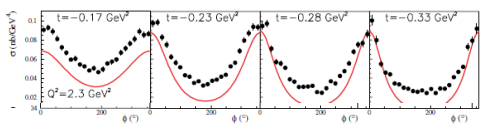


POETIC6, 07/09/2015

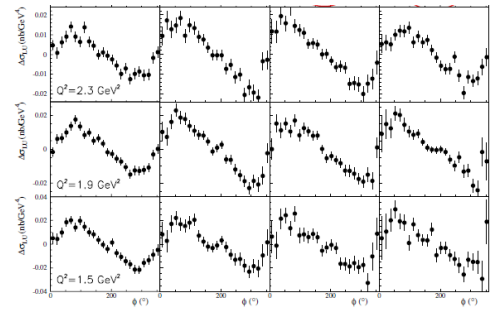
Fits of Compton Form Factors

Michel Guidal (IPN Orsay)

# JLab Hall A

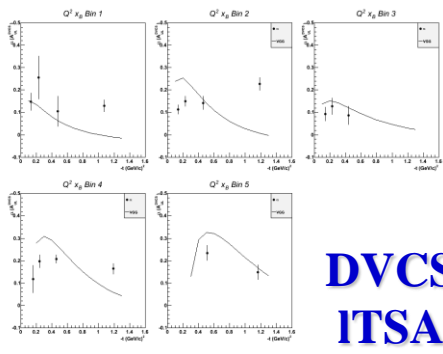
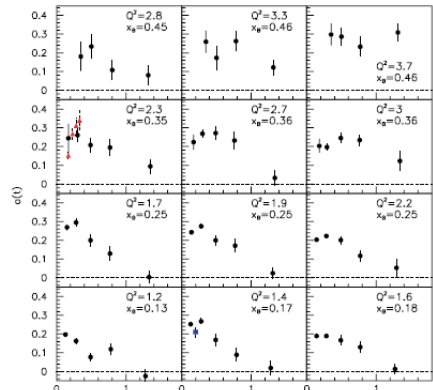


**DVCS**  
unpol. X-section

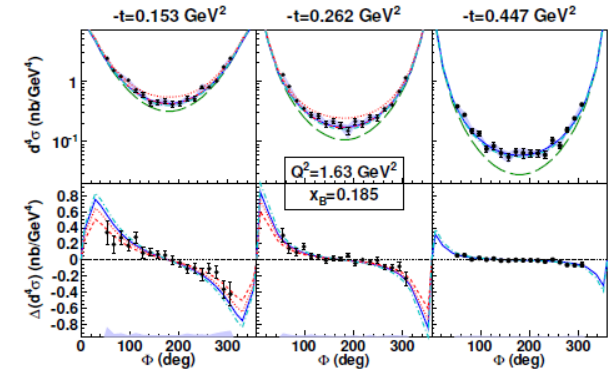


**DVCS**  
B-pol. X-section

# JLab CLAS DVCS BSA

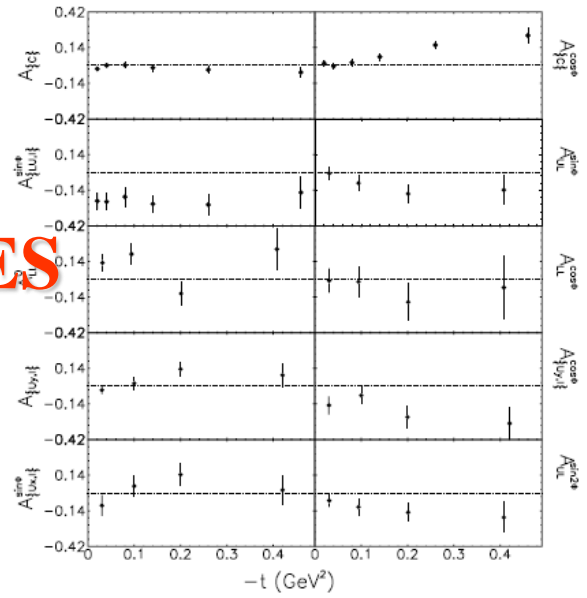


**DVCS**  
ITSA

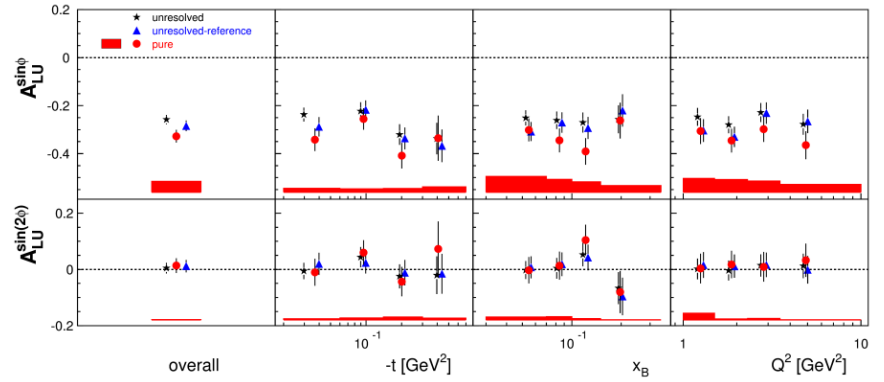


**DVCS unpol. and**  
B-pol. X-sections

# HERMES

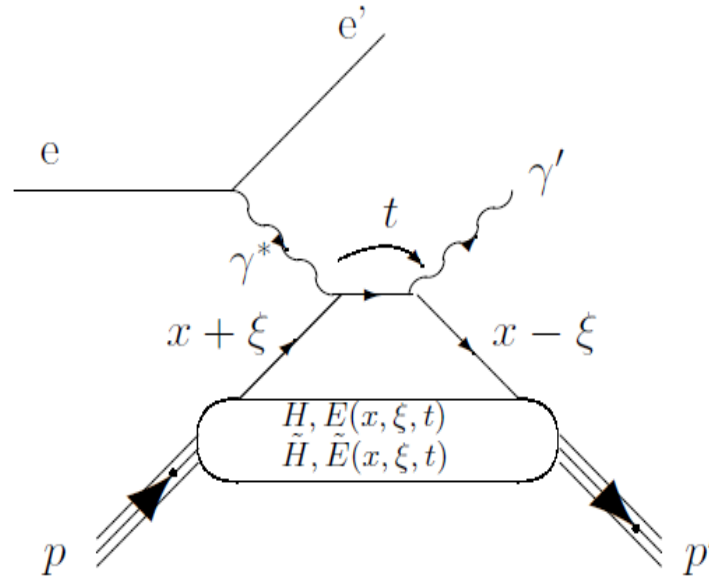


**DVCS**  
BSA, ITSA, tTSA, BCA



**DVCS**  
BSA, ITSA, tTSA, BCA

$H^q(\mathbf{x}, \xi, t)$  but only  $\xi$  and  $t$  experimentally accessible



$$T^{DVCS} \sim \int_{-1}^{+1} \frac{H(x, \xi, t)}{x \pm \xi + i\epsilon} dx + \dots \sim P \int_{-1}^{+1} \frac{H(x, \xi, t)}{x \pm \xi} dx - i\pi H(\pm \xi, \xi, t) + \dots$$

# In general, **8** GPD quantities accessible (sub-)Compton Form Factors

$$H_{\text{Re}}(\xi, t) \equiv \mathcal{P} \int_0^1 dx [H(x, \xi, t) - H(-x, \xi, t)] C^+(x, \xi)$$

$$E_{\text{Re}}(\xi, t) \equiv \mathcal{P} \int_0^1 dx [E(x, \xi, t) - E(-x, \xi, t)] C^+(x, \xi)$$

$$\tilde{H}_{\text{Re}}(\xi, t) \equiv \mathcal{P} \int_0^1 dx [\tilde{H}(x, \xi, t) + \tilde{H}(-x, \xi, t)] C^-(x, \xi)$$

$$\tilde{E}_{\text{Re}}(\xi, t) \equiv \mathcal{P} \int_0^1 dx [\tilde{E}(x, \xi, t) + \tilde{E}(-x, \xi, t)] C^-(x, \xi)$$

$$H_{\text{Im}}(\xi, t) \equiv H(\xi, \xi, t) - H(-\xi, \xi, t)$$

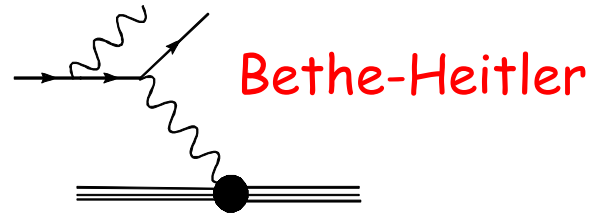
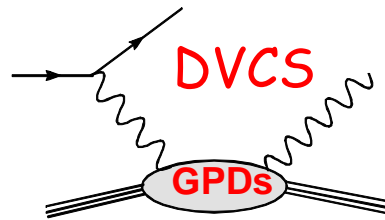
$$E_{\text{Im}}(\xi, t) \equiv E(\xi, \xi, t) - E(-\xi, \xi, t)$$

$$\tilde{H}_{\text{Im}}(\xi, t) \equiv \tilde{H}(\xi, \xi, t) + \tilde{H}(-\xi, \xi, t)$$

$$\tilde{E}_{\text{Im}}(\xi, t) \equiv \tilde{E}(\xi, \xi, t) + \tilde{E}(-\xi, \xi, t)$$

**with**  $C^\pm(x, \xi) = \frac{1}{x - \xi} \pm \frac{1}{x + \xi}$

Given the well-established **LT-LO DVCS+BH** amplitude



Can one recover the **8 CFFs** from the DVCS observables?

$$\text{Obs} = \text{Amp}(\text{DVCS+BH}) \otimes \text{CFFs}$$

Two (quasi-) model-independent approaches to extract, at fixed  $x_B$ ,  $t$  and  $Q^2$  (« local » fitting), the CFFs from the DVCS observables

# 1/ Mapping and linearization

If enough observables measured, one has a system of 8 equations with 8 unknowns

Given reasonable approximations (leading-twist dominance, neglect of some  $1/Q^2$  terms,...), the system can be linear (practical for the error propagation)

$$\begin{pmatrix} A_{LU,I}^{\sin(1\phi)} \\ A_{UL,+}^{\sin(1\phi)} \\ A_{UT,I}^{\sin(\varphi)\cos(1\phi)} \\ A_{UT,I}^{\cos(\varphi)\sin(1\phi)} \end{pmatrix} \Rightarrow \Im \begin{pmatrix} \mathcal{H} \\ \tilde{\mathcal{H}} \\ \mathcal{E} \\ \bar{\mathcal{E}} \end{pmatrix}, \quad \begin{pmatrix} A_C^{\cos(1\phi)} \\ A_{LL,+}^{\cos(1\phi)} \\ A_{LT,I}^{\sin(\varphi)\sin(1\phi)} \\ A_{LT,I}^{\cos(\varphi)\cos(1\phi)} \end{pmatrix} \Rightarrow \Re \begin{pmatrix} \mathcal{H} \\ \tilde{\mathcal{H}} \\ \mathcal{E} \\ \bar{\mathcal{E}} \end{pmatrix}$$

$$\Delta\sigma_{LU} \sim \sin\phi \operatorname{Im}\{F_1\mathcal{H} + \xi(F_1+F_2)\tilde{\mathcal{H}} - kF_2\mathcal{E}\}d\phi$$

$$\Delta\sigma_{UL} \sim \sin\phi \operatorname{Im}\{F_1\tilde{\mathcal{H}} + \xi(F_1+F_2)(\mathcal{H} + x_B/2\mathcal{E}) - \xi kF_2\bar{\mathcal{E}} + \dots\}d\phi$$

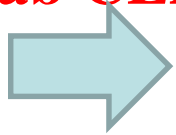
## 2/ «Brute force » fitting

$\chi^2$  minimization (with **MINUIT + MINOS**) of the available DVCS observables at a given  $\mathbf{x}_B$ ,  $t$  and  $Q^2$  point by varying the CFFs within a limited hyper-space (e.g. **5xVGG**)

The problem can be (largely) underconstrained:

**JLab Hall A:** pol. and unpol. X-sections

**JLab CLAS:** BSA + TSA



2 constraints and 8 parameters !

However, as some observables are largely dominated by a single or a few CFFs, there is a convergence (i.e. a well-defined minimum  $\chi^2$ ) for these latter CFFs.

The contribution of the non-converging CFF enters in the error bar of the converging ones. For instance (naive):

$$3 = y + 0.001x \quad \text{If } -10 < x < 10: \quad 3 = y \pm 0.01 \quad (\text{or } y = 3 \pm 0.01)$$

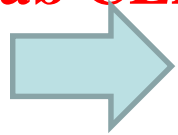
## 2/ «Brute force » fitting

$\chi^2$  minimization (with **MINUIT + MINOS**) of the available DVCS observables at a given  $\mathbf{x}_B$ ,  $t$  and  $Q^2$  point by varying the CFFs within a limited hyper-space (e.g. **5xVGG**)

The problem can be (largely) underconstrained:

**JLab Hall A:** pol. and unpol. X-sections

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2 constraints and 8 parameters !

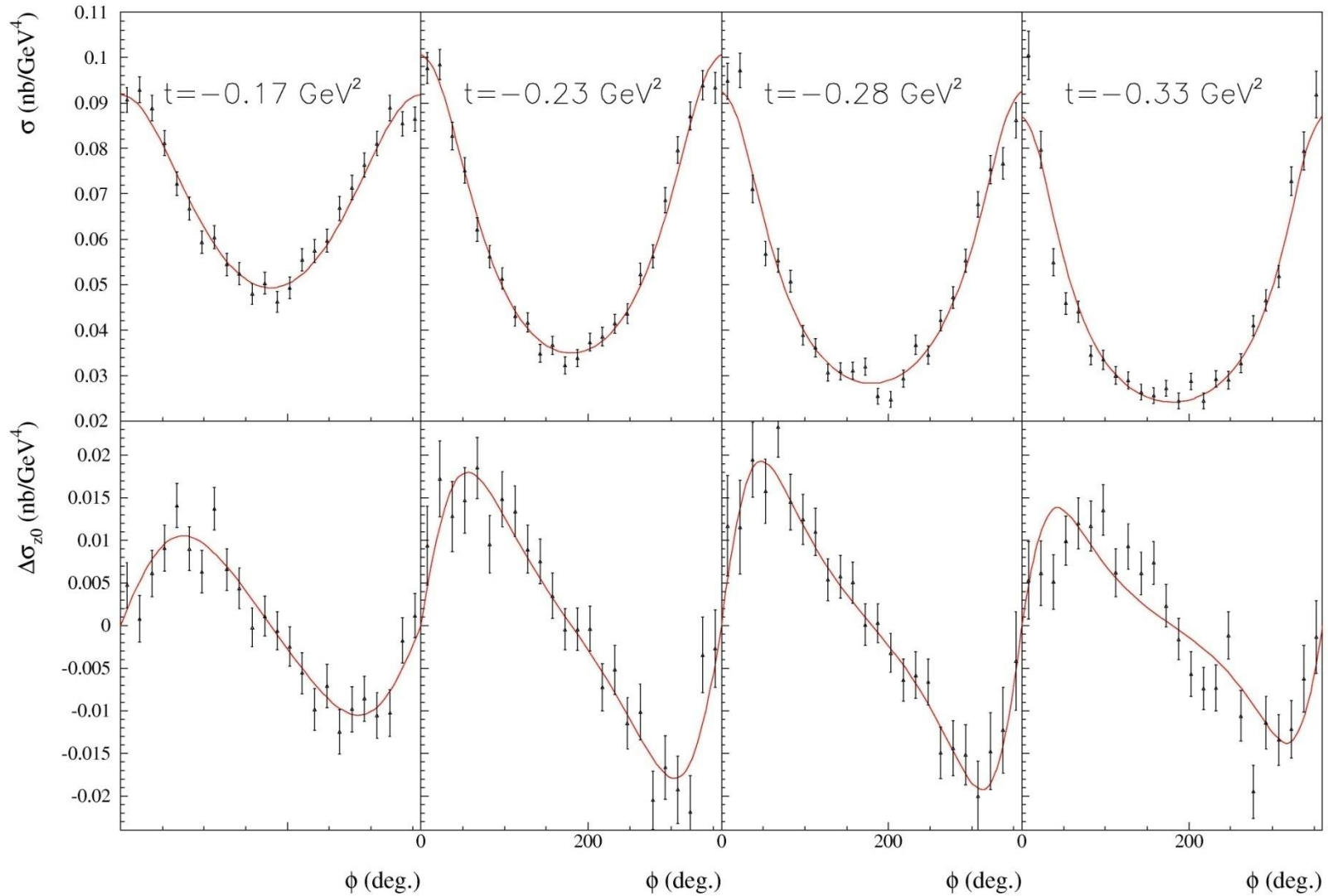
However, as some observables are largely dominated by a single or a few CFFs, there is a convergence (i.e. a well-defined minimum  $\chi^2$ ) for these latter CFFs.

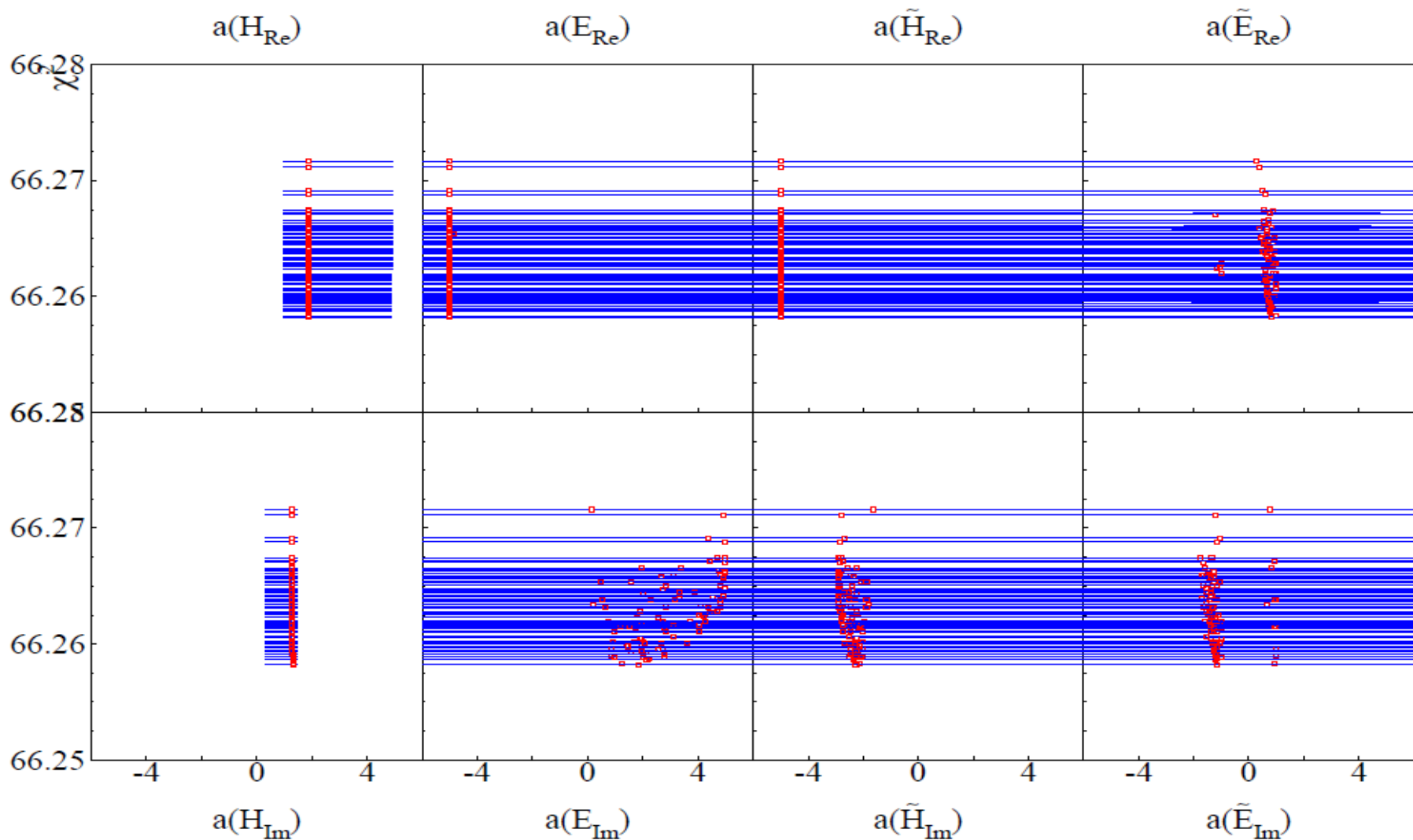
The contribution of the non-converging CFF enters in the error bar of the converging ones.



# Hall A : $\sigma$ & $\Delta\sigma_{LU}$ , $x_B=0.36, Q^2=2.3, t=.17,.23,.28,.33$

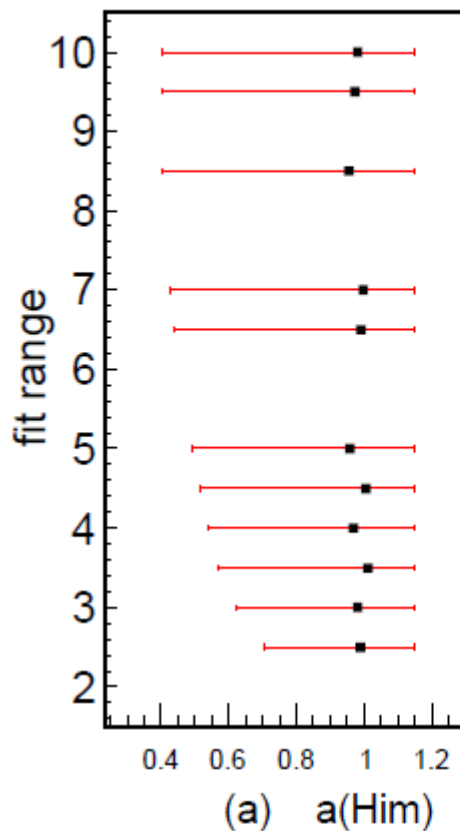
$E_e=5.75$  GeV,  $x_B=0.36, Q^2=2.3$  GeV<sup>2</sup>





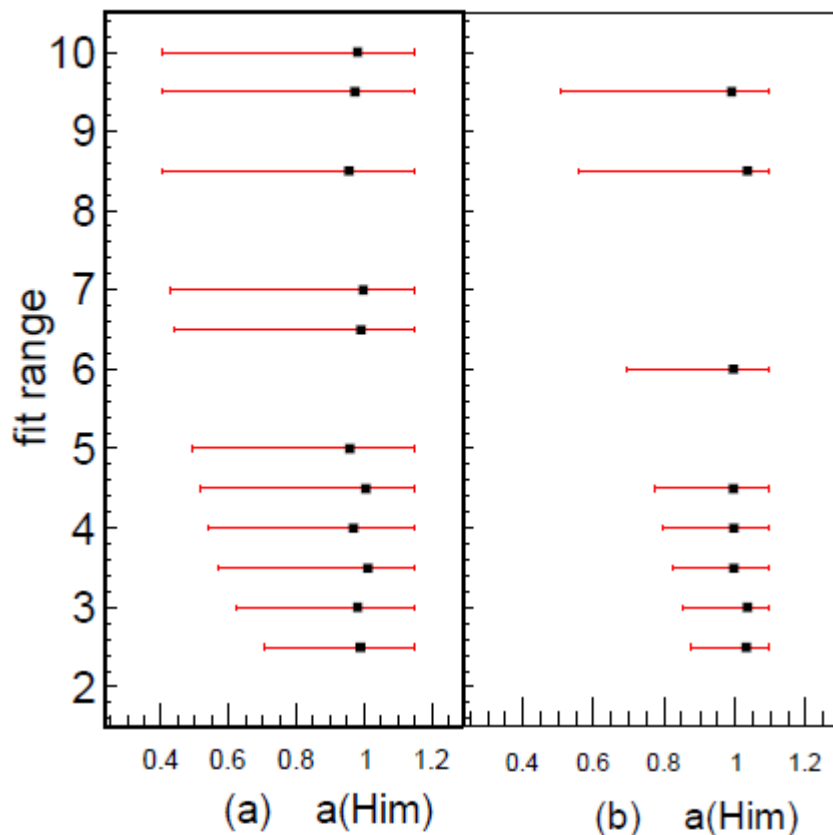
**M. Boer, MG J.Phys. G42 (2015) 3, 034023**

**Figure 3.** Result of the fits for the 8 CFF multipliers  $a(\dots)$  (central value in red and error bar in blue on the  $x$ -axis) and the corresponding  $\chi^2$  value ( $y$ -axis), for the first tens of several hundreds of trials differing only by the starting values of the 8 CFF multipliers. This example is for the third  $t$ -bin of the JLab Hall A data.



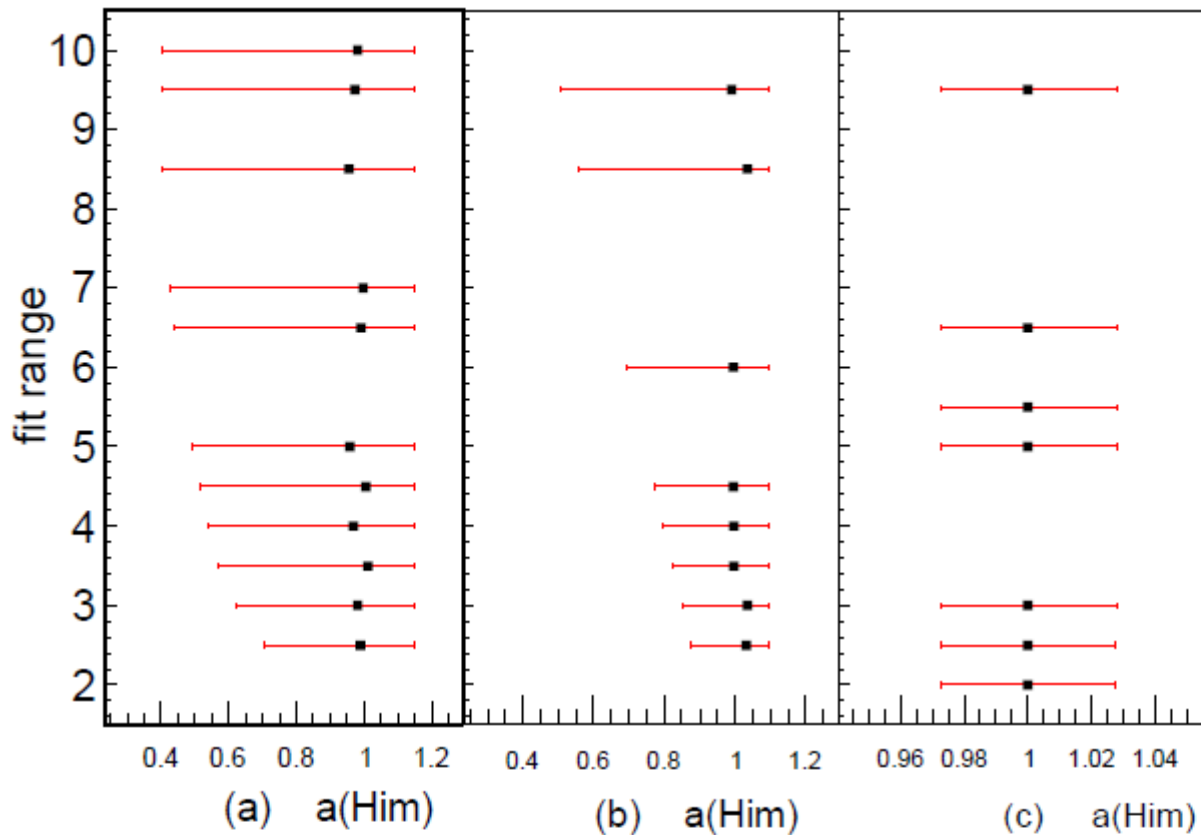
### M. Boer, MG J.Phys. G42 (2015) 3, 034023

Figure 5. Results of fits based on simulations for the CFF multiplier  $a(H_{Im})$  and its error bar ( $x$ -axis) for different maximum ranges of the domain of variation allowed for the CFFs ( $y$ -axis, in units of VGG CFFs) and for different sets of observables fitted: (a) the unpolarized cross section  $\sigma$  and the beam-polarized cross section  $\Delta\sigma_{LU}$  (like the Hall A data), (b)  $\sigma$ ,  $\Delta\sigma_{LU}$  and  $\Delta\sigma_{Uz}$ , (c)  $\sigma$ ,  $\Delta\sigma_{LU}$ ,  $\Delta\sigma_{Uz}$ ,  $\Delta\sigma_{Ux}$  and  $\Delta\sigma_{Uy}$ , (d)  $\sigma$  and all single and double polarization observables. The pseudo-data that were fitted were generated with the VGG values, so that the multipliers of all CFFs should be 1. In this example, the kinematics is approximately similar to the JLab Hall A data:  $x_i=0.2$ ,  $Q^2=3 \text{ GeV}^2$  and  $-t=0.4 \text{ GeV}^2$ .



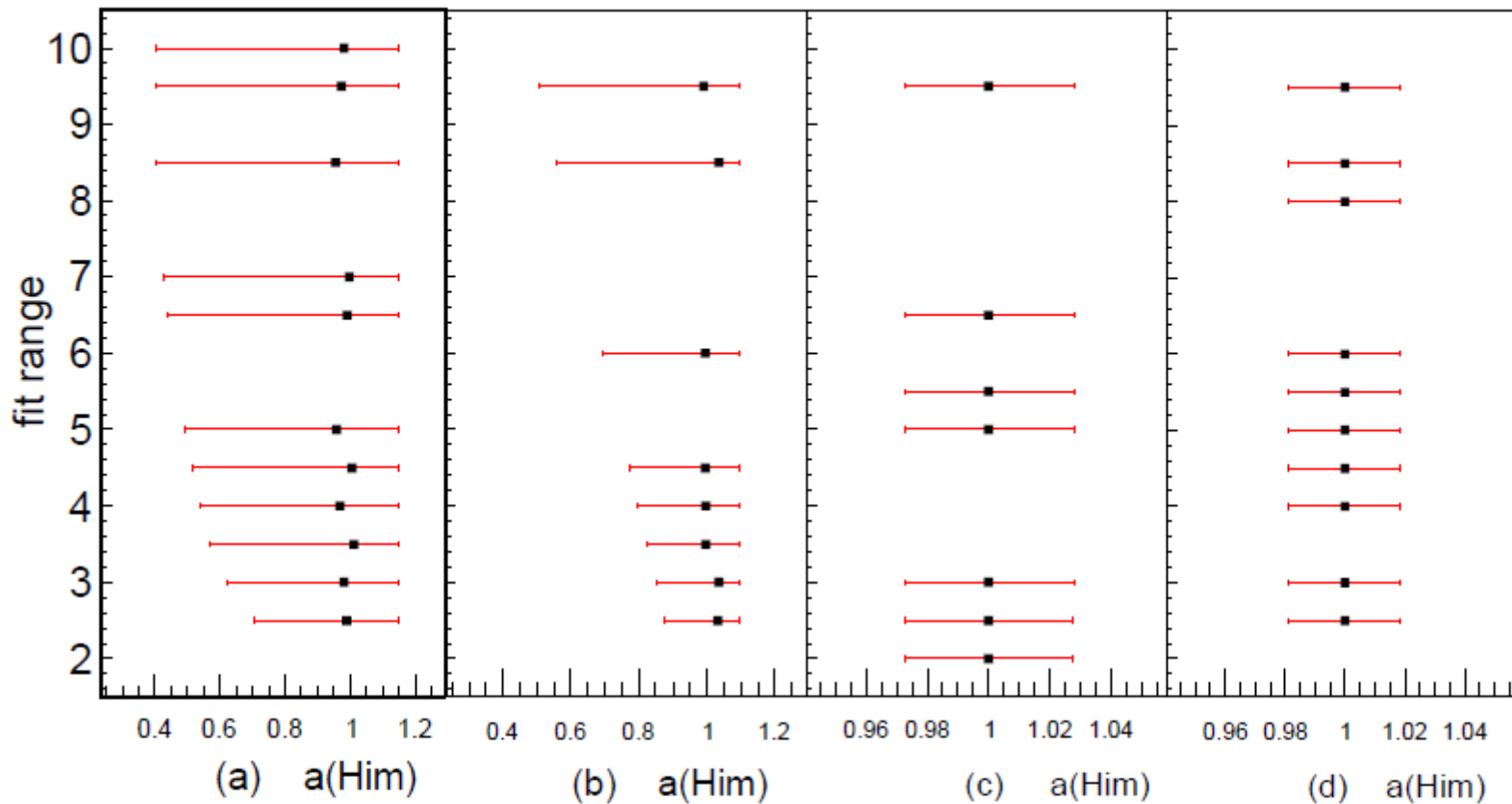
**M. Boer, MG J.Phys. G42 (2015) 3, 034023**

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**M. Boer, MG J.Phys. G42 (2015) 3, 034023**

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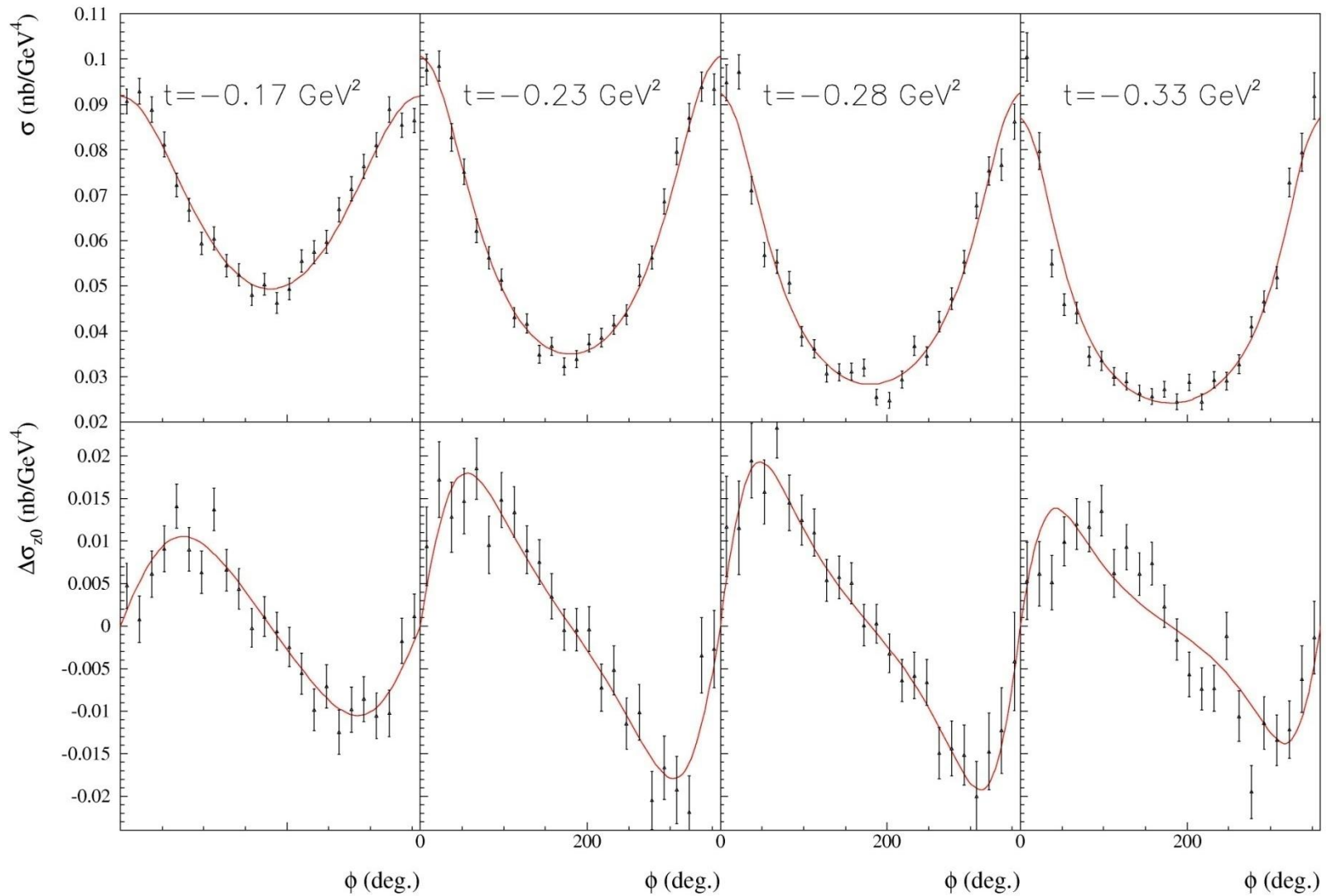


**M. Boer, MG J.Phys. G42 (2015) 3, 034023**

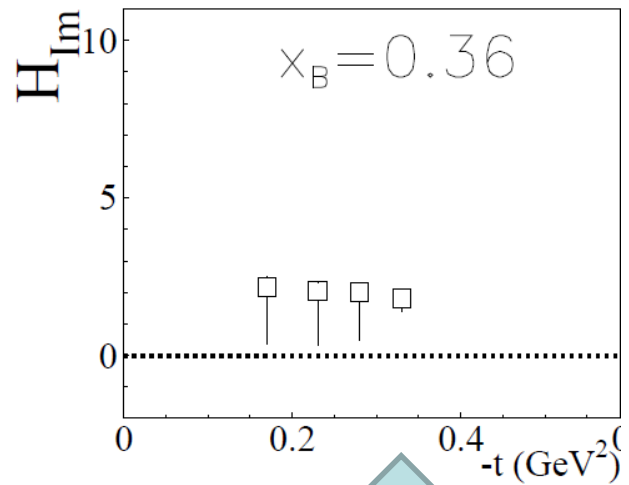
Figure 5. Results of fits based on simulations for the CFF multiplier  $a(H_{Im})$  and its error bar ( $x$ -axis) for different maximum ranges of the domain of variation allowed for the CFFs ( $y$ -axis, in units of VGG CFFs) and for different sets of observables fitted: (a) the unpolarized cross section  $\sigma$  and the beam-polarized cross section  $\Delta\sigma_{LU}$  (like the Hall A data), (b)  $\sigma$ ,  $\Delta\sigma_{LU}$  and  $\Delta\sigma_{Uz}$ , (c)  $\sigma$ ,  $\Delta\sigma_{LU}$ ,  $\Delta\sigma_{Uz}$ ,  $\Delta\sigma_{Ux}$  and  $\Delta\sigma_{Uy}$ , (d)  $\sigma$  and all single and double polarization observables. The pseudo-data that were fitted were generated with the VGG values, so that the multipliers of all CFFs should be 1. In this example, the kinematics is approximately similar to the JLab Hall A data:  $x_i=0.2$ ,  $Q^2=3 \text{ GeV}^2$  and  $-t=0.4 \text{ GeV}^2$ .

# Hall A : $\sigma$ & $\Delta\sigma_{LU}$ , $x_B=0.36, Q^2=2.3, t=.17,.23,.28,.33$

$E_e=5.75$  GeV,  $x_B=0.36, Q^2=2.3$  GeV<sup>2</sup>



# JLab Hall A



**unpol.sec.eff.**

+

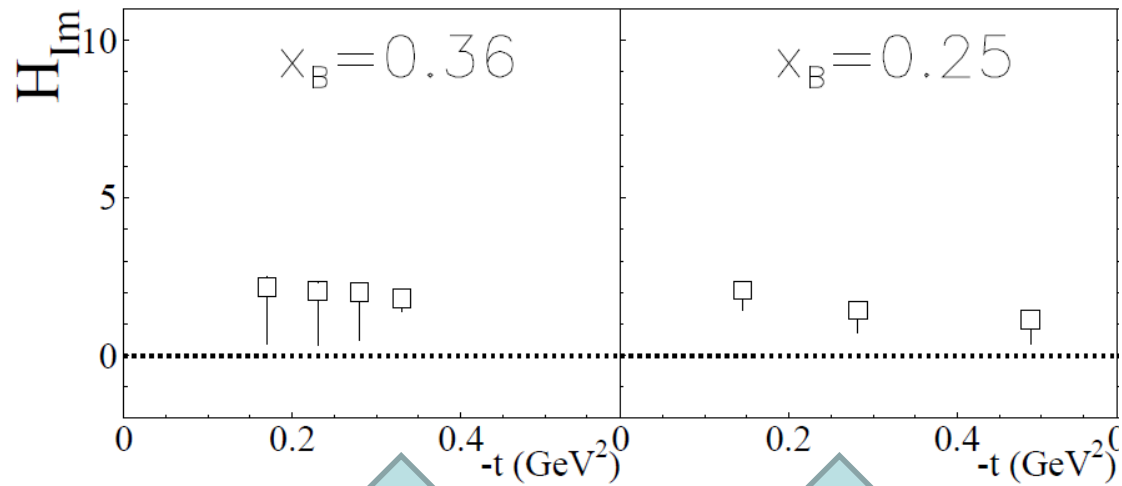
**beam pol.sec.eff.**

□  $\chi^2$  minimization



JLab Hall A

JLab CLAS



**unpol.sec.eff.**

**beam spin asym.**

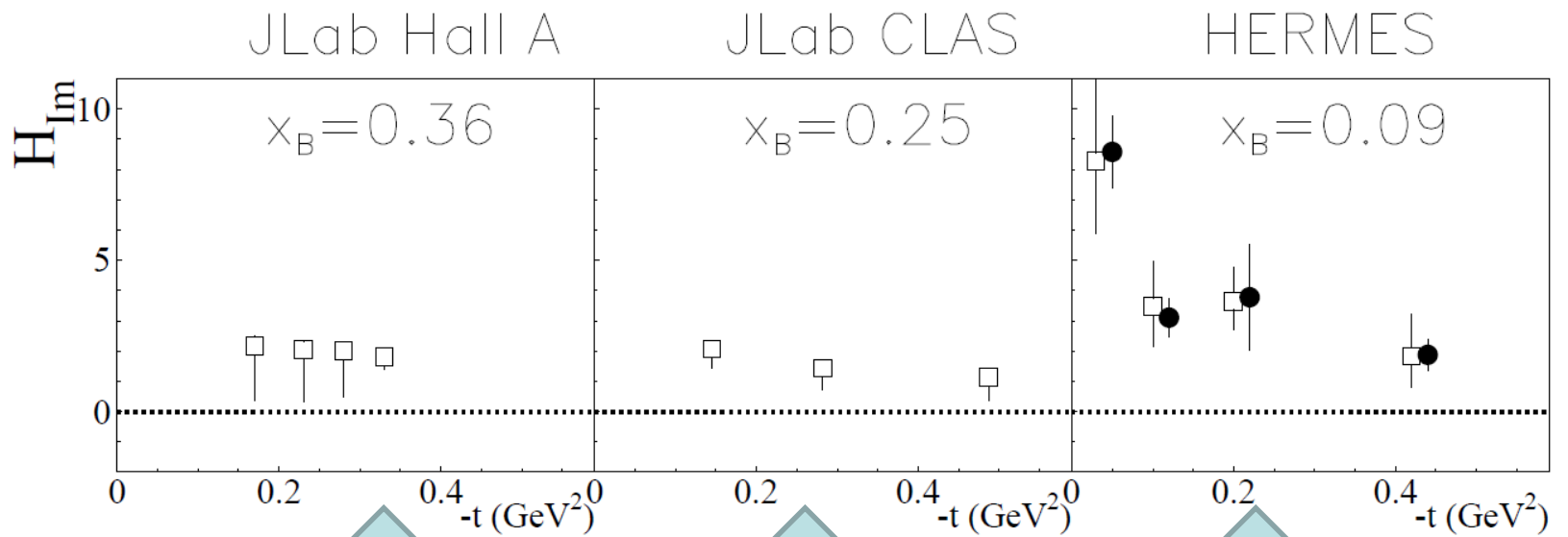
+

+

**beam pol.sec.eff.**

**long. pol. tar. asym**

□  $\chi^2$  minimization



**unpol.sec.eff.**

+

**beam pol.sec.eff.**

**beam spin asym.**

+

**long. pol. tar. asym**

**beam charge asym.**

+

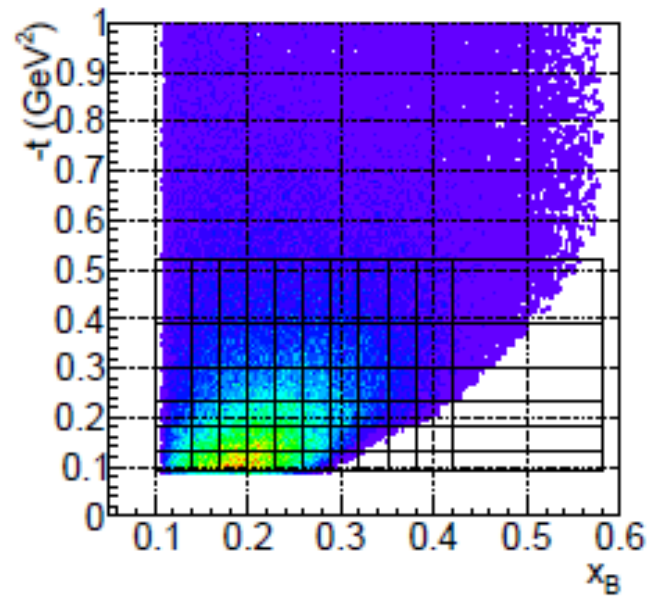
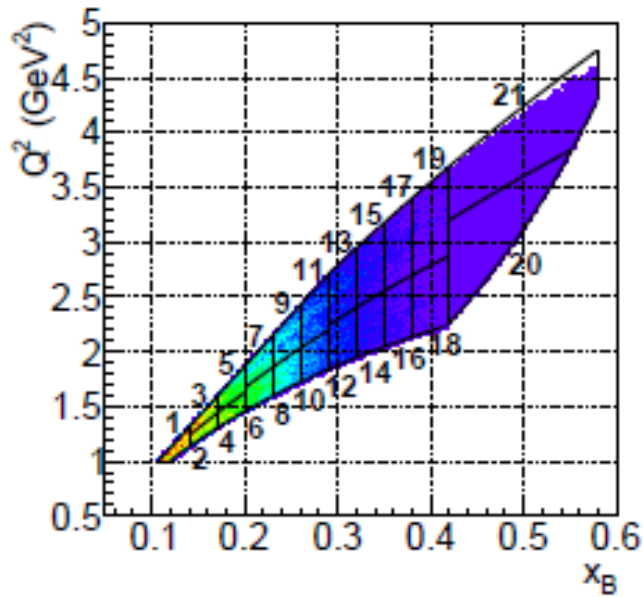
**beam spin asym**

+

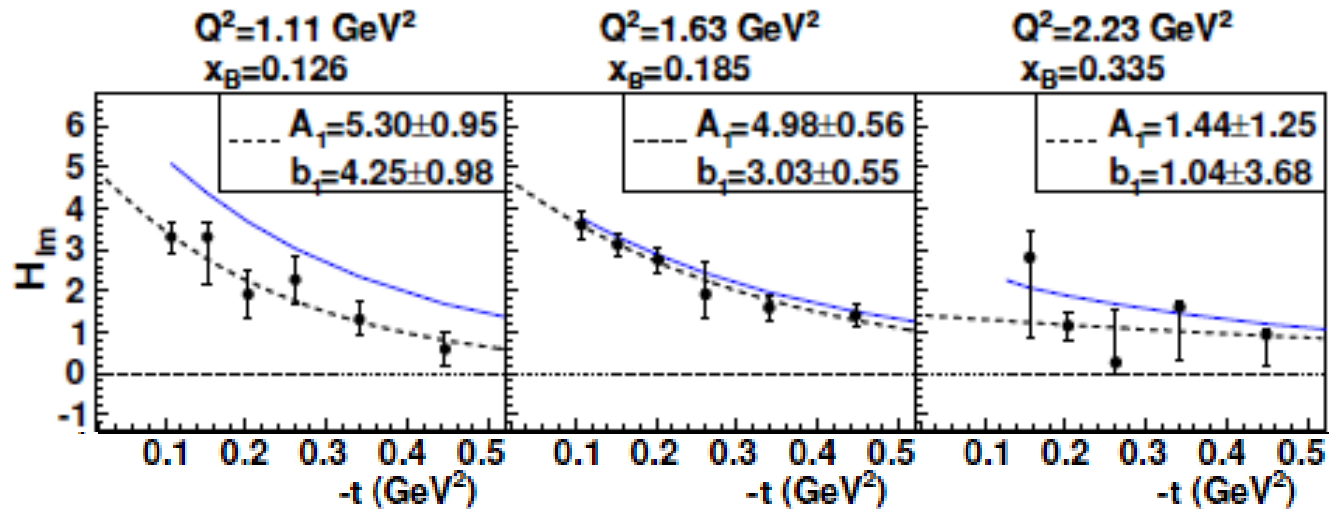
...

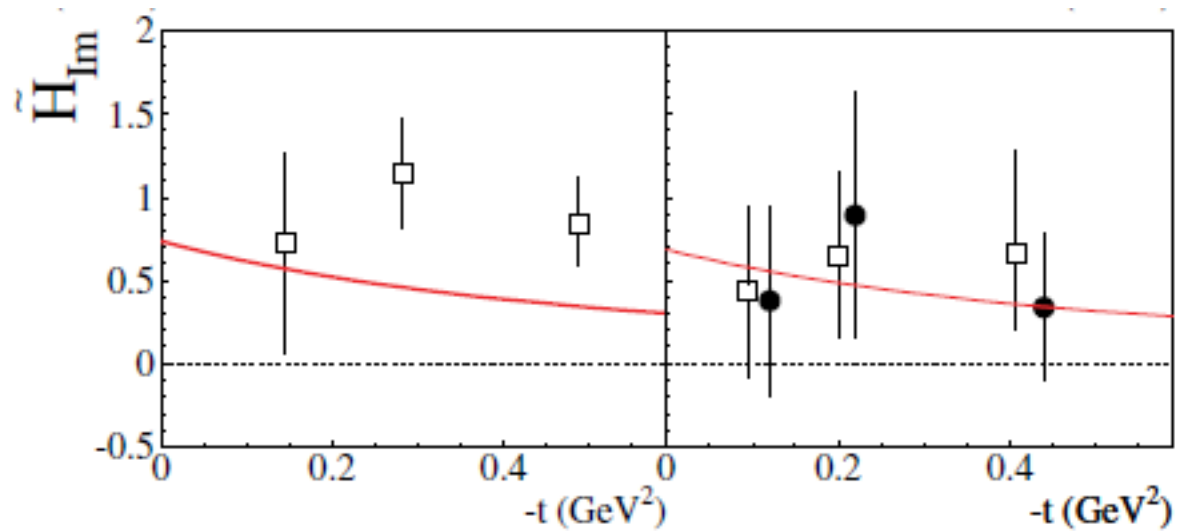
□  $\chi^2$  minimization

● linearization



[CLAS coll. arXiv:1504.02009](https://arxiv.org/abs/1504.02009) [hep-ex]

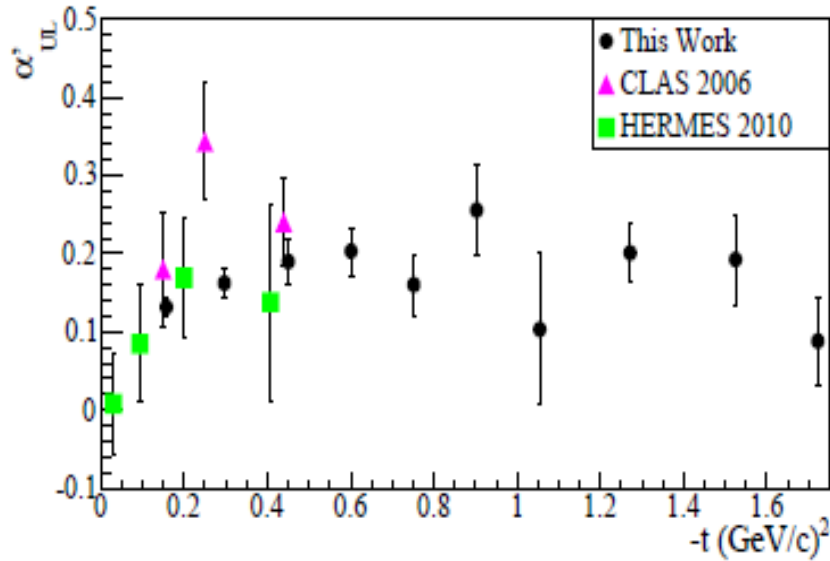




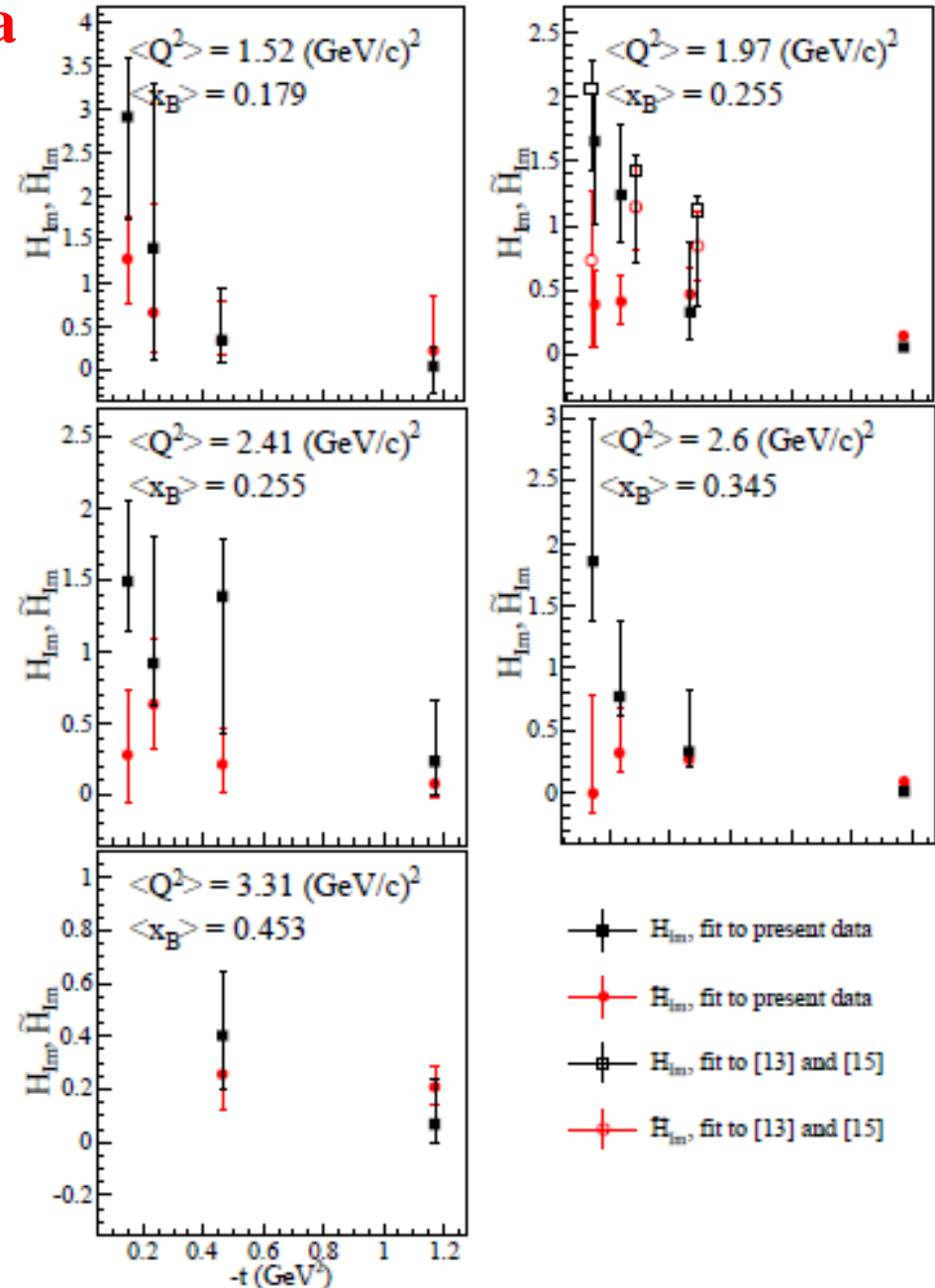
The **axial** charge ( $\sim \tilde{H}_{im}$ ) appears to be more « concentrated » than the **electromagnetic** charge ( $\sim H_{im}$ )

# New A\_UL and A\_LL CLAS data

Phys.Rev. D91 (2015) 5, 052014



Confirms the apparent  
 «  $x_B$ -independence » and  
 flatter «  $t$ -dependence » of  $\tilde{H}_{Im}$



# From CFFs to spatial densities

How to go from momentum coordinates (**t**)  
to space-time coordinates (**b**) ?  
(with error propagation)

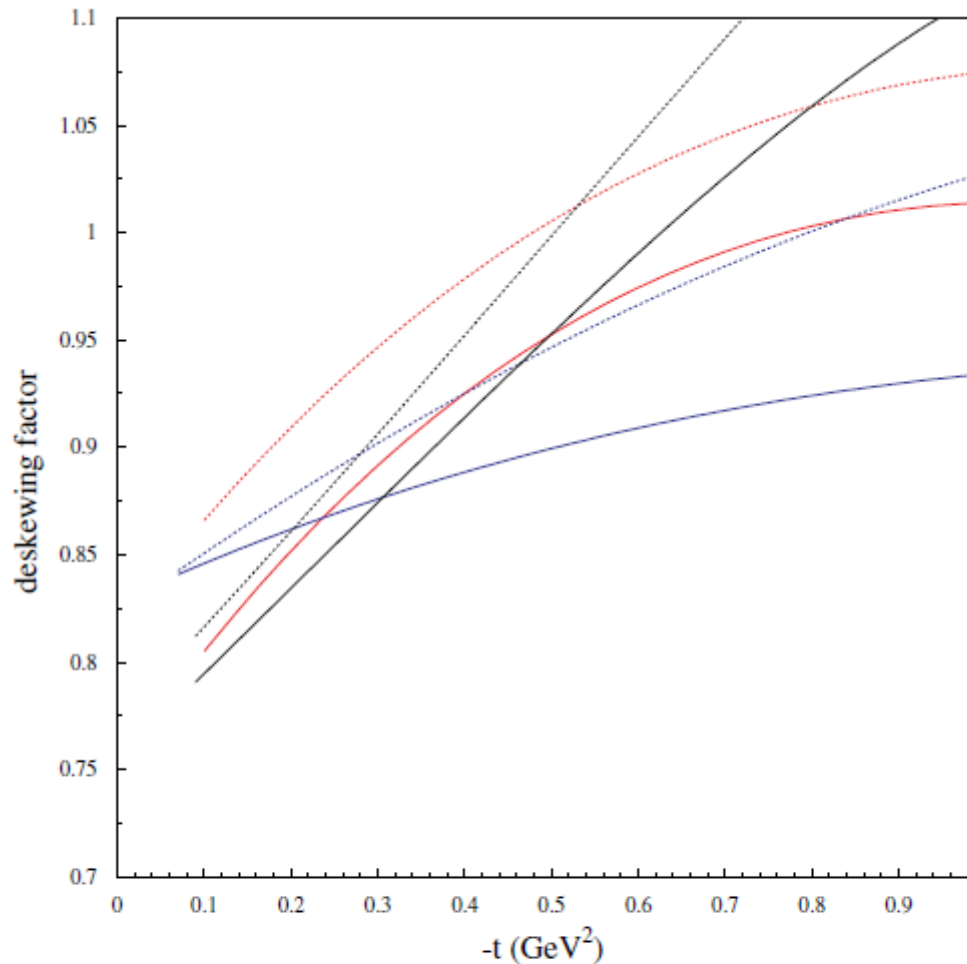
$$H_{\text{Im}}(\xi, t) \equiv H(\xi, \xi, t) - H(-\xi, \xi, t)$$

$$H(x, b_{\perp}) = \int_0^{\infty} \frac{d\Delta_{\perp}}{2\pi} \Delta_{\perp} J_0(b_{\perp} \Delta_{\perp}) H(x, 0, -\Delta_{\perp}^2) \quad \text{Burkardt (2000)}$$

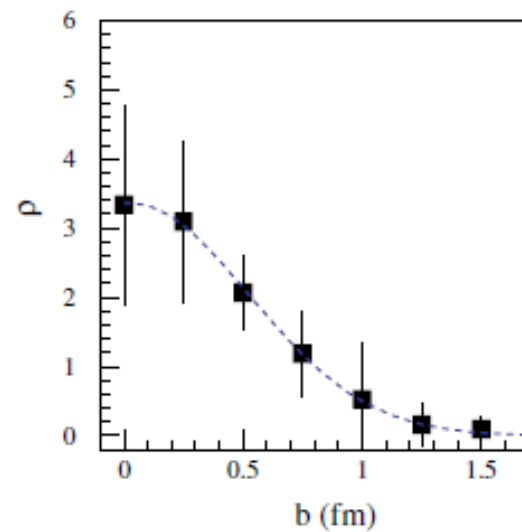
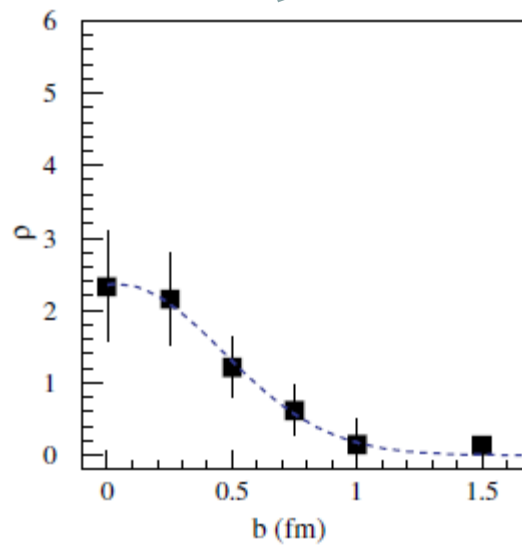
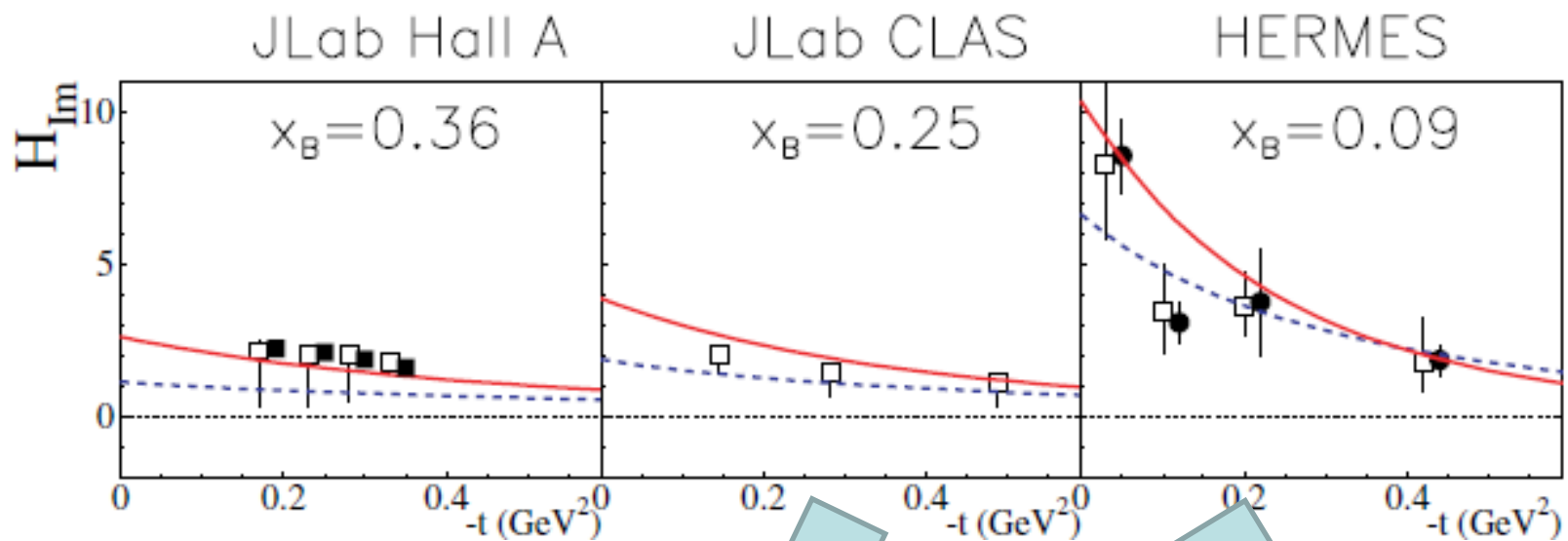
Applying a (model-dependent) “deskewing” factor:

$$\frac{H(\xi, 0, t)}{H(\xi, \xi, t)}$$

and, in a first approach, neglecting the sea contribution

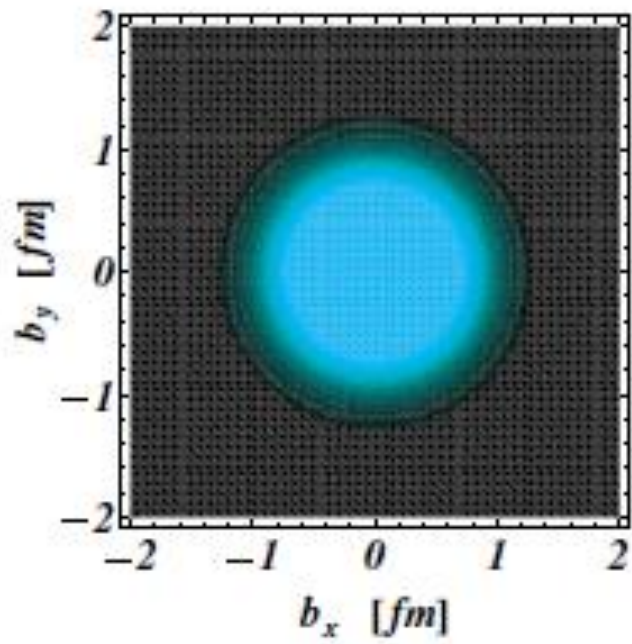


‘Deskewing’ factor  $H(\xi, 0, t)/H(\xi, \xi, t)$  as a function of  $-t$  for the VGG model (red curves), the GK model (blue curves) and the dual model (black curves). The solid curves correspond to  $x_B = 0.1$  (HERMES kinematics) and the dashed ones to  $x_B = 0.25$  (CLAS kinematics).

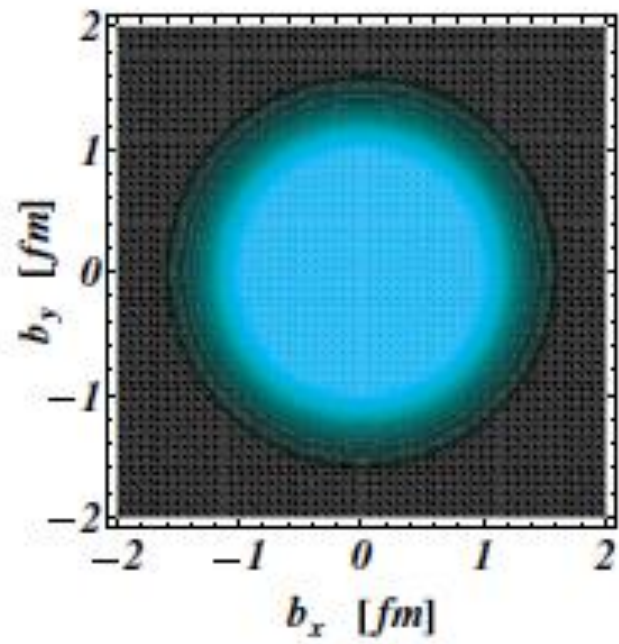




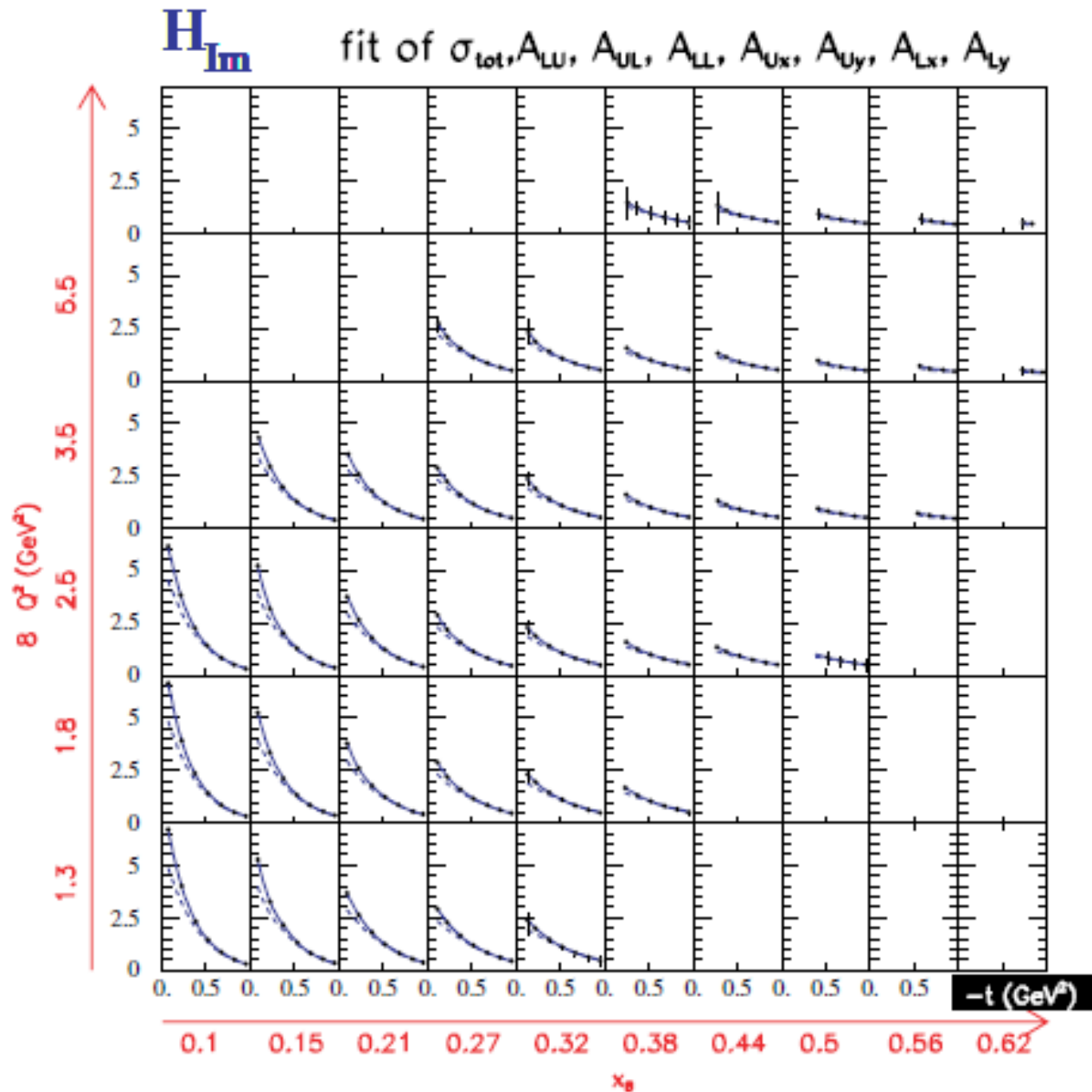
$x_B=0.25$



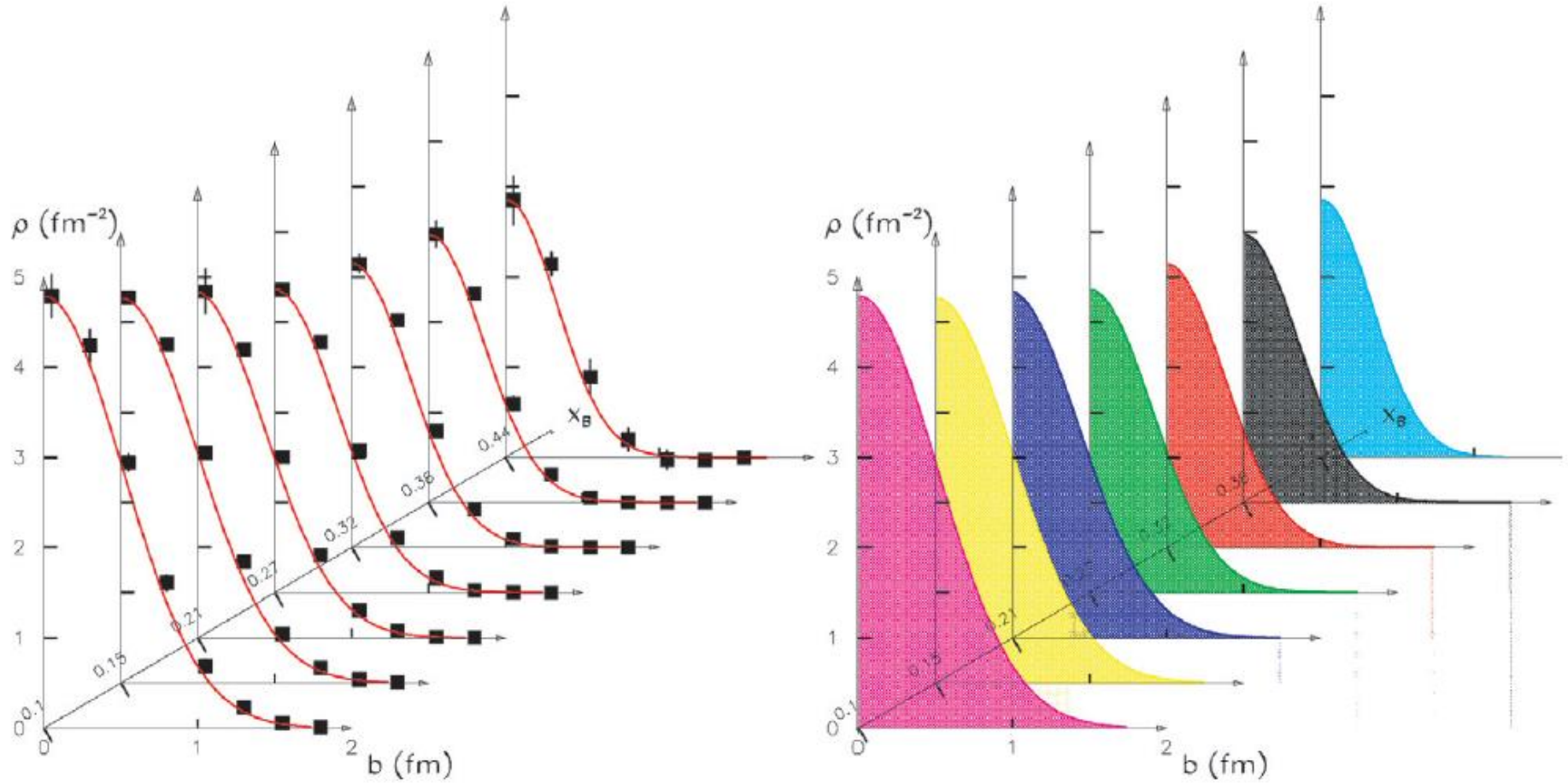
$x_B=0.09$

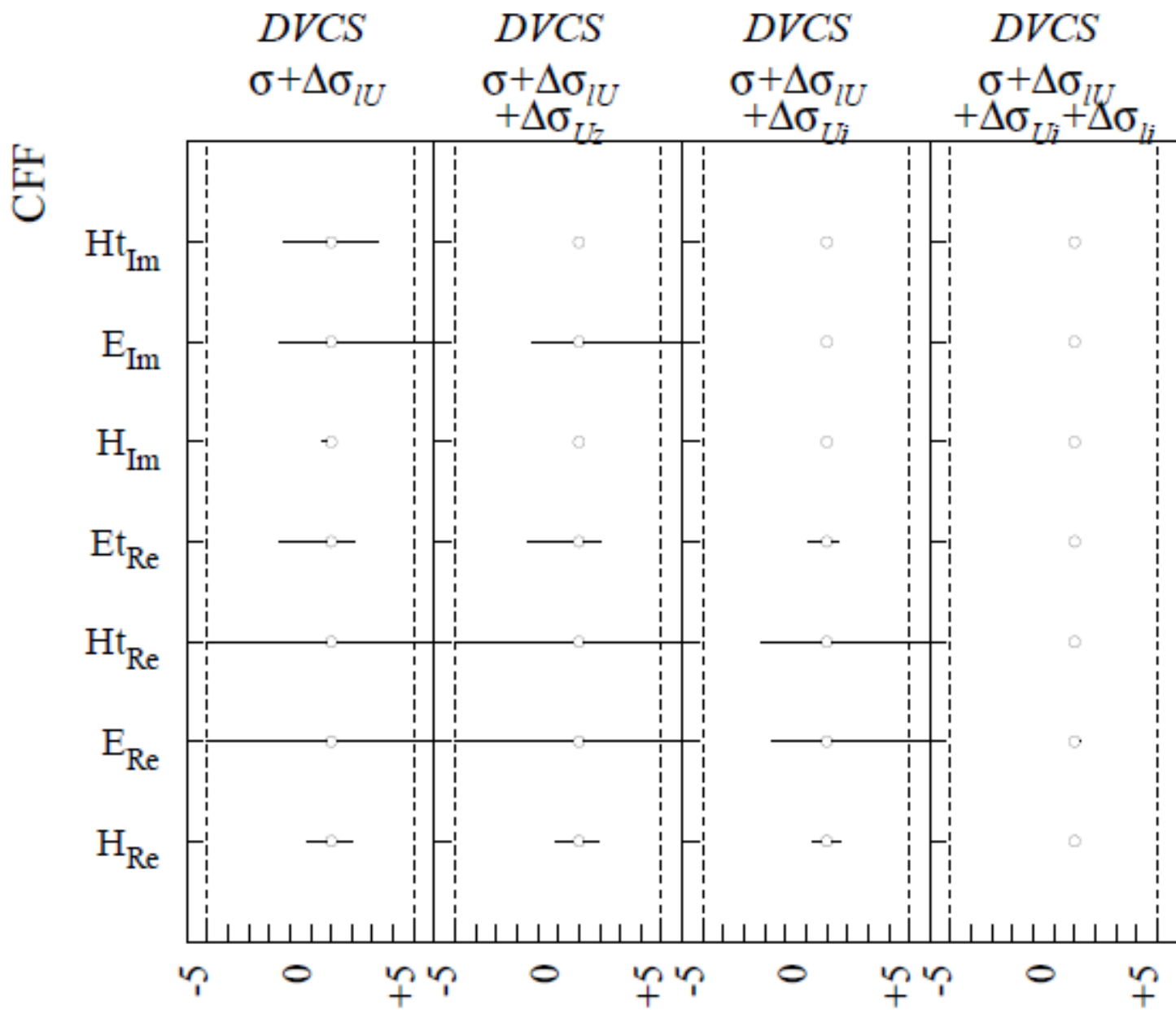


# Projections for CLAS12 for $H_{Im}$



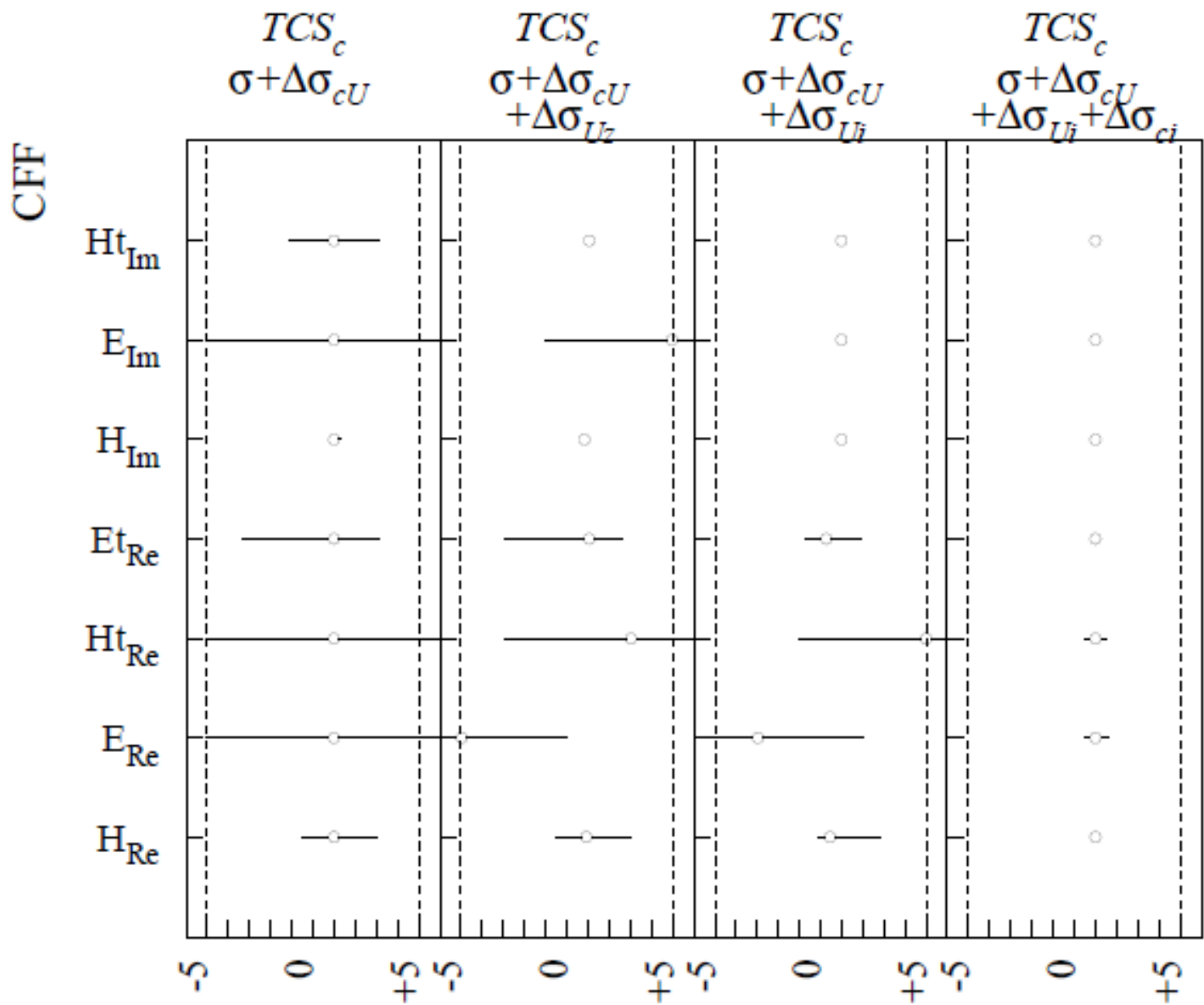
# Corresponding spatial densities





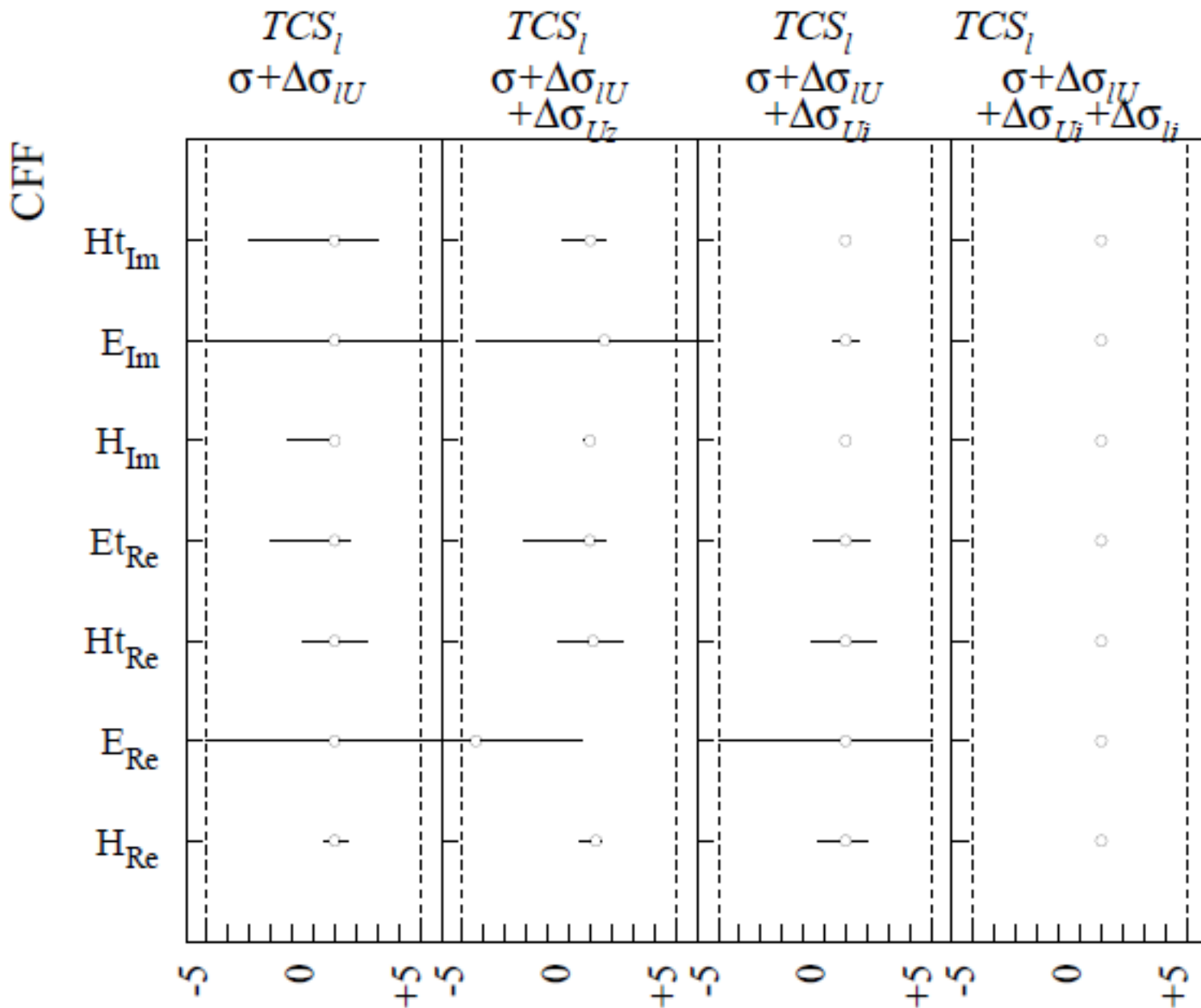
$E_e=11 \text{ GeV}, x_B=0.33, Q^2=3 \text{ GeV}^2, t=-.4 \text{ GeV}^2$

a)



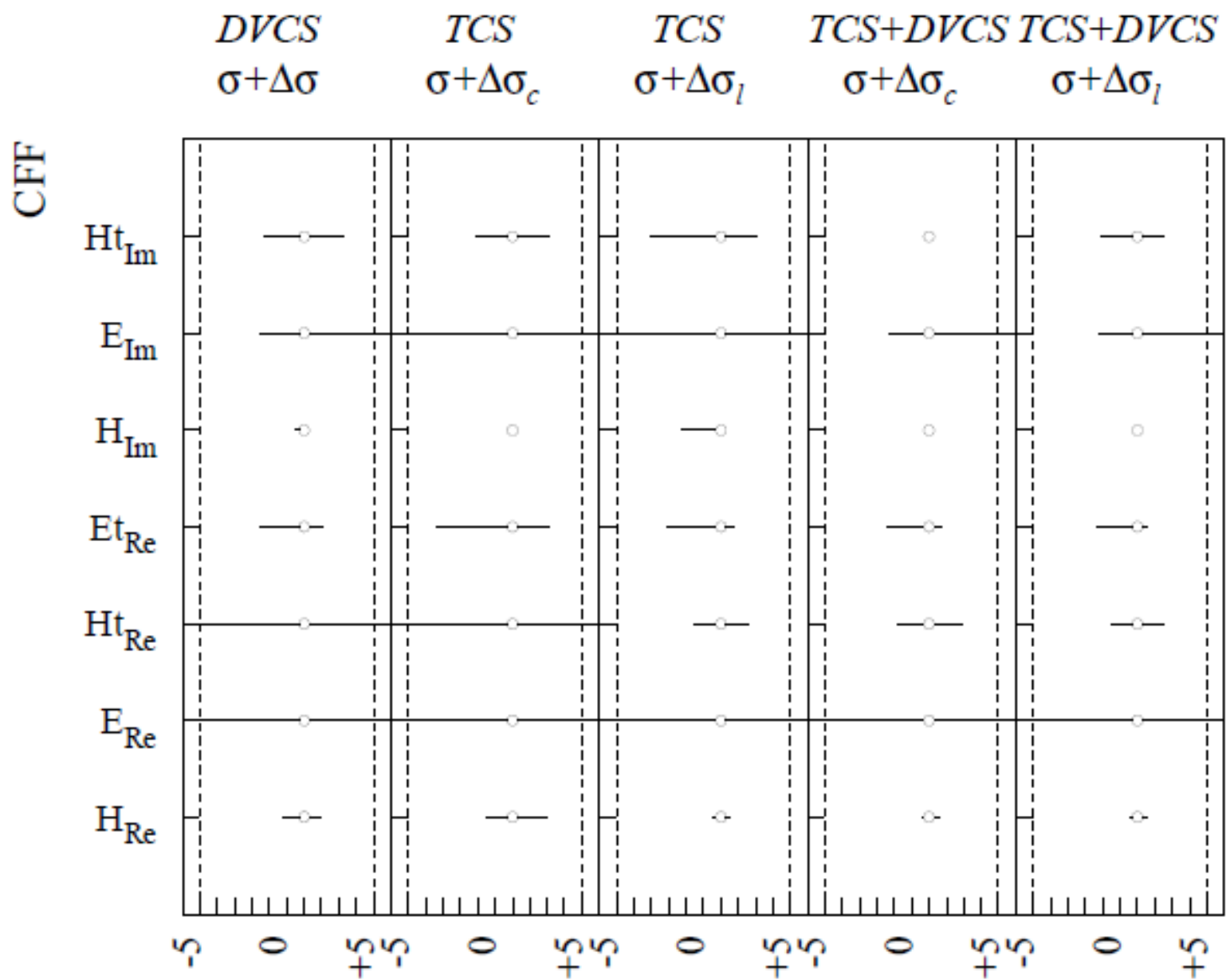
$\xi=0.2, Q'^2=7 \text{ GeV}^2, t=-.4 \text{ GeV}^2, \theta_{cm}=90\text{deg}$

a)



$\xi=0.2, Q'^2=7 \text{ GeV}^2, t=-.4 \text{ GeV}^2, \theta_{cm}=90\text{deg}$





a0



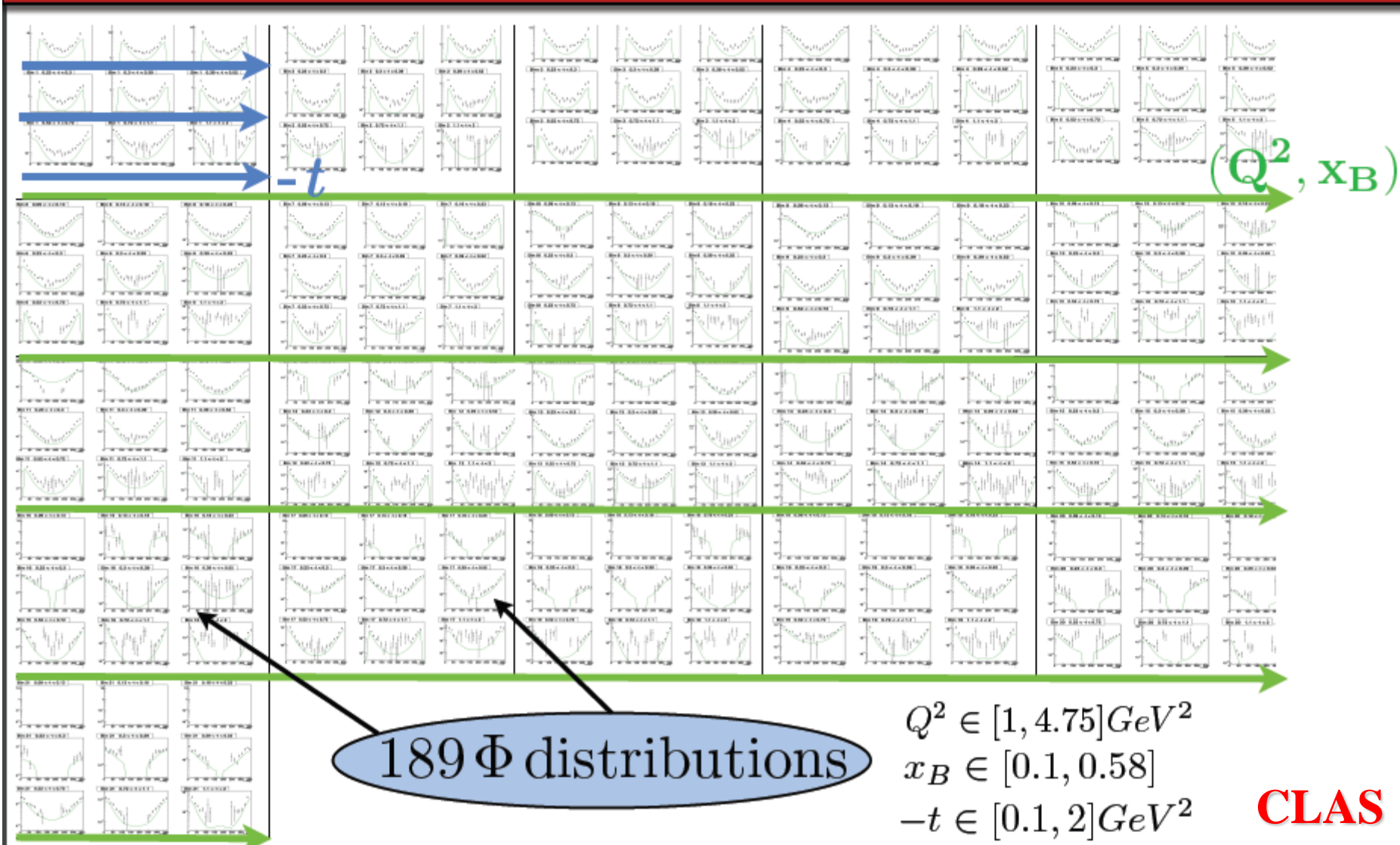
a)



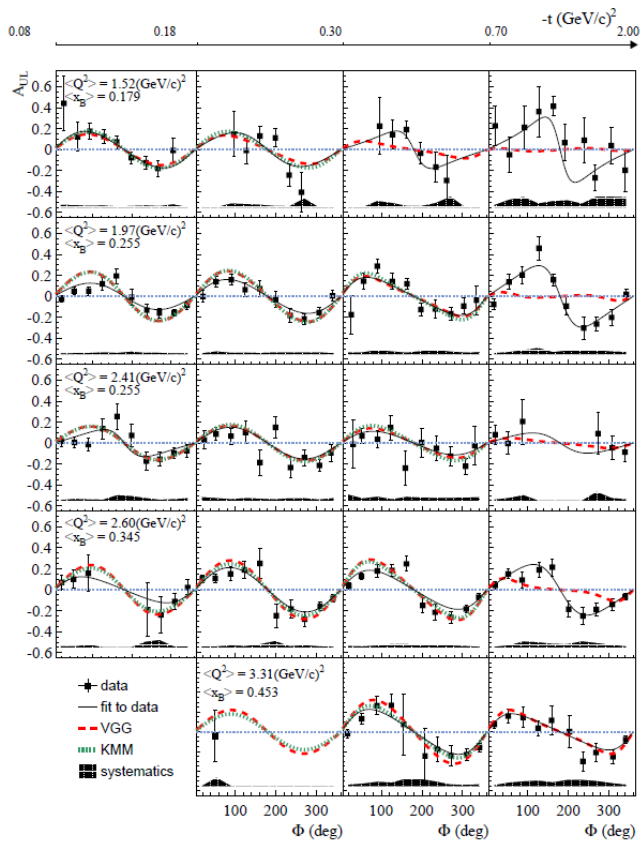


-  **GPDs contain a wealth of information on nucleon structure and dynamics: space-momentum quark correlation, orbital momentum, pion cloud, pressure forces within the nucleon,...**
-  **They are complicated functions of 3 variables  $x, \xi, t$ , depend on quark flavor, correction to leading-twist formalism,...: extraction of GPDs from precise data and numerous observables, global fitting, model inputs,...**
-  **First new insights on nucleon structure already emerging from current data with new fitting algorithms**
-  **Large flow of new observables and data expected soon (JLab6, JLab12, COMPASS) will allow a precise nucleon tomography in the valence region**

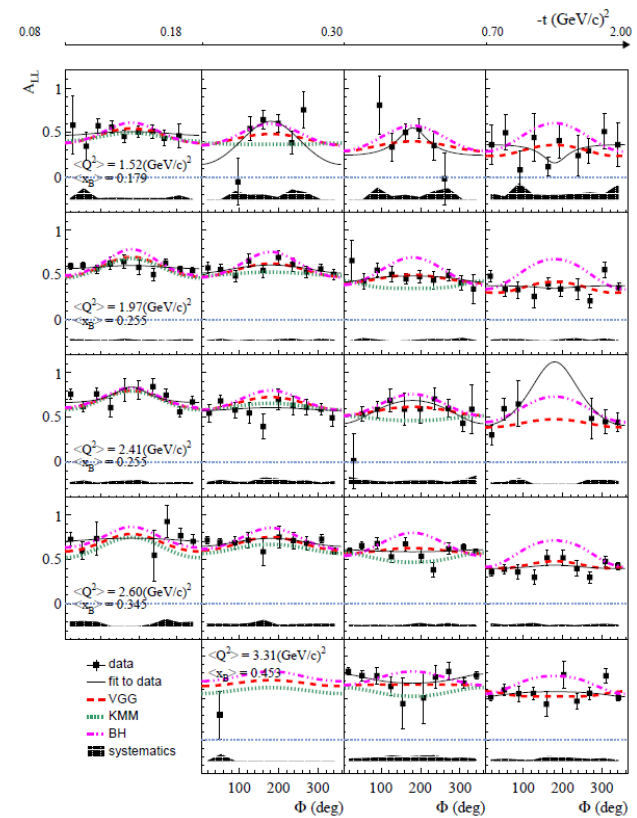
# DVCS differential cross section



To be published in 2014



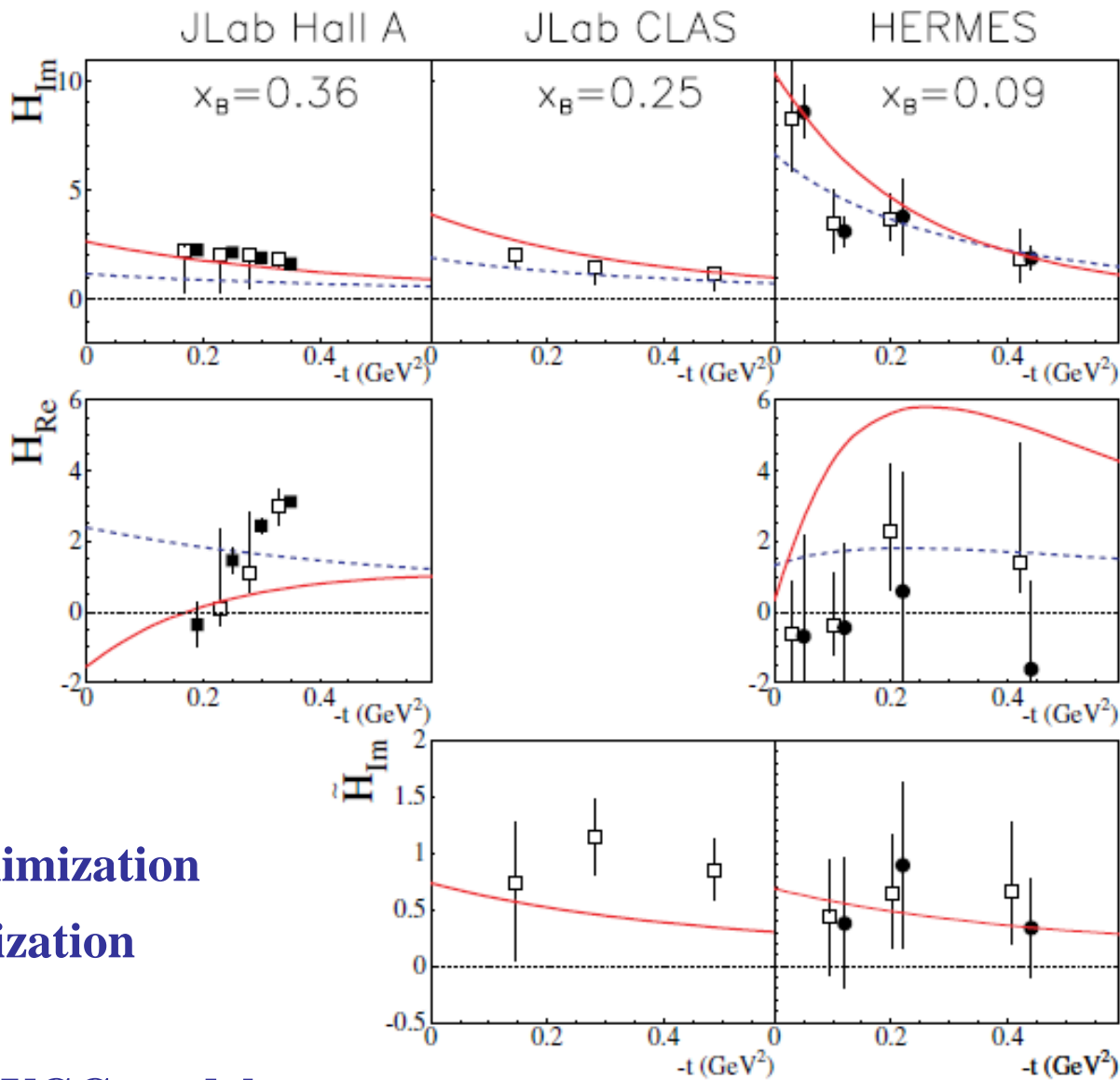
Long. target spin asym.



Double beam-target spin asym.

**CLAS**  
**(courtesy S. Niccolai)**

To be published in 2014



□  $\chi^2$  minimization

● linearization

— VGG model

- - - KM10 model/fit

■ Moutarde 10 model/fit

# Other approach:

Assume a functional shape and fit some parameters

\*D. Mueller & K. Kumericki  
 \*H. Moutarde  
 \*VGG by the HERMES and n-DVCS Hall A coll.

(slide from K. Kumericki, Photons11)

$$\Im\mathcal{H}(\xi, t) = \pi \left[ \frac{4}{9} H^{u_{\text{val}}}(\xi, \xi, t) + \frac{1}{9} H^{d_{\text{val}}}(\xi, \xi, t) + \frac{2}{9} H^{\text{sea}}(\xi, \xi, t) \right]$$

- Valence quarks model:

$$H(x, x, t) = n r 2^\alpha \left( \frac{2x}{1+x} \right)^{-\alpha(t)} \left( \frac{1-x}{1+x} \right)^b \frac{1}{\left( 1 - \frac{1-x}{1+x} \frac{t}{M^2} \right)^p}.$$

- Fixed:  $n$  (from PDFs),  $\alpha(t)$  (eff. Regge),  $p$  (counting rules)

$$\alpha^{\text{val}}(t) = 0.43 + 0.85 t/\text{GeV}^2 \quad (\rho, \omega)$$

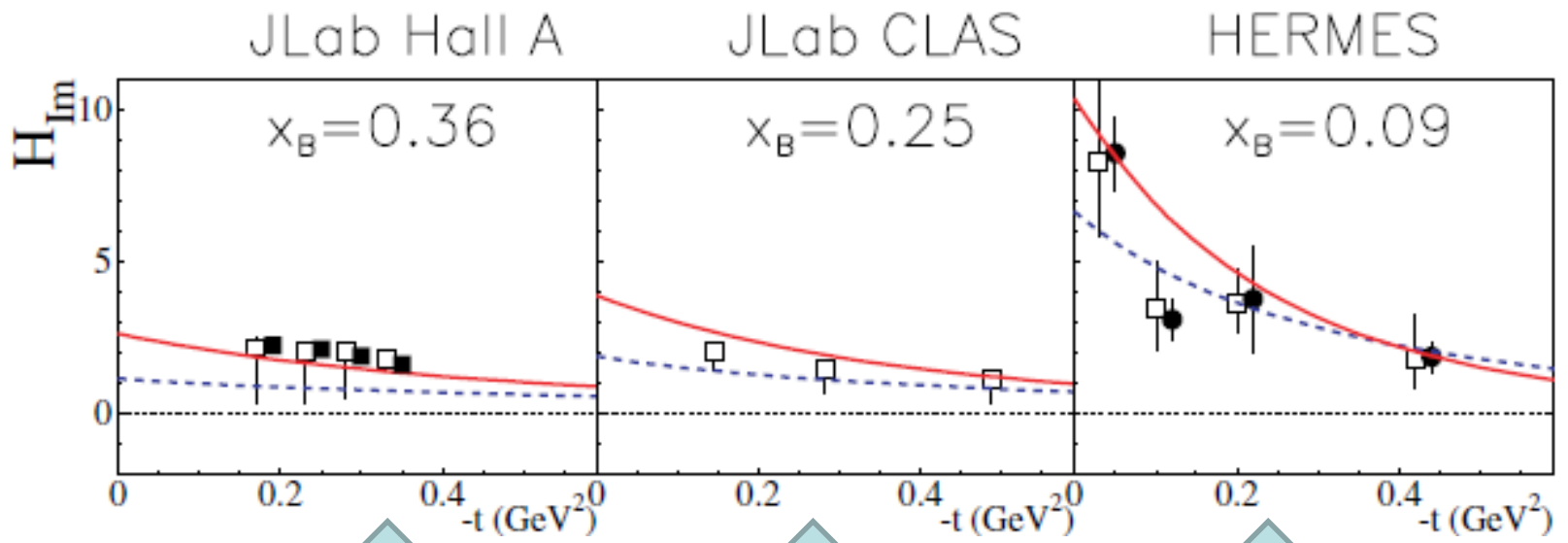
- Sea partons modelled in conformal moment space.

- $\Re\mathcal{H}$  determined by dispersion relations

$$\Re\mathcal{H}(\xi, t, Q^2) = \frac{1}{\pi} \text{PV} \int_0^1 d\xi' \left( \frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \Im\mathcal{H}(\xi', t, Q^2) - \frac{C}{\left( 1 - \frac{t}{M_C^2} \right)^2}$$

- Typical set of free parameters:

$M_0^{\text{sea}}, s_{\text{sea}}, s_G$	sea <sup>1</sup> quarks and gluons $H$
$r^{\text{val}}, M^{\text{val}}, b^{\text{val}}$	valence $H$
$C, M_C$	subtraction constant ( $H, E$ )
$(\tilde{r}^{\text{val}}, \tilde{M}^{\text{val}}, \tilde{b}^{\text{val}})$	valence $\tilde{H}$ (if needed)



↑  
unpol.sec.eff.

+

beam pol.sec.eff.

↑  
beam spin asym.

+

long. pol. tar. asym

↑  
beam charge asym.

+

beam spin asym

+

...

□  $\chi^2$  minimization

● linearization

■ Moutarde 10 model/fit

— VGG model

- - - KM10 model/fit