Thermalization in confining geometries

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3. **Soon at IKOS CIF, LTD**

Outline

- I. Injecting energy/thermalization in confining theories; analytical approach
- Slow Vs fast injection times
- II. Injecting energy/thermalization in confining theories; exact (numerical) methods
- III. On going work/extensions
- IV. Hall conductivities and a flavor of confining theories in magnetic fields (if time allows)

I. Holographic thermalization in confining theories from fast/slow injection times

Arxiv:1311.7560

B. Craps, E. Kiritsis, J. Lindgren, C. Rosen, J. Vanhoof, H. Zhang

The model

$$\mathcal{S} = \frac{1}{2\kappa^2}\int \mathrm{d}^{d+1}\mathbf{x}\sqrt{-g}\left(R + \frac{d(d-1)}{L^2} - \frac{1}{2}(\partial a)^2\right)$$

According to the AdS/CFT dictionary, it allows the computation of:

g_mn←→T_mn and a←→Tr[F^2]=Tr[E^2+B^2]

The ansatz

$$ds^2 = -g(r,v)dv^2 + 2drdv + f^2(r,v)d\vec{x}^2 \qquad \text{and} \qquad a = a(r,v)$$

UV Boundary conditions (r=oo is UV, v=t-L^2/r)

$$\lim_{r\to\infty}\frac{f(r,v)}{r}=1 \quad , \quad \lim_{r\to\infty}\frac{g(r,v)}{r^2}=1 \quad \text{and} \quad \lim_{r\to\infty}a(r,v)=a_s(v)$$

$$r\to r+h(v)$$

$$f(r,v)=r\left(1+\mathcal{O}\left(\frac{1}{r^2}\right)\right),$$
$$g(r,v)=r^2\left(1+\mathcal{O}\left(\frac{1}{r^2}\right)\right),$$
$$a(r,v)=a_s(v)+\mathcal{O}\left(\frac{1}{r}\right).$$

Introduce confinement; hard wall at r=ro.

IR BCs: Nuemann or Dirichlet (or even mixed)

$$0=\left.(n^\mu\partial_\mu)a(r,v)\right|_{r=r_0}=\left.\left(\sqrt{g}\frac{\partial}{\partial r}+\frac{1}{\sqrt{g}}\frac{\partial}{\partial v}\right)a(r,v)\right|_{r=r_0}$$

$$0=\left.a(r,v)\right|_{r=r_0}$$

Initial conditions and the v<0 situation

$$f(r,v) = r \quad , \quad g(r,v) = r^2 \quad \text{and} \quad a(r,v) = 0$$

$$\begin{aligned} a_s(v) &= 0 && (v < 0) \\ a_s(v) &= \epsilon\, a_0(v) && (0 < v < \delta t) \\ a_s(v) &= 0 && (\delta t < v) \end{aligned}$$

Perturbative expansion around empty AdS

$$f(r,v) = r + \sum_{n=1} \epsilon^n f_n(r,v),$$

$$g(r,v) = r^2 + \sum_{n=1}^{\infty} \epsilon^n g_n(r,v)$$

$$a(r,v) = \sum_{n=1}^{\infty} \epsilon^n a_n(r,v).$$

Initial and BCs (r=oo, UV) on the expansion

$$g_n(r,v) \leqslant \mathcal{O}(1) \quad , \quad f_n(r,v) \leqslant \mathcal{O}\left(\frac{1}{r}\right) \begin{cases} a_n(r,v) \leqslant \mathcal{O}\left(\frac{1}{r}\right) & \text{for } n > 1 \\ \lim_{r\to\infty} a_1(r,v) = a_0(v). \end{cases}$$

Lowest order Results of the expansion and right background:

$$ds^2 = -r^2\left(1 - \frac{M(v)}{r^d}\right)dv^2 + 2drdv + r^2 d\vec{x}^2 \quad M(v) \sim \frac{\epsilon^2}{(\delta t)^d}.$$

Remark: The model involves two parameters: Ɛ and δt

Ɛ : strength of the scalar (here Ɛ <<1)

δt: compact support of time interval of scalar

Fact: successive terms, altough smaller in Ɛ grow large at late times => pert. Theory breaks unless…

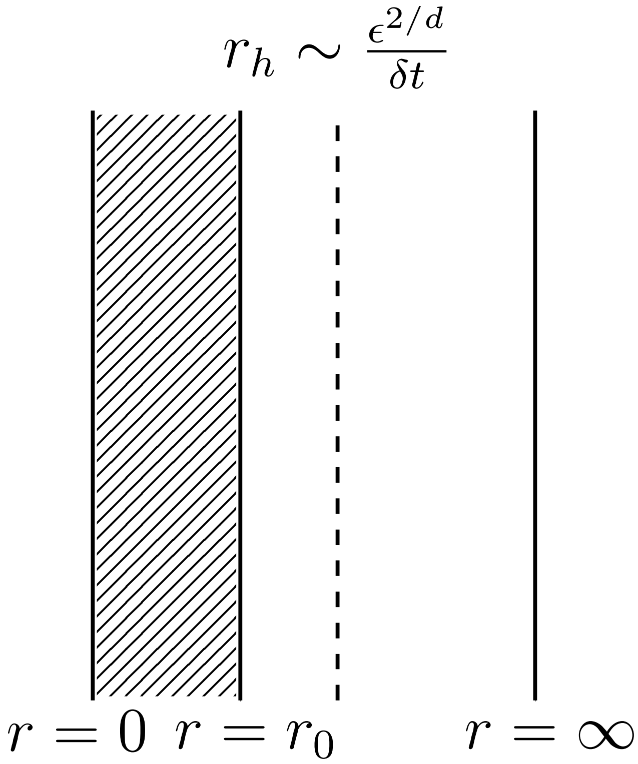
...unless one chooses the Vadya BH background to do the expansion. Then the problematic terms resum and organize themselves in a well-defined expansion around Vadya. [Bhattacharyya-Minwalla]

Since the lowest contribution to the BH mass is $M(v) \sim \frac{\epsilon^2}{(\delta t)^d}$

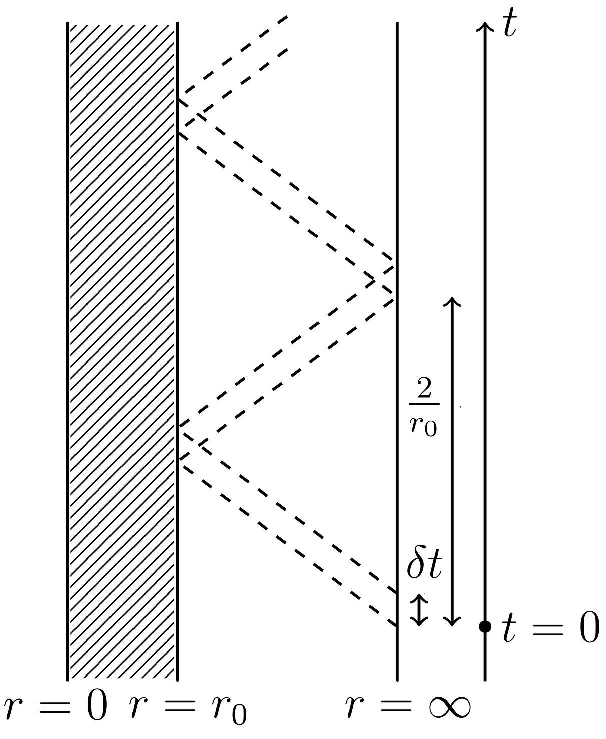
In the BH phase => $r_h \sim \frac{\epsilon^{2/d}}{\delta t}$ => $r_0 \lesssim \frac{\epsilon^{2/d}}{\delta t}$

BH Vs Scattering solution

BH solution



Scattering solution



*Both the geometry and the scalar oscillate in the scattering solution

Scattering solution formula for scalar

(lowest order; NBC, d=3)

$$a_1(r,v) = \sum_{m=1}^{\infty} \left(a_0\left(v - \tfrac{2(m-1)}{r_0}\right) + \frac{\dot{a}_0\left(v - \tfrac{2(m-1)}{r_0}\right)}{r} - a_0\left(v + \tfrac{2}{r} - \tfrac{2m}{r_0}\right) + \frac{\dot{a}_0\left(v + \tfrac{2}{r} - \tfrac{2m}{r_0}\right)}{r} \right)$$

- Recall that current analysis is restricted for ε<<1.
- <u>Case I, slow injection times</u> $r_0 \delta t \gg 1$
  No extra condition required to have such a scattering solution.
- <u>Case II, fast injection times</u> $r_0 \delta t \ll 1$
  Additional condition $\epsilon \ll (r_0 \delta t)^{\frac{5}{2}}$ required=>even smaller energy injection compared to case I.

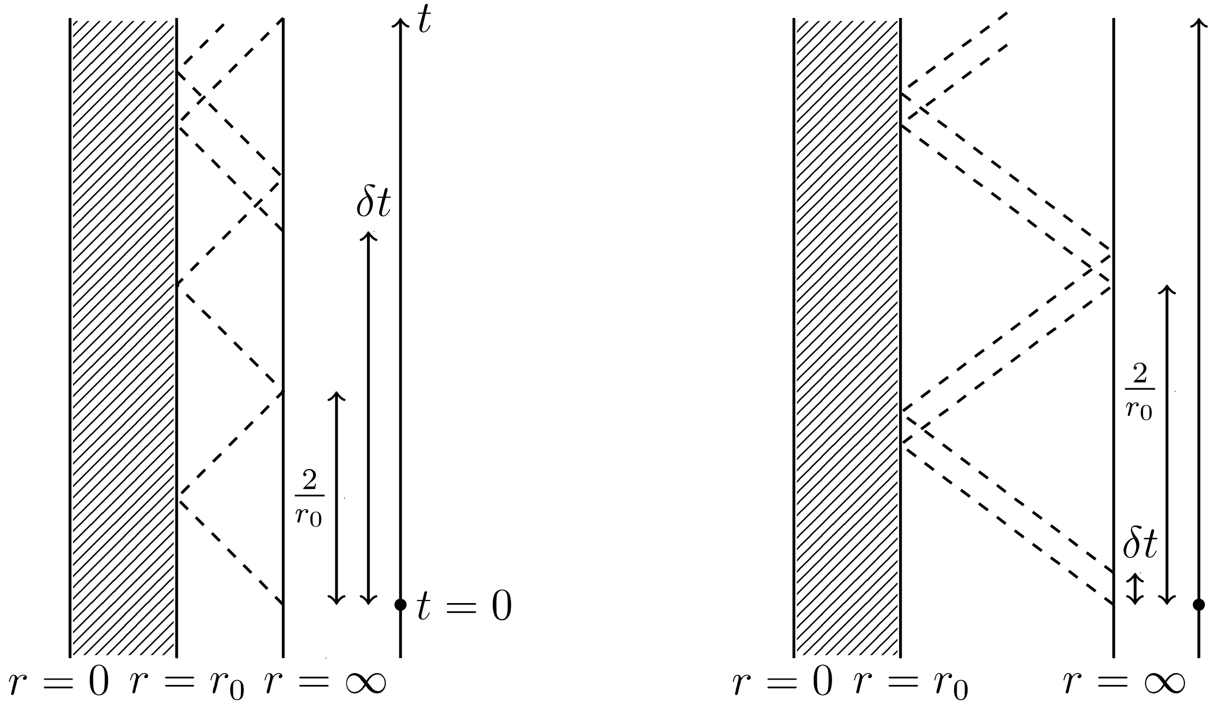
Slow Vs Short injection times

Slow injection times

$r_0 \delta t \gg 1$

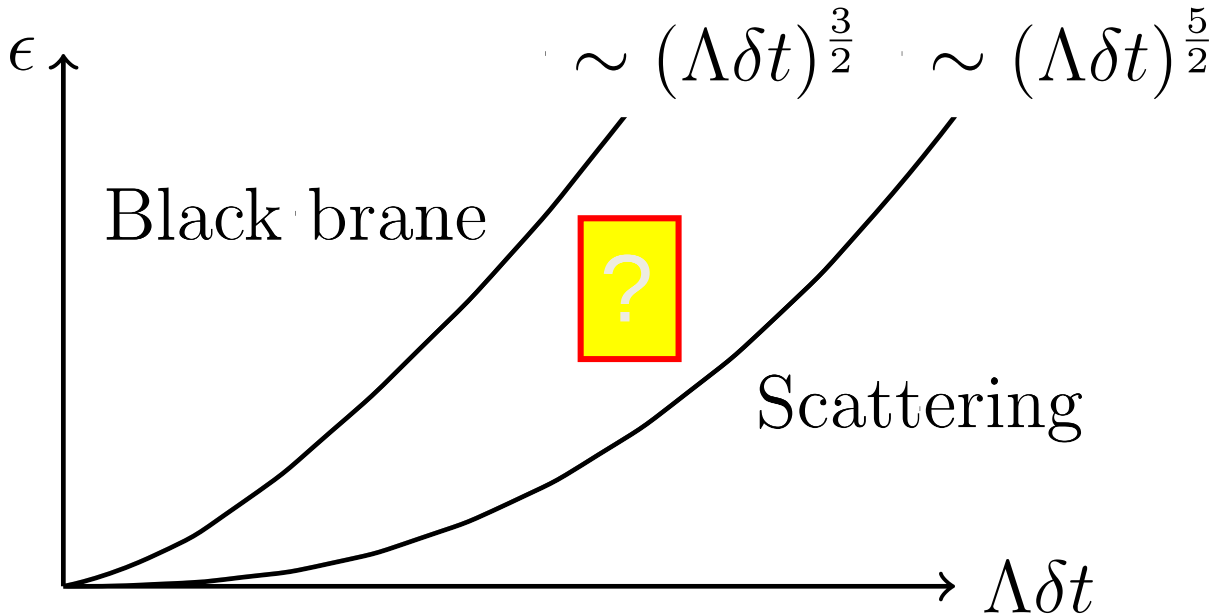
Fast injection times

$r_0 \delta t \ll 1$



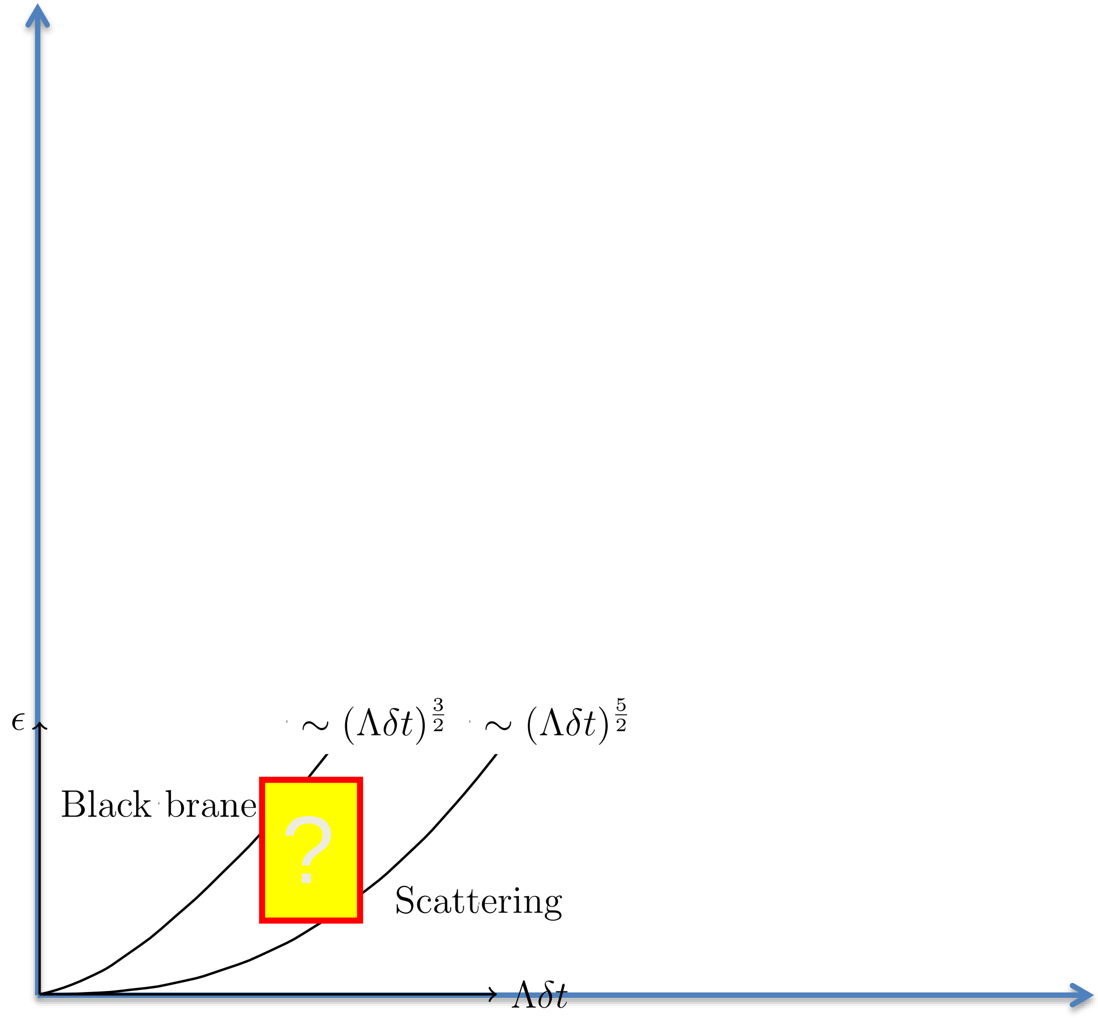
$\epsilon \ll (r_0 \delta t)^{\frac{5}{2}}$

Figure 5: *(Left) Scattering with $r_0 \delta t \gg 1$. (Right) Scattering with $r_0 \delta t \ll 1$.*



ϵ
$\sim (\Lambda\delta t)^{\frac{3}{2}}$
$\sim (\Lambda\delta t)^{\frac{5}{2}}$
Black brane
?
Scattering
$\Lambda\delta t$

Conclusions



II. Injecting energy/thermalization in confining theories; exact (numerical) methods

Arxiv: 1406.1454

Craps, Lindgren, Taliotis, Vanhoof, Zhang

Introduction: SABR model

- SABR model is defined by

$$dS_t = \sigma_t \phi(S_t) dW_t, \quad [dW_t] = dt,$$

$$d\sigma_t = \sigma_t \gamma dB_t, \quad [dB_t] = dt.$$

$$[dB_t, dW_t] = \rho dt$$

where ρ correlates the two Brownian motions Wt and Bt, volatility is stochastic, $\phi(S)=S^{\beta}$ for standard SABR and St is the stock price absorbed at zero (see later). Interest rates are assumed zero.

Just checking if you are still with me

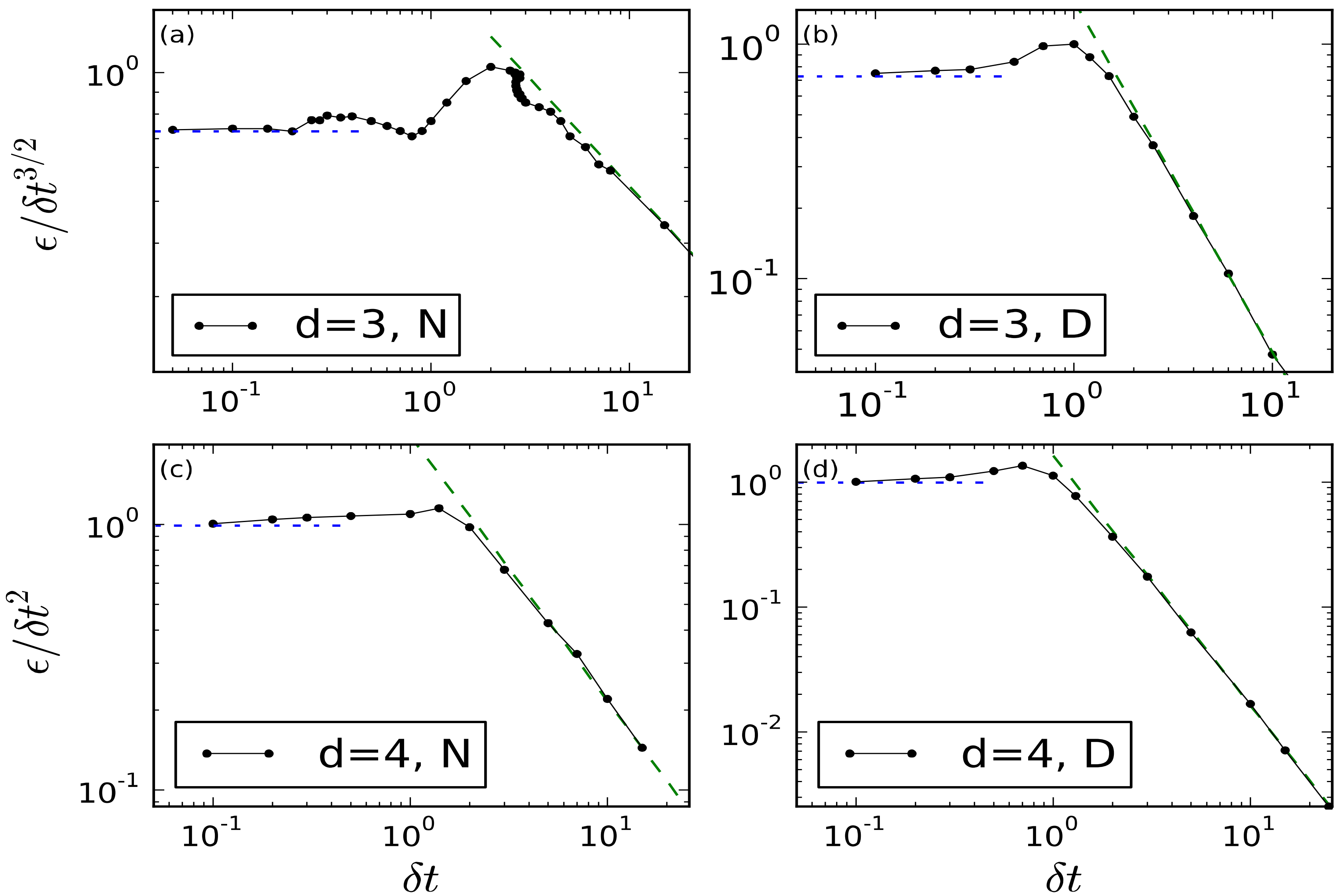
- Existing literature: $\phi(S)=1/S$ => lognormal.
- B&T: $\phi(S)=1$ => normal.

  Advantage: no negative densities => no arbitrage

☺

Motivating a numerical approach

- Investigate for any ε in [0,oo).
- See what happens in the intermediate region we could not study analytically; complete the phase diagram.
- Cross-Checks: are the analytical results reproduced by the numerics?.
- Any interesting features could not see analytically?

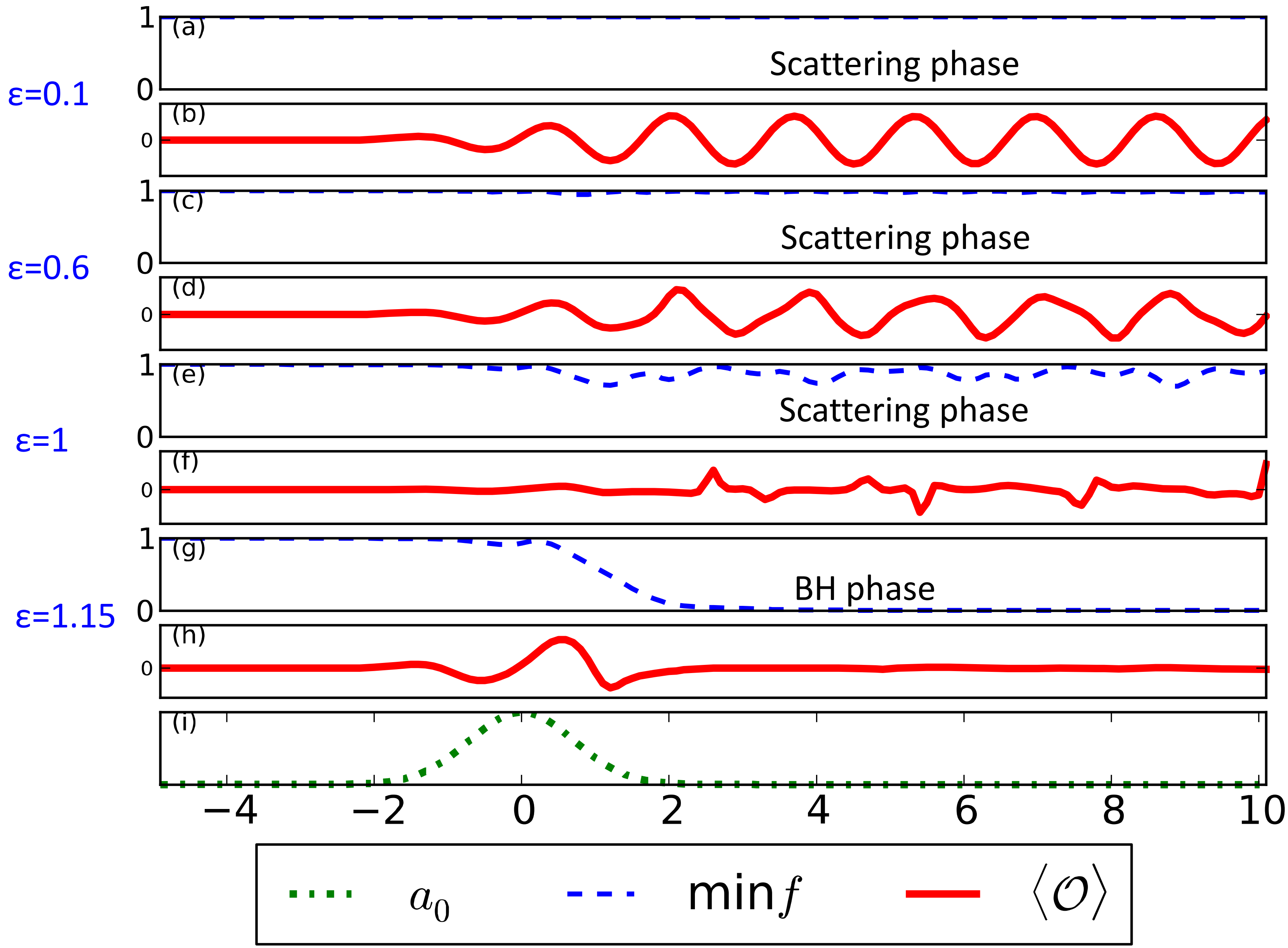


***Phase Diagram (log scale)**: Upper (lower) phase d=3 (d=4). Left (right) uses NBCs (DBCs).

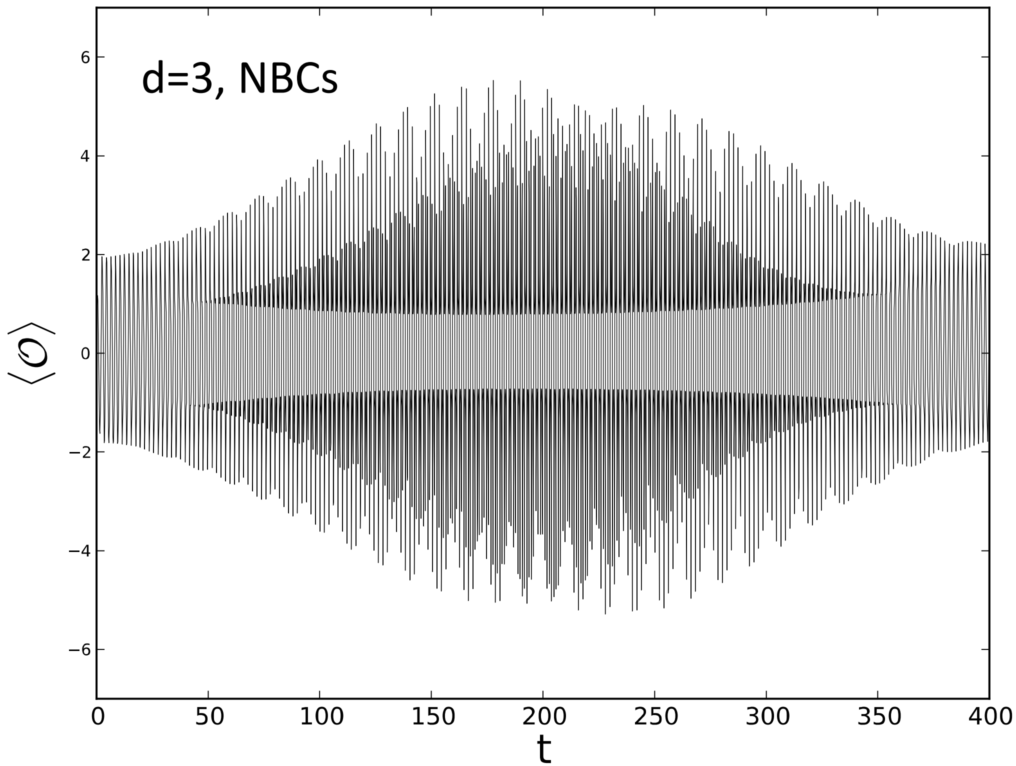
(i) Below (above) the curve is the scattering (BH) phase.

(ii) The green (blue) dashed line reproduces our analytical long (short) injection times

Time evolution of fields; δt=zo=1; d=3



Modulation on <0>



(i) Are the observed oscillations the dual counterpart of the quantum revivals? [Mas, Abajo-Arrastia, da Silva, Lopez, Serantes]. Has our model advantage for such a study in the sense that its boundary field theory is not compact compared to global AdS? Is our model a good candidate for related CM applications?

(ii) d=3 NBCs: the scattering phase persists for (ever) as long as we could follow numerically. … <0> modulation.

(iii) A major difference with existing results is that previous point (ii) refers **to infinite volume** QFTs Vs **finite volumes** (global AdS).

(iv) … no collapse seems to exist. For ε<<1 eigenvalues … transcendental character forbids resonances [Maliborski, [illegible], Bizon]. In fact, we have an exact non-linear argument that applies for any ε.

This is the most important result of the paper that goes against current intuition.

Extracting QFT Data and CFT anomaly

d=4: After adding CT to remove UV divs get scheme dependent expectations.

$$\langle \mathcal{O} \rangle = 4a^{(4)}(t) + \frac{(19-24\beta)}{48}(\dot{a}_0(t))^2\ddot{a}_0(t) - \frac{(3-4\alpha)}{16}\ddddot{a}_0(t)$$

$$\langle T_{tt} \rangle = -\frac{3}{2}f^{(4)}(t) - \frac{(11-24\beta)}{384}(\dot{a}_0(t))^4 - \frac{(1-3\alpha)}{16}(\ddot{a}_0(t))^2,$$

$$\langle T_{x_i x_i} \rangle = -\frac{1}{2}f^{(4)}(t) - \frac{(1-8\beta)}{384}(\dot{a}_0(t))^4 + \frac{\alpha}{16}(\ddot{a}_0(t))^2$$

a^(4) and f^(4) encoded in bulk dynamics. For ε<<1 have analytic expressions; i.e. NBCs,d=3.

$$\langle T_{tt} \rangle = -\frac{1}{2}\int_0^t \mathrm{d}\tau \Big(\dot{a}_0(\tau)\dddot{a}_0(\tau) + 2\sum_{m=1}^{\infty} \dot{a}_0(\tau)\dddot{a}_0(\tau - 2mz_0)\Big) + \mathcal{O}(\epsilon^4)$$

$$\langle \mathcal{O}(t) \rangle = \dddot{a}_0(t) + 2\sum_{m=1}^{\infty}\dddot{a}_0(t-2mz_0) + \mathcal{O}(\epsilon^3)$$

Back to d=4: Observe (i) scheme dependence disappears for t>δt.

(ii) Trace anomaly & the late time restoration.

$$\langle \mathrm{Tr}\,(T_{\mu\nu}) \rangle = -\langle T_{tt} \rangle + 3\langle T_{x_i x_i} \rangle = \frac{1}{16}(\ddot{a}_0(t))^2 + \frac{1}{48}(\dot{a}_0(t))^4$$

QFT interpretations and possible scenarios

- Showed the scattering solutions never collapse, corresponding to QFT states that never thermalize.

- How about <O>=Tr[E^2-B^2]? In BH phase have <O>=0, hence Tr[E^2]=Tr[B^2]. Could this be expected based on equipartition between E and B gluon polarizations during the thermal phase?

- How about when <O>=/=0? Can be interpreted as conversions of glueballs into different glueballs (and back)?

Artifact of the model or qualitatively universal results?

- It is emphasized that HW model not as sick as one expects.
- Variational principle is well-defined at HW if suitable GH terms added-have computed these terms.
- Worked out already the AdS-soliton ala cigar geometry (E. Witten) where space ends smooth.
- Used AdS6. Seen qualitatively similar results: scattering for $\varepsilon \ll 1$ & δt long injection and BH otherwise [work in progress].
- Suggest that other geometries ala IHQCD [Kirists,Gursoly,Nitti. Mazzanti ,...] yield similar results.

Cyrille, how are we doing with time?

Holographic Hall conductivities from dyonic backgrounds

Lindgren, Papadimitriou, Taliotis, Vanhoof, work expected
e-Print: arXiv:1505.04131 [hep-th]

Set up and goals to study

- The background:

$$S=\frac{1}{2\kappa^2}\int \mathrm{d}^{d+1}\mathbf{x}\sqrt{-g}\,(R[g]-\partial_\mu\phi\partial^\mu\phi-Z(\phi)\partial_\mu\chi\partial^\mu\chi-V(\phi,\chi)-\Sigma(\phi)F_{\mu\nu}F^{\mu\nu})$$

$$S_{CS}=-\frac{1}{2\kappa^2}\int \mathrm{d}^4\mathbf{x}\sqrt{-g}\Pi(\chi)\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- The ansatz (scalars run):

$$ds_B^2=dr^2+e^{2A(r)}\left(-f(r)dt^2+dx^2+dy^2\right),$$

$$A_B=\alpha(r)dt+\frac{H}{2}(xdy-ydx),$$

$$\phi_B=\phi_B(r),\qquad \chi_B=\chi_B(r),$$

- The possibilities

-study QCD-like theories in a magnetic field.

-Study (S,T duality) SL(2,Z) covariant theories (FQHE, [Lippert, Meyer, Taliotis, arxiv: 1409.1369]) with mass gap.

Universal Results

- Set up machinery in computing the 2-point functions/transport coefficients for any such theories. Expressed all QFT quantities in terms of one function: the response function Ro(w).
- Derived holographically the small and large frequency asymptotic behavior of the transport coefficients for all these theories. Obtain uiversal results. <u>In particular, obtained a linear spectrum at large w</u>.
- Generalized [Hartnoll,Herzog] the linear relation between 2-point GFs that is inherited to the transport coefficients (electric/thermoelectric conductivities).
- Constructed exact magnetic and confining backgrounds to be used for follow up works in QCD or CM at strong magnetic fields.

Thank you

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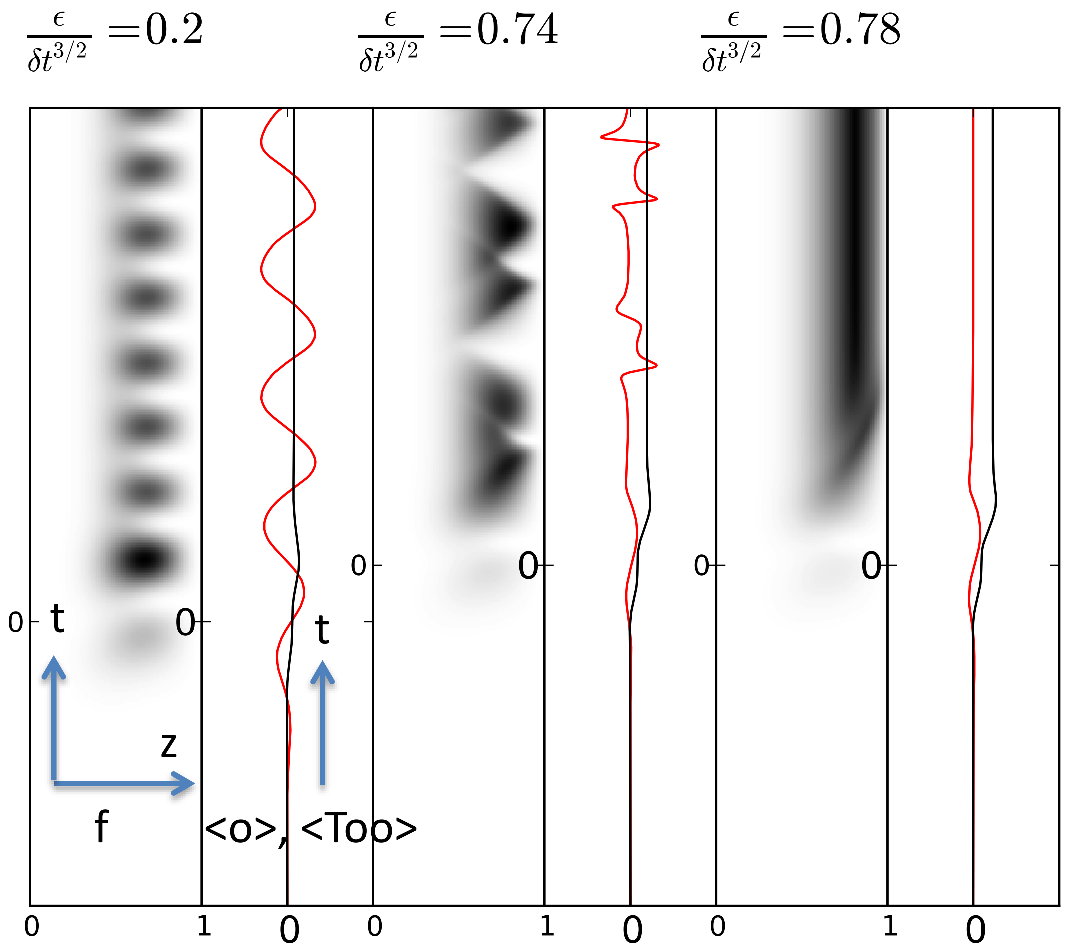
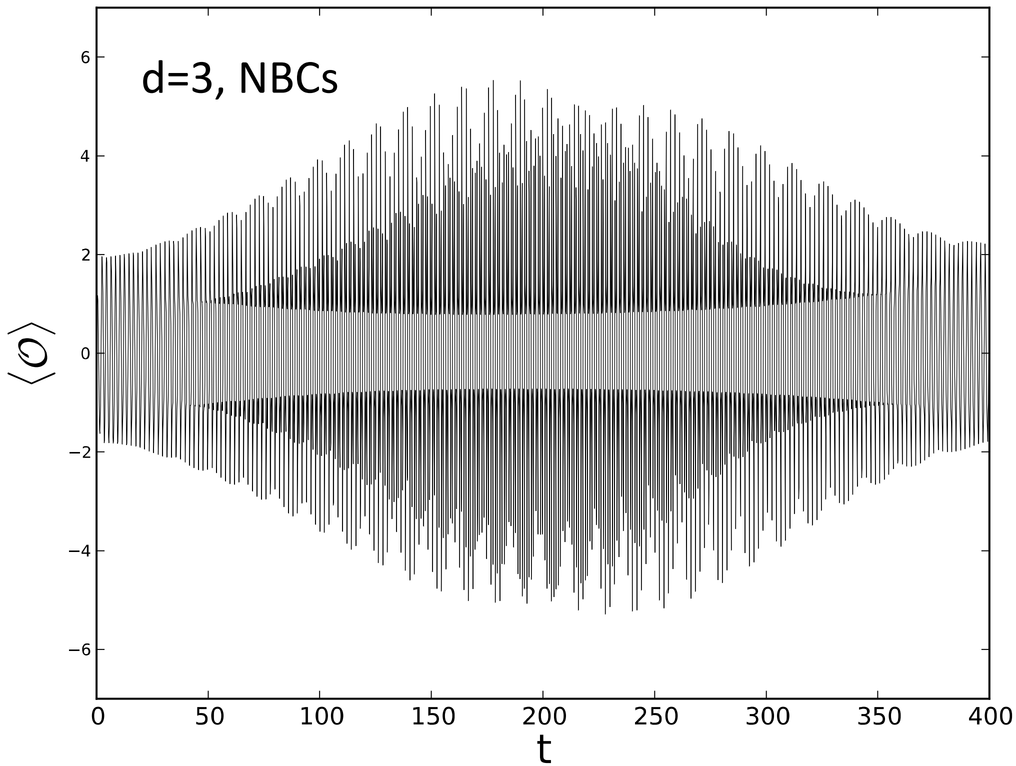
Extracting the QFT data (δt=zo=1; d=3)
Time evolution of fields



*Darker (white) areas show f is close (far from) to 0.
Hence, darker (white) areas show BH (scattering) phase

Modulation on <0>



(I)Case d=/3 and comparison with [Maliborski ,Rostworowski, Bizon].

(ii) Case d=3 and NB-moduation. Most Important result of paper.

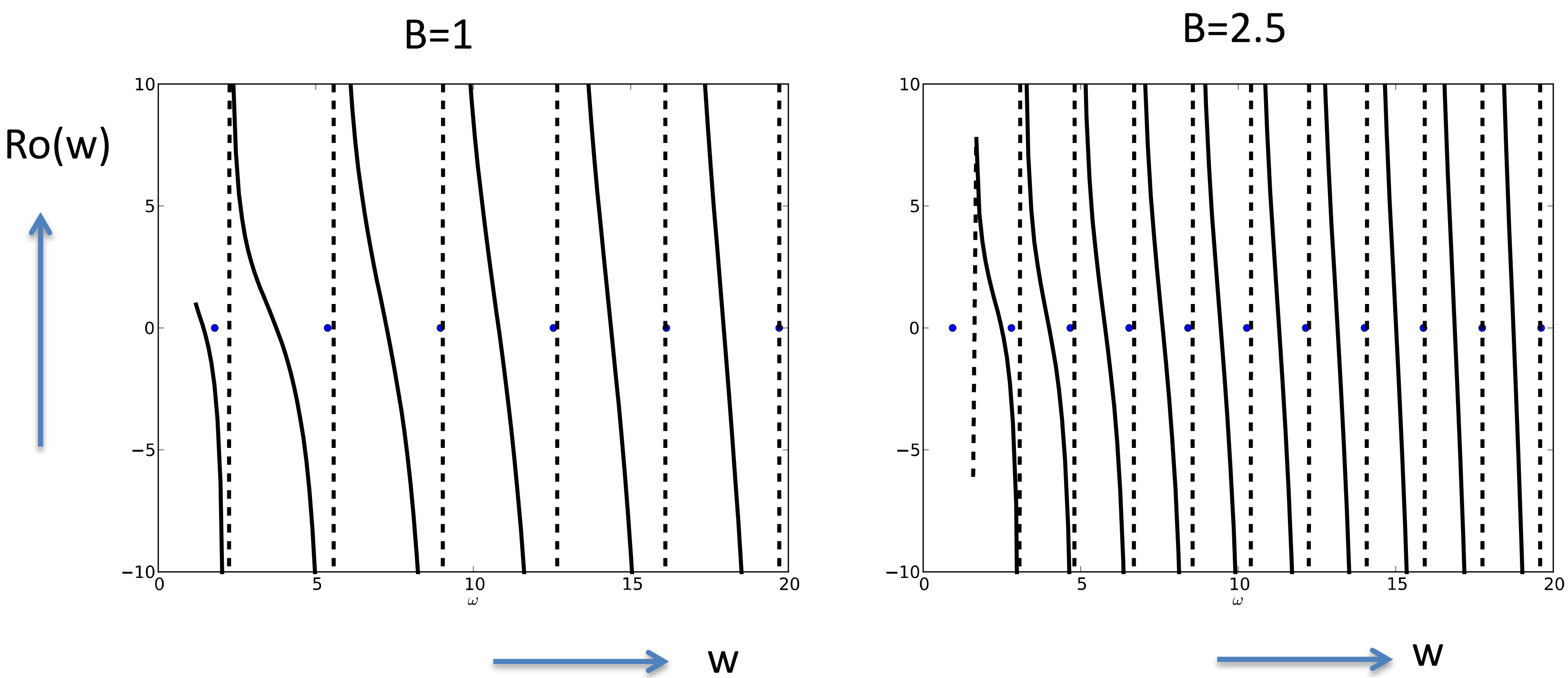
In fact, we have an exact non-linear argument tha applies for any ε.

(iii) Major diffrenece our set up: infinite Vs finite volumes.

(iv) Are the observed oscillations the dual counterpart of the quantum Revivals? Can we use our model in CM systems? [Mas,Abajo-Arrastia,da Silva,López]

Back-Up Slides I.

Preview of confinement and in presence of B-Field



(i) Have analytic power for w>>B.
(ii) In this limit, spectrum is linear.
(iii) As B increases, the spacing of spectrum becomes smaller.
(iv) There is a critical Bc s.t. the spectrum becomes continuous.