



# Radiative decays of resonances in lattice QCD

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Palaiseau, September 8, 2015

AA, V. Bernard, U.-G. Meißner, A. Rusetsky, Nucl. Phys. B 886 (2014)



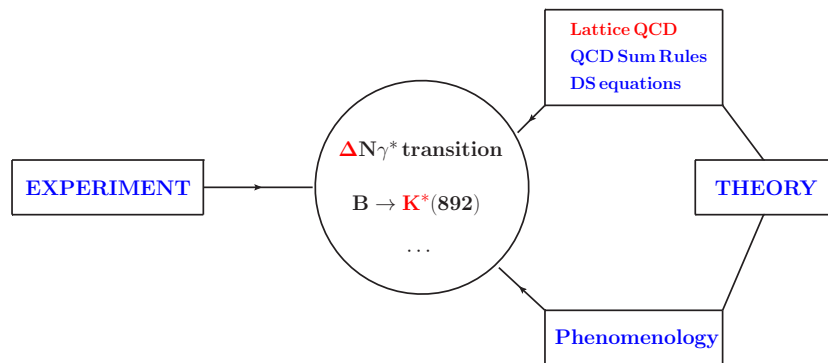
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# Outline

- ▶ Introduction: the  $\Delta N \gamma^*$  transition
- ▶ Motivation
- ▶ Goals, results
- ▶ The  $B \rightarrow K^* \gamma^*$ : work in progress
- ▶ Outlook

# Introduction



- $\gamma^* N \rightarrow \pi N$  near the  $\Delta(1232)$ : study of the **hadron deformation**
- $B \rightarrow K^* \gamma^*$ ,  $B \rightarrow K^* l^+ l^-$  with  $K^*(892) \rightarrow K \pi$ : sensitive to **NP**

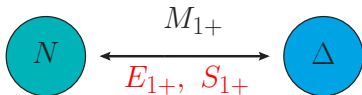
$$\gamma^* N \rightarrow \Delta$$

$$\gamma^* p \rightarrow \Delta^+(1232) \rightarrow p\pi^0 \quad (66\%)$$

$$\gamma^* p \rightarrow \Delta^+(1232) \rightarrow n\pi^+ \quad (33\%)$$

$$\gamma^* p \rightarrow \Delta^+(1232) \rightarrow p\gamma \quad (0.56\%)$$

↔ study of the de-excitation radiation pattern



- Selection rules  $\Rightarrow M_{1+}, E_{1+}, S_{1+}$  **multipole amplitudes**

I. G. Aznauryan, V. D. Burkert and T. -S. H. Lee, arXiv:0810.099

- A challenge: determine the quadrupoles  $E_{1+}, S_{1+}$  ( $\ll M_{1+}$ )

- The electromagnetic transition matrix element:

$$\langle \Delta(P, \lambda) | J_\mu(0) | N(Q, \epsilon) \rangle = \left( \frac{2}{3} \right)^{1/2} \bar{u}^\sigma(P, \lambda) \mathcal{O}_{\sigma\mu} u(Q, \epsilon),$$

with the Lorentz-structure

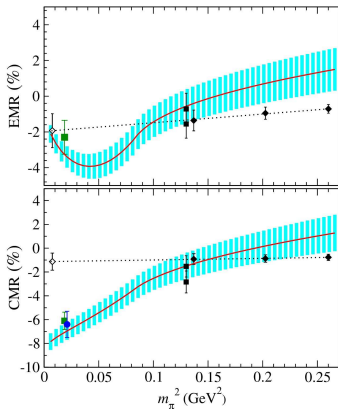
$$\mathcal{O}_{\sigma\mu} = G_M(Q^2) K_{\sigma\mu}^{M1} + G_E(Q^2) K_{\sigma\mu}^{E2} + G_C(Q^2) K_{\sigma\mu}^{C2}$$

H. F. Jones and M. D. Scadron, Ann. Phys. **81** (1973) 1

$$\text{EMR} \equiv \frac{\text{Im} E_{1+}^{3/2}}{\text{Im} M_{1+}^{3/2}} = -\frac{G_E(Q^2)}{G_M(Q^2)}, \quad \text{CMR} \equiv \frac{\text{Im} L_{1+}^{3/2}}{\text{Im} M_{1+}^{3/2}} = -\frac{|\vec{Q}|}{2m_\Delta} \frac{G_C(Q^2)}{G_M(Q^2)}$$

↔ extracted from experiment      calculated on the lattice ↔

# The values of EMR and CMR



V. Pascalutsa, M. Vanderhaeghen, Phys. Rev. D **73** (2006) 034003

- The  $\Delta$  is treated as a **stable** particle!

$\Leftrightarrow$  *Note:* the chiral extrapolation is not reliable for  $m_\pi \gtrsim 300$  MeV!

# Motivation

*The largest conceptual question as we enter the chiral regime in full QCD, is how to fully incorporate the physical effect of the decay of the  $\Delta$  into a pion and nucleon on the transition form factors.*

C. Alexandrou et al., Phys. Rev. D **77** (2008) 085012

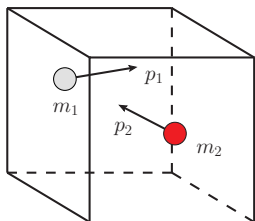
## Methods

- How to study resonances/bound states on the lattice?  
→ **Lüscher approach**
- How to find electroweak process amplitude/form factors?  
→ **Lellouch-Lüscher method**
- How to build a bridge: two particle spectrum  $\leftrightarrow$  scattering sector? → **non-relativistic EFT**/Bethe-Salpeter equation

# Lüscher approach

- scattering phase shift  $\leftrightarrow$  finite volume energy spectrum

M. Lüscher, Nucl. Phys. B **354** (1991) 531.



$$\cot \delta_0(p) = -\cot \phi(q) \quad (\text{Lüscher equation})$$

$\phi(q)$  – known function

$$q = \frac{pL}{2\pi}, \quad p^2 = \lambda(s, m_1^2, m_2^2)/4s$$

$$p_R \cot \delta_0(p_R) = -\frac{1}{a_0} + \frac{1}{2} r_0 p_R^2 + \dots = -ip_R$$

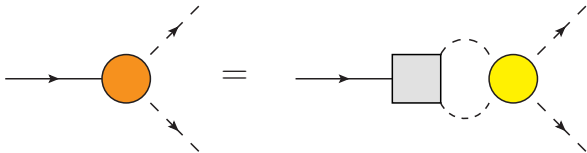
- ▷ lattice data  $\Rightarrow$  Lüscher equation  $\Rightarrow$  scattering phase
- ▷ fit effective-range expansion parameters
- ▷ analytic continuation to the resonance position  $p_R$



# Lellouch-Lüscher method

- Seminal work on  $K \rightarrow \pi\pi$  by

L. Lellouch and M. Lüscher, *Commun.Math.Phys.* **219**, 31 (2001)



$$|\langle \pi\pi | H_{\text{weak}} | K \rangle|_{\text{finite volume}} \propto |A(K \rightarrow \pi\pi)| \times \left( \frac{p^2}{\delta'(p) + \phi'(q)} \right)^{-1/2}$$

↔ *Note*: both methods are valid **only**

- ▷ for sufficiently **big** volumes  $L > 2R_{\text{int}}$  ( $m_\pi L \gtrsim 4$ )
- ▷ **below** 3(4)-particle threshold (low 3-momenta)

## The framework: non-relativistic EFT

- Ideally suited for the problem we study:
  - ▷ the total number of heavy particles is conserved
  - ▷ manifestly Lorentz-invariant formulation is possible
  - ▷ the theory is matched to the full QFT (e.g., ChPT)



Bubble-chain diagrams

$$T \propto 1 + cJ + c^2 J^2 + \dots = \frac{1}{1 - cJ}$$

- G. Colangelo, J. Gasser, B. Kubis, A. Rusetsky, Phys. Lett. B **638** (2006) 187  
J. Gasser, B. Kubis, A. Rusetsky, Nucl. Phys. B **850** (2011) 96

- A bridge: finite volume spectrum  $\leftrightarrow$  scattering sector

# Goals, results

- Previous work:

D. Hoja, U.-G. Meißner, A. Rusetsky, JHEP 1004 (2010) 050

V. Bernard, D. Hoja, U.-G. Meißner, A. Rusetsky, JHEP 1209 (2012) 023

↔ scalar resonance form factor in the external scalar field (analog:  $\Delta\Delta\gamma^*$ )

- The  $\Delta N\gamma^*$  transition:

- ▷ inclusion of *spin*;

- ▷ generalization to *transition* form factors.

# Kinematics

- The  $\Delta$  is at rest  $\mathbf{P} = 0$  and nucleon momentum  $\mathbf{Q}$  along 3-axis
- rotational symmetry  $\rightarrow$  **cubic** symmetry: choose some irrep(s)
- Irreps  $G_1, G_2$ : no  $S - P$ -wave mixing (the  $P_{31}$  wave is small)

$$p \cot \delta(p) = \frac{2}{\sqrt{\pi}L} \left\{ \hat{Z}_{00}(1; q^2) \pm \frac{1}{\sqrt{5}q^2} \hat{Z}_{20}(1; q^2) \right\} \equiv -p \cot \phi(q), \quad q = \frac{pL}{2\pi}$$

$\hookrightarrow$  Lüscher eq. for the  $P_{33}$  wave

M. Göckeler et al., Phys. Rev. D **86** (2012) 094513

- *Note*: in practice, the case  $\mathbf{P} \neq 0$  could be preferable  
 $\hookrightarrow$  must be considered separately

# Spin

- Introduce  $\mathcal{O}^\mu(\mathbf{x}, t)$  - the  $\Delta$  field,  $\psi(\mathbf{x}, t)$  - the nucleon field
- Spin-projecting local operators:

$$G_2 \quad \mathcal{O}_{3/2}(t) = \sum_{\mathbf{x}} \frac{1}{2} (1 + \Sigma_3) \frac{1}{2} (1 + \gamma_4) \frac{1}{\sqrt{2}} (\mathcal{O}^1(\mathbf{x}, t) - i\Sigma_3 \mathcal{O}^2(\mathbf{x}, t)),$$

$$G_2 \quad \mathcal{O}_{1/2}(t) = \sum_{\mathbf{x}} \frac{1}{2} (1 - \Sigma_3) \frac{1}{2} (1 + \gamma_4) \frac{1}{\sqrt{2}} (\mathcal{O}^1(\mathbf{x}, t) + i\Sigma_3 \mathcal{O}^2(\mathbf{x}, t)),$$

$$G_1 \quad \tilde{\mathcal{O}}_{1/2}(t) = \sum_{\mathbf{x}} \frac{1}{2} (1 + \Sigma_3) \frac{1}{2} (1 + \gamma_4) \mathcal{O}^3(\mathbf{x}, t),$$

$$\bar{\psi}_{\pm 1/2}^{\mathbf{Q}}(t) = \sum_{\mathbf{x}} e^{i\mathbf{Q}\mathbf{x}} \bar{\psi}(\mathbf{x}, t) \frac{1}{2} (1 \pm \Sigma_3) \frac{1}{2} (1 + \gamma_4), \quad \Sigma_3 = \text{diag}(\sigma_3, \sigma_3)$$

$\Rightarrow$  three  $\Delta N \gamma^*$  form factors are **separately** projected out

$\hookrightarrow$  similar procedure can be applied to fields with other spin

# Extraction of the form factors

- on the **real** energy axis  $\rightarrow$  model- and process-dependent

I. G. Aznauryan, V. D. Burkert and T. -S. H. Lee, arXiv:0810.099

D. Drechsel, O. Hanstein, S.S. Kamalov, L. Tiator, Nucl. Phys. A 645 (1999) 145

- at the **resonance pole**  $\rightarrow$  *process-independent*  $\Rightarrow$  favourable!

▷ **definition** of the resonance matrix element:

$$\langle P, \text{resonance} | J(0) | Q, \text{stable} \rangle = \lim_{P^2 \rightarrow s_R, Q^2 \rightarrow M^2} Z_R^{-1/2} Z^{-1/2} (s_R - P^2)(M^2 - Q^2) F(P, Q)$$

$\hookrightarrow$  **analytic continuation** to the pole is unavoidable

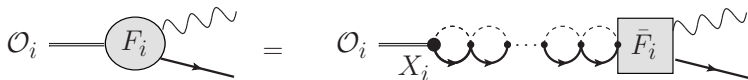
S. Mandelstam, Proc. Roy. Soc. Lond. A **233** (1955) 248.

- infinitely narrow resonances  $\rightarrow$  identical results

$\hookrightarrow$  however, discrepancies in the results could be sizable:

R. L. Workman, L. Tiator and A. Sarantsev, Phys. Rev. C **87** (2013) 6, 068201

## $\Delta N \gamma^*$ vertex: real axis



- ▷ The  $F_i = F_i(p_n, |\mathbf{Q}|)$ ,  $i=1, 2, 3 \rightarrow G_M, G_E, G_C$  form factors, are measured on *the lattice*
- ▷ The  $\bar{F}_i(p_n, |\mathbf{Q}|)$  are *volume-independent* irreducible amplitudes
- The  $\bar{F}_i$  are related to the  $\gamma^* N \rightarrow \pi N$  multipole amplitudes

$$\mathcal{A}_i(p, |\mathbf{Q}|) = e^{i\delta(p)} \cos \delta(p) \bar{F}_i(p, |\mathbf{Q}|)$$

$\hookrightarrow$  Watson's theorem

$$\mathcal{A}_i(p_n, |\mathbf{Q}|) = e^{i\delta(p_n)} V^{1/2} \left( \frac{1}{|\delta'(p_n) + \phi'(q_n)|} \frac{p_n^2}{2\pi} \right)^{-1/2} |F_i(p_n, |\mathbf{Q}|)|$$

$\hookrightarrow$  LL equation for the photoproduction amplitude in the elastic region

- The  $\mathcal{A}_i \leftrightarrow$  multipoles:

$$\tilde{\mathcal{A}}_{1/2} = -16\pi i E \sqrt{2} \frac{\sqrt{Q^2}}{|Q|} S_{1+},$$

$$\mathcal{A}_{1/2} = -\frac{1}{2}(3E_{1+} + M_{1+})(-16\pi i E),$$

$$\mathcal{A}_{3/2} = \frac{\sqrt{3}}{2}(E_{1+} - M_{1+})(-16\pi i E).$$

- The narrow width approximation:

$$|\text{Im } \mathcal{A}_i(p_A, |Q|)| = \sqrt{\frac{8\pi}{p_A \Gamma}} |F_i^A(p_A, |Q|)|,$$

$p = p_A$  - Breit-Wigner pole,  $F_i^A(p_A, |Q|) \rightarrow \Delta N \gamma^*$  form factors

I. G. Aznauryan, V. D. Burkert and T. -S. H. Lee, arXiv:0810.099



## Complex plane

- The  $\Delta N \gamma^*$  matrix elements, evaluated at the pole  $p = p_R$ :

$$F_i^R(p_R, |\mathbf{Q}|) = Z_R^{1/2} \bar{F}_i(p_R, |\mathbf{Q}|)$$

$$Z_R = \left( \frac{p_R}{8\pi E_R} \right)^2 \left( \frac{16\pi p_R^3 E_R^3}{w_{1R} w_{2R} (2p_R h'(p_R^2) + 3ip_R^2)} \right), \quad w_{iR} = \sqrt{m_i^2 + p_R^2}$$

$$p^3 \cot \delta(p) \doteq h(p^2) = -\frac{1}{a} + \frac{1}{2} r p^2 + \dots, \quad h(p_R^2) = -ip_R^3$$

- ▷ The narrow width approximation ( $p_R \rightarrow p_n$ ):

$$F_i^R(p_R, |\mathbf{Q}|) \rightarrow F_i^A(p_A, |\mathbf{Q}|) \quad \text{as} \quad p_R \rightarrow p_A!$$

$$F_i^R(p_n, |\mathbf{Q}|) = V^{1/2} \left( \frac{E_n}{2w_{1n} w_{2n}} \right)^{1/2} F_i(p_n, |\mathbf{Q}|)$$

↪ proper normalization of states

## Prescription on the lattice

- Measure the  $F_i(p, |\mathbf{Q}|)$  at different values of  $p$  with  $|\mathbf{Q}|$  fixed.
- Real energy: extract the multipole amplitudes (see above).
- Extraction of the matrix elements at the **resonance pole**:
  1. fit the functions  $p^3 \cot \delta(p) \bar{F}_i(p, |\mathbf{Q}|)$

$$p^3 \cot \delta(p) \bar{F}_i(p, |\mathbf{Q}|) = A_i(|\mathbf{Q}|) + p^2 B_i(|\mathbf{Q}|) + \dots$$

2. evaluate the resonance matrix elements by substitution

$$F_i^R(p_R, |\mathbf{Q}|) = i p_R^{-3} Z_R^{1/2} (A_i(|\mathbf{Q}|) + p_R^2 B_i(|\mathbf{Q}|) + \dots)$$

↪ *Note*: no additional computation to find the  $F_i^R(p_R, |\mathbf{Q}|)$

## $B \rightarrow K^* \gamma^*$ : work in progress

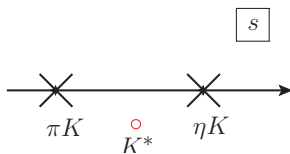
- Recent lattice measurement:

R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, Phys. Rev. D **89**, 094501 (2014)

$\leftrightarrow$  consistent with LCSR determinations

- Related work: multichannel LL equation, kinematics

R. A. Briceño et al., Phys.Rev. D **91** (2015) 3, 034501



- The 2-channel Lellouch-Lüscher formula is reproduced
- The form factor at the pole is extracted in the 2-channel problem
- The infinitely narrow resonance limit is considered

# Outlook

- Complete the work on the  $B \rightarrow K^* \gamma^*$  process:
  - ▶ extraction at the pole position
  - ▶ moving frames
  
- Briceño et al., arXiv:1507.06622, considered the  $\pi \gamma^* \rightarrow \rho \rightarrow \pi \pi$  form factor at  $m_\pi \approx 400$  MeV
  - ↪  $\Delta N \gamma^*$ ,  $B \rightarrow K^* \gamma^*$ : tasks for the near future!

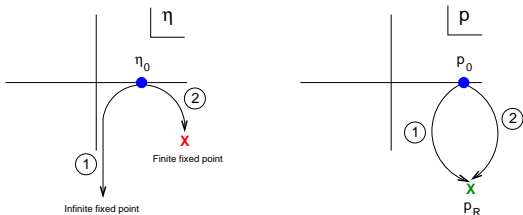
**Thank you!**

## Finite fixed points ( $\Delta\Delta\gamma^*, \dots$ )

- The corresponding integral  $I$ : V. Bernard, et. al., JHEP 1209 (2012)  
023

$$I = I_\infty + \frac{1}{32\pi E p} \left( 1 + \cot^2 \delta(p) \right) \eta \phi'(\eta) \Big|_{p=p_n}, \quad \eta = \frac{pL}{2\pi}$$

- $p \rightarrow p_R$ :  $Z_{00}(1; \eta_R^2) + i\pi^{3/2}\eta_R = 0 \rightarrow \exists \eta_R : |\text{Im } \eta_R| < \infty$

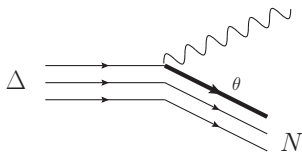


- Now,  $\cot \phi(\eta) + i \propto \eta - \eta_R$  and  $\phi(\eta) \propto \ln(\eta - \eta_R) \Rightarrow$

$$(\cot \phi(\eta) + i)\phi'(\eta) \rightarrow \text{const} \neq 0$$

- Solution: perform the measurement at **two** energy levels

# Kinematics



- The  $\Delta$  is at rest  $\mathbf{P} = 0$  and nucleon momentum  $\mathbf{Q}$  along 3-axis.
- To perform the fit (see below): vary  $p$ , while  $|\mathbf{Q}|$  fixed.
  - ▷ (partially) twisted boundary conditions;
- ↪ nucleon form factors: M. Göckeler et al., PoS LATTICE 2008 (2008) 138
  - ▷ asymmetric boxes  $L \times L \times L'$ . Alexandru, Döring
- Note: in practice, the case  $\mathbf{P} \neq 0$  could be preferable.

## Lattice correlators

- The two-point functions:

$$\tilde{D}_{1/2}(t) = \text{Tr} \langle 0 | \tilde{\mathcal{O}}_{1/2}(t) \bar{\tilde{\mathcal{O}}}_{1/2}(0) | 0 \rangle,$$

$$D_{1/2}(t) = \text{Tr} \langle 0 | \mathcal{O}_{1/2}(t) \bar{\mathcal{O}}_{1/2}(0) | 0 \rangle,$$

$$D_{3/2}(t) = \text{Tr} \langle 0 | \mathcal{O}_{3/2}(t) \bar{\mathcal{O}}_{3/2}(0) | 0 \rangle,$$

$$D_{\mathbf{Q}}^{\pm}(t) = \text{Tr} \langle 0 | \psi_{\pm 1/2}^{\mathbf{Q}}(t) \bar{\psi}_{\pm 1/2}^{\mathbf{Q}}(0) | 0 \rangle.$$

- The three-point functions:

$$\tilde{R}_{1/2}(t', t) = \langle 0 | \tilde{\mathcal{O}}_{1/2}(t') J^3(0) \bar{\psi}_{1/2}^{\mathbf{Q}}(t) | 0 \rangle,$$

$$R_{1/2}(t', t) = \langle 0 | \mathcal{O}_{1/2}(t') J^+(0) \bar{\psi}_{-1/2}^{\mathbf{Q}}(t) | 0 \rangle,$$

$$R_{3/2}(t', t) = \langle 0 | \mathcal{O}_{3/2}(t') J^+(0) \bar{\psi}_1^{\mathbf{Q}}(t) | 0 \rangle,$$

where

$$J^+(0) = \frac{1}{\sqrt{2}} (J^1(0) + iJ^2(0)).$$

$\Rightarrow$



- In the limit  $t' \rightarrow +\infty$ ,  $t \rightarrow -\infty$ :

$$\mathcal{N} \frac{\text{Tr}(\tilde{R}_{1/2}(t', t))}{\tilde{D}_{1/2}(t' - t)} \left( \frac{D_{\mathbf{Q}}^+(t') \tilde{D}_{1/2}(-t) \tilde{D}_{1/2}(t' - t)}{\tilde{D}_{1/2}(t') D_{\mathbf{Q}}^+(-t) D_{\mathbf{Q}}^+(t' - t)} \right)^{1/2} \rightarrow \langle 1/2 | J^3(0) | 1/2 \rangle,$$

$$-\mathcal{N} \frac{\text{Tr}(R_{1/2}(t', t))}{D_{1/2}(t' - t)} \left( \frac{D_{\mathbf{Q}}^-(t') D_{1/2}(-t) D_{1/2}(t' - t)}{D_{1/2}(t') D_{\mathbf{Q}}^-(-t) D_{\mathbf{Q}}^-(t' - t)} \right)^{1/2} \rightarrow \langle 1/2 | J^+(0) | -1/2 \rangle,$$

$$-\mathcal{N} \frac{\text{Tr}(R_{3/2}(t', t))}{D_{3/2}(t' - t)} \left( \frac{D_{\mathbf{Q}}^+(t') D_{3/2}(-t) D_{3/2}(t' - t)}{D_{3/2}(t') D_{\mathbf{Q}}^+(-t) D_{\mathbf{Q}}^+(t' - t)} \right)^{1/2} \rightarrow \langle 3/2 | J^+(0) | 1/2 \rangle,$$

where  $\mathcal{N} = (4E \sqrt{m_N^2 + \mathbf{Q}^2})^{1/2}$ .

||

$F_i$ ,  $i=1, 2, 3$

$\Rightarrow$  three  $\Delta N \gamma^*$  form factors are **separately** projected out.

$\hookrightarrow$  similar procedure can be applied to fields with other spin

## Real axis

- The  $\bar{F}_i$  are related to the  $\gamma^* N \rightarrow \pi N$  multipole amplitudes

$$\mathcal{A}_i(p, |\mathbf{Q}|) = e^{i\delta(p)} \cos \delta(p) \bar{F}_i(p, |\mathbf{Q}|)$$

↪ Watson's theorem

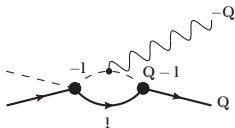
- $\mathcal{A}_i(p_A, |\mathbf{Q}|) = 0$  at  $\delta(p_A) = \pi/2$  ? **No!**

$$\mathcal{A}_i(p, |\mathbf{Q}|) = \underbrace{\frac{e^{i\delta(p)}}{p^3} \sin \delta(p)}_{\text{potential}} \underbrace{p^3 \cot \delta(p)}_{\text{zero at } p^2 = p_A^2} \underbrace{\bar{F}_i(p, |\mathbf{Q}|)}_{\text{diverges at } p^2 = p_A^2}$$

$$\mathcal{A}_i(p_n, |\mathbf{Q}|) = e^{i\delta(p_n)} V^{1/2} \left( \frac{1}{|\delta'(p_n) + \phi'(q_n)|} \frac{p_n^2}{2\pi} \right)^{-1/2} |F_i(p_n, |\mathbf{Q}|)|$$

↪ **LL equation** for the photoproduction amplitude in the elastic region

# Analytic continuation



A potentially dangerous diagram in the  $\bar{F}_i$

$$I = \frac{1}{V} \sum_{\mathbf{l}} \frac{1}{8w_1(\mathbf{l})w_2(-\mathbf{l})w_2(\mathbf{Q}-\mathbf{l})} \frac{1}{(w_1(\mathbf{l}) + w_2(-\mathbf{l}) - E_n)(w_1(\mathbf{l}) + w_2(\mathbf{Q}-\mathbf{l}) - Q^0)}$$

• After some transformations  $\Rightarrow$

$$I = \underbrace{\frac{1}{V} \sum_{\mathbf{l}} \frac{1}{2E_n(\mathbf{l}^2 - p_n^2)}}_{\propto p_n \cot \delta(p_n)} \frac{1}{2} \underbrace{\int_{-1}^1 dy \frac{1}{2\hat{w}_2(\hat{w}_1 + \hat{w}_2 - Q^0)}}_{\text{polynomial in } p_n^2}$$

$$\hat{w}_1 = \sqrt{m_N^2 + p_n^2}, \quad \hat{w}_2 = \sqrt{M_\pi^2 + p_n^2 + Q^2 - 2|\mathbf{Q}|p_n y}.$$

$\Rightarrow p^2 I$  is a low-energy **polynomial** in  $p^2$