Forward di-jet production in dilute-dense collisions

Elena Petreska

École Polytechnique and Universidade de Santiago de Compostela

Collaboration with Piotr Kotko, Krzysztof Kutak, Cyrille Marquet,

Sebastian Sapeta and Andreas van Hameren

arXiv:1503.03421 and preliminary work

POETIC, École Polytechnique

September 7-11, 2015

Forward di-jet production in dilute-dense collisions

- Color Glass Condensate at small x

- High-Energy Factorization

$$Q_s \ll k_t \sim P_t$$

- Transverse Momentum Dependent factorization

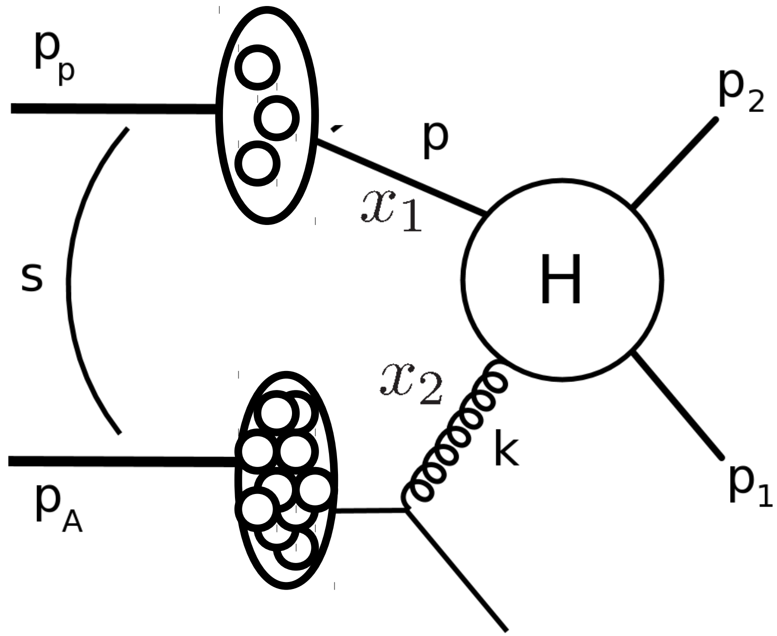
$$k_t \sim Q_s \ll P_t$$

- Collinear factorization

$$Q_s \ll k_t \ll P_t$$

Forward di-jet production in dilute-dense collisions

$$p(p_p) + A(p_A) \to j_1(p_1) + j_2(p_2) + X$$



$$x_1 = \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \sim 1$$

➢ Dilute

$$x_2 = \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \ll 1$$

➢ Dense

$$|k_t|^2 = |p_{1t} + p_{2t}|^2$$

➔ Momentum imbalance

Forward di-jet production in dilute-dense collisions

- Color Glass Condensate at small x

- High-Energy Factorization

$$Q_s \ll k_t \sim P_t$$

- Transverse Momentum Dependent factorization

$$k_t \sim Q_s \ll P_t$$

- Collinear factorization

$$Q_s \ll k_t \ll P_t$$

Forward di-jet production in dilute-dense collisions

• Color Glass Condensate

- Small-x limit;
  - Saturation and multigluon distributions;
    - No k_t factorization.

• High-Energy Factorization

$Q_s \ll k_t \sim P_t$

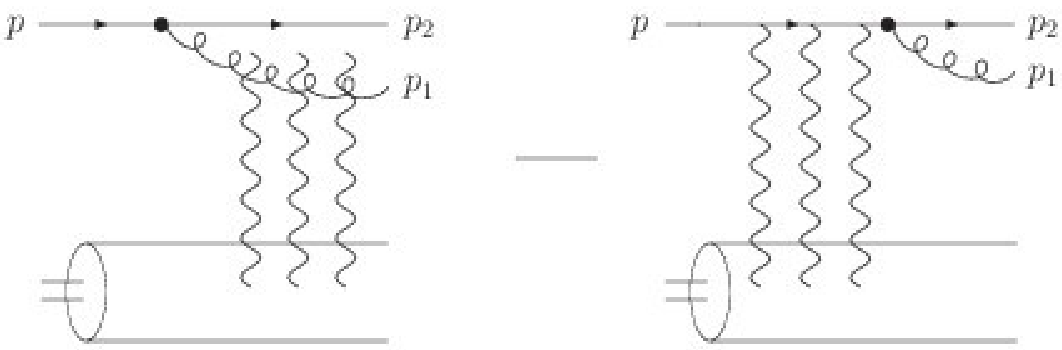
$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{1}{16\pi^3 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}_{g/A}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

K. Kutak and S. Sapeta (2012)

- Parton distributions of collinear factorization for the large-x projectile;
  - k_t dependent unintegrated gluon distribution for the small-x target;
    - Off-shell matrix elements.

Forward di-jet production in dilute-dense collisions

CGC



$$\frac{d\sigma(pA \to qgX)}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \alpha_s C_F (1-z) p_1^+ x_1 f_{q/p}(x_1,\mu^2)\, |\mathcal{M}|^2$$

$$|\mathcal{M}|^2 = \int \frac{d^2\mathbf{x}}{(2\pi)^2}\frac{d^2\mathbf{x}'}{(2\pi)^2}\frac{d^2\mathbf{b}}{(2\pi)^2}\frac{d^2\mathbf{b}'}{(2\pi)^2} e^{-ip_{1t}\cdot(\mathbf{x}-\mathbf{x}')} e^{-ip_{2t}\cdot(\mathbf{b}-\mathbf{b}')} \sum_{\lambda\alpha\beta} \phi^{\lambda^*}_{\alpha\beta}(p,p_1^+,\mathbf{x}'-\mathbf{b}')\phi^{\lambda}_{\alpha\beta}(p,p_1^+,\mathbf{x}-\mathbf{b})$$

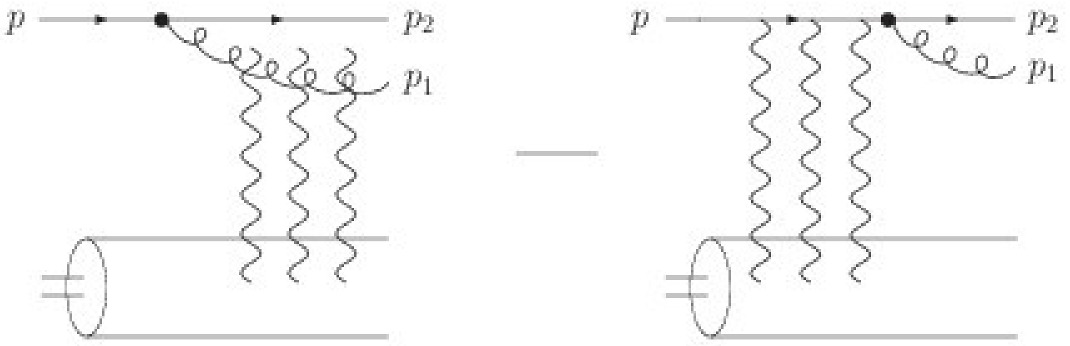
$$\times \Big\{ S^{(4)}_{qg\bar{q}g}[\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_2] - S^{(3)}_{qg\bar{q}}[\mathbf{b},\mathbf{x},\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]$$

$$-S^{(3)}_{qg\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{x}',\mathbf{b}';x_2] + S^{(2)}_{q\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2] \Big\}$$

C. Marquet (2007)

Forward di-jet production in dilute-dense collisions

CGC



$$U(\mathbf{x}) = \mathcal{P}\exp\left[ig\int dx^+ A_a^-(x^+,\mathbf{x})t^a\right]$$

$$V(\mathbf{x}) = \mathcal{P}\exp\left[ig\int dx^+ A_a^-(x^+,\mathbf{x})T^a\right]$$

$$\frac{d\sigma(pA\to qgX)}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \alpha_s C_F (1-z) p_1^+ x_1 f_{q/p}(x_1,\mu^2)\,|\mathcal{M}|^2$$

$$|\mathcal{M}|^2 = \int \frac{d^2\mathbf{x}}{(2\pi)^2}\frac{d^2\mathbf{x}'}{(2\pi)^2}\frac{d^2\mathbf{b}}{(2\pi)^2}\frac{d^2\mathbf{b}'}{(2\pi)^2} e^{-ip_{1t}\cdot(\mathbf{x}-\mathbf{x}')} e^{-ip_{2t}\cdot(\mathbf{b}-\mathbf{b}')} \sum_{\lambda\alpha\beta} \phi_{\alpha\beta}^{\lambda^*}(p,p_1^+,\mathbf{x}'-\mathbf{b}')\phi_{\alpha\beta}^{\lambda}(p,p_1^+,\mathbf{x}-\mathbf{b})$$

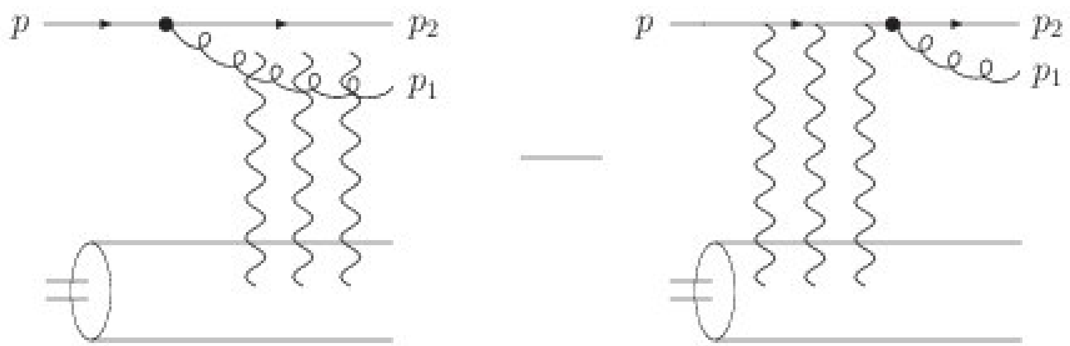
$$\times\left\{S_{qg\bar{q}g}^{(4)}[\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_2] - S_{qg\bar{q}}^{(3)}[\mathbf{b},\mathbf{x},\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]\right.$$

$$\left.-S_{qg\bar{q}}^{(3)}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{x}',\mathbf{b}';x_2] + S_{q\bar{q}}^{(2)}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]\right\}$$

C. Marquet (2007)

Forward di-jet production in dilute-dense collisions

CGC



$$S^{(2)}_{q\bar{q}}(\mathbf{z},\mathbf{z}') = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{z})U^{\dagger}(\mathbf{z}')\right)\right\rangle$$

$$S^{(3)}_{qg\bar{q}}(\mathbf{b},\mathbf{x},\mathbf{z}') = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(U^{\dagger}(\mathbf{z}')t^c U(\mathbf{b})t^d\right) V^{cd}(\mathbf{x})\right\rangle$$

$$S^{(4)}_{qg\bar{q}g}(\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}) = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{b})U^{\dagger}(\mathbf{b}')t^d t^c\right)\left[V(\mathbf{x})V^{\dagger}(\mathbf{x}')\right]^{cd}\right\rangle$$

$$|\mathcal{M}|^2 = \int \frac{d^2\mathbf{x}}{(2\pi)^2}\frac{d^2\mathbf{x}'}{(2\pi)^2}\frac{d^2\mathbf{b}}{(2\pi)^2}\frac{d^2\mathbf{b}'}{(2\pi)^2} e^{-ip_{1t}\cdot(\mathbf{x}-\mathbf{x}')} e^{-ip_{2t}\cdot(\mathbf{b}-\mathbf{b}')} \sum_{\lambda\alpha\beta} \phi^{\lambda^*}_{\alpha\beta}(p,p_1^+,\mathbf{x}'-\mathbf{b}')\phi^{\lambda}_{\alpha\beta}(p,p_1^+,\mathbf{x}-\mathbf{b})$$

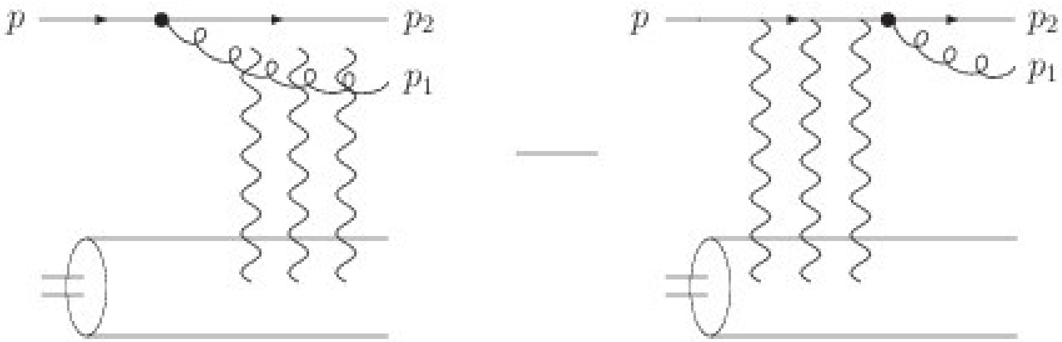
$$\times\left\{ S^{(4)}_{qg\bar{q}g}[\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_2] - S^{(3)}_{qg\bar{q}}[\mathbf{b},\mathbf{x},\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]\right.$$

$$\left. - S^{(3)}_{qg\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{x}',\mathbf{b}';x_2] + S^{(2)}_{q\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]\right\}$$

C. Marquet (2007)

Forward di-jet production in dilute-dense collisions

CGC



$$\sum_{\lambda\alpha\beta} \phi^{\lambda^*}_{\alpha\beta}(p, p_1^+, \mathbf{u}')\phi^{\lambda}_{\alpha\beta}(p, p_1^+, \mathbf{u}) = \frac{8\pi^2}{p_1^+}\frac{\mathbf{u}\cdot\mathbf{u}'}{|\mathbf{u}|^2|\mathbf{u}'|^2}(1+(1-z)^2)$$

$$|\mathcal{M}|^2 = \int \frac{d^2\mathbf{x}}{(2\pi)^2}\frac{d^2\mathbf{x}'}{(2\pi)^2}\frac{d^2\mathbf{b}}{(2\pi)^2}\frac{d^2\mathbf{b}'}{(2\pi)^2} e^{-ip_{1t}\cdot(\mathbf{x}-\mathbf{x}')} e^{-ip_{2t}\cdot(\mathbf{b}-\mathbf{b}')} \sum_{\lambda\alpha\beta} \phi^{\lambda^*}_{\alpha\beta}(p, p_1^+, \mathbf{x}'-\mathbf{b}')\phi^{\lambda}_{\alpha\beta}(p, p_1^+, \mathbf{x}-\mathbf{b})$$

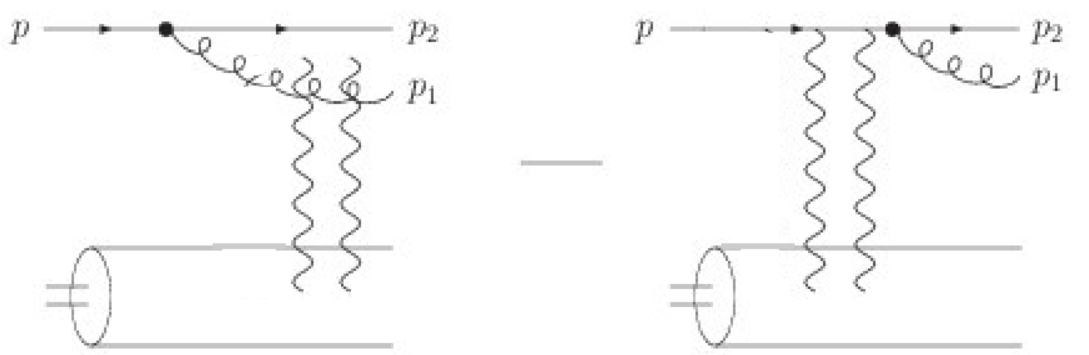
$$\times\left\{ S^{(4)}_{qg\bar{q}g}[\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_2] - S^{(3)}_{qg\bar{q}}[\mathbf{b},\mathbf{x},\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]\right.$$

$$\left. -S^{(3)}_{qg\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{x}',\mathbf{b}';x_2] + S^{(2)}_{q\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_2]\right\}$$

C. Marquet (2007)

Forward di-jet production in dilute-dense collisions

CGC in the limit $Q_s \ll k_t \sim P_t$



$$U(\mathbf{x}) \approx 1 + ig\int dx^+ A^-(x^+,\mathbf{x}) - \frac{g^2}{2}\int dx^+ dy^+ \mathcal{P}\left\{A^-(x^+,\mathbf{x})A^-(y^+,\mathbf{x})\right\} + \mathcal{O}(A^3)$$

$$S^{(4)}_{qg\bar{q}g}(\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}') = S^{(2)}(\mathbf{b}-\mathbf{b}') - \frac{C_A}{C_F}S^{(2)}(\mathbf{x}-\mathbf{x}') - \frac{C_A}{C_F}$$

$$-\frac{C_A}{2C_F}\left[S^{(2)}(\mathbf{x}'-\mathbf{b}) + S^{(2)}(\mathbf{x}-\mathbf{b}') - S^{(2)}(\mathbf{x}-\mathbf{b}) - S^{(2)}(\mathbf{x}'-\mathbf{b}')\right]$$

$$S^{(3)}_{qg\bar{q}}(\mathbf{b},\mathbf{x},\mathbf{v}') = \frac{C_A}{2C_F}\left[S^{(2)}(\mathbf{b}-\mathbf{x}) + S^{(2)}(\mathbf{x}-\mathbf{v}')\right] - \frac{1}{2C_AC_F}S^{(2)}(\mathbf{b}-\mathbf{v}') - \frac{C_A}{2C_F}$$

$$S^{(2)}_{q\bar{q}}(\mathbf{z},\mathbf{z}') = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{z})U^\dagger(\mathbf{z}')\right)\right\rangle$$

Forward di-jet production in dilute-dense collisions

CGC in the limit $Q_s \ll k_t \sim P_t$

- One gluon distribution (dipole distribution)

$$x_2 G^{\rm dipole}(x_2, k_t) = \frac{N_c S_\perp}{2\pi^2 \alpha_s} k_t^2 \int \frac{d^2 r_t}{(2\pi)^2} e^{-ik_t\cdot r_t} \frac{1}{N_c} \left\langle {\rm Tr} U(0_t) U^\dagger(r_t) \right\rangle$$

High-Energy Factorization $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma^{pA\to {\rm dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{1}{16\pi^3 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1, \mu^2) \, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}_{g/A}(x_2, k_t) \frac{1}{1+\delta_{cd}}$$

$$\mathcal{F}_{g/A}(x_2, k_t) = \pi x_2 G^{\rm dipole}(x_2, k_t)$$

Forward di-jet production in dilute-dense collisions

CGC in the limit $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma(pA\to qgX)}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{2\pi} x_1 f_{q/p}(x_1,\mu^2) z(1-z)\hat{P}_{gq}(z)\left[1+\frac{(1-z)^2 p_{1t}^2}{P_t^2} - \frac{1}{N_c^2}\frac{z^2 p_{2t}^2}{P_t^2}\right]\frac{\mathcal{F}_{g/A}(x_2,k_t)}{p_{1t}^2\, p_{2t}^2}$$

$$\frac{d\sigma(pA\to q\bar{q}X)}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{4C_F\pi} x_1 f_{g/p}(x_1,\mu^2) z(1-z)\hat{P}_{qg}(z)\left[-\frac{1}{N_c^2}+\frac{(1-z)^2 p_{1t}^2 + z^2 p_{2t}^2}{P_t^2}\right]\frac{\mathcal{F}_{g/A}(x_2,k_t)}{p_{1t}^2\, p_{2t}^2}$$

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

$$\frac{d\sigma(pA\to ggX)}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2 N_c}{\pi C_F} x_1 f_{g/p}(x_1,\mu^2) z(1-z)\hat{P}_{gg}(z)\left[1+\frac{(1-z)^2 p_{1t}^2 + z^2 p_{2t}^2}{P_t^2}\right]\frac{\mathcal{F}_{g/A}(x_2,k_t)}{p_{1t}^2\, p_{2t}^2}$$

E. Iancu and J. Laidet (2013)

High-Energy Factorization $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{1}{16\pi^3(x_1 x_2 s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\,|\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}_{g/A}(x_2,k_t)\frac{1}{1+\delta_{cd}}$$

Forward di-jet production in dilute-dense collisions

High-Energy Factorization is equivalent to the CGC theory in the limit

$$Q_s \ll k_t \sim P_t$$

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

Forward di-jet production in dilute-dense collisions

- Color Glass Condensate at small x

- High-Energy Factorization

$$Q_s \ll k_t \sim P_t$$

- Transverse Momentum Dependent factorization

$$k_t \sim Q_s \ll P_t$$

- Collinear factorization

$$Q_s \ll k_t \ll P_t$$

Forward di-jet production in dilute-dense collisions

• Color Glass Condensate

- Small-x limit;
  - Saturation and multigluon distributions;
    - No k_t factorization.

• Transverse Momentum Dependent factorization $k_t \sim Q_s \ll P_t$

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1x_2s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

F. Dominguez, C. Marquet, B. Xiao and F. Yuan (2011)

- Large N_c limit; Equivalence with CGC;
  - Parton distributions of collinear factorization for the large-x projectile;
    - Five k_t dependent unintegrated gluon distributions for the small-x target;
      - On-shell hard factors.

Forward di-jet production in dilute-dense collisions

- Improved Transverse Momentum Dependent factorization

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

- Includes all finite N_c corrections;
- Eight unintegrated gluon distributions, two independent per channel;
- Valid for an arbitrary value of k_t.

➢ *Sebastian's talk*

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization

$$\frac{d\sigma^{pA\to\text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} \propto x_1 f_{a/p}(x_1,\mu^2) \otimes H_{ag\to cd} \otimes \mathcal{F}_{ag}(x_2,k_t)$$

$$\mathcal{F}(x_2,k_t) \overset{\text{naive}}{=} 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \langle A|\text{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}\right) F^{i-}(0)\right]|A\rangle$$

- Gauge links in the $A^+ = 0$ gauge:

$$\mathcal{U}^{[\pm]} = U(0,\pm\infty;\mathbf{0})U(\pm\infty,\xi^+;\boldsymbol{\xi})$$

$$\mathcal{U}^{[\Box]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

$$U(a,b;\mathbf{x}) = \mathcal{P}\exp\left[ig\int_a^b dx^+ A_a^-(x^+,\mathbf{x})t^a\right]$$

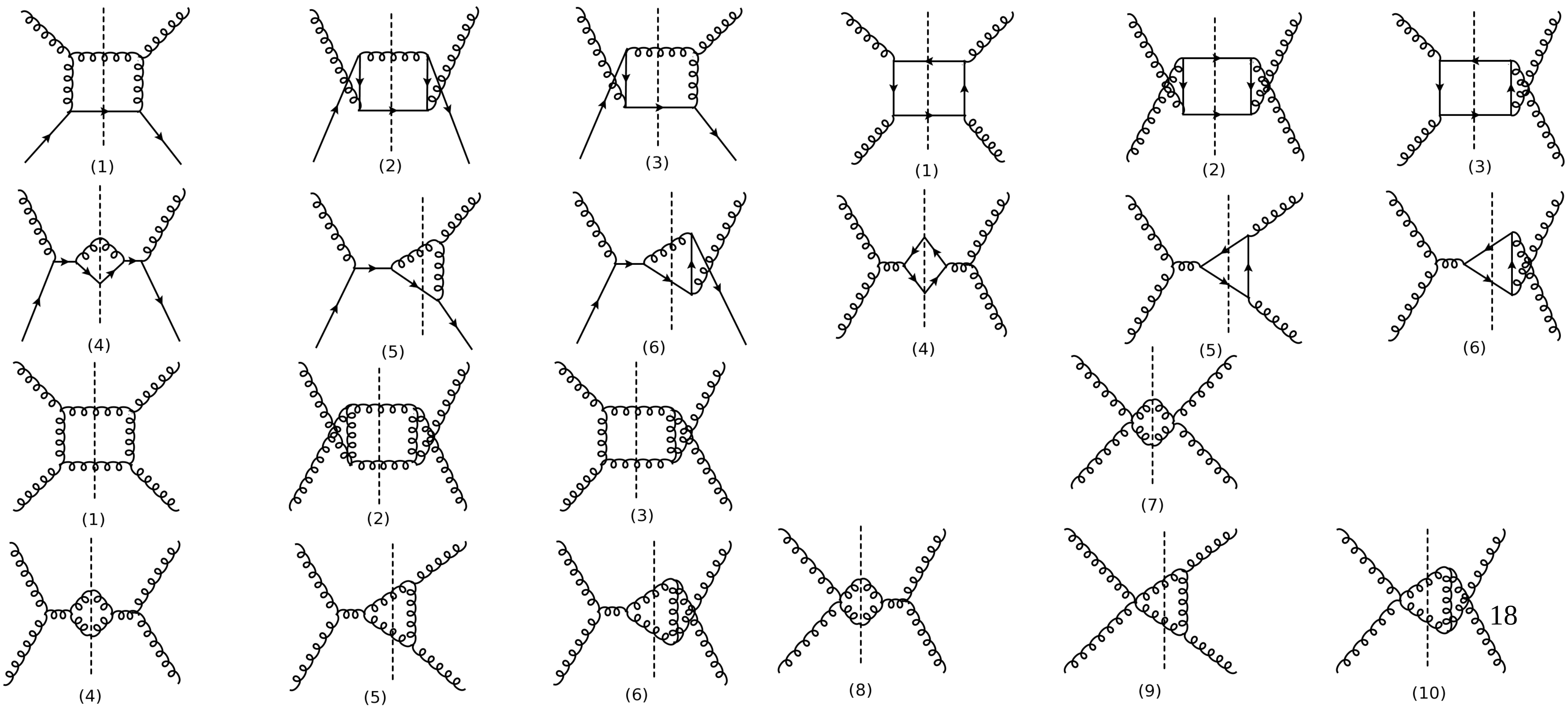
- TMD gluon distributions are gauge invariant, but process dependent.

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} \propto x_1 f_{a/p}(x_1,\mu^2) \otimes H_{ag\to cd} \otimes \mathcal{F}_{ag}(x_2,k_t)$$

C. J. Bomhof, P. J. Mulders and F. Pijlman (2006)



Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization at finite N_c

$$\frac{d\sigma^{pA\to\text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag\to cd} \Phi^{(i)}_{ag\to cd} \frac{1}{1+\delta_{cd}}$$

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

$$\mathcal{F}^{(1)}_{qg} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \left\langle \text{Tr}\left[F(\xi)\,\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}^{(2)}_{qg} \propto \left\langle \text{Tr}\left[F(\xi)\,\frac{\text{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}^{(1)}_{gg} \propto \left\langle \text{Tr}\left[F(\xi)\,\frac{\text{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}^{(4)}_{gg} \propto \left\langle \text{Tr}\left[F(\xi)\,\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

$$\mathcal{F}^{(2)}_{gg} \propto \frac{1}{N_c}\left\langle \text{Tr}\left[F(\xi)\,\mathcal{U}^{[\Box]\dagger}\right] \text{Tr}\left[F(0)\,\mathcal{U}^{[\Box]}\right]\right\rangle$$

$$\mathcal{F}^{(5)}_{gg} \propto \left\langle \text{Tr}\left[F(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}^{(3)}_{gg} \propto \left\langle \text{Tr}\left[F(\xi)\,\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}^{(6)}_{gg} \left\langle \text{Tr}\left[F(\xi)\,\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right] \frac{\text{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c} \frac{\text{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\right\rangle$$

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization at finite N_c

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag\to cd} \Phi^{(i)}_{ag\to cd} \, \frac{1}{1+\delta_{cd}}$$

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

	$K^{(1)}_{ag\to cd}$	$K^{(2)}_{ag\to cd}$
$qg\to qg$	$-\frac{\hat{s}^2+\hat{u}^2}{2\hat{t}^2\hat{s}\hat{u}}\left[\hat{u}^2+\frac{\hat{s}^2-\hat{t}^2}{N_c^2}\right]$	$-\frac{C_F}{N_c}\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{\hat{t}^2\hat{u}}$
$gg\to q\bar{q}$	$\frac{1}{2N_c}\frac{(\hat{t}^2+\hat{u}^2)^2}{\hat{s}^2\hat{t}\hat{u}}$	$-\frac{1}{2C_F N_c^2}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$
$gg\to gg$	$\frac{2N_c}{C_F}\frac{(\hat{s}^2-\hat{t}\hat{u})^2(\hat{t}^2+\hat{u}^2)}{\hat{t}^2\hat{u}^2\hat{s}^2}$	$\frac{2N_c}{C_F}\frac{(\hat{s}^2-\hat{t}\hat{u})^2}{\hat{t}\hat{u}\hat{s}^2}$

Forward di-jet production in dilute-dense collisions

Work in progress

- Gluon distributions in the Golec-Biernat-Wusthoff model

K. Golec-Biernat and M. Wusthoff (1998)

$$S(\mathbf{r}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(\mathbf{r})U^\dagger(\mathbf{0})\right]\right\rangle = \exp\left[-\frac{\mathbf{r}^2 Q_s^2}{4}\right]$$

$$\mathcal{F}_{qg}^{(1)}(x_2,k_t) = \frac{N_c S_\perp}{2\pi^3\alpha_s Q_s^2(x_2)} k_t^2 \exp\left[-\frac{k_t^2}{Q_s^2(x_2)}\right]$$

$$\mathcal{F}_{qg}^{(2)}(x_2,k_t) = \frac{N_c S_\perp}{2\pi^3\alpha_s} \exp\left[-\frac{k_t^2}{Q_s^2(x_2)}\right] \int_1^\infty \frac{dt}{t(t+2)} \exp\left[\frac{2k_t^2}{(t+2)Q_s^2(x_2)}\right]$$

$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = \frac{N_c S_\perp}{16\pi^3\alpha_s} \exp\left[-\frac{k_t^2}{2Q_s^2(x_2)}\right] \left(2+\frac{k_t^2}{Q_s^2(x_2)}\right)$$

$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = \frac{N_c S_\perp}{16\pi^3\alpha_s} \exp\left[-\frac{k_t^2}{2Q_s^2(x_2)}\right] \left(2-\frac{k_t^2}{Q_s^2(x_2)}\right)$$

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) = \frac{N_c S_\perp}{4\pi^3\alpha_s} \exp\left[-\frac{k_t^2}{2Q_s^2(x_2)}\right] \int_1^\infty \frac{dt}{t(t+1)} \exp\left[\frac{k_t^2}{2(t+1)Q_s^2(x_2)}\right]$$

Forward di-jet production in dilute-dense collisions

Work in progress

- Gluon distributions in the Golec-Biernat-Wusthoff model

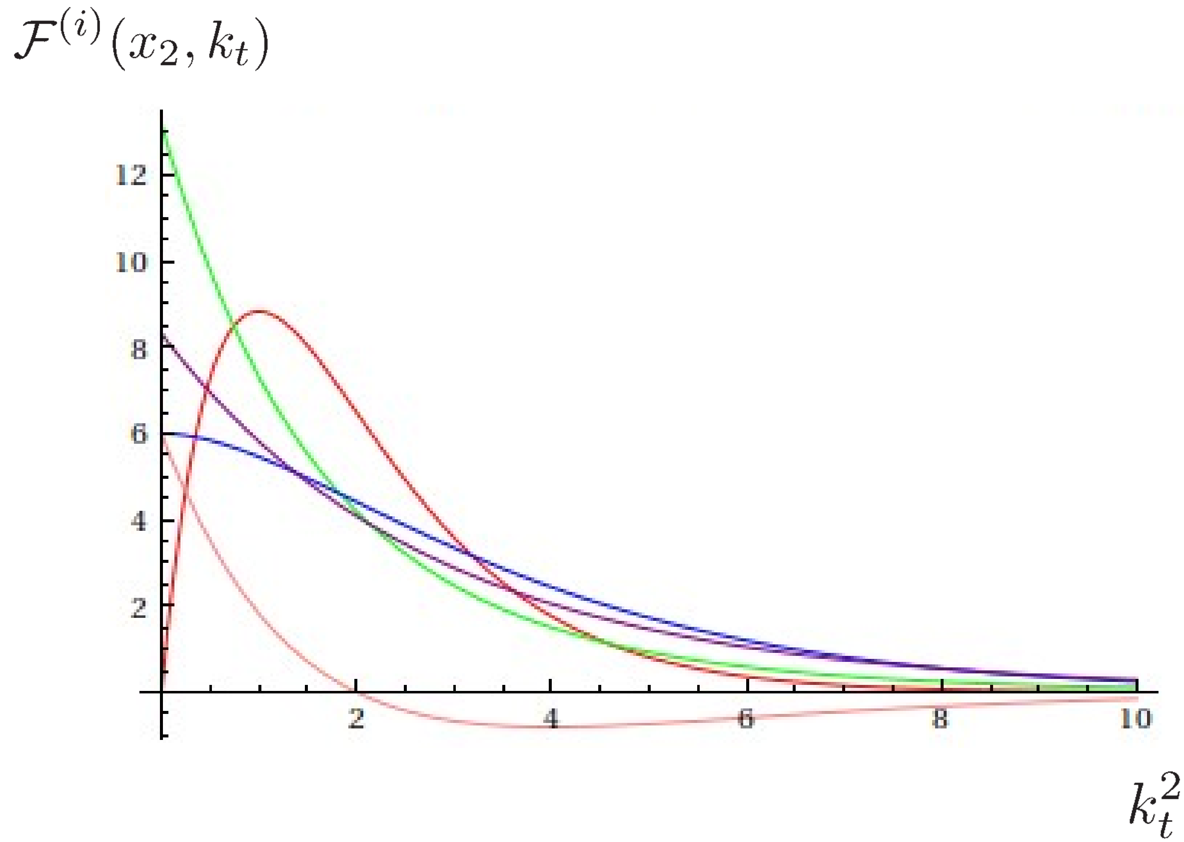
$\mathcal{F}_{qg}^{(1)}(x_2, k_t)$

$\mathcal{F}_{qg}^{(2)}(x_2, k_t)$

$\mathcal{F}_{gg}^{(1)}(x_2, k_t)$

$\mathcal{F}_{gg}^{(2)}(x_2, k_t)$

$\mathcal{F}_{gg}^{(3)}(x_2, k_t)$



Forward di-jet production in dilute-dense collisions

• Conclusions

- The High-Energy Factorization framework can be derived from CGC in the dilute target limit; $Q_s \ll k_t \sim P_t$
  - TMD factorization formula can be written at finite N_c with two k_t dependent gluon distributions per channel;
    - Improved TMD factorization unifies the two regimes.

➢ *Sebastian's talk*

• Outlook

- Phenomenology;
  - Model expressions for all gluon TMD's present at finite N_c ;
    - Beyond GBW: MV, rcBK...

Back up

Forward di-jet production in dilute-dense collisions

Work in progress

- Gluon distributions in the Golec-Biernat-Wusthoff model

$$S(\mathbf{r}) = \frac{1}{N_c} \left\langle \mathrm{Tr}\left[U(\mathbf{r})U^\dagger(\mathbf{0})\right]\right\rangle = \exp\left[-\frac{\mathbf{r}^2 Q_s^2}{4}\right]$$

- At large N_c there are two fundamental gluon distributions:
  - Dipole distribution $$\mathcal{F}_{qg}^{(1)} = \frac{N_c\, k_t^2\, S_\perp}{2\pi^2\alpha_s} \int \frac{d^2 r}{(2\pi)^2} e^{-i\mathbf{k}\cdot\mathbf{r}} \frac{1}{N_c} \left\langle \mathrm{Tr}\left[U(\mathbf{r})U^\dagger(\mathbf{0})\right]\right\rangle$$

- Weizsäcker-Williams distribution $$x_2 G_1(x_2, k_t) = \frac{C_F}{2\alpha_s \pi^4} \int d^2 b \int \frac{d^2 r}{r^2}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, N_A(x, r, \mathbf{b})$$

$$\mathcal{F}_{qg}^{(2)}(x_2, k_t) = \int d^2 q_t\, x_2 G_1(x_2, q_t) F(x_2, k_t - q_t)$$

$$\mathcal{F}_{gg}^{(1)}(x_2, k_t) = \int d^2 q_t\, x_2 G_2(x_2, q_t) F(x_2, k_t - q_t)$$

$$\mathcal{F}_{gg}^{(2)}(x_2, k_t) = -\int d^2 q_t \frac{(k_t - q_t)\cdot q_t}{q_t^2} x_2 G_2(x_2, q_t) F(x_2, k_t - q_t)$$

$$\mathcal{F}_{gg}^{(3)}(x_2, k_t) = \int d^2 q_t d^2 q_t'\, x_2 G_1(x_2, q_t) F(x_2, q_t') F(x_2, k_t - q_t - q_t')$$

Forward di-jet production in dilute-dense collisions

$$\mathcal{A}_3 = \left(\frac{p\,k}{p\,p_A}\right)^2 \frac{(p_2\,p_1)(p\,p_A)+(p\,p_1)(p_A\,p_2)+(p_A\,p_1)(p\,p_2)}{(p_A\,p_1)(p\,p_1)(p_2\,p_1)(p\,p_A)(p_A\,p_2)(p\,p_2)} \left[(p_A\,p_1)^4+(p\,p_A)^4+(p_A\,p_2)^4\right]$$

$$\overline{|\mathcal{M}_{qg\to qg}|}^2 = C_1\mathcal{A}_1^{(ab)} + \overline{C}_1\mathcal{A}_1^{(nab)} \;,\quad \overline{|\mathcal{M}_{gg\to q\bar{q}}|}^2 = C_2\mathcal{A}_2^{(ab)} + \overline{C}_2\mathcal{A}_2^{(nab)} \;,\quad \overline{|\mathcal{M}_{gg\to gg}|}^2 = C_3\mathcal{A}_3$$

$$\mathcal{A}_1^{(ab)} = \left(\frac{p\,k}{p\,p_A}\right)^2 \frac{(p\,p_A)^2+(p_A\,p_2)^2}{(p\,p_1)\;(p_2\,p_1)} \quad \mathcal{A}_2^{(nab)} = \left(\frac{p\,k}{p\,p_A}\right)^2 \frac{(p_A\,p_2)^2+(p_A\,p_1)^2}{(p\,p_1)\;(p\,p_2)} \left(\frac{(p\,p_1)\;(p_A\,p_2)}{(p_2\,p_1)\,(p\,p_A)} + \frac{(p\,p_2)\;(p_A\,p_1)}{(p_2\,p_1)\,(p\,p_A)} -\right.$$

$$\mathcal{A}_1^{(nab)} = \left(\frac{p\,k}{p\,p_A}\right)^2 \frac{(p\,p_A)^2+(p_A\,p_2)^2}{(p\,p_1)\;(p_2\,p_1)} \left(\frac{(p_2\,p_1)\;(p\,p_A)}{(p\,p_2)\,(p_A\,p_1)} + \frac{(p\,p_1)\;(p_A\,p_2)}{(p\,p_2)\,(p_A\,p_1)} - 1\right)$$

$$\mathcal{A}_2^{(ab)} = \left(\frac{p\,k}{p\,p_A}\right)^2 \frac{(p_A\,p_2)^2+(p_A\,p_1)^2}{(p\,p_1)\;(p\,p_2)}$$

High-Energy Factorization $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{1}{16\pi^3(x_1x_2s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\, \overline{|\mathcal{M}_{ag^*\to cd}|}^2 \mathcal{F}_{g/A}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

TMD factorizaion in the large *Nc* limit

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_i^n H^{(i)}_{ag\to cd}\mathcal{F}^{(i)}_{ag}(x_2,k_t)\frac{1}{1+\delta_{cd}}$$

Dominguez, Marquet, Xiao and Yuan (2011)

$$qg \to qg: \qquad n=2$$

$$gg \to q\bar{q}: \qquad n=2$$

$$gg \to gg: \qquad n=3$$

$$\mathcal{F}^{(1)}_{qg} = xG^{(2)}(x,q_\perp),\ \ \mathcal{F}^{(2)}_{qg} = \int xG^{(1)}(q_1)\otimes F(q_2)\ ,$$

$$\mathcal{F}^{(1)}_{gg} = \int xG^{(2)}(q_1)\otimes F(q_2),\ \ \mathcal{F}^{(2)}_{gg} = -\int \frac{q_{1\perp}\cdot q_{2\perp}}{q_{1\perp}^2} xG^{(2)}(q_1)\otimes F(q_2)\ ,$$

$$\mathcal{F}^{(3)}_{gg} = \int xG^{(1)}(q_1)\otimes F(q_2)\otimes F(q_3)\ ,$$

TMD factorizaion at finite *Nc*

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i^n H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

$$qg \to qg : \qquad n = 2$$

$$gg \to q\bar{q} : \qquad n = 3$$

$$gg \to gg : \qquad n = 6$$

Kotko, Kutak, Marquet, EP,
Sapeta, van Hameren (2015)

Not all the hard factors are independent

$\Leftrightarrow$

Only two independent gluon distributions

TMD factorizaion at finite *Nc* and arbitrary k_t

$$\frac{d\sigma^{pA\to\text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag\to cd}\Phi^{(i)}_{ag\to cd}\,\frac{1}{1+\delta_{cd}}$$

Effective procedure for calculating off-shell matrix elements:

Stanfard Feynman rules with a gauge vector defined by the target four-momentum

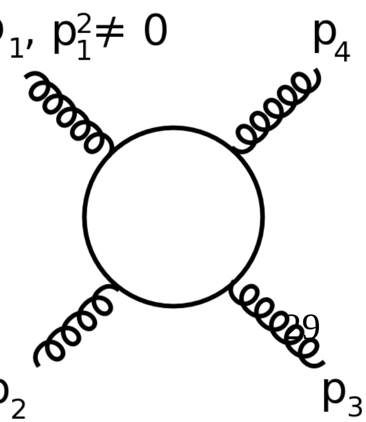
+ longitudinal polarization vector for the off-shell gluon

TMD's and hard factors from color ordered amplitudes:

$gg \to gg$

$$\mathcal{M}^{a_1a_2a_3a_4}_{gg^*\to gg}\left(n_1,\varepsilon_2^{\lambda_2},\varepsilon_3^{\lambda_3},\varepsilon_4^{\lambda_4}\right) = f_{a_1a_2c}f_{ca_3a_4}\,\mathcal{M}_{gg^*\to gg}\left(1^*,2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}\right)$$

$$+f_{a_1a_3c}f_{ca_2a_4}\,\mathcal{M}_{gg^*\to gg}\left(1^*,3^{\lambda_3},2^{\lambda_2},4^{\lambda_4}\right)$$



TMD factorizaion at finite *Nc* and arbitrary k_t

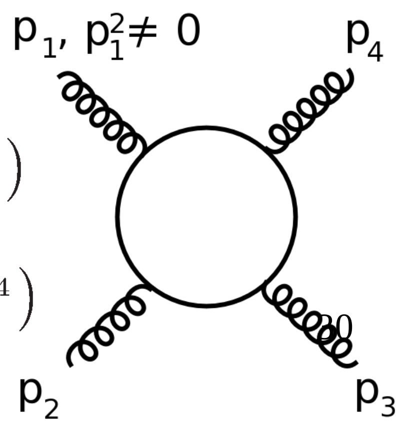
$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag\to cd}\Phi^{(i)}_{ag\to cd}\,\frac{1}{1+\delta_{cd}}$$

colour-ordered amplitude squared	gluon TMD
$\lvert\mathcal{M}_{gg^*\to gg}(1^*,2,3,4)\rvert^2$ $\lvert\mathcal{M}_{gg^*\to gg}(1^*,3,2,4)\rvert^2$	$\Phi^{(1)}_{gg\to gg} = \frac{1}{2N_c^2}\big(N_c^2\mathcal{F}^{(1)}_{gg} - 2\mathcal{F}^{(3)}_{gg}$ $+\mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\big)$
$\mathcal{M}_{gg^*\to gg}(1^*,2,3,4)\,\mathcal{M}^*_{gg^*\to gg}(1^*,3,2,4)$ $\mathcal{M}^*_{gg^*\to gg}(1^*,2,3,4)\,\mathcal{M}_{gg^*\to gg}(1^*,3,2,4)$	$\Phi^{(2)}_{gg\to gg} = \frac{1}{N_c^2}\big(N_c^2\mathcal{F}^{(2)}_{gg} - 2\mathcal{F}^{(3)}_{gg}$ $+\mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\big)$

TMD's and hard factors from color ordered amplitudes:

$gg \to gg$

$$\mathcal{M}^{a_1a_2a_3a_4}_{gg^*\to gg}\left(n_1,\varepsilon_2^{\lambda_2},\varepsilon_3^{\lambda_3},\varepsilon_4^{\lambda_4}\right) = f_{a_1a_2c}f_{ca_3a_4}\,\mathcal{M}_{gg^*\to gg}\left(1^*,2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}\right)$$
$$+f_{a_1a_3c}f_{ca_2a_4}\,\mathcal{M}_{gg^*\to gg}\left(1^*,3^{\lambda_3},2^{\lambda_2},4^{\lambda_4}\right)$$



Improved TMD factorizaion

$$\frac{d\sigma^{pA\to\text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag\to cd}\Phi^{(i)}_{ag\to cd}\frac{1}{1+\delta_{cd}}$$

i	1	2
$K^{(i)}_{gg^*\to gg}$	$\frac{N_c}{C_F}\frac{(\bar{s}^4+\bar{t}^4+\bar{u}^4)(\bar{u}\hat{u}+\bar{t}\hat{t})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$	$-\frac{N_c}{2C_F}\frac{(\bar{s}^4+\bar{t}^4+\bar{u}^4)(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$
$K^{(i)}_{gg^*\to q\bar{q}}$	$\frac{1}{2N_c}\frac{(\bar{t}^2+\bar{u}^2)(\bar{u}\hat{u}+\bar{t}\hat{t})}{\bar{s}\hat{s}\hat{t}\hat{u}}$	$\frac{1}{4N_c^2C_F}\frac{(\bar{t}^2+\bar{u}^2)(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s})}{\bar{s}\hat{s}\hat{t}\hat{u}}$
$K^{(i)}_{qg^*\to qg}$	$-\frac{\bar{u}(\bar{s}^2+\bar{u}^2)}{2\bar{t}\hat{t}\hat{s}}\left(1+\frac{\bar{s}\hat{s}-\bar{t}\hat{t}}{N_c^2\;\bar{u}\hat{u}}\right)$	$-\frac{C_F}{N_c}\frac{\bar{s}(\bar{s}^2+\bar{u}^2)}{\bar{t}\hat{t}\hat{u}}$

Modified Mandelstam variables:

$$\bar{s} = (x_2p_A+p)^2$$

$$\bar{t} = (x_2p_A-p_1)^2$$

$$\bar{u} = (x_2p_A-p_2)^2$$

Kotko, Kutak, Marquet, EP, Sapeta, van Hameren (2015)