Non-dipolar Wilson links for TMD pion wave functions

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Explore the hadronic structure exclusively

- Collinear factorization as a “standard" tool.
  - Factorization properties of hard exclusive processes better understood.
    - ★ $\gamma^* \gamma \to \pi^0$ [hard contribution at LP, this talk]
    - ★ $\gamma^* \pi \to \pi$ [hard contribution at LP]
    - ★ $B \to \pi \ell \nu$ decays [end-point divergence, interpretations]
  - Diagrammatical factorization versus SCET factorization.
  - One-dimensional profiles of hadrons.

- TMD factorization highly nontrivial.
  - Complicated definitions of TMD wave functions.
  - k_T resummation and threshold resummation.
  - Diagrammatical factorization.
  - 3D images of hadrons.

QCD factorization for $\gamma^* \gamma \to \pi^0$

- Collinear factorization [Brodsky and Lepage; Efremov and Radyushkin]:

$$F_\pi(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 dx\, H(x, Q^2, \mu)\, \phi_\pi(x, \mu)\,.$$

  - Collinear divergences absorbed in $\phi_\pi(x, \mu)$:

$$\langle 0 | \bar{q}(0)\, W_{n_-}(0, t n_-)\, \not{n}_- \gamma_5\, q(t n_-) | \pi^+(p) \rangle = i f_\pi\, p_+ \int_0^1 dx\, e^{-ixtp_+}\, \phi_\pi(x, \mu)\,.$$

  - End-point regions need to be considered separately.

- TMD factorization[Li and Sterman, 1992; Botts and Sterman, 1989]:

$$F_\pi(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 dx \int d^2\vec{b}\, H(x, \vec{b}, Q^2, \mu)\, \phi_\pi(x, \vec{b}, \mu)\, e^{-S(x, \vec{b}, Q)}\,.$$

  - Transverse momentum dependence becomes important in the end-point regions.
  - Naive definition of TMD pion wave function:

$$\phi_\pi^{\text{naive}}(x, \vec{k}_T, \mu) \overset{?}{=} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2}\, e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)}$$
$$\times \langle 0 | \bar{q}(0) W_{n_-}^\dagger(+\infty, 0)\, \not{n}_- \gamma_5\, [\text{tr. link}]\, W_{n_-}(+\infty, z)\, q(z) | \pi^+(p) \rangle\,.$$

Light-cone singularity

- Rapidity divergence in the infrared subtraction:

$$\phi_\pi^{(1)} \otimes H^{(0)} \supset \int [dl] \frac{1}{[(k+l)^2+i0)][l_+ + i0][l^2+i0]} \times \left[H^{(0)}(x+l_+/p_+, \vec{k}_T + \vec{l}_T) - H^{(0)}(x, \vec{k}_T) \right].$$

  - Rapidity divergence due to the Eikonal propagator.
  - No rapidity divergence for a k_T independent hard function.
  - Rapidity singularity of TMDs yields ill-defined hard functions.

- Regularization of the rapidity divergence [Collins, 2003].
  - Rotating the gauge links away from the light-cone ($u = (u_+, u_-, \vec{0}_T)$):

$$\phi_\pi(x, \vec{k}_T, y_u, \mu) \stackrel{?}{=} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)} \times \langle 0 | \bar{q}(0) W_u^\dagger(+\infty, 0) \not{n}_- \gamma_5 \, [\text{tr. link}] \, W_u(+\infty, z) \, q(z) | \pi^+(p) \rangle .$$

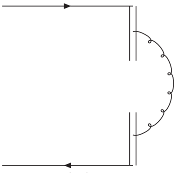
  - Introducing soft subtractions:

$$\phi_\pi(x, \vec{k}_T, y_u, \mu) \stackrel{?}{=} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)} \times \frac{\langle 0 | \bar{q}(0) W_{n_-}^\dagger(+\infty, 0) \not{n}_- \gamma_5 \, [\text{tr. link}] \, W_{n_-}(+\infty, z) \, q(z) | \pi^+(p) \rangle}{\langle 0 | W_{n_-}^\dagger(+\infty, 0) W_u(+\infty, 0) \, [\text{tr. link}] \, W_{n_-}(+\infty, z) \, W_u^\dagger(+\infty, z) | 0 \rangle} .$$

Pinch singularity

- Singularity from Wilson line self energies [Bacchetta et al, 2008].
  - Pinch singularity only appears in a TMD parton density with $u^2 < 0$.
  - Pinch singularity appears in the TMD wave functions for any off-light-cone u.

$$\phi_\pi \supset \int [dl] \frac{u^2}{[l+i0)][u\cdot l + i0][u \cdot l - i0]} \times \delta(x' - x + l_+/p_+)\delta^{(2)}(\vec{k}'_T - \vec{k}_T + \vec{l}_T)\,.$$



  - Pinch singularity corresponds to the linear divergence in the length of the Wilson line in the coordinate space.

- Off-light-cone Wilson lines regularize rapidity divergence, at the price of introducing unwanted pinch singularity.

Collins' modification

- New definition without pinch singularity [Collins, 2011]:

$$\phi_\pi^C(x,\vec{k}_T,y_2,\mu) = \lim_{\substack{y_1\to+\infty \\ y_u\to-\infty}} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} e^{i(xp_+ z_- - \vec{k}_T\cdot\vec{z}_T)}$$
$$\times \langle 0|\bar{q}(0) W_u^\dagger(+\infty,0) \not{n}_- \gamma_5 \,[\text{tr. link}]\, W_u(+\infty,z)\, q(z)|\pi^+(p)\rangle$$
$$\times \sqrt{\frac{S(z_T;y_1,y_2)}{S(z_T;y_1,y_u)\,S(z_T;y_2,y_u)}}\,.$$

↑ rapidity of the gauge vector $n_2 = (e^{y_2}, e^{-y_2}, \vec{0}_T)$

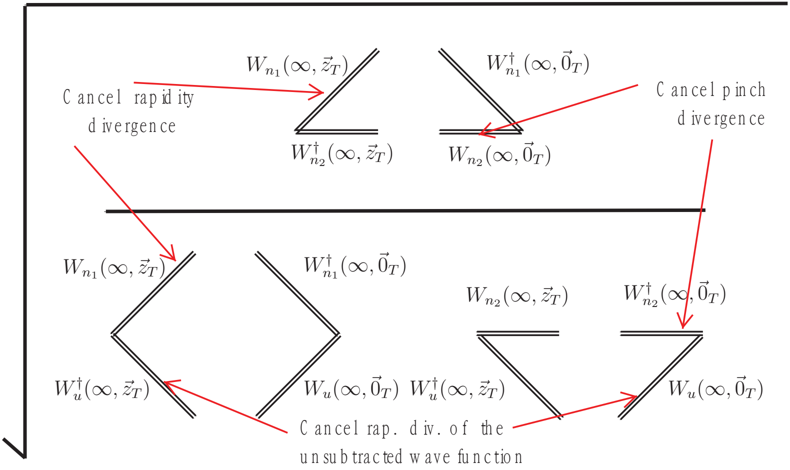
  - Soft function:

$$S(z_T;y_A,y_B) = \frac{1}{N_c}\langle 0|W_{n_B}^\dagger(\infty,\vec{z}_T)_{ca}\, W_{n_A}(\infty,\vec{z}_T)_{ad}\, W_{n_B}(\infty,0)_{bc}\, W_{n_A}^\dagger(\infty,0)_{db}|0\rangle.$$

- General properties of the new definition:
  - The unsubtracted wave function only involves light-cone Wilson lines.
  - Each soft factor has at most one off-light-cone Wilson line.
  - No rapidity divergences and no pinch singularities.

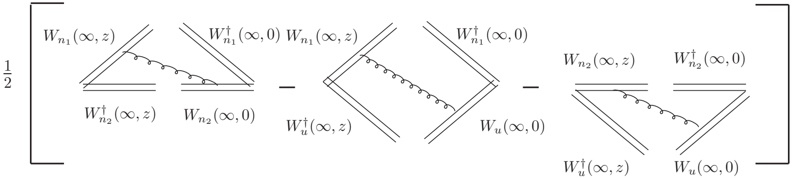
Why the new definition works?

- Cancellation mechanism ($y_1 \to +\infty$, $y_u \to -\infty$):

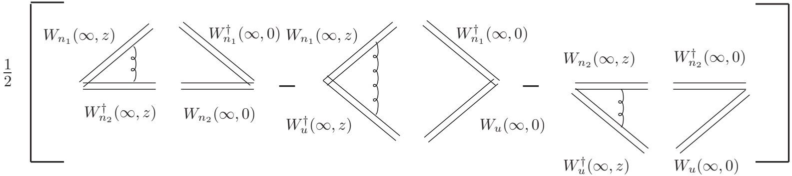


One-loop computations

- Soft function at one loop:



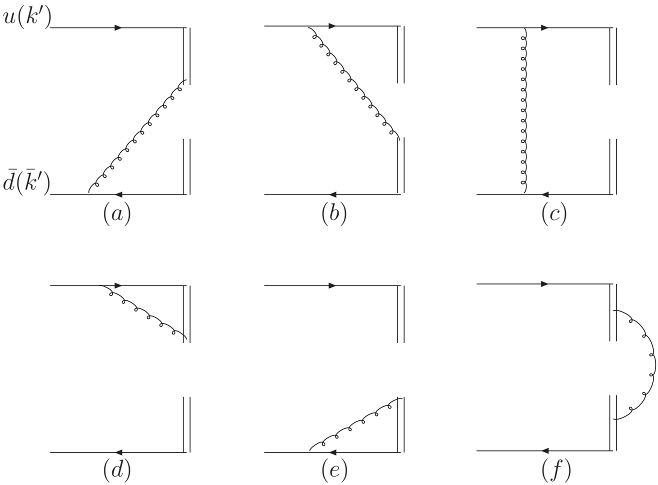
(a)



(b)

One-loop computations

- Unsubtracted TMD wave function at one loop:



One-loop computations

- Order of taking various limits [Collins, 2011]:
  - Compute the unsubtracted wave function and the soft function in D dimension.
  - Take the limits of infinite Wilson-line rapidities $y_1 \to +\infty$ and $y_u \to -\infty$.
  - Add the UV counterterms and remove the UV regulator, i.e., take $D \to 4$.

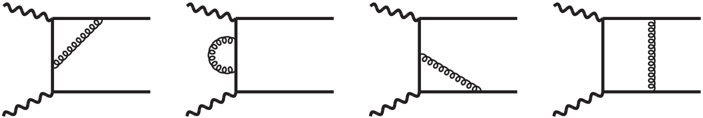
- Rapidity divergence cancels between the soft factor and the unsubtracted wave function:

$$
\begin{aligned}
\phi_{\pi,a}^{C(1)} + \phi_{\pi,b}^{C(1)} + S_a^{(1)} = -\frac{\alpha_s C_F}{2\pi} \Bigg[& 2 \ln \frac{\mu^2}{k_T^2} + \ln^2 \left(\frac{k_+}{m_g} \right) + \ln^2 \left(\frac{\bar{k}_+}{m_g} \right) \\
& + \ln \left(\frac{k_T^2}{m_g^2} \right) \cdot \ln \left(\frac{k_T^2}{k_+ \bar{k}_+} \right) - \ln \left(2\, e^{-2y_2} \right) \cdot \ln \left(\frac{k_+ \bar{k}_+}{m_g^2} \right) \Bigg] \\
& \times \delta(k_+ - k_+') \, \delta(\vec{k}_T - \vec{k}_T') + \text{finite terms} \, .
\end{aligned}
$$

- Soft divergence cancels between $\phi_{\pi,a}^{C(1)} + \phi_{\pi,b}^{C(1)} + S_a^{(1)}$ and $\phi_{\pi,d}^{C(1)} + \phi_{\pi,e}^{C(1)} + S_b^{(1)}$.

TMD factorization for $\gamma^* \gamma \to \pi^0$

- One-loop QCD diagrams [Nandi and Li, 2007]:



$$G^{(1)} = -\frac{\alpha_s C_F}{4\pi} [2 \ln x + 3] \ln \left(\frac{k_T^2}{Q^2} \right) H^{(0)}(x, k_T) + \cdots .$$

- $\phi_\pi^{C(1)} \otimes H^{(0)}$ reproduces the collinear logarithm of QCD diagrams:

$$\phi_\pi^{C(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{4\pi} [2 \ln x + 3] \ln \left(\frac{k_T^2}{Q^2} \right) H^{(0)}(x, k_T) + \cdots .$$

- Rapidity evolution equation [soft K function]:

$$\frac{d}{dy_2} \ln \phi_\pi^C(x, \vec{k}_T, y_2, \mu) = \underbrace{\frac{\alpha_s C_F}{\pi} \left[\ln \left(\frac{x \bar{x} p_+^2}{\mu^2} \right) + 2 y_2 \right]}_{\text{only from the soft subtraction}} .$$

Simplified definitions of TMDs possible?

- Treatment of rapidity and pinch singularities:

Light-cone Wilson lines

$\downarrow$ TMD hard function

Rapidity divergence

$\downarrow$

Off-light-cone Wilson lines

$\downarrow$ Dipolar structure

Pinch singularity

$\swarrow$ $\searrow$

Nontrivial soft subtraction | Non-dipolar Wilson lines

$\downarrow$

Simper soft subtraction

TMDs with non-dipolar Wilson lines

- New definition:

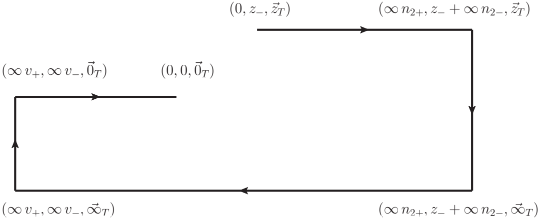
$$\phi_\pi^{\rm new}(x,\vec{k}_T,y_2,\mu) = \int \frac{dz_-}{2\pi}\int\frac{d^2z_T}{(2\pi)^2}\,e^{i(xp_+z_- -\vec{k}_T\cdot\vec{z}_T)} \times \frac{\langle 0|\bar{q}(0)W^\dagger_{n_2}(+\infty,0)\,\not{n}_-\,\gamma_5\,[\text{links}@\infty]\,W_v(+\infty,z)\,q(z)|\pi^+(p)\rangle}{[\text{color}]\,\langle 0|W^\dagger_{n_2}(+\infty,0)\,[\text{links}@\infty]\,W_v(+\infty,0)|0\rangle}\,.$$

$$n_2=(e^{y_2},e^{-y_2},\vec{0}_T)\,,\qquad v=(e^{y_v},e^{-y_v},\vec{0}_T)\,.$$

  - Key point: pinched divergence alleviated to soft divergence.

$$\phi_\pi^{\rm new}\supset\int[dl]\frac{n_2\cdot v}{[l+i0)][n_2\cdot l+i0][v\cdot l-i0]}\,\delta(x'-x+l_+/p_+)\,\delta^{(2)}(\vec{k}'_T-\vec{k}_T+\vec{l}_T)\,.$$

  - Wilson-line path of the unsubtracted wave function:



TMDs with non-dipolar Wilson lines

- Orthogonal Wilson lines ($n_2 \cdot v = 0$):

$$\phi_\pi^{\rm I}(x, \vec{k}_T, y_2, \mu) = \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2}\, e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)} \times \langle 0 | \bar{q}(0)\, W_{n_2}^\dagger(+\infty, 0)\, \not{n}_-\, \gamma_5\, [\text{links}@\infty]\, W_v(+\infty, z)\, q(z) | \pi^+(p) \rangle \,.$$

$$\uparrow \qquad\qquad \uparrow$$

$$n_2 = (e^{y_2}, e^{-y_2}, \vec{0}_T)\,, \qquad v = (-e^{y_2}, e^{-y_2}, \vec{0}_T)\,.$$

  - Wilson line self energies vanish in Feynman gauge.
    $\Rightarrow$ Soft subtraction not needed.
  - $\phi_\pi^{\rm I} \otimes H^{(0)}$ reproduces the collinear logarithm of QCD diagrams:

$$\phi_\pi^{\rm I} \otimes H^{(0)} = -\frac{\alpha_s C_F}{4\pi}\, [2 \ln x + 3]\, \ln\left(\frac{k_T^2}{Q^2}\right) H^{(0)}(x, k_T) + \cdots \,.$$

- Antiparallel Wilson lines:

$$\phi_\pi^{\rm II}(x, \vec{k}_T, y_2, \mu) = \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2}\, e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)} \times \frac{\langle 0 | \bar{q}(0)\, W_{n_2}^\dagger(+\infty, 0)\, \not{n}_-\, \gamma_5\, [\text{links}@\infty]\, W_{n_2}(-\infty, z)\, q(z) | \pi^+(p) \rangle}{[\text{color}]\, \langle 0 | W_{n_2}^\dagger(+\infty, 0)\, [\text{links}@\infty]\, W_{n_2}(-\infty, 0) | 0 \rangle} \,.$$

Equivalence of TMD definitions

- Recover the naive definition in the limit of vanishing regulators.
  - Collins' definition ($y_2 \to -\infty \Rightarrow y_2 = y_u$):

$$\sqrt{\frac{S(z_T; y_1, y_2)}{S(z_T; y_1, y_u)\, S(z_T; y_2, y_u)}} \to 1\,.$$

  - Our definition ($y_2 \to -\infty \Rightarrow n_2 = v = n_-$):

$$\phi_\pi^{\mathrm{I}}(x, \vec{k}_T, y_2, \mu) \to \phi_\pi^{\mathrm{naive}}(x, \vec{k}_T, \mu)\,.$$

  - The same collinear divergence in the limit $y_2 = y_u \to -\infty$.

- Aim: To show that the collinear behaviors for an arbitrary rapidity y_2 are the same.
  $\Rightarrow$ Equivalent rapidity evolution equations.

Rapidity evolution equation of ϕ_π^C

- ϕ_π^C depends on the Lorenz invariant $(n_2 \cdot p)^2/(n_2^2 k^2)$.
- Chain rule for the rapidity derivative:

$$\frac{d}{dy_2}\phi_\pi^C = \frac{n_2^2}{2p\cdot n_2} p^\alpha \frac{d}{dn_2^\alpha}\phi_\pi^C .$$

- Rapidity derivative of the Wilson-line Feynman rule:

$$\frac{n_2^2}{2p\cdot n_2} p^\alpha \frac{d}{dn_2^\alpha}\frac{n_2^\mu}{n_2\cdot l} = \frac{\hat{n}_2^\mu}{n_2\cdot l}, \qquad \underset{\uparrow}{\hat{n}_2^\mu} = \frac{n_2^2}{2p\cdot n_2}\left(p^\mu - \frac{p\cdot l}{n_2\cdot l} n_2^\mu\right) .$$

suppress the collinear dynamics

- Rapidity evolution equation:

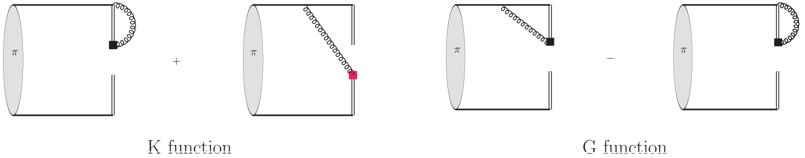
$$\frac{d}{dy_2}\phi_\pi^C = \lim_{\substack{y_1\to+\infty \\ y_u\to-\infty}} \frac{1}{2}\left[\underbrace{\frac{S'(z_T;y_1,y_2)}{S(z_T;y_1,y_2)}}_{K(z_T;y_1,y_2)} - \underbrace{\frac{S'(z_T;y_2,y_u)}{S(z_T;y_2,y_u)}}_{K(z_T;y_2,y_u)\,.}\right]\phi_\pi^C \approx \lim_{y_1\to+\infty} K(z_T;y_1,y_2)\,\phi_\pi^C .$$

Rapidity evolution equation of $\phi_\pi^{\rm I}$

- $\phi_\pi^{\rm I}$ depends on the Lorenz invariants $(n_2 \cdot p)^2/(n_2^2 k^2)$ and $(v \cdot p)^2/(v_2^2 k^2)$.
- Rapidity derivative:

$$\frac{d}{dy_2} \phi_\pi^{\rm I} \equiv \left[\frac{n_2^2}{2p \cdot n_2} p^\alpha \frac{d}{dn_2^\alpha} + \frac{v^2}{2p \cdot v} p^\alpha \frac{d}{dv^\alpha} \right] \phi_\pi^{\rm I} .$$

- Both special vertices $\hat{n}_2^\mu$ and $\hat{v}^\mu$ suppress the collinear dynamics.
- Both soft and hard gluon radiations contribute to the rapidity evolution kernel.



- Rapidity evolution equation:

$$\frac{d}{dy_2} \phi_\pi^{\rm I} = \lim_{y_1 \to +\infty} \left[K(z_T; y_1, y_2) + G(y_2) \right] \phi_\pi^{\rm I} .$$

↑ independent of z_T

Summary and outlook

- TMD factorization for hard exclusive processes non-trivial.
  - Infrared subtraction can induce additional singularities.
  - Remove unwanted rapidity and pinch divergences.
- New proposal for the definition of TMD pion wave functions.
  - Non-dipolar Wilson lines with simper soft subtraction.
  - Reproduce the collinear dynamics of full QCD diagrams for $\gamma^* \gamma \to \pi^0$.
  - Equivalent rapidity evolution equations.
- Future developments:
  - Phenomenological applications in demand.
  - Extensions of the new definition to more complicated processes.
  - Systematic power counting scheme, factorization at operator level.