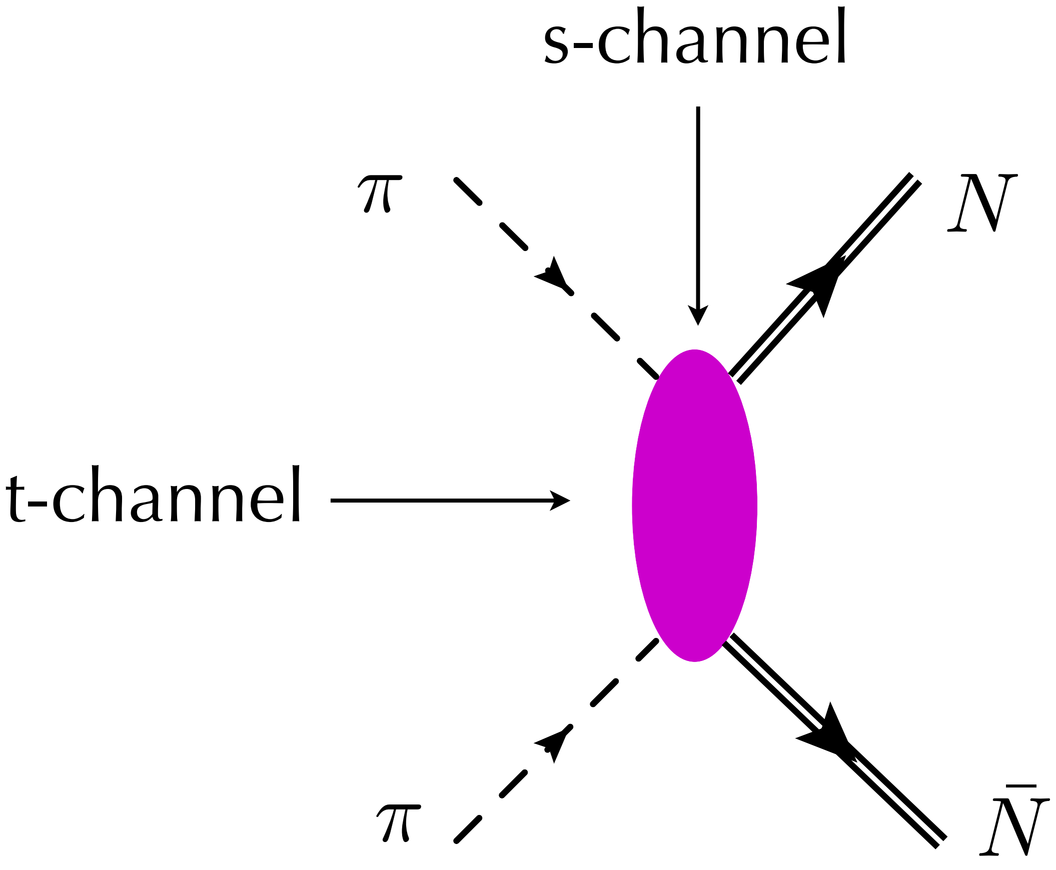
$\pi\pi \to N\bar{N}$ scattering amplitudes



analytical continuation of s-channel partial-wave helicity amplitudes calculated from DRs

G. Hoehler, 1983

$\pi\pi \to N\bar{N}$ scattering amplitudes



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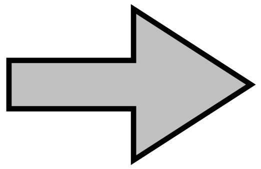
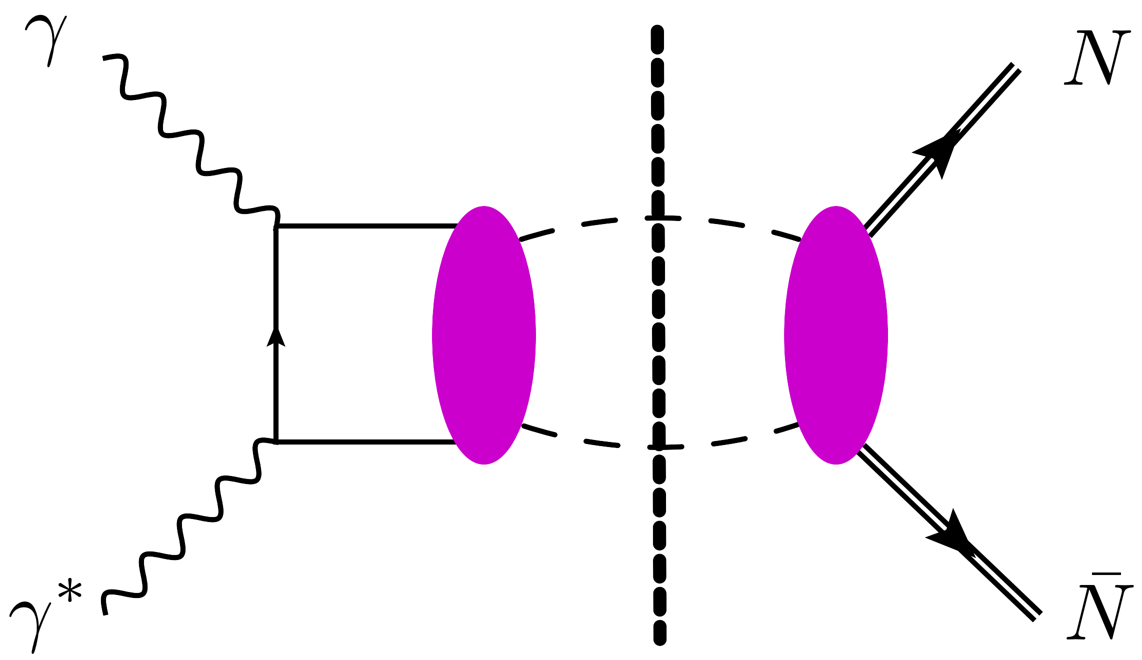

input → $\pi\pi$ phase shifts

G. Hoehler, 1983

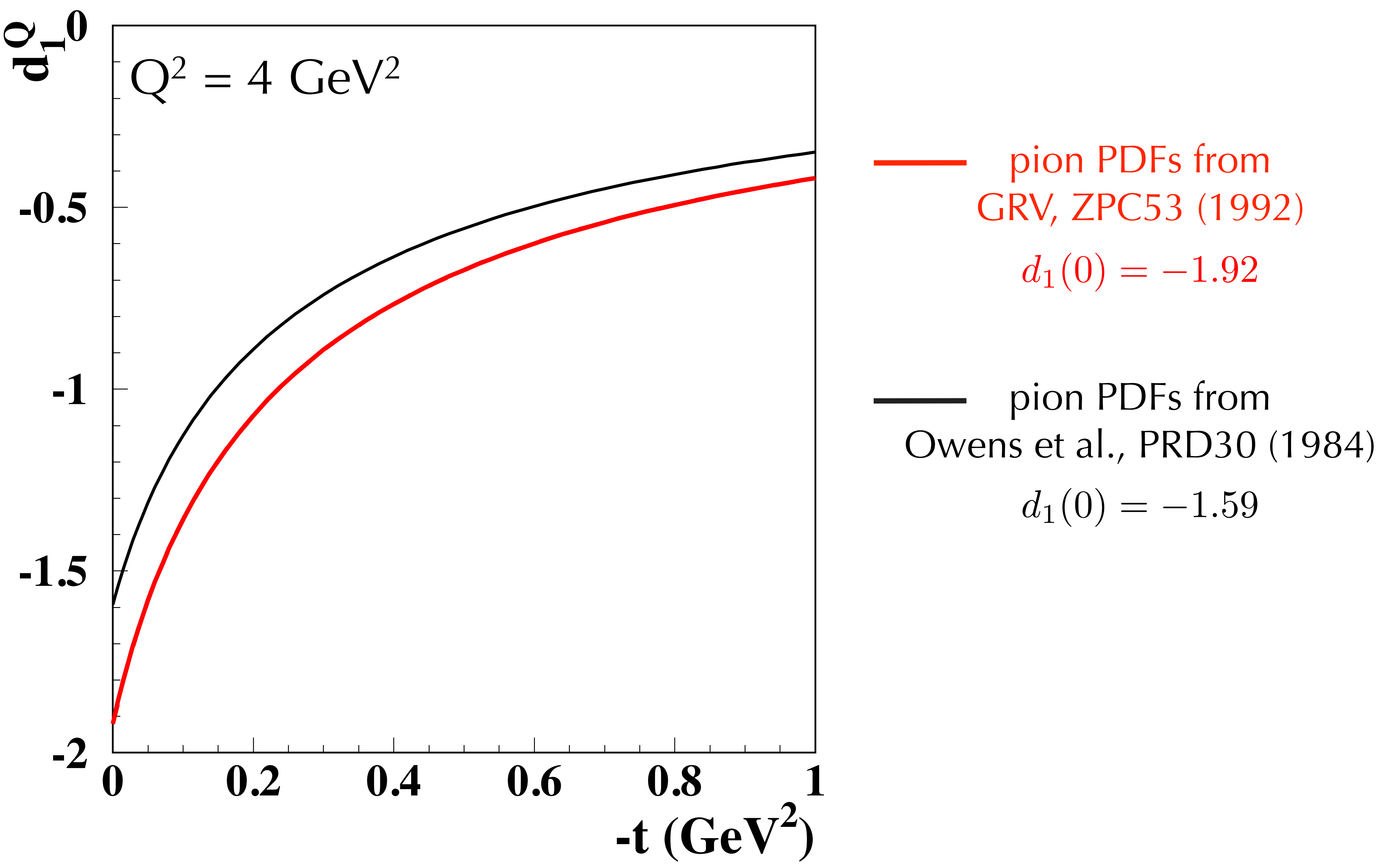
Intermediate summary: input for t-channel DRs



- two pion intermediate states $\longrightarrow$ partial wave expansion and take $I = 0,\ J = 0, 2$
- expansion in Gegenbauer polynomials: $D(t) = \sum_{\{n\,odd\}} d_n(t)$ $\longrightarrow$ DRs for $d_1(t)$
- $\gamma^*\gamma \to \pi\pi$: GDAs with input from pion PDFs and $\pi\pi$ phase shifts
- $\pi\pi \to N\bar{N}$: analytical continuations of pion-nucleon scattering amplitudes with input from $\pi\pi$ phase shifts

DR Results for D-term Form Factor

Q= u + d



χQSM

$d_1^Q(0) = -2.35$

Schweitzer et al., (2007)

Skyrme model

$d_1^Q(0) = -4.48$

Schweitzer et al., (2007)

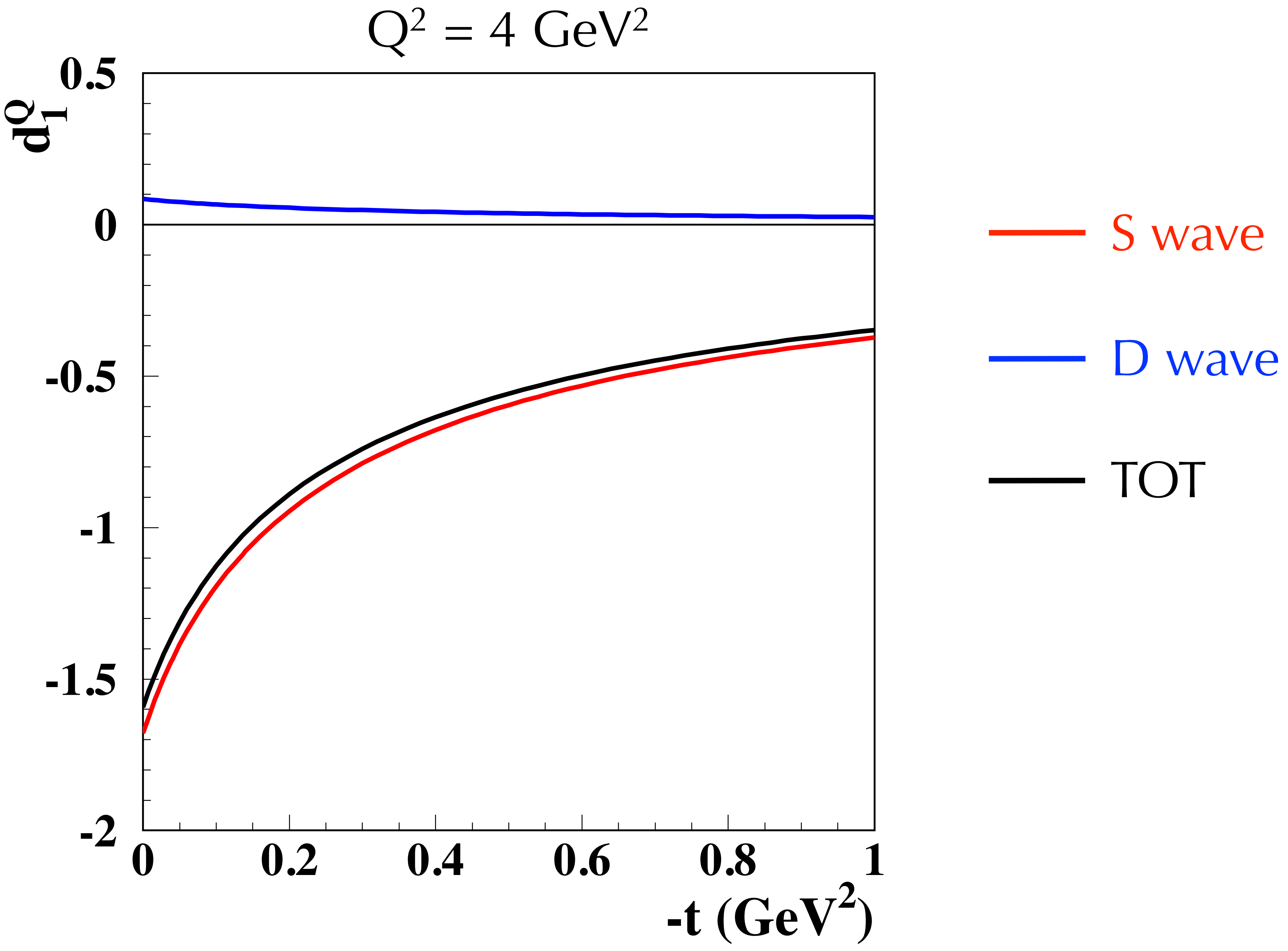
Effective LFWFs

$d_1^Q(0) = -2.01$

Mueller and Hwang, (2014)

DR Results for D-term Form Factor

$Q = u + d$



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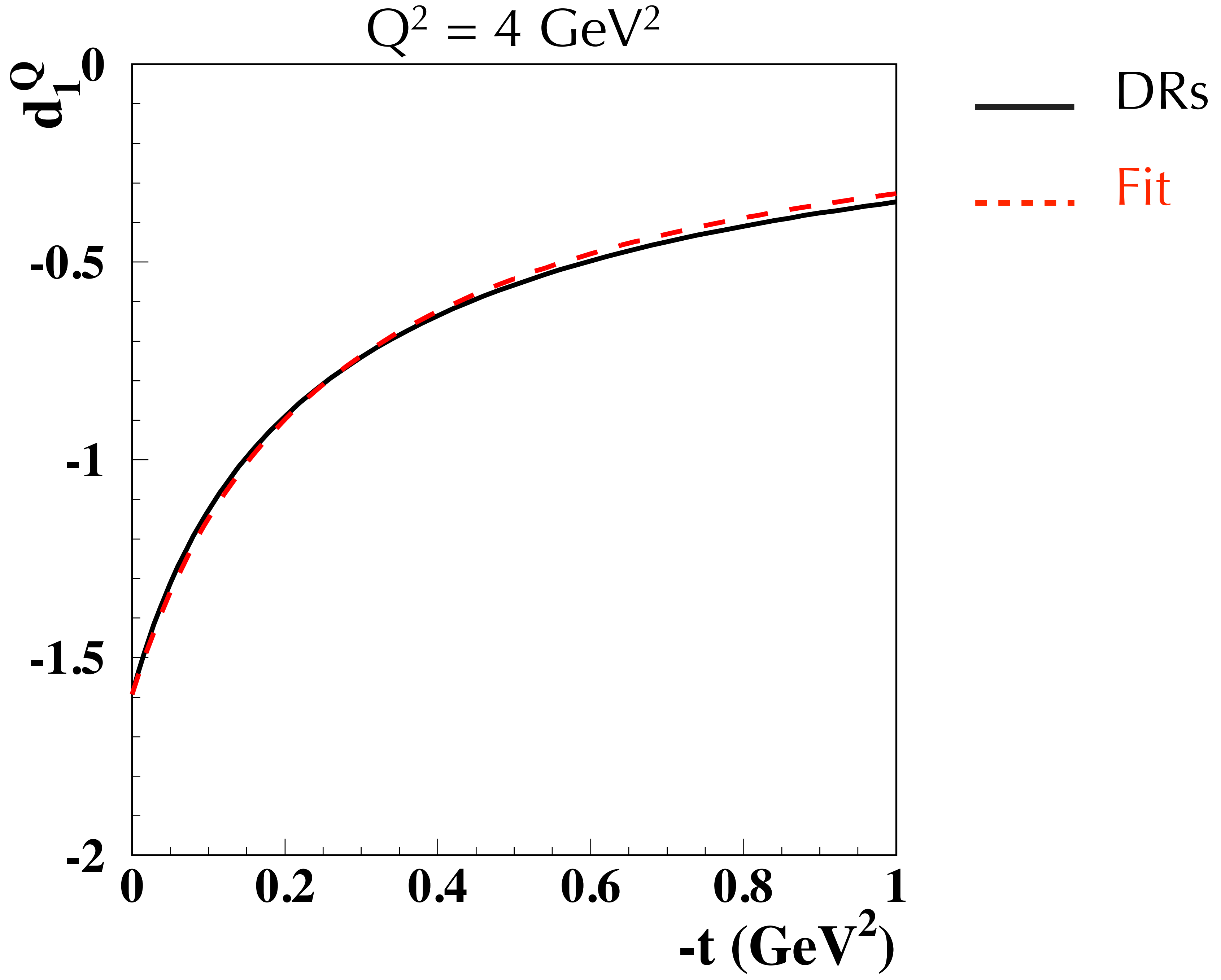
D-term Form Factor: t-dependence



Fit: $F^Q(t) = \dfrac{d_1^Q(0)}{[1 - t/(\alpha M_D^2)]^\alpha}$ with $M_D = 0.487\,\mathrm{GeV}$

$\alpha = 0.841$

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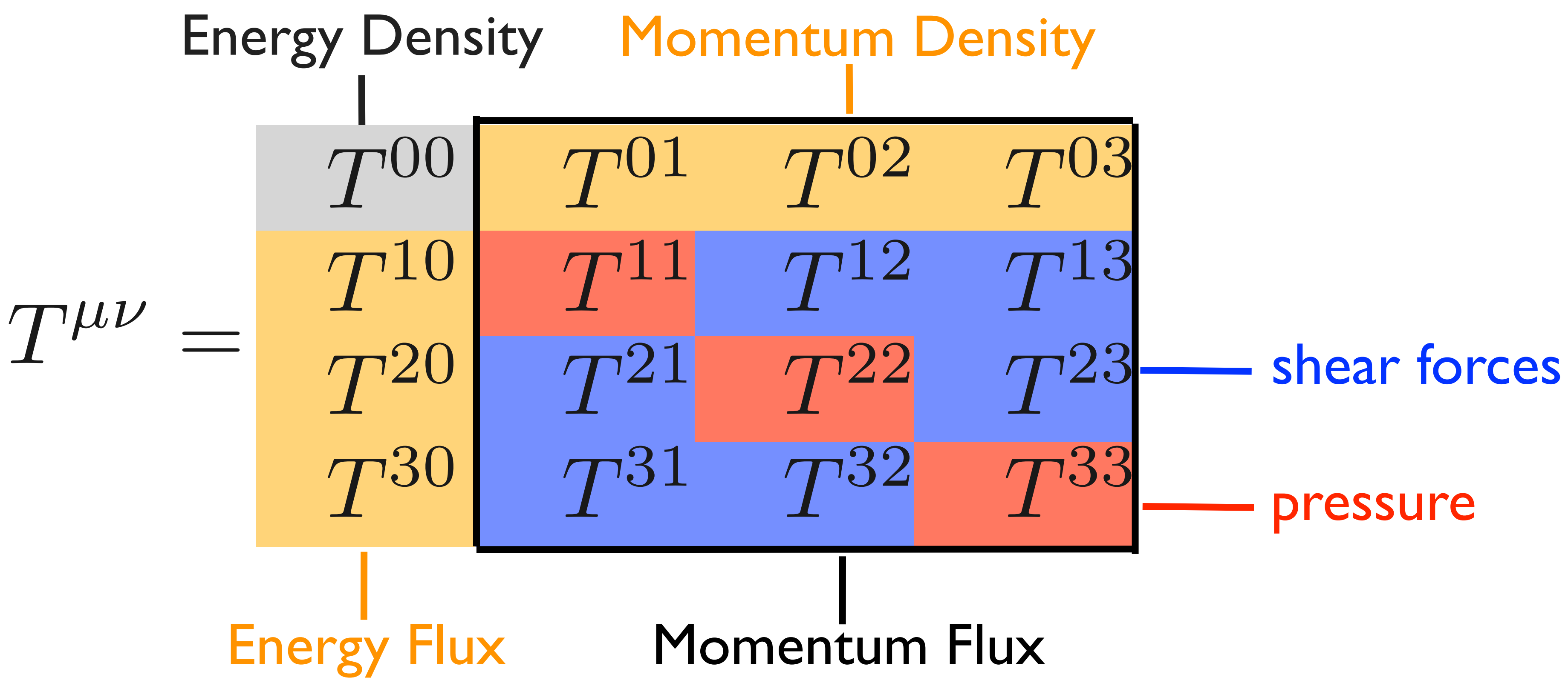


Energy Density

Momentum Density

$$T^{\mu\nu} = \begin{array}{cccc} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{array}$$

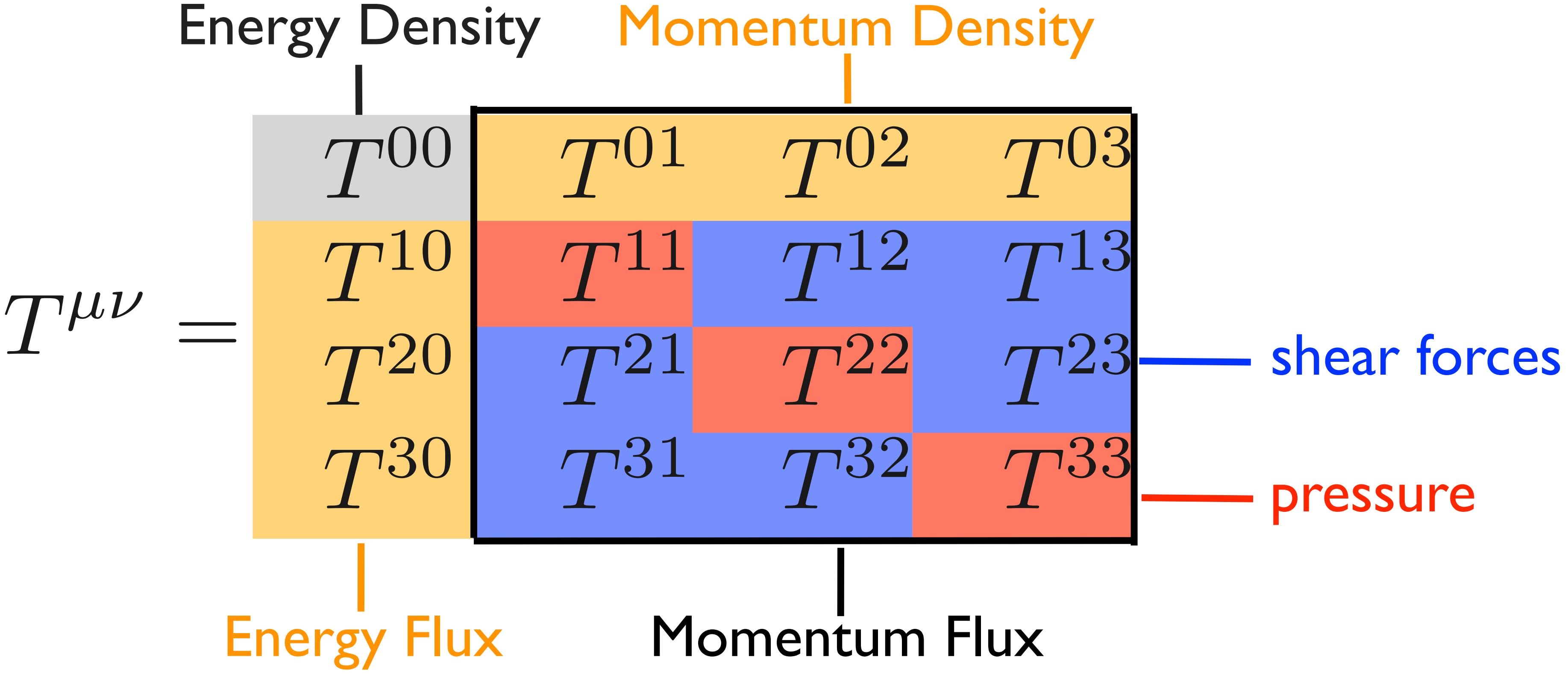
shear forces

pressure

Energy Flux

Momentum Flux

$$\langle P'|T^{Q,G}_{\mu\nu}|P\rangle = \bar{u}(P')[M_2^{Q,G}(t)\,\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}]u(P)$$



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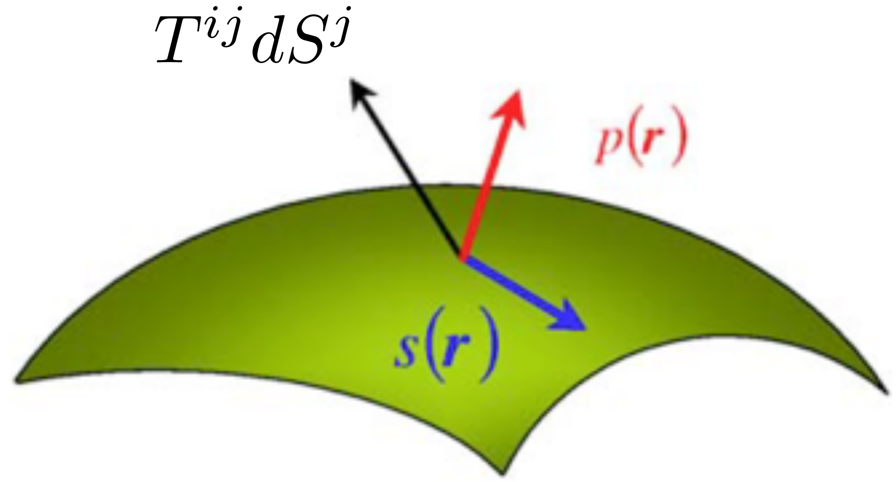
$\Downarrow$ FT in $\vec{r}$

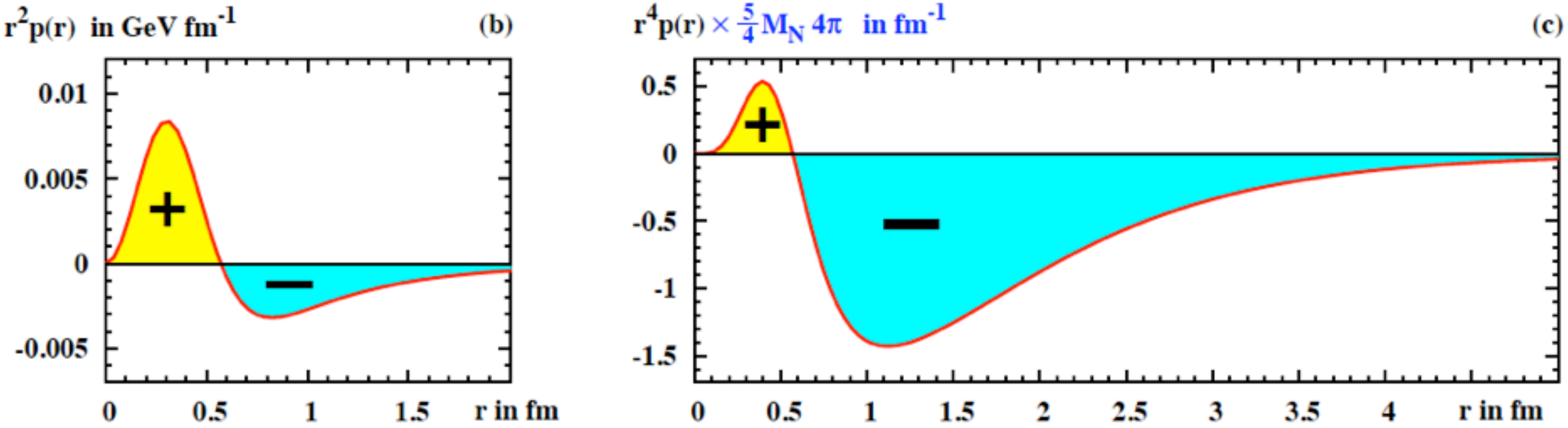
$$T^Q_{ij}(\vec{r}) = \underset{\substack{\downarrow \\ \text{shear forces}}}{s(\vec{r})}\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + \underset{\substack{\downarrow \\ \text{pressure}}}{p(\vec{r})}\,\delta_{ij}$$

"mechanical properties" of nucleon



M. Polyakov, 2003

Stability and Sign of D-term



χQSM: *Schweitzer et al., 2007*

$$d_1(t) = \frac{15 M_N}{2t} \int \mathrm{d}^3 r \, r^2 j_0(r\sqrt{-t}) \, p(r)$$

conservation of EMT

$$\int_0^\infty \mathrm{d}r \, r^2 \, p(r) = 0$$

stability condition

$$\int_0^\infty \mathrm{d}r \, r^4 \, p(r) < 0$$

$$\boxed{d_1(0) = 5\pi M_N \int_0^\infty \mathrm{d}r \, r^4 \, p(r) < 0}$$

Summary

- Dispersion Relations for DVCS amplitudes

  constraints from analyticity, crossing, built in

- Subtraction functions for twist-2 DVCS amplitudes

  for $H + E$ and $\tilde{H}$: no subtractions

  for $\tilde{E}$: pseudoscalar meson poles

  for E : D-term

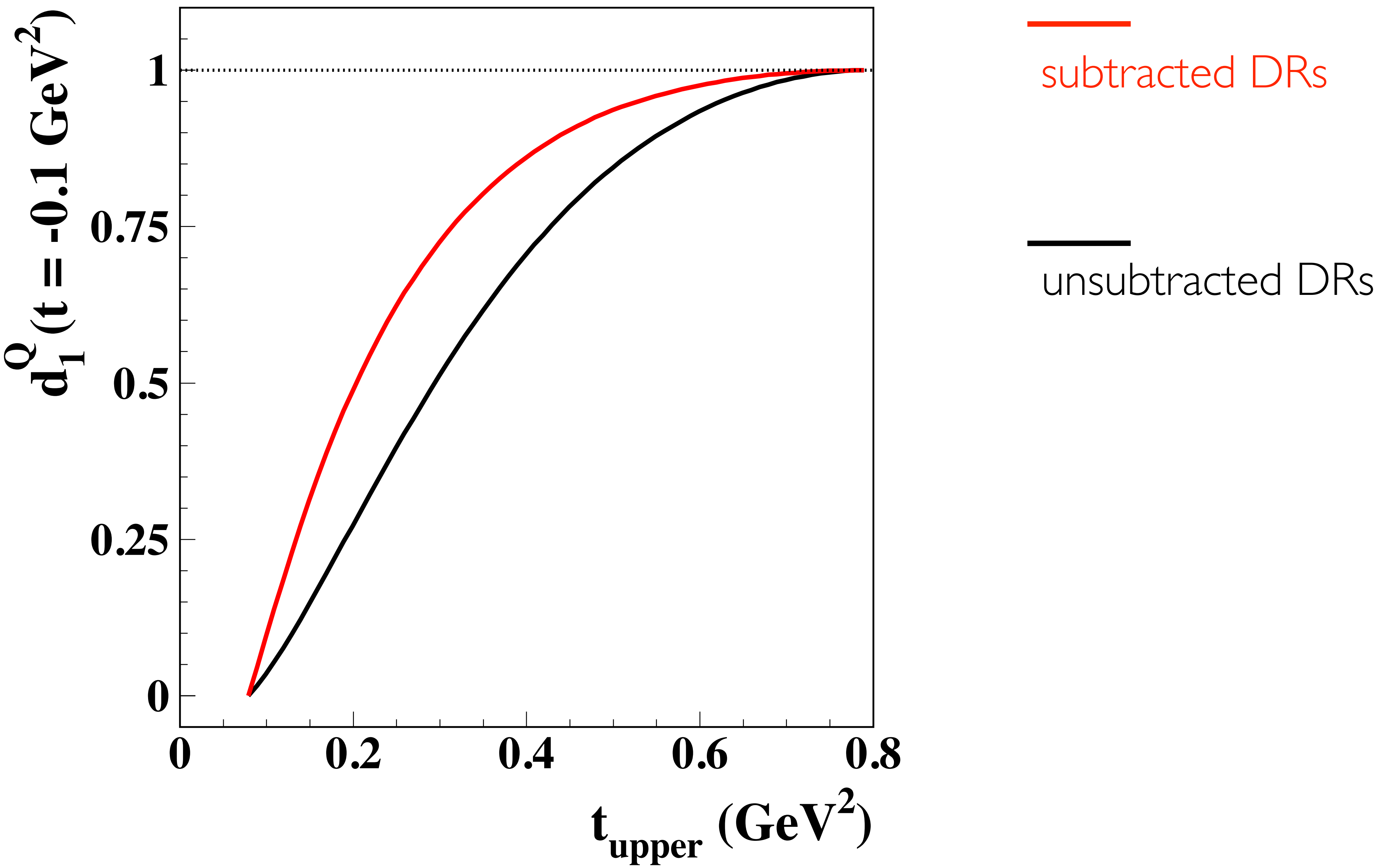
- D-term from t-channel Dispersion Relations

  D-term $\longrightarrow$ two-pion correlated state with I=0, J=0, 2

  model independent representation
  with input from two-pion GDAs and pion-nucleon scattering

Backup Slides

Convergence of DRs



subtracted DRs: $d_1^Q(t) = \quad d_1^Q(0) - \frac{t}{\pi}\int_{4m_\pi^2}^{+\infty} \mathrm{d}t' \, \frac{\mathrm{Im}_t A_2(0,t',Q^2)}{t'(t'-t)}$

↓

subtraction constant to be fitted to data