

PRECISION MEASUREMENT OF $\sin^2 \theta_w$ AT MESA

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H. Spiesberger

PRISMA Cluster of Excellence,
Institut für Physik,

Johannes Gutenberg-Universität Mainz



Electron proton / ion scattering at low and high Q^2

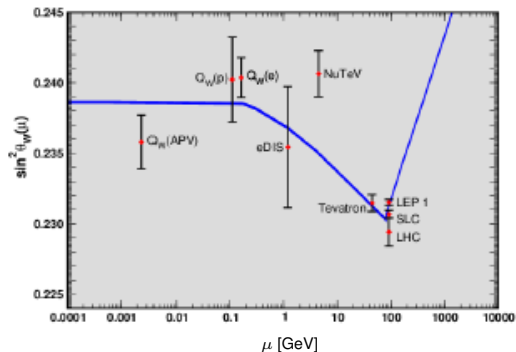
Goals:

- High-precision measurements of the nucleon structure
- Search for new physics
- Electroweak physics:
 - Test of the standard model, and after LHC discoveries
 - Test of the standard model extended with new physics
- Key parameter of the standard model: $\sin^2 \theta_w$

This talk:

- The MESA project at Mainz University
- Theory for precision measurements of $\sin^2 \theta_w$ in low-energy elastic ep scattering

The Running Weak Mixing Angle: Present Status



PDG 2014

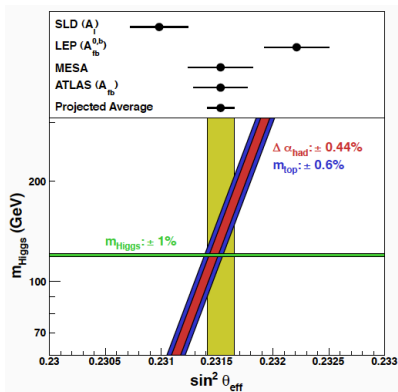
- $Q_W(APV)$: atomic parity violation (Cs)
- $Q_W(p)$: Q_{Weak}
- $Q_W(e)$: Moller scattering
- NuTeV: Neutrino scattering (re-analysis needed)

Z-pole measurements:

- LEP1 and SLC
- Tevatron
- LHC: CMS and ATLAS

- Most precise single measurements disagree (3σ)
- very different implications for new physics

Standard Model Relation: Higgs Boson Mass versus $\sin^2 \theta_W$





Combination of precision measurements at the Z-pole

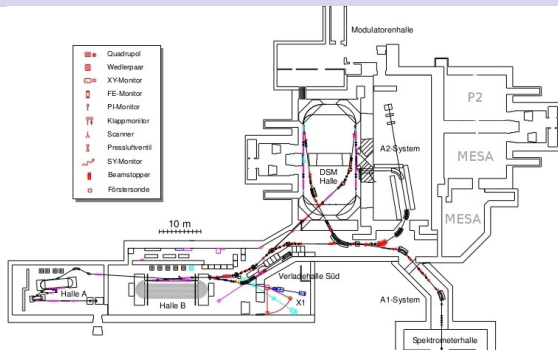
→ M_{Higgs} - $\sin^2 \hat{\theta}_W(\mu)$ SM relation (red-blue band)

Precision measurement of $\sin^2 \hat{\theta}_W(\mu)$ has provided indirect evidence for the allowed range of M_{Higgs}

Combination of measurements provide strong tests of the SM,
... and maybe evidence for new physics

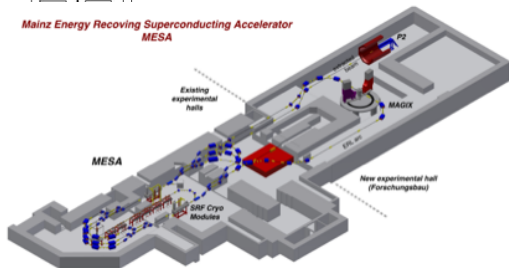
- **MESA =**
Mainz Energy-recovering Superconducting Accelerator
A small superconducting accelerator for particle and nuclear physics
- Funded by PRISMA - Cluster of Excellence and Collaborative Research Center 1044
German Science Foundation (DFG)
 PRISMA
 SFB 1044
THE SYNERGIC POWER
OF THE STANDARD MODEL
- **P2** (Project Precision 2):
Parity-violating electron proton scattering
- Other Projects: Search for a dark photon,
Nuclear physics program
- Commissioning planned for 2019
- cf. Qweak at LJAB

MESA Layout



MAMI (Mainzer Mikrotron)
and MESA

Mainz Energy Recovering Superconducting Accelerator
MESA



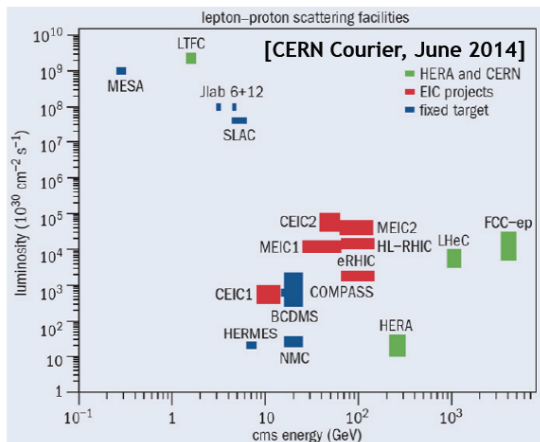
MESA Parameters

Parameter	P2 (Mainz)
Beam energy	155 MeV
Central scattering angle θ_e	35°
Detector acceptance for θ_e	25° to 45°
Azimuthal detector acceptance ϕ_e	2π
Central Q^2	$\simeq 0.0071 \text{ (GeV}/c)^2$
Averaged Q^2	$\simeq 0.0045 \text{ (GeV}/c)^2$
Polarization	$(85 \pm 0.5) \%$
Beam current	$150 \mu\text{A}$
ℓ H ₂ Target length	60 cm
A_{exp}	-28.35 ppb
$\Delta A(G_A)$ (0.4 %)	± 0.11 ppb
$\Delta A(\gamma Z \text{ box})$ (0.3 %)	± 0.09 ppb
ΔA_{stat} (1.3 %)	± 0.38 ppb
ΔA_{sys} (0.6 %)	± 0.17 ppb
ΔA_{tot} (1.5 %)	± 0.44 ppb
$\sin^2 \theta_W$	0.238
$\Delta \sin^2 \theta_W$	$3.1 \cdot 10^{-4}$
$\Delta \sin^2 \theta_W / \sin^2 \theta_W$	0.13 %

LHeC / FCC-he Context

(10,000 h)
data taking →

$$\int \mathcal{L} dt \simeq 8.6 \text{ ab}^{-1}$$



P. Newman

Measure the (tiny) difference between cross sections for electrons with positive and negative helicity to filter out the weak interaction

$$A_{LR} = \frac{\sigma(e_{\downarrow}) - \sigma(e_{\uparrow})}{\sigma(e_{\downarrow}) + \sigma(e_{\uparrow})} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} (Q_W(\mathcal{N}) - F(Q^2))$$

Weak charge of the proton:

$$Q_W(p) = 1 - 4 \sin^2 \theta_W$$

$$\frac{\Delta \sin^2 \theta_W}{\sin^2 \theta_W} = \frac{1 - 4 \sin^2 \theta_W}{4 \sin^2 \theta_W} \frac{\Delta Q_W(p)}{Q_W(p)}$$

1.5% precision for $Q_W(p)$ corresponds to 0.13% precision for $\sin^2 \theta_W$

Measurement errors from: statistics, polarization ($A_{exp} = P_e A_{LR}$), systematic effects and required hadronic physics: form factors

$$F(Q^2) = F_{\text{EMFF}}(Q^2) + F_{\text{Axial}}(Q^2) + F_{\text{Strangeness}}(Q^2)$$

$$A_{LR} = A_{Q_{\text{weak}}} + A_{\text{EMFF}} + A_{\text{Axial}} + A_{\text{Strangeness}}$$

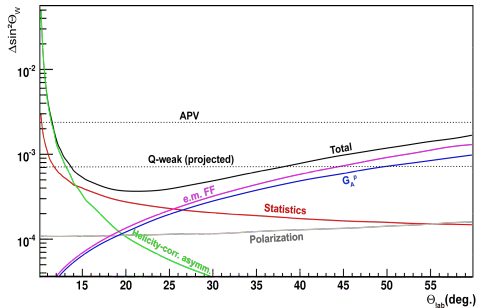
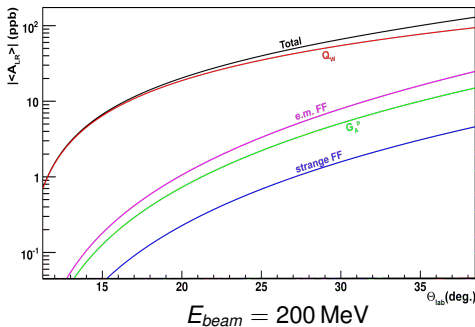
$$F_{\text{EMFF}}(Q^2) = -\frac{\epsilon G_E^p G_E^n + \tau G_M^p G_M^n}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2},$$

$$F_{\text{Axial}}(Q^2) = -\frac{(1 - 4 \sin^2 \theta_W) \sqrt{1 - \epsilon^2} \sqrt{\tau(1 + \tau)} G_M^p G_A^p}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2},$$

$$F_{\text{Strangeness}}(Q^2) = -\frac{\epsilon G_E^p G_E^s + \tau G_M^p G_M^s}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2} - \frac{\epsilon G_E^p G_E^{ud} + \tau G_M^p G_M^{ud}}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2},$$

$$\epsilon = [1 + 2(1 + \tau) \tan^2(\theta/2)]^{-1}, \quad \tau = Q^2/4m_p^2$$

Contributions to the PV Asymmetry and Expected Error



→ Optimal measurement for $E = 155$ MeV, $\theta_e = 35^\circ \pm 10^\circ$

$$\langle Q \rangle = 0.067 \text{ GeV}$$

SM prediction: $A_{LR} = -2.8 \times 10^{-8}$, precision goal: 1.5%

$$\Delta \sin^2 \theta_w = \pm 0.00031, \text{ i.e. } 0.13 \%$$

Polarization asymmetry including higher-order corrections:

$$A_{LR} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \left(Q_W(\mathcal{N})(1 + \delta_1) - \tilde{F}(Q^2) \right)$$

$$Q_W(\mathcal{N})(1 + \delta_1) = (\rho_{\text{NC}} + \Delta_e) (1 - 4\kappa \sin^2 \theta_W + \Delta'_e) + \delta_{\text{Box}}$$

Universal corrections: Δ_e and κ from loop diagrams:
scale-dependence ($\mu \rightarrow Q$) \rightarrow

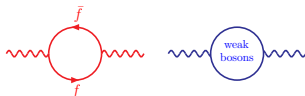
$$\sin^2 \theta_{\text{eff}}(\mu^2) = \kappa(\mu^2) \sin^2 \theta_W$$

and scheme-dependent

δ_{Box} from box graphs for WW , ZZ , $Z\gamma$ exchange,
non-universal vertex correction Δ'_e , QED corrections
Match complete 1-loop corrections to definition of $\sin^2 \theta_W$

Including quantum corrections, universal contributions at one-loop:

- Self energy diagrams of the exchanged boson (γ and Z)



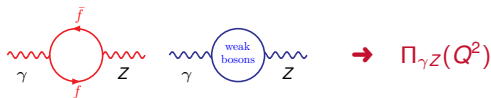
at large Q^2 : $\propto \log \frac{Q^2}{m_f^2}$ small, $O(1\%)$

- Photon self energy = vacuum polarization, absorbed in the running fine structure constant:

$$\alpha \rightarrow \alpha(Q^2) = \frac{\alpha}{1 - \Pi_\gamma(Q^2)}$$

One-loop Corrections: Scale-Dependent $\sin^2 \theta_W$

- Photon-Z mixing



can be absorbed into **effective**, running, **scale-dependent weak mixing angle**

Definitions of the weak mixing angle, $\kappa(Q^2) \sin^2 \theta_W$:

- On-shell definition: $\cos \theta_W = \frac{m_W}{m_Z}$ (fixed)
- $\sin^2 \theta_{\text{eff}}(Q^2)$ absorbs $\Pi_{\gamma Z}(Q^2)$, usually together with parts of vertex corrections (e.g. Czarnecki, Marciano)
- **$\overline{\text{MS}}$ scheme:** $\sin \hat{\theta}_W(\mu)$ (via $\tan \hat{\theta}_W(\mu) = g_1/g_2$ and RGE)
less sensitive to m_{top} , suited for comparisons with extensions of the SM

→ Relation

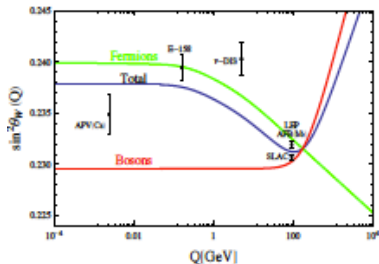
$$\sin^2 \hat{\theta}_W(\mu = M_Z) = \left(1 + \frac{\rho_t}{\tan^2 \theta_W} + \dots \right) \sin^2 \theta_W$$

with $\rho_t = 3G_\mu m_{\text{top}}^2 / 8\sqrt{2}\pi^2 = 0.00939 (m_{\text{top}}/173 \text{ GeV})^2$

→ Additional uncertainty: $\Delta \sin^2 \hat{\theta}_W \simeq \pm 0.0006$ for a 1% error on m_{top}

$\sin^2 \hat{\theta}_W(Q)$: Uncertainties

- Use running $\sin^2 \hat{\theta}_W(Q)$ to compare different measurements
- Match definition of $\sin^2 \hat{\theta}_W(Q)$ with the complete 1-loop corrections



Czarnecki-Marciano, Jegerlehner

- Scheme dependence, partly compensated by $\delta_{non-universal}$
- Include higher orders
→ 2-loop corrections
- Parameter dependence? m_t, m_H, \dots
- Hadronic contribution!
need data and assumptions ...

Prescriptions are known, with small uncertainties ...

Higher-Order Corrections: Hadronic Contributions

$\Pi^{\gamma\gamma}$ and $\Pi^{\gamma Z}$ are sensitive to low-scale hadronic physics

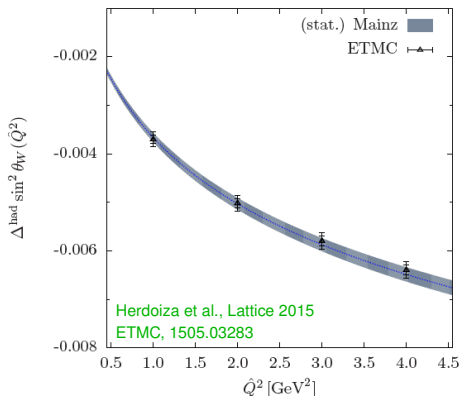
→ Use dispersion relation, e.g.

$$\Delta\alpha(q^2) = \frac{q^2}{12\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} \frac{R^{\gamma\gamma}(s)}{s - q^2} \quad \text{with} \quad R^{\gamma\gamma}(s) = \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{4\pi\alpha^2/3s}$$

→ Similar approach for $\Pi^{\gamma Z}$ requires data for $\sigma_{\text{tot}}(\nu\bar{\nu} \rightarrow \text{hadrons})$ or use flavor-separated e^+e^- data, **isospin symmetry** and **OZI-rule**

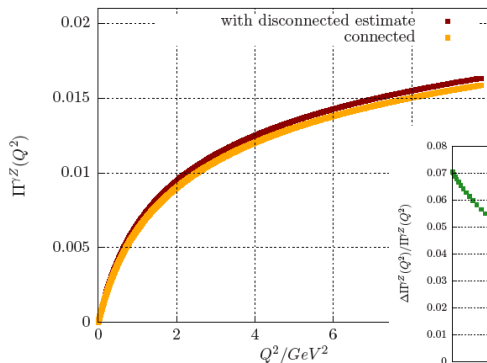
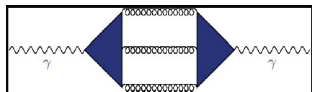
→ Use lattice techniques

First results available,
errors start to be competitive

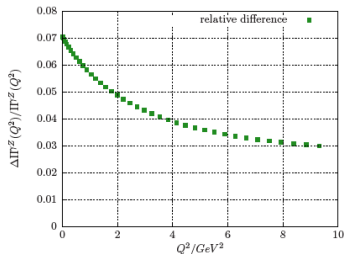


Higher-Order Corrections: Hadronic Contributions

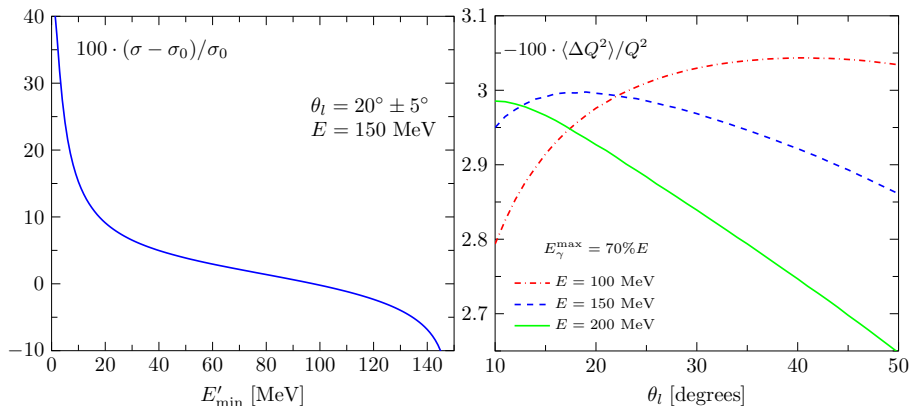
- OZI-rule violating contributions?
- Lattice QCD
- still with large statistical errors
- but obtain an upper limit



Gülpers et al., Lattice 2015

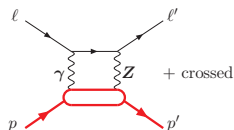


Higher-Order Corrections: QED



- Straightforward to calculate, but need flexible MC simulation
- QED does not violate parity symmetry, but real photon emission leads to a shift of Q^2

Higher-Order Corrections: Box Graphs



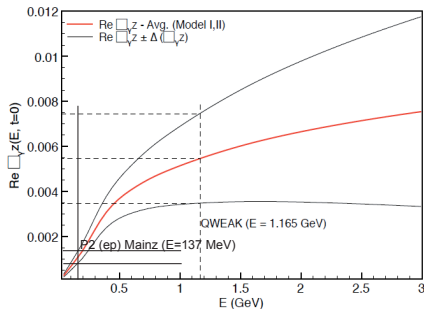
γZ box graphs

Sensitivity to hadronic physics at low $Q^2 \rightarrow$ an important source of error

3 groups with independent analyses agree in size, but disagree on errors

Hall et al.; Carlson and Rislow;
Gorchtein et al.

Gorchtein, Horowitz, Ramsey-Musolf

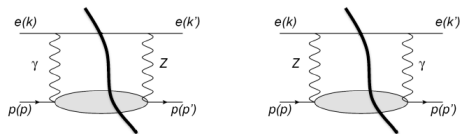


Advantage at MESA/P2: low energy

$$\Delta A_{LR}^{box} / A_{LR} = \pm 0.4 \%$$

More work needed to reduce the error ...

Box Graphs: Dispersion Relations



Optical theorem and dispersion relations:

$$\text{Im}\square_{\gamma Z}(E) = \frac{\alpha}{(s - M_Z^2)^2} \int_{W_\pi^2}^s dW^2 \int_0^{Q_{\max}^2} dQ^2 \frac{M_Z^2}{Q^2 + M_Z^2} \\ \times \left\{ F_1^{\gamma Z}(x, Q^2) + AF_2^{\gamma Z}(x, Q^2) + \frac{g_V^e}{g_A} BF_3^{\gamma Z} \right\}$$

Separated into vector and axial-vector parts of the proton current:

$$\text{Re}\square_{\gamma Z}^V(E) = \frac{2E}{\pi} \int_{\nu_\pi}^{\infty} \frac{dE'}{E'^2 - E^2} \text{Im}\square_{\gamma Z}^V(E')$$

$$\text{Re}\square_{\gamma Z}^A(E) = \frac{2}{\pi} \int_{\nu_\pi}^{\infty} \frac{E' dE'}{E'^2 - E^2} \text{Im}\square_{\gamma Z}^A(E')$$

Box Graphs: Sensitivity to Hadronic Input

Different calculations due to different assumptions about the structure function input (regions, parametrizations)

Q^2_{\max} \ W_{\max}	2 GeV	4 GeV	∞
∞	1%	1%	3%
2 GeV ²	3%	2%	1%
1 GeV ²	76%	10%	2%

Contributions to $\text{Re}\square_{\gamma Z}^V$
 $\text{Re}\square_{\gamma Z}^V = 0.00104$

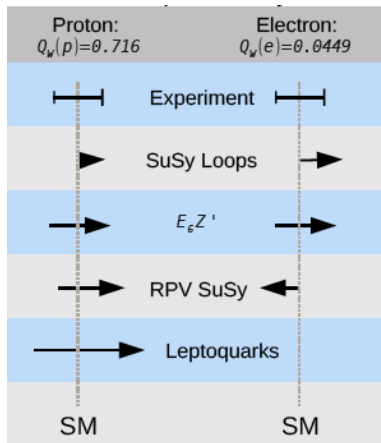
Q^2_{\max} \ W_{\max}	2 GeV	4 GeV	∞
∞	0.5%	0.5%	4.6%
2 GeV ²	6.7%	6.7%	0.5%
1 GeV ²	60%	20%	0.5%

Contributions to the uncertainty of $\text{Re}\square_{\gamma Z}^V$
 $\Delta\text{Re}\square_{\gamma Z}^V = 0.00015$

$(Q_W(p) = 0.0712 \pm 0.0007,$
 $E = 150 \text{ MeV}, \theta_e = 0)$

M. Gorshteyn

Beyond the Standard Model: New Physics in $Q_W(p)$



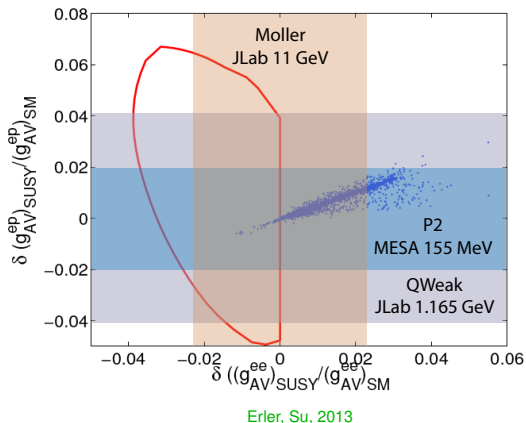
Searches based on

- model-dependent analyses
- effective quark couplings
- contact interactions

Characteristic shifts of Q_W predicted by extensions of the Standard Model

Complementarity between elastic ep ($Q_W(p)$) and Moller ($Q_W(e)$) scattering

Supersymmetric Models and the Weak Charge



Example: supersymmetric models with and without R -parity violation

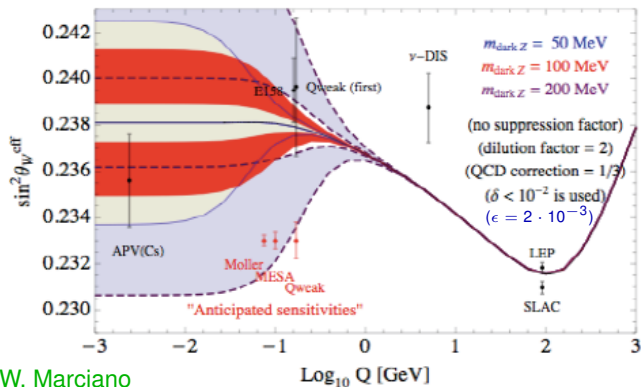
Also precision measurements at low-energy are sensitive to TeV-scale physics

Perspective will shift after LHeC discoveries

Dark Photon and $\sin^2 \hat{\theta}_W(Q)$

Dark matter and extra $U(1)$ symmetry with kinetic mixing (B. Holdom)

- Dark photon with very small mass: $m_D = O(50)$ MeV
- Free mixing parameter $\epsilon = O(10^{-3})$, possibly generated by loop effects
- Model with **parity violation**, like ordinary Z, but suppressed by ϵZ
- Combine kinetic and mass mixing → shift $\Delta \sin^2 \theta_w(Q^2) \simeq 0.42 \epsilon \frac{\delta m_Z^2}{Q^2 + m_D^2}$



W. Marciano

$$\mathcal{L}_{\text{eq}} = \left(\frac{G_F}{\sqrt{S}} g_{VA}^{\text{eq}}(\text{SM}) + \frac{g^2}{\Lambda^2} \right) \bar{e} \gamma_\mu e \bar{q} \gamma^\mu \gamma_5 q$$

Convention: $g^2 = 4\pi$

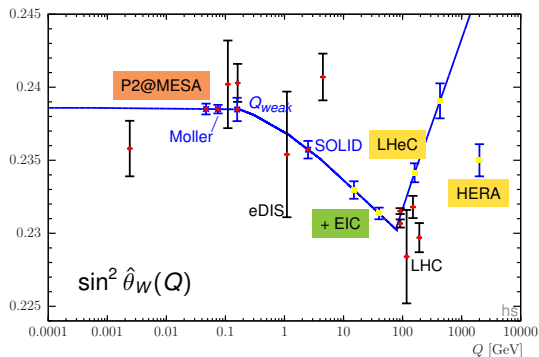
MESA-P2 probes
 Λ up to $\simeq 50$ TeV

comparable with
 LHC (300 fb^{-1})

	precision	$\Delta \sin^2 \bar{\theta}_{\text{W}}(0)$	Λ_{new} (expected)
APV Cs	0.58 %	0.0019	32.3 TeV
E158	14 %	0.0013	17.0 TeV
Qweak I	19 %	0.0030	17.0 TeV
Qweak final	4.5 %	0.0008	33 TeV
PVDIS	4.5 %	0.0050	7.6 TeV
SoLID	0.6 %	0.00057	22 TeV
MOLLER	2.3 %	0.00026	39 TeV
P2	2.0 %	0.00036	49 TeV
PVES ^{12}C	0.3 %	0.0007	49 TeV

J. Erler

Scale Dependence of $\sin^2 \hat{\theta}_W(Q)$: Present and Future



Future additions to the PDG

Expected from low-energy:

- Mainz: P2@MESA
- Moller at JLAB
- Q_{weak} at JLAB
- SOLID at JLAB

HERA estimate
combined analysis

LHeC: from A_{LR} and σ_{NC}/σ_{CC} ,
energy range 10 - 1000 GeV

EIC: see K.Kumar's talk

- Present measurements of the weak mixing angle need improvement
 - MESA, a new high-precision measurement of $\sin^2 \theta_w$ from parity-violating electron scattering
 - Combined with possible future measurements at low energies, EIC, LHeC:
- Precision measurements
will cover a wide range of energy scales
- Still interesting theory work to be completed ...