JIMWLK: From concepts to observables

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POETIC 2015

Outline

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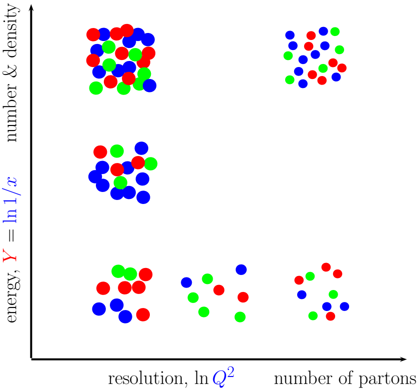
Large energies mean large densities



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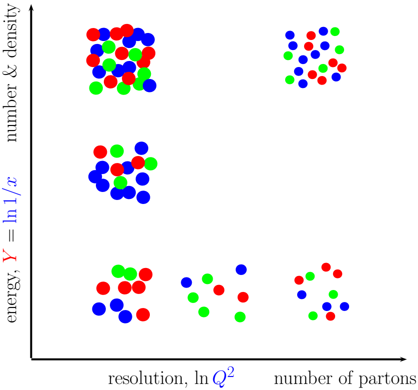

- density ➡ nonlinear effects
- finite correlation length R_s

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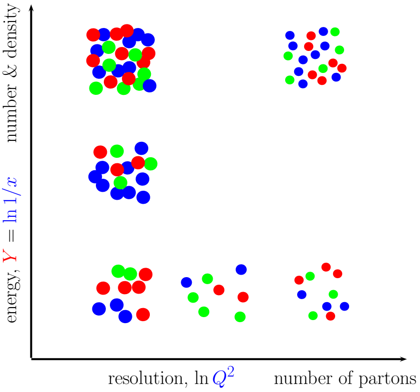

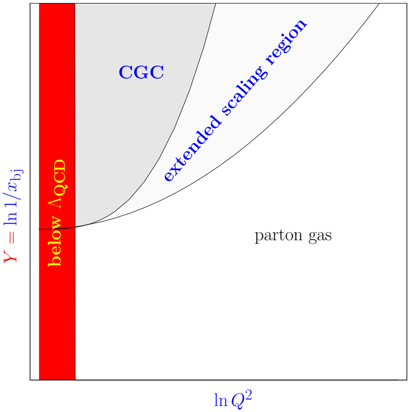

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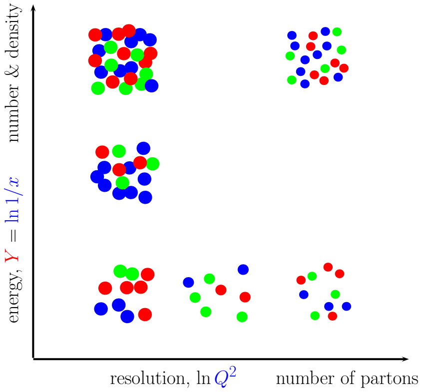

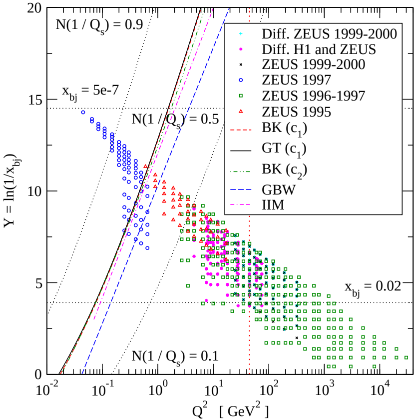

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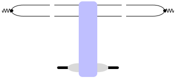
- Real world example: HERA ep

Outline

Total cross section (zeroeth order in $\alpha^m(\alpha_s \ln(1/x))^n$)

$\sigma_{\text{DIS}}(Y, Q^2) = 2\text{Im}$ 

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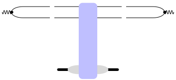
$\sigma_{\rm DIS}(Y, Q^2) = 2\,\mathrm{Im}$  $= \int d^2 r \;\; |\psi^2|(r^2 Q^2)$

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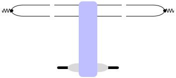
$\sigma_{\mathrm{DIS}}(Y, Q^2) = 2\mathrm{Im}$  $= \int d^2r \; |\psi^2|(r^2 Q^2) \; 2\int d^2b \, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})}{N_c} \rangle (Y)$

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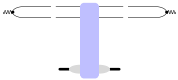
$\sigma_{\rm DIS}(Y, Q^2) = 2\,\mathrm{Im}$  $= \int d^2r\ |\psi^2|(r^2Q^2)\ 2\int d^2b\ \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)}{N_c}\rangle(Y)$

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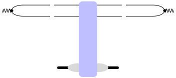
$\sigma_{\text{DIS}}(Y, Q^2) = 2\text{Im}$  $= \int d^2 r \; |\psi^2|(r^2 Q^2) \; 2 \int d^2 b \, \langle \frac{\text{tr}(1 - U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)}{N_c} \rangle (Y)$

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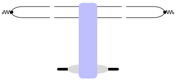
$$\sigma_{\text{DIS}}(Y, Q^2) = 2\text{Im}$$  $$= \int d^2 r \, |\psi^2|(r^2 Q^2) \; 2\int d^2 b \, \langle \frac{\text{tr}(1 - U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)}{N_c} \rangle(Y)$$

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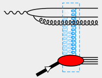

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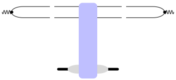
$$\sigma_{\rm DIS}(Y,Q^2) = 2\text{Im}$$  $$= \int d^2r\ |\psi^2|(r^2Q^2)\ 2\int d^2b\ \langle\frac{\text{tr}(1-U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})}{N_c}\rangle(Y)$$

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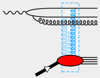
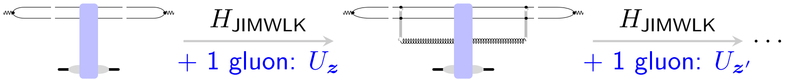


- Bookkeeping device: $\langle\ldots\rangle(Y) = \int \hat{D}[U]\ \ldots \hat{Z}_Y[U]$

The JIMWLK evolution equation



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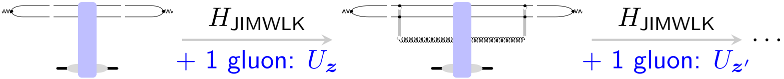
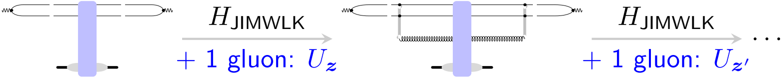


- $$\frac{d}{dY} Z_Y[U] = -H_{\text{JIMWLK}}[U] \;\; Z_Y[U]$$

Heribert Weigert **Nucl. Phys. A703**, 2002, 823

explicit form

The JIMWLK evolution equation



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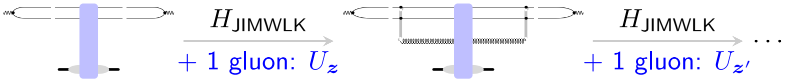

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▸ explicit form

- resums all $\sim [\alpha_s \ln(1/x)]^n$ (at LO)
- ➡ energy dependence of $\langle \ldots \rangle(Y)$

Saturation scale and cross section

- $\langle \ldots \rangle(Y) \quad \Longrightarrow \quad \langle \frac{\text{tr}(1 - U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)}{N_c} \rangle(Y) =: N_Y(\boldsymbol{r})$

Saturation scale and cross section

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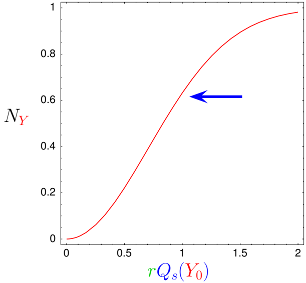
- qualitative expectation:



$$R_s(Y) \sim \frac{1}{Q_s(Y)}$$

$R_s(Y) \equiv$ correlation length
$Q_s(Y) \equiv$ saturation scale

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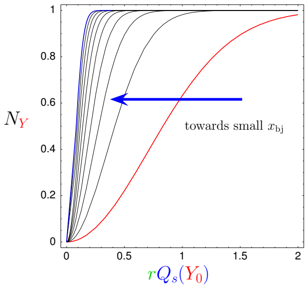
correlation length shrinks:



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NLO-corrections

LO: $[\alpha_s \ln(1/x)]^n$; NLO: $[\alpha_s]^n[\ln(1/x)]^{n-1}$

- Corrections to evolution:

- Corrections to wave functions/impact factors

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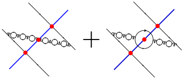

Gardi, Kuokkanen, Rummukainen, Weigert
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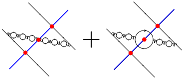

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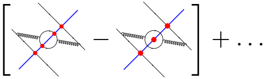

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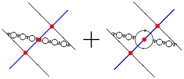

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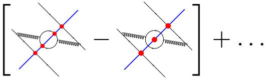

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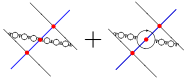

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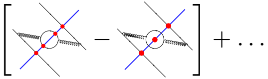

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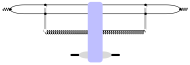

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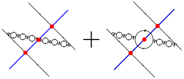

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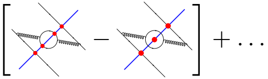

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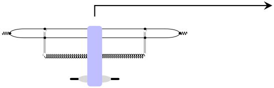
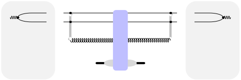
LO JIMWLK evolution ($\sim [\alpha_s \ln(1/x)]^1$)





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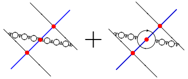

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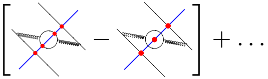

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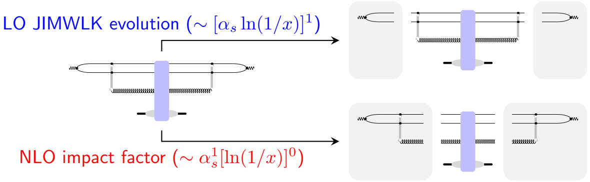


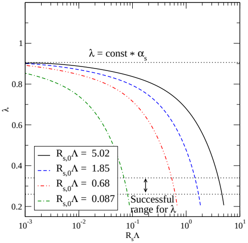
✓

Balitsky, Chirilli (2011)
not yet included

Effects of NLO-corrections

- NLO evolution: speed reduced

$$\lambda(Y) := \frac{d}{dY} \ln Q_s^2(Y)$$



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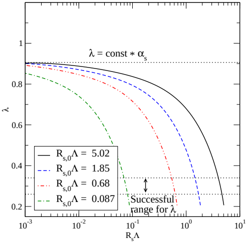

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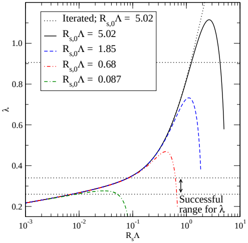

+ running coupling

- remarkable slowdown
- fits become possible

large effect expected

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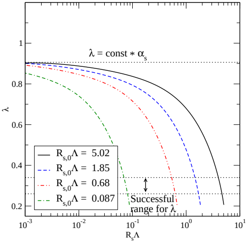

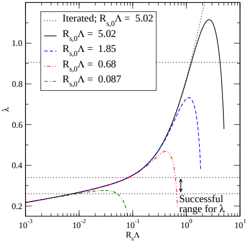

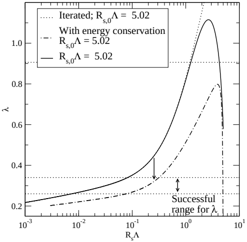

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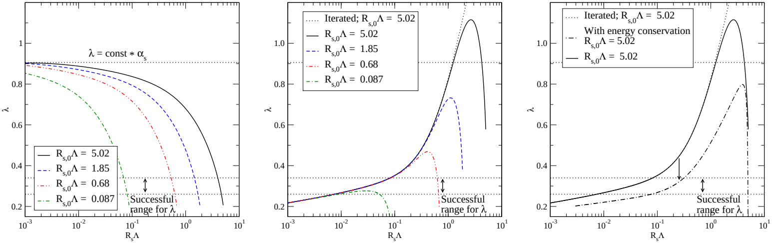

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- Effect of NLO impact factors? yet unknown

Fit to HERA data

- Total cross section:

- Rapidity gap events (diffractive events):

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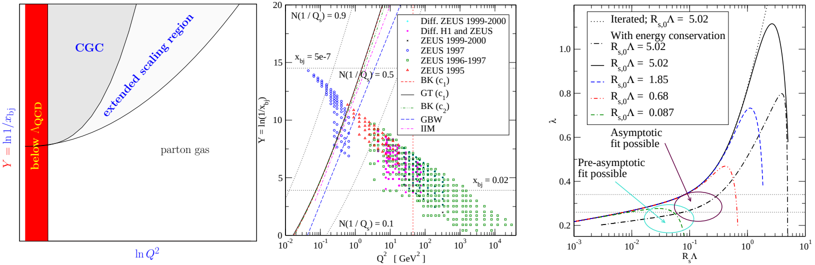

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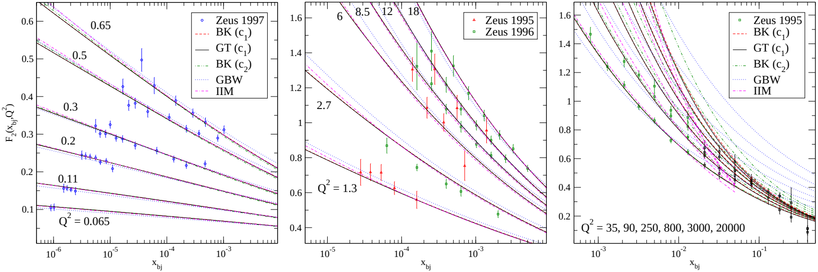

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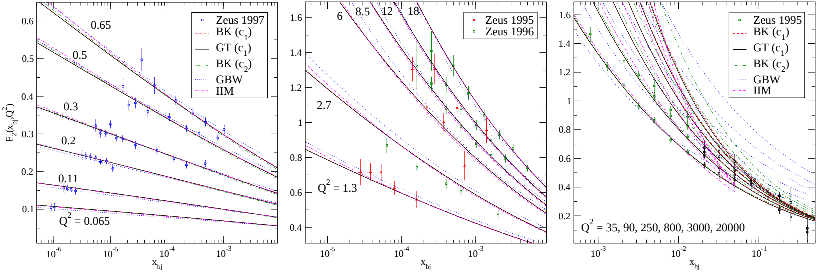
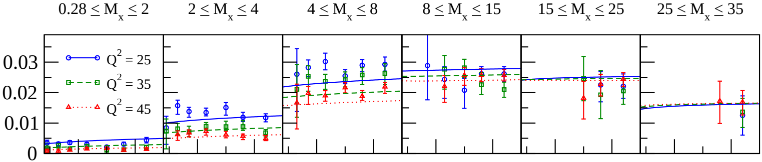


- Rapidity gap events (diffractive events): $\chi^2/\text{dof} \sim 1.3$

ratios diffractive/total cross sections (sample only):

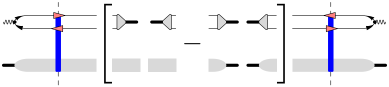


- Lack of NLO impact factors: predictive power down!

Outline

Not everything is made of dipoles ...

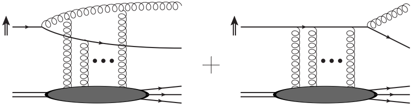
- Compare vector meson production with and without a rapidity gap



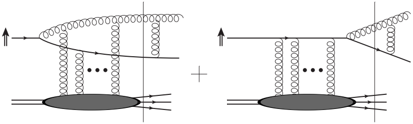
$$4\pi\frac{d\sigma_{T,L}}{dt} = \int\limits_0^1 d\alpha d\alpha' \int d^2x d^2x' d^2y d^2y' e^{-i\boldsymbol{l}\cdot[(\alpha\boldsymbol{x}+(1-\alpha)\boldsymbol{y})-(\alpha'\boldsymbol{x}'+(1-\alpha')\boldsymbol{y}')]}$$
$$\times \Psi^*_{T,L}(\alpha', \boldsymbol{x}'-\boldsymbol{y}', Q^2)\Psi_{T,L}(\alpha, \boldsymbol{x}-\boldsymbol{y}, Q^2)$$
$$\times \left[\langle \mathrm{tr}(U_{\boldsymbol{y}'}U^\dagger_{\boldsymbol{x}'})\mathrm{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle_Y - \langle \mathrm{tr}(U_{\boldsymbol{y}'}U^\dagger_{\boldsymbol{x}'})\rangle_Y \langle \mathrm{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle_Y\right]/N_c^2$$

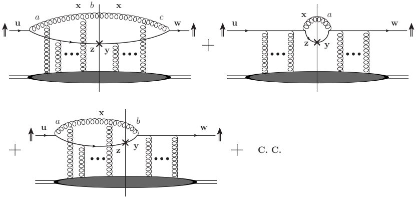
Marquet and Weigert, Nucl. Phys. A **843** (2010) 68 [arXiv:1003.0813 [hep-ph]].

Single spin asymmetries and the pomeron



Kovchegov and Sievert, [arXiv:1201.5890 [hep-ph]].





Sievers effect

$$\mathcal{I}^{(q)} = \left\langle \frac{\mathrm{tr}(U_{\boldsymbol{z}} U^\dagger_{\boldsymbol{y}})}{N_c} + \frac{\mathrm{tr}(U_{\boldsymbol{u}} U^\dagger_{\boldsymbol{w}})}{N_c} - \frac{1}{d_A} U^{ab}_{\boldsymbol{x}} \mathrm{tr}(t^a U_{\boldsymbol{z}} t^b U^\dagger_{\boldsymbol{w}}) - \frac{1}{d_A} U^{ab}_{\boldsymbol{x}} \mathrm{tr}(t^a U_{\boldsymbol{u}} t^b U^\dagger_{\boldsymbol{y}}) \right\rangle$$

STSA: odd under $\boldsymbol{z} \leftrightarrow \boldsymbol{y}$, driven by imaginary part of $\mathcal{I}^{(q)}$.

Group constraints and gauge invariance – examples

- Operator relations – manifestations of gauge invariance

$$\tilde{U}_{\boldsymbol{z}}^{ab}\text{tr}(t^a U_{\boldsymbol{x}} t^b U_{\boldsymbol{y}}^\dagger) = \text{tr}(U_{\boldsymbol{z}} t^a U_{\boldsymbol{z}}^\dagger U_{\boldsymbol{x}} t^a U_{\boldsymbol{y}}^\dagger)$$

- $\boldsymbol{z} \mapsto \boldsymbol{x}$ or $\boldsymbol{y}$: $\quad C_\text{f}\text{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)$, then $\boldsymbol{y} \mapsto \boldsymbol{x}$: $\quad 2N_c C_\text{f}$
- $\boldsymbol{y} \mapsto \boldsymbol{x}$: $\quad \frac{1}{2}\tilde{\text{tr}}(\tilde{U}_{\boldsymbol{z}} \tilde{U}_{\boldsymbol{x}}^\dagger)$, then $\boldsymbol{z} \mapsto \boldsymbol{x}$: $\quad 2N_c C_\text{f}$
- Fierz:

$$\frac{1}{2}\text{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{z}}^\dagger)\text{tr}(U_{\boldsymbol{z}} U_{\boldsymbol{y}}^\dagger) - \frac{1}{2N_c}\text{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)$$

then $\boldsymbol{y} \mapsto \boldsymbol{x}$:

$$\frac{1}{2}(|\text{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{z}}^\dagger|^1 - 1) = \frac{1}{2}\tilde{\text{tr}}(\tilde{U}_{\boldsymbol{z}} \tilde{U}_{\boldsymbol{x}}^\dagger)$$

- $\langle \ldots \rangle$ ➡ more correlators than degrees of freedom
- g^2 correlator $\in \mathbb{R}$
- all other possibly $\in \mathbb{C}$

Gauge invariant truncations of JIMWLK

- Parametrize $\langle\ldots\rangle(Y)$ via n-point functions

$$\langle\ldots\rangle(Y) = \exp\Big\{\int^Y dy\Big[-\frac{1}{2}\int_{\boldsymbol{uv}} G_{\boldsymbol{uv}}(y) i\nabla^a_{\boldsymbol{u}} i\nabla^a_{\boldsymbol{v}} - \frac{1}{3!}\int_{\boldsymbol{uvw}} G_{\boldsymbol{uvw}}(y)\ d^{abc} i\nabla^a_{\boldsymbol{u}} i\nabla^b_{\boldsymbol{v}} i\nabla^c_{\boldsymbol{w}} + \text{etc.}\Big]\Big\}\ldots$$

  - gauge invariant at any order
  - only fully symmetric $G_{\boldsymbol{u}_1\ldots\boldsymbol{u}_n}$ necessary: $[i\nabla^a_{\boldsymbol{u}}, i\nabla^b_{\boldsymbol{v}}] = i f^{abc} i\nabla^c_{\boldsymbol{v}} \delta^{(2)}_{\boldsymbol{u},\boldsymbol{v}}$
  - Result:

$$\langle\ldots\rangle(Y) \mapsto \langle\ldots\rangle[G_{\boldsymbol{u}_1\boldsymbol{u}_2}, G_{\boldsymbol{u}_1\boldsymbol{u}_2\boldsymbol{u}_3}, \text{etc}](Y)$$

Gauge invariant truncations of JIMWLK

- Examples:

$$\frac{1}{N_c}\langle \text{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle = e^{-C_f(\mathcal{P}+i\mathcal{O})_{\boldsymbol{xy}}}$$

$$\frac{1}{2N_cC_f}\langle U^{ab}_{\boldsymbol{z}}\text{tr}(t^aU_{\boldsymbol{x}}t^bU^\dagger_{\boldsymbol{y}})\rangle = e^{-\left\{\left[\frac{N_c}{2}((\mathcal{P}+i\mathcal{O})_{\boldsymbol{xz}}+(\mathcal{P}+i\mathcal{O})_{\boldsymbol{zy}})-(\mathcal{P}+i\mathcal{O})_{\boldsymbol{xy}}\right)-C_{\mathsf{f}}(\mathcal{P}+i\mathcal{O})_{\boldsymbol{xy}}\right\}(Y)}$$

where

$$\mathcal{P}_{\boldsymbol{xy}} = \int^Y dy\left(G_{\boldsymbol{xy}} - \frac{1}{2}(G_{\boldsymbol{xx}}+G_{\boldsymbol{yy}}\right) = \mathcal{P}_{\boldsymbol{yx}} \qquad i\mathcal{O}_{\boldsymbol{xy}} := \frac{C_d}{4}\int^Y dy\,(G_{\boldsymbol{yxx}} - G_{\boldsymbol{yyx}}) = -i\mathcal{O}_{\boldsymbol{yx}}$$

- Identification with pomeron and odderon:

$$\frac{1}{N_c}\langle \text{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle(Y) = 1 - P_{\boldsymbol{xy}}(Y) + iO_{\boldsymbol{xy}}(Y) = e^{-C_{\mathsf{f}}(\mathcal{P}_{\boldsymbol{xy}}(Y)+i\mathcal{O}_{\boldsymbol{xy}})(Y)}$$

$$\left(1 - P_{\boldsymbol{xy}}(Y) + iO_{\boldsymbol{xy}}(Y)\right)^* = \frac{1}{N_c}\langle \text{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})^*\rangle(Y) = \frac{1}{N_c}\langle \text{tr}(U_{\boldsymbol{y}}U^\dagger_{\boldsymbol{x}})\rangle(Y) = 1 - P_{\boldsymbol{yx}}(Y) + iO_{\boldsymbol{yx}}(Y)$$

  - complex conjugation $= \boldsymbol{x} \leftrightarrow \boldsymbol{y}$
  - $P_{\boldsymbol{xy}}(Y) = P_{\boldsymbol{yx}}(Y)$ ➡ pomeron = Re
  - $O_{\boldsymbol{xy}}(Y) = -O_{\boldsymbol{yx}}(Y)$ ➡ odderon = Im

Gauge invariant truncations of JIMWLK

- Evolution is imposed by JIMWLK via Balitsky hierarchy:

$$\frac{d}{dY}\langle \mathrm{tr}(U_{\boldsymbol{x}}U_{\boldsymbol{y}}^\dagger)\rangle(Y) = \frac{\alpha_s}{\pi^2}\int d^2z\,\mathcal{K}_{\boldsymbol{xzy}}\langle U_{\boldsymbol{z}}^{ab}\mathrm{tr}(t^aU_{\boldsymbol{x}}t^bU_{\boldsymbol{y}}^\dagger) - C_f\mathrm{tr}(U_{\boldsymbol{x}}U_{\boldsymbol{y}}^\dagger)\rangle(Y)$$

$$\frac{d}{dY}\langle U_{\boldsymbol{z}}^{ab}\mathrm{tr}(t^aU_{\boldsymbol{x}}t^bU_{\boldsymbol{y}}^\dagger)\rangle(Y) = \ldots$$

$$\vdots$$

- truncate at $G_{\_,\_,\_}$

$$\frac{d}{dY}N_c e^{-C_f(\mathcal{P}+i\mathcal{O})_{\boldsymbol{xy}}} = \frac{\alpha_s}{\pi^2}\int d^2z\,\mathcal{K}_{\boldsymbol{xzy}}N_cC_f e^{-\left\{\left[\frac{N_c}{2}((\mathcal{P}+i\mathcal{O})_{\boldsymbol{xz}}+(\mathcal{P}+i\mathcal{O})_{\boldsymbol{zy}})-(\mathcal{P}+i\mathcal{O})_{\boldsymbol{xy}}\right)-C_\mathrm{f}(\mathcal{P}+i\mathcal{O})_{\boldsymbol{xy}}\right\}(Y)} - C_fN_ce^{-C_f(\mathcal{P}+i\mathcal{O})_{\boldsymbol{xy}}}$$

- solve real and imaginary part for $\mathcal{P}$ and $\mathcal{O}$
- manifestly gauge invariant

$\boldsymbol{z}\mapsto\boldsymbol{x}$ or $\boldsymbol{y}$ → $C_\mathrm{f}\mathrm{tr}(U_{\boldsymbol{x}}U_{\boldsymbol{y}}^\dagger)$ → ($\boldsymbol{y}\mapsto\boldsymbol{x}$) → $2N_cC_\mathrm{f}$

$$U_{\boldsymbol{z}}^{ab}tr(t^aU_{\boldsymbol{x}}t^bU_{\boldsymbol{y}}^\dagger) = \mathrm{tr}(U_{\boldsymbol{z}}t^aU_{\boldsymbol{z}}^\dagger U_{\boldsymbol{x}}t^aU_{\boldsymbol{y}}^\dagger)$$

$\boldsymbol{y}\mapsto\boldsymbol{x}$ → $\frac{1}{2}\tilde{\mathrm{tr}}(\tilde{U}_{\boldsymbol{z}}\tilde{U}_{\boldsymbol{x}}^\dagger)$ → ($\boldsymbol{z}\mapsto\boldsymbol{x}$) → $2N_cC_\mathrm{f}$

Correlators – theory expectations

- Examples:

$$\frac{1}{N_c}\langle \text{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle = e^{-C_f(\mathcal{P}+i\mathcal{O})_{\boldsymbol{xy}}}$$

$$\frac{1}{2N_cC_f}\langle U^{ab}_{\boldsymbol{z}}\text{tr}(t^aU_{\boldsymbol{x}}t^bU^\dagger_{\boldsymbol{y}})\rangle = e^{-\left\{\left[\frac{N_c}{2}((\mathcal{P}+i\mathcal{O})_{\boldsymbol{xz}}+(\mathcal{P}+i\mathcal{O})_{\boldsymbol{zy}})-(\mathcal{P}+i\mathcal{O})_{\boldsymbol{xy}}\right)-C_f(\mathcal{P}+i\mathcal{O})_{\boldsymbol{xy}}\right\}(Y)}$$

  - fixed symmetry properties

$$\mathcal{P}_{\boldsymbol{xy}} = \int^Y dy\left(G_{\boldsymbol{xy}} - \frac{1}{2}(G_{\boldsymbol{xx}}+G_{\boldsymbol{yy}}\right) = \mathcal{P}_{\boldsymbol{yx}} \qquad i\mathcal{O}_{\boldsymbol{xy}} := \frac{C_d}{4}\int^Y dy\,(G_{\boldsymbol{yxx}} - G_{\boldsymbol{yyx}}) = -i\mathcal{O}_{\boldsymbol{yx}}$$

    ⟶ $\mathcal{O} \propto \boldsymbol{x}-\boldsymbol{y}$ & no external index ⟶ need other vector $\boldsymbol{s}$

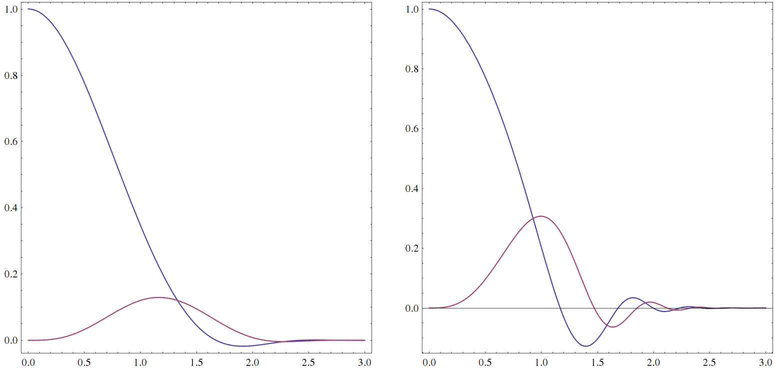
  - need to break translational/rotational inv., example: STSA
  - short distance perturbative limits

$$\mathcal{P}_{\boldsymbol{xy}} \propto (\boldsymbol{x}-\boldsymbol{y})^2 \qquad \mathcal{O}_{\boldsymbol{xy}} \propto (\boldsymbol{x}-\boldsymbol{y}^2)(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{s} = |\boldsymbol{x}-\boldsymbol{y}^2|^3\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{s}}$$

Correlators – theory expectations

- extrapolate pert. result for $\langle \mathrm{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger \rangle$

Re & Im of $e^{-r^2+i\frac{1}{3}r^3}$ Re & Im of $e^{-r^2+i.98r^3}$

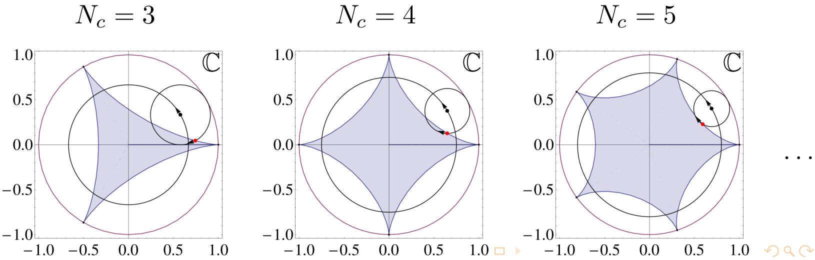


- think initial conditions. . .

- what else?

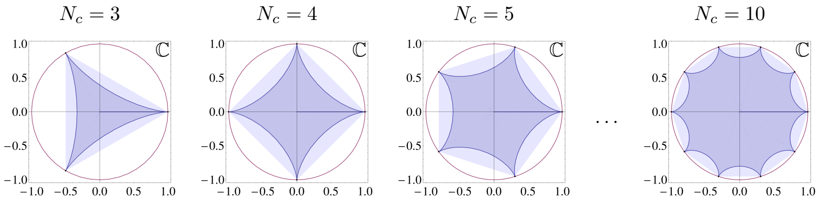
Correlators – theory expectations

- $U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger \in \mathsf{SU}(3)$
  - N_c eigenvalues $e^{i\phi_i}$
  - $1 = \det(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger) = e^{i\sum_{i=1}^{N_c}\phi_i} \Leftrightarrow \sum_{i=1}^{N_c}\phi_i = 2\pi n; n \in \mathbb{Z}$
- $\mathrm{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)/N_c = \frac{1}{N_c}\left(\sum_{i=1}^{N_c-1} e^{i\phi_i} + e^{-i\sum_{i=1}^{N_c-1}\phi_i}\right)$ falls inside hypocycloid in $\mathbb{C}$



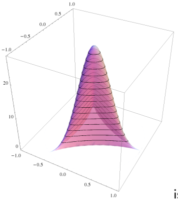
Correlators – theory expectations

- $\langle \mathrm{tr}(U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})/N_c \rangle$ slightly less constrained

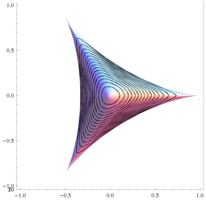


- unbiased distributions determined by Haar measure

$$\prod_{i>j}^{N_c} |e^{i\phi_i} - e^{i\phi_j}|^2 \Big|_{\sum_{k=1}^{N_c} \phi_i = 0} \xrightarrow{N_c \to 3} 64 \sin^2\left(\frac{\phi_2-\phi_1}{2}\right) \sin^2\left(\frac{\phi_3-\phi_1}{2}\right) \sin^2\left(\frac{\phi_3-\phi_2}{2}\right)\Big|_{\phi_3 = -(\phi_1+\phi_2)}$$



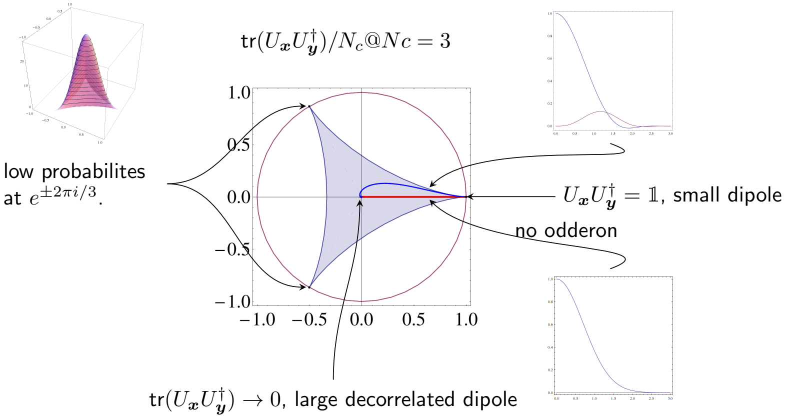
isometric



top down

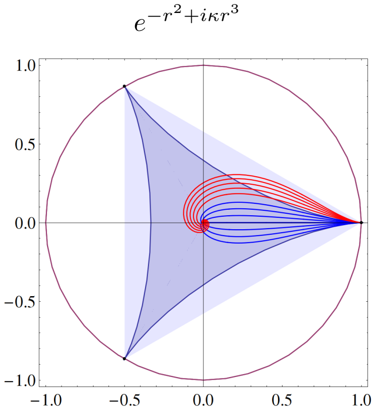
Correlators – theory expectations

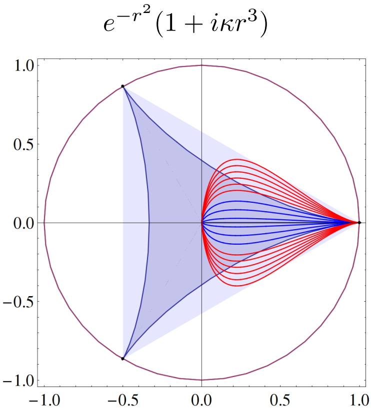
- pert. corr. and trace constraints



Correlators – theory expectations

- (pert) initial conditions vs trace constratints – from short dist!



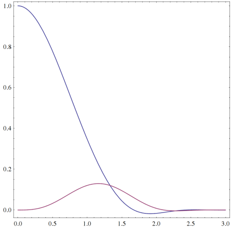


- hypocycloid $|\kappa| < 1/3$, polygon $|\kappa| < .98$

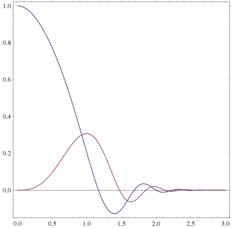
Correlators – theory expectations

- severe (pert. driven) limitations on $O_{\boldsymbol{xy}}$

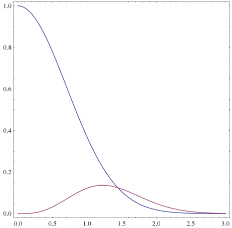
Re & Im of $e^{-r^2+i\frac{1}{3}r^3}$



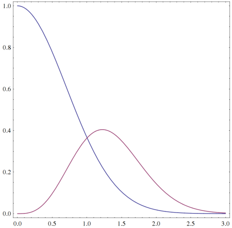
Re & Im of $e^{-r^2+i.98r^3}$



Re& Im of $e^{-r^2}(1+i\frac{1}{3}r^3)$

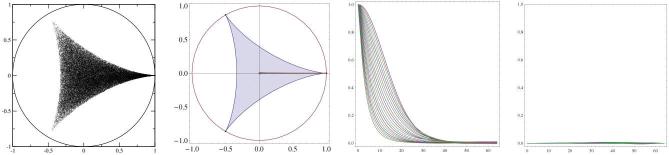


Re & Im of $e^{-r^2}(1+i.98r^3)$

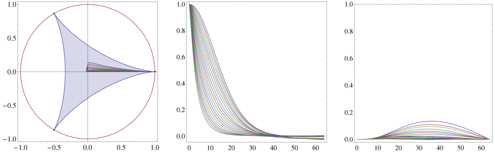


The pomeron in JIMWLK (explorations)

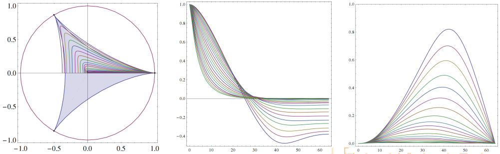
- pure pomeron ensemble (GBW initial cond)



- small odderon component (distorted GBW ensemble)



- mathematically maxed odderon (no physical motivation) $\leq \mathrm{Im} e^{2\pi i/3} = \sqrt{3}/2 \approx .866$



The pomeron in JIMWLK (explorations)

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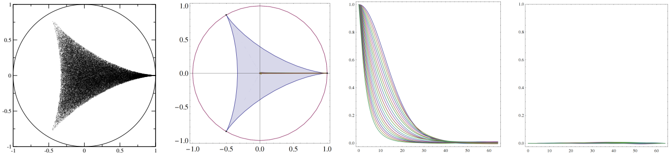
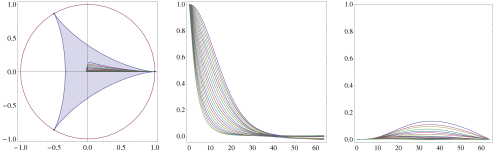


- small odderon component (distorted GBW ensemble)

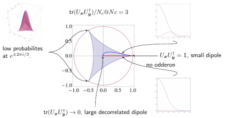


- observations
  - pomeron decorrelates, develops (pseudo) scaling
  - pomeron evolution speed largely unaffected by presence of odderon
  - odderon allows anticorrelations
  - odderon decays in place

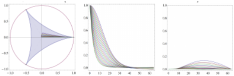
Outline

Perspectives

- New observables need new tools to analyze
- Gauge invariant truncations (generalize GT) allow to access
  - pomeron, odderon ... (pomeron hierarchy)
- Group theory constrains contributions



  - constrains odderon max size $\sqrt{32}$, likely *smaller* .2
- JIMWLK or B-hierarchy



  - pomeron decorrelates, develops (pseudo) scaling
  - pomeron evolution speed largely unaffected by presence of odderon
  - odderon allows anticorrelations
  - odderon decays in place
- To do list
  - exten to NLO
  - become quantitative
  - cross sections