The gauge-invariant canonical energy-momentum tensor

Based on : [C.L., JHEP 1508 (2015) 045]

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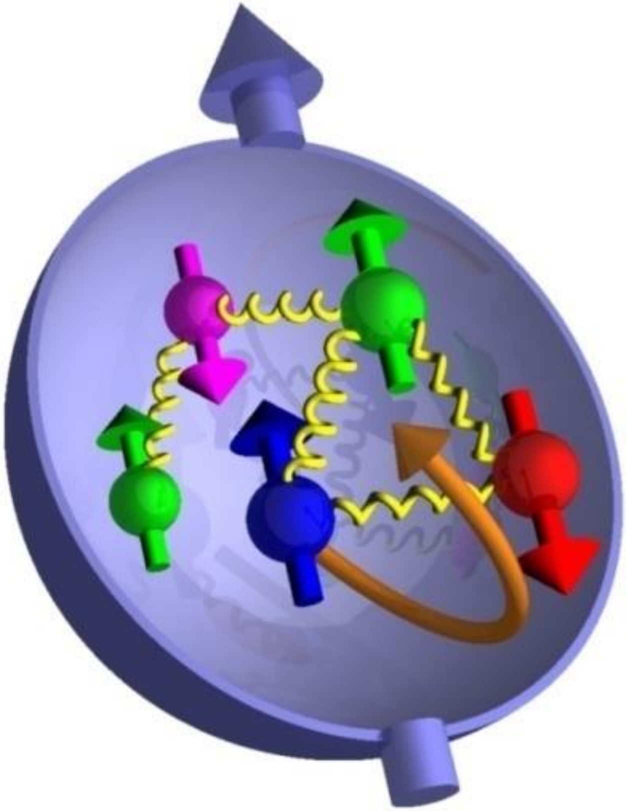
Outline



- Energy-momentum tensors
- General parametrization
- Relations to parton distributions
- Conclusions

Energy-momentum tensor

A lot of interesting physics is contained in the EMT

[Polyakov, Shuvaev (2002)]
[Polyakov (2003)]
[Goeke *et al.* (2007)]
[Cebulla *et al.* (2007)]

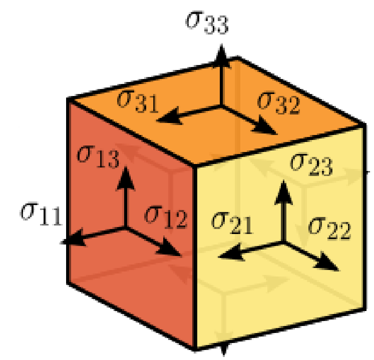
Energy density **Momentum density**

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Shear stress

Normal stress (pressure)

Energy flux **Momentum flux**



E.g.

$$M = \int \mathrm{d}^3 r \, T^{00}(\vec{r})$$

$$L^i = \int \mathrm{d}^3 r \, \epsilon^{ijk} r^j T^{0k}(\vec{r})$$

Energy-momentum tensor

Canonical EMT

[Jaffe, Manohar (1990)]

$$T^{\mu\nu}_{\text{can}} = \overline{\psi}\gamma^\mu \frac{i}{2}\overset{\leftrightarrow}{\partial^\nu}\psi - 2\mathrm{Tr}[G^{\mu\alpha}\partial^\nu A_\alpha] - g^{\mu\nu}\mathcal{L}_{\text{QCD}}$$

Gauge transformation

$$T^{\mu\nu}_{\text{can}} \mapsto T^{\mu\nu}_{\text{can}} - \underbrace{\frac{2i}{g}\partial_\alpha \mathrm{Tr}[G^{\alpha\mu}U^{-1}\partial^\nu U]}_{\partial_\alpha X^{[\alpha\mu]\nu}}$$

Superpotential

Freedom of the EM distribution

$$T'^{\mu\nu} = T^{\mu\nu} + \partial_\alpha X^{[\alpha\mu]\nu}$$

satisfies

$$\partial_\mu T'^{\mu\nu} = \partial_\mu T^{\mu\nu} = 0$$

$$\int \mathrm{d}^3 r\, T'^{0\nu} = \int \mathrm{d}^3 r\, T^{0\nu}$$

➡ **Total EM is gauge invariant and conserved**

Energy-momentum tensor

Ji EMT

QCD EOM $\mathcal{D}_\alpha G^{\alpha\mu a} = -g\overline{\psi}\gamma^\mu t^a \psi$

$$T^{\mu\nu}_{\text{Ji}} = T^{\mu\nu}_{\text{can}} - 2\partial_\alpha \text{Tr}[G^{\alpha\mu}A^\nu]$$
$$= \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu \psi - 2\text{Tr}[G^{\mu\alpha}G^\nu{}_\alpha] - g^{\mu\nu}\mathcal{L}_{\text{QCD}}$$

Gauge invariant

[Ji (1997)]

Belinfante-Rosenfeld EMT

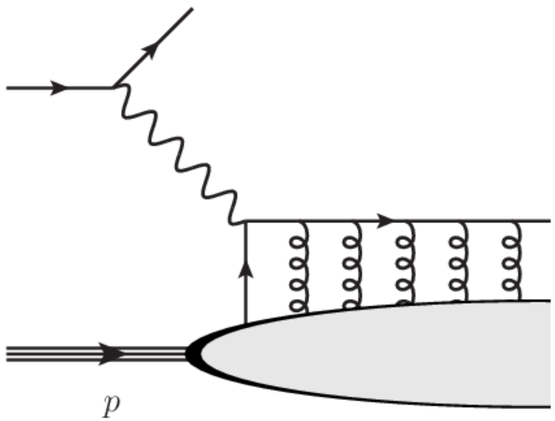
QCD identity $\overline{\psi}\gamma^{[\mu}i\overleftrightarrow{D}^{\nu]}\psi = -\partial_\alpha(\epsilon^{\alpha\mu\nu\rho}\overline{\psi}\gamma_\rho\gamma_5\psi)$

$$T^{\mu\nu}_{\text{Bel}} = T^{\mu\nu}_{\text{Ji}} + \tfrac{1}{4}\partial_\alpha(\epsilon^{\alpha\mu\nu\rho}\overline{\psi}\gamma_\rho\gamma_5\psi)$$
$$= \overline{\psi}\gamma^{\{\mu}\tfrac{i}{4}\overleftrightarrow{D}^{\nu\}}\psi - 2\text{Tr}[G^{\mu\alpha}G^\nu{}_\alpha] - g^{\mu\nu}\mathcal{L}_{\text{QCD}}$$

Gauge invariant and symmetric

Factorization and background field

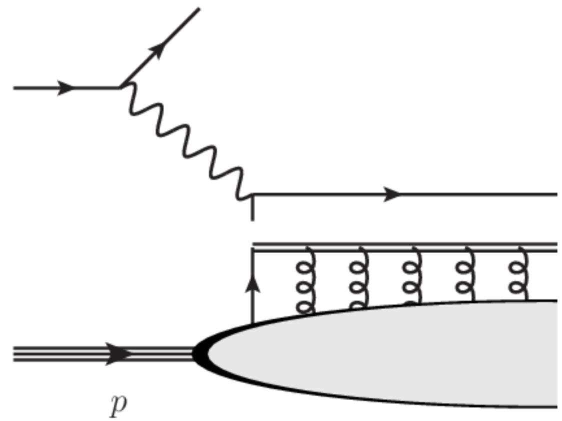
System



Local

~

« Factorized » system



Non-local

Wilson line

~ background field

$A_\mu^{\text{pure}} = \mathcal{W}\frac{i}{g}\partial_\mu\mathcal{W}^{-1}$

Pure gauge !

Chen *et al.* approach **(similar to Background Field Method)**

$$A_\mu = A_\mu^{\text{pure}} + A_\mu^{\text{phys}}$$

Background **Dynamical**

[Chen *et al.* (2008-09)]
[Wakamatsu (2010-11)]
[C.L. (2013-14)]

Energy-momentum tensor

Canonical EMT

[Jaffe, Manohar (1990)]

$$T^{\mu\nu}_{\text{can}} = \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{\partial}^\nu\psi - 2\text{Tr}[G^{\mu\alpha}\partial^\nu A_\alpha] - g^{\mu\nu}\mathcal{L}_{\text{QCD}}$$

Gauge-invariant canonical EMT

$$\begin{aligned} T^{\mu\nu}_{\text{gic}} &= T^{\mu\nu}_{\text{can}} - 2\partial_\alpha \text{Tr}[G^{\alpha\mu} A^\nu_{\text{pure}}] \\ &= \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu_{\text{pure}}\psi - 2\text{Tr}[G^{\mu\alpha}\mathcal{D}^\nu_{\text{pure}} A^{\text{phys}}_\alpha] - g^{\mu\nu}\mathcal{L}_{\text{QCD}} \end{aligned}$$

Gauge-invariant kinetic EMT

$$\begin{aligned} T^{\mu\nu}_{\text{gik}} &= T^{\mu\nu}_{\text{Ji}} + 2\partial_\alpha \text{Tr}[G^{\alpha\mu} A^\nu_{\text{phys}}] \\ &= \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu\psi - 2\text{Tr}[G^{\mu\alpha}\mathcal{D}^\nu_{\text{pure}} A^{\text{phys}}_\alpha - (\mathcal{D}_\alpha G^{\alpha\mu}) A^\nu_{\text{phys}}] - g^{\mu\nu}\mathcal{L}_{\text{QCD}} \end{aligned}$$

[Wakamatsu (2010-11)]
[C.L. (2013-14)]
[Leader, C.L. (2014)]

Energy-momentum tensor

Family	Energy-momentum tensor	Gauge invariant	Local	Symmetric
Kinetic	Belinfante-Rosenfeld [1–3]	✓	✓	✓
	Ji [9]	✓	✓	–
	gikWakamatsu [41]	✓	–	–
Canonical	Jaffe-Manohar [8]	–	✓	–
	gicChen *et al.* [15]	✓	–	–

A convenient gauge-invariant basis

$$T_1^{\mu\nu} = \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu\psi$$

$$T_2^{\mu\nu} = -2\mathrm{Tr}[G^{\mu\alpha}G^\nu{}_\alpha] + \tfrac{1}{2}g^{\mu\nu}\mathrm{Tr}[G^{\rho\sigma}G_{\rho\sigma}]$$

$$T_3^{\mu\nu} = -\overline{\psi}\gamma^\mu g A^\nu_{\text{phys}}\psi$$

$$T_4^{\mu\nu} = \tfrac{1}{4}\partial_\alpha(\epsilon^{\alpha\mu\nu\rho}\overline{\psi}\gamma_\rho\gamma_5\psi)$$

$$T_5^{\mu\nu} = 2\partial_\alpha\mathrm{Tr}[G^{\alpha\mu}A^\nu_{\text{phys}}]$$

Kinetic

$$T^{\mu\nu}_{\text{Bel},q} = T_1^{\mu\nu} + T_4^{\mu\nu} \qquad T^{\mu\nu}_{\text{Bel},G} = T_2^{\mu\nu}$$

$$T^{\mu\nu}_{\text{Ji},q} = T_1^{\mu\nu} \qquad T^{\mu\nu}_{\text{Ji},G} = T_2^{\mu\nu}$$

$$T^{\mu\nu}_{\text{gik},q} = T_1^{\mu\nu} \qquad T^{\mu\nu}_{\text{gik},G} = T_2^{\mu\nu} + T_5^{\mu\nu}$$

Canonical

$$T^{\mu\nu}_{\text{gic},q} = T_1^{\mu\nu} + T_3^{\mu\nu} \qquad T^{\mu\nu}_{\text{gic},G} = T_2^{\mu\nu} - T_3^{\mu\nu} + T_5^{\mu\nu}$$

[C.L. (2015)]

Matrix elements

Belinfante-Rosenfeld EMT $X_a \equiv X_a(\Delta^2) \in \mathbb{R}$ $a = q, G$ **[Ji (1997)]**

$$\langle p', S'|T^{\mu\nu}_{\text{Bel},a}(0)|p, S\rangle = \overline{u}(p', S')\Gamma^{\mu\nu}_{\text{Bel},a}(P,\Delta)u(p,S)$$

$$\Gamma^{\mu\nu}_{\text{Bel},a}(P,\Delta) = \frac{P^{\{\mu}\gamma^{\nu\}}}{2}A_a + \frac{P^{\{\mu}i\sigma^{\nu\}\Delta}}{4M}B_a + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C_a + Mg^{\mu\nu}\bar{C}_a$$

$$A_q + A_G = 1 \qquad B_q + B_G = 0 \qquad \bar{C}_q + \bar{C}_G = 0$$

Ji EMT **[Bakker *et al.* (2004)]**

$$\langle p', S'|T^{\mu\nu}_{\text{Ji},a}(0)|p, S\rangle = \overline{u}(p', S')\Gamma^{\mu\nu}_{\text{Ji},a}(P,\Delta)u(p,S)$$

$$\Gamma^{\mu\nu}_{\text{Ji},a}(P,\Delta) = \Gamma^{\mu\nu}_{\text{Bel},a}(P,\Delta) + \frac{P^{[\mu}\gamma^{\nu]}}{2}D_a$$

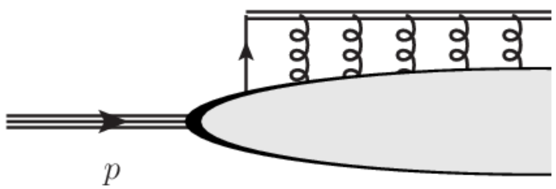
$$D_G = 0$$

Angular momentum relations ⚠ **Kinetic version !**

$$J^a_z = \tfrac{1}{2}(A_a + B_a) \qquad L^q_z = \tfrac{1}{2}(A_q + B_q + D_q) \qquad S^q_z = -\tfrac{1}{2}D_q$$

[Ji (1997)]
[Shore, White (2000)]

Matrix elements



$n \qquad n^2 = 0 \qquad N = \frac{M^2\, n}{P\cdot n}$

Light-front EMT $\qquad N\cdot A_{\text{phys}} = 0 \qquad X_a \equiv X_a(\xi,\Delta^2;\eta) \in \mathbb{C} \qquad a = 1,\cdots,5$

$$\langle p',S'|T^{\mu\nu}_a(0)|p,S\rangle = \overline{u}(p',S')\Gamma^{\mu\nu}_a(P,\Delta,N;\eta)u(p,S)$$

$$\begin{aligned}
\Gamma^{\mu\nu}_a(P,\Delta,N;\eta) &= M g^{\mu\nu} A^a_1 + \frac{P^\mu P^\nu}{M}\,A^a_2 + \frac{\Delta^\mu\Delta^\nu}{M}\,A^a_3 + \frac{P^\mu i\sigma^{\nu\Delta}}{2M}\,A^a_4 + \frac{P^\nu i\sigma^{\mu\Delta}}{2M}\,A^a_5\\
&+ \frac{N^\mu N^\nu}{M}\,B^a_1 + \frac{P^\mu N^\nu}{M}\,B^a_2 + \frac{P^\nu N^\mu}{M}\,B^a_3 + \frac{N^\mu i\sigma^{\nu\Delta}}{2M}\,B^a_4 + \frac{N^\nu i\sigma^{\mu\Delta}}{2M}\,B^a_5 + \frac{\Delta^\mu i\sigma^{\nu N}}{2M}\,B^a_6 + \frac{\Delta^\nu i\sigma^{\mu N}}{2M}\,B^a_7\\
&+ \left[M g^{\mu\nu} B^a_8 + \frac{P^\mu P^\nu}{M}\,B^a_9 + \frac{\Delta^\mu\Delta^\nu}{M}\,B^a_{10} + \frac{N^\mu N^\nu}{M}\,B^a_{11} + \frac{P^\mu N^\nu}{M}\,B^a_{12} + \frac{P^\nu N^\mu}{M}\,B^a_{13}\right]\frac{i\sigma^{N\Delta}}{2M^2}\\
&+ \frac{P^\mu\Delta^\nu}{M}\,B^a_{14} + \frac{P^\nu\Delta^\mu}{M}\,B^a_{15} + \frac{\Delta^\mu N^\nu}{M}\,B^a_{16} + \frac{\Delta^\nu N^\mu}{M}\,B^a_{17} + \frac{M}{2}\,i\sigma^{\mu\nu}B^a_{18} + \frac{\Delta^\nu i\sigma^{\mu\Delta}}{2M}\,B^a_{19}\\
&+ \frac{P^\mu i\sigma^{\nu N}}{2M}\,B^a_{20} + \frac{P^\nu i\sigma^{\mu N}}{2M}\,B^a_{21} + \frac{N^\mu i\sigma^{\nu N}}{2M}\,B^a_{22} + \frac{N^\nu i\sigma^{\mu N}}{2M}\,B^a_{23}\\
&+ \left[\frac{P^\mu\Delta^\nu}{M}\,B^a_{24} + \frac{P^\nu\Delta^\mu}{M}\,B^a_{25} + \frac{\Delta^\mu N^\nu}{M}\,B^a_{26} + \frac{\Delta^\nu N^\mu}{M}\,B^a_{27}\right]\frac{i\sigma^{N\Delta}}{2M^2}
\end{aligned}$$

Angular momentum relations $\qquad$ **Kinetic and canonical versions !**

$$J^a_z = \tfrac{1}{2}\Re\mathrm{e}[A^a_4 + A^a_5] \qquad L^a_z = \Re\mathrm{e}[A^a_4] \qquad S^q_z = -\tfrac{1}{2}\Re\mathrm{e}[A^1_4 - A^1_5] \qquad S^G_z = -\Re\mathrm{e}[A^5_4]$$

[C.L. (2015)]

Link with vector GPDs

$$\int \mathrm{d}x\, x\, F^{[\gamma^\mu]}_{S'S}(P,x,\Delta,N) = \frac{1}{M^2}\,\langle p',S'|T_1^{\mu N}(0)|p,S\rangle$$

Twist 2

$$\int \mathrm{d}x\, x\, H^q(x,\xi,t) = A_q(t) + 4\xi^2\, C_q(t)$$

$$\int \mathrm{d}x\, x\, E^q(x,\xi,t) = B_q(t) - 4\xi^2\, C_q(t)$$

Twist 3

$$\int \mathrm{d}x\, x\, H^q_{2T}(x,\xi,t) = 0$$

$$\int \mathrm{d}x\, x\, E^q_{2T}(x,\xi,t) = 0$$

$$\int \mathrm{d}x\, x\, \tilde{H}^q_{2T}(x,\xi,t) = -2\xi\, C_q(t)$$

$$\int \mathrm{d}x\, x\, \tilde{E}^q_{2T}(x,\xi,t) = -\tfrac{1}{2}[A_q(t) + B_q(t) - D_q(t)]$$

Twist 4

$$\int \mathrm{d}x\, x\, H^q_3(x,\xi,t) = \tfrac{1}{2}A_q(t) + \bar{C}_q(t) - 2\xi^2\,\tfrac{P^2}{M^2}\,C_q(t) + \tfrac{t}{8M^2}\,[B_q(t) - 8C_q(t) - D_q(t)]$$

$$\int \mathrm{d}x\, x\, [H^q_3(x,\xi,t) + E^q_3(x,\xi,t)] = \tfrac{P^2}{2M^2}\, D_q(t)$$

Similar to Ji's sum rule !

[C.L. (2015)]

Link with axial-vector FFs

$$-\frac{i}{2}\,\epsilon^{\mu\nu\Delta\alpha}\int \mathrm{d}x\, F^{[\gamma_\alpha\gamma_5]}_{S'S}(P,x,\Delta,N) = \langle p',S'|T_1^{[\mu\nu]}(0)|p,S\rangle$$

$$\overline{\psi}\gamma^{[\mu}i\overleftrightarrow{D}^{\nu]}\psi = -\partial_\alpha(\epsilon^{\alpha\mu\nu\rho}\overline{\psi}\gamma_\rho\gamma_5\psi)$$

$$G_A^q(t) = -D_q(t)$$

Consistent with spin sum rule

$$J_z^a = \tfrac{1}{2}(A_a + B_a) \qquad L_z^q = \tfrac{1}{2}(A_q + B_q + D_q) \qquad S_z^q = -\tfrac{1}{2}\,D_q$$

[Shore, White (2000)]

Link with vector TMDs

$$\int dx\, d^2k_T\, x\, \Phi^{[\gamma^\mu]}_{S'S}(P,x,k_T,N;\eta) = \frac{1}{M^2}\langle p,S'|T_1^{\mu N}(0)|p,S\rangle$$

Twist 2 $$\int dx\, d^2k_T\, x\, f_1^q(x,k_T^2) = A_q(0)$$

Twist 3 $$\int dx\, d^2k_T\, x\, f_T^q(x,k_T^2) = 0$$

Twist 4 $$\int dx\, d^2k_T\, x\, f_3^q(x,k_T^2) = \frac{1}{2}A_q(0) + \bar{C}_q(0)$$

$$\int dx\, d^2k_T\, k_T^\alpha\, \Phi^{[\gamma^\mu]}_{S'S}(P,x,k_T,N;\eta) = \delta^\alpha_{T\nu}\langle p,S'|T_{1+3}^{\mu\nu}(0)|p,S\rangle$$

Twist 2 $$\int dx\, d^2k_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp q}(x,k_T^2;\eta) = -\frac{1}{2}\Im\mathrm{m}\left[B^3_{18}(0,0) - B^3_{20}(0,0)\right]$$

Twist 3 $$\int dx\, d^2k_T\, \frac{k_T^2}{2M^2}\, f^{\perp q}(x,k_T^2) = \bar{C}_q(0) + \Re\mathrm{e}[A^3_1(0,0)]$$

$$\int dx\, d^2k_T\, \frac{k_T^2}{2M^2}\, f_L^{\perp q}(x,k_T^2;\eta) = \frac{1}{2}\Im\mathrm{m}[B^3_{18}(0,0)]$$

$$X^G = -X^q$$

Twist 4 $$\int dx\, d^2k_T\, \frac{k_T^2}{2M^2}\, f_{3T}^{\perp q}(x,k_T^2;\eta) = \frac{1}{4}\Im\mathrm{m}\left[B^3_{18}(0,0) + B^3_{20}(0,0) + 2B^3_{22}(0,0)\right]$$

[C.L. (2015)]

Sum rules

Transverse momentum conservation

$$\sum_{a=q,G} \int \mathrm{d}x\, \mathrm{d}^2 k_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp a}(x, k_T^2) = 0$$

[Burkardt (2004)]

Vanishing net transverse momentum flux

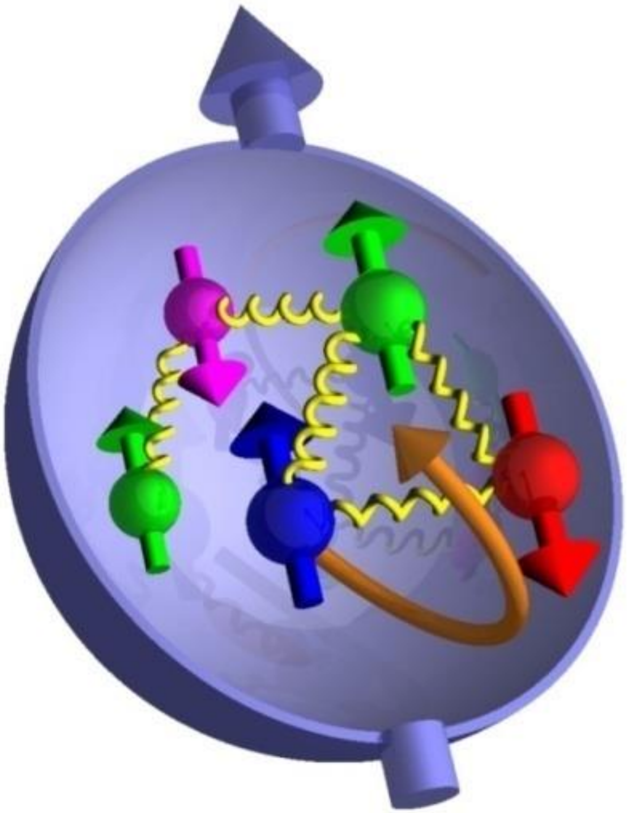
$$\sum_{a=q,G} \int \mathrm{d}x\, \mathrm{d}^2 k_T\, \frac{k_T^2}{2M^2}\, f^{\perp a}(x, k_T^2) = 0$$

$$\sum_{a=q,G} \int \mathrm{d}x\, \mathrm{d}^2 k_T\, \frac{k_T^2}{2M^2}\, f_L^{\perp a}(x, k_T^2) = 0$$

[C.L. (2015)]

$$\sum_{a=q,G} \int \mathrm{d}x\, \mathrm{d}^2 k_T\, \frac{k_T^2}{2M^2}\, f_{3T}^{\perp a}(x, k_T^2) = 0$$

Conclusions



- Definition of EMT density is not unique
- General parametrization involves 32 scalar functions
- 9 of them are directly related to 2-parton GPDs and TMDs
- Simpler derivation of Burkardt sum rule + new sum rules for higher-twist TMDs

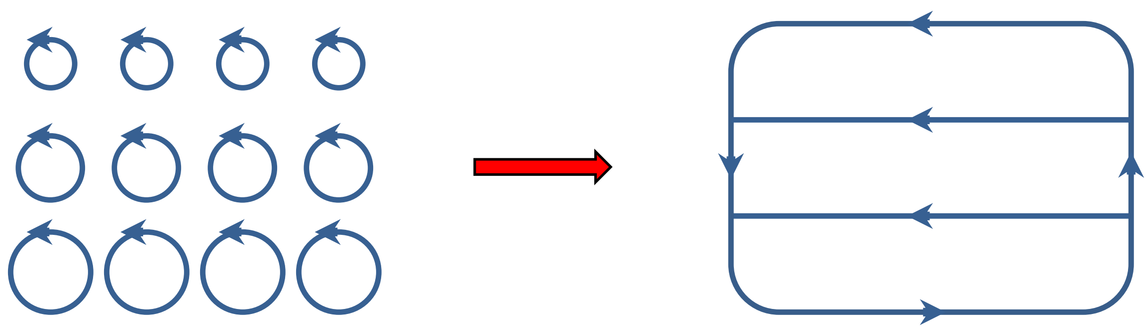
Backup slides

Energy-momentum tensor

In presence of spin density $T^{0i} \neq T^{i0}$

Belinfante « improvement »

$$T_B^{\mu\nu} \equiv T^{\mu\nu} + \tfrac{1}{2}\partial_\lambda[S^{\lambda\mu\nu} + S^{\mu\nu\lambda} + S^{\nu\mu\lambda}] \\ = T_B^{\nu\mu}$$



Spin density gradient **Four-momentum circulation**

In rest frame

$$M = \int \mathrm{d}^3r\, T_B^{00}(\vec{r})$$

$$J^i = \int \mathrm{d}^3r\, \epsilon^{ijk} r^j T_B^{0k}(\vec{r})$$

No « spin » contribution !

Link with GPDs

« Trick »

Local — **Non-local**

$$\overline{\psi}\gamma^\mu iD^+\psi = \int\frac{\mathrm{d}z^-}{2\pi}\,2\pi\,\delta(z^-)\,\overline{\psi}(-\tfrac{z^-}{2})\gamma^\mu\mathcal{W}[-\tfrac{z^-}{2},0]i\overleftrightarrow{D}^+\mathcal{W}[0,\tfrac{z^-}{2}]\psi(\tfrac{z^-}{2})$$

$$=\int\frac{\mathrm{d}z^-}{2\pi}\int\mathrm{d}x P^+\,e^{ixP^+z^-}\,\overline{\psi}(-\tfrac{z^-}{2})\gamma^\mu\mathcal{W}[-\tfrac{z^-}{2},0]i\overleftrightarrow{D}^+\mathcal{W}[0,\tfrac{z^-}{2}]\psi(\tfrac{z^-}{2})$$

$$=P^+\int\mathrm{d}x\int\frac{\mathrm{d}z^-}{2\pi}\,e^{ixP^+z^-}\,i\partial_z^+\left[\overline{\psi}(-\tfrac{z^-}{2})\gamma^\mu\mathcal{W}[-\tfrac{z^-}{2},\tfrac{z^-}{2}]\psi(\tfrac{z^-}{2})\right]$$

$$=2(P^+)^2\int\mathrm{d}x\,x\,\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}\,e^{ixP^+z^-}\,\overline{\psi}(-\tfrac{z^-}{2})\gamma^\mu\mathcal{W}[-\tfrac{z^-}{2},\tfrac{z^-}{2}]\psi(\tfrac{z^-}{2})$$

GPD operator

Twist-2 $\mu=+$

$$A(t)+B(t)=\int\mathrm{d}x\,x\,[H(x,\xi,t)+E(x,\xi,t)]$$

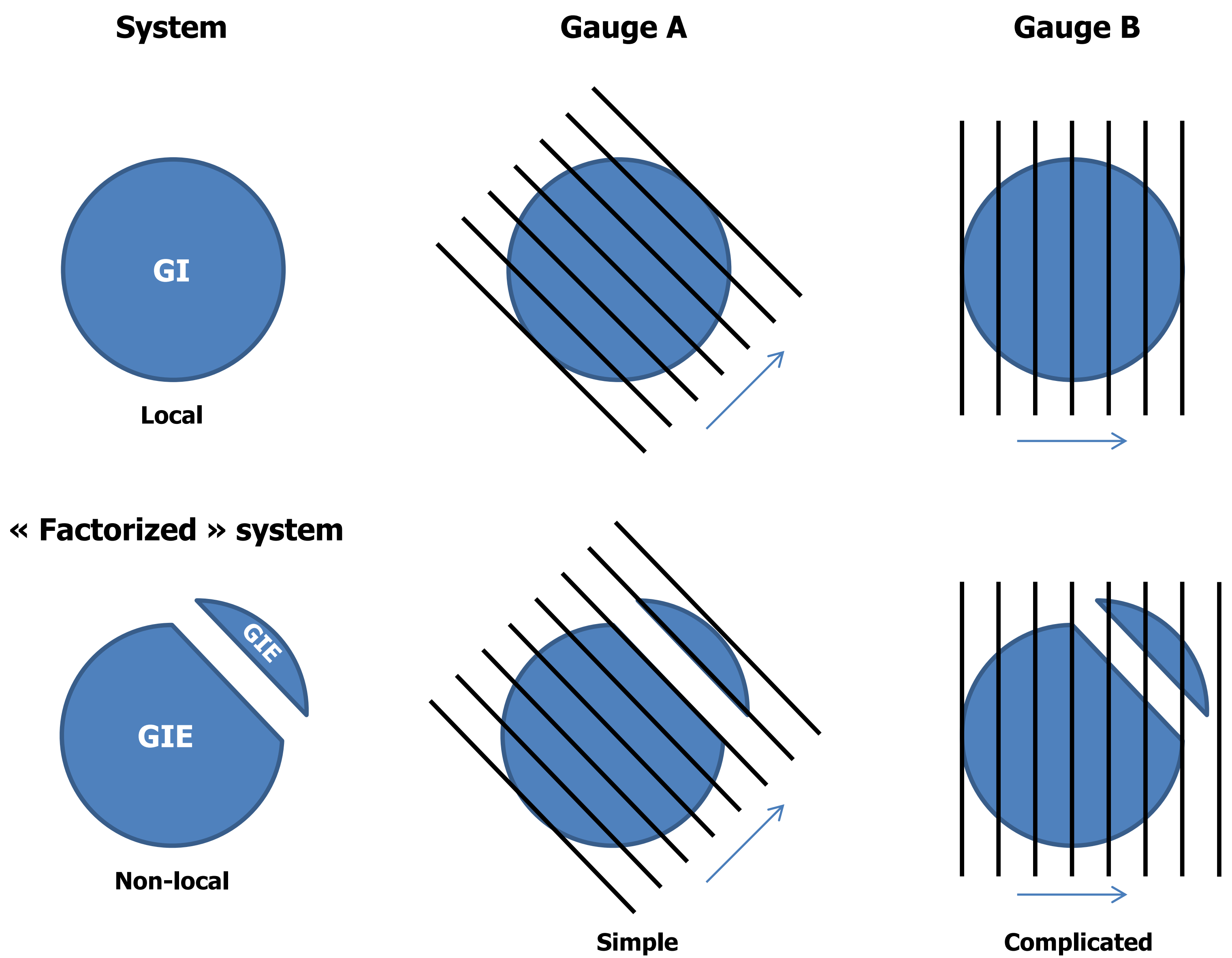
[Ji (1997)]

Twist-3 $\mu=\perp$

$$A(t)+B(t)+D(t)=-2\int\mathrm{d}x\,x\,G_2(x,\xi,t)$$

[Penttinen *et al.* (2000)]
[Kiptily, Polyakov (2004)]
[Hatta, Yoshida (2012)]

Gauge invariance *vs* locality



[C.L. (2013-14)]

Bonus : AM correlations

Wigner distribution

ρ_X	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

[C.L., Pasquini (in preparation)]

GPDs

	U	L	T
U	H		$\mathcal{E}_T$
L		$\widetilde{H}$	
T	E		$H_T, \widetilde{H}_T$

TMDs

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$