Status of DVCS fits after 2015 JLab data

Krešimir Kumerički

Physics Department
University of Zagreb, Croatia

In collaboration with Dieter Müller

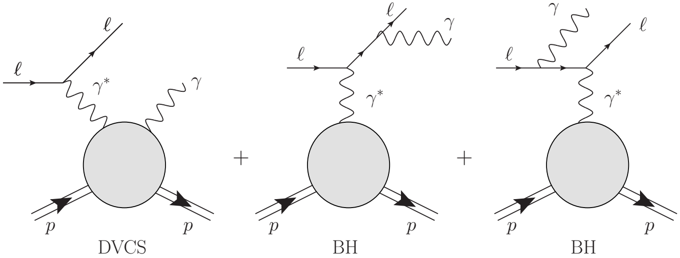
6th International Conference on Physics Opportunities at an Electron-Ion Collider (POETIC6)
Palaiseau, France, 7–11 September 2015

Introduction - DVCS

- Deeply virtual Compton scattering (DVCS) is hoped to be a "gold plated" process of exclusive physics (as advertised in previous talk [Braun])
- Where are we now?

Introduction - DVCS

- Deeply virtual Compton scattering (DVCS) is hoped to be a "gold plated" process of exclusive physics (as advertised in previous talk [Braun])
- Where are we now?
- Brief reminder: DVCS is measured via leptoproduction of a photon



- Interference with Bethe-Heitler process gives unique access to both real and imaginary part of DVCS amplitude.

DVCS cross-section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + \mathcal{I} + |\mathcal{T}_{\rm DVCS}|^2$$

$$|\mathcal{T}_{\rm BH}|^2 \propto \frac{1}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH}\cos(n\phi) + s_1^{\rm BH}\sin\phi \right\}$$

$$\mathcal{I} \propto \frac{1}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3}\left[c_n^{\mathcal{I}}\cos(n\phi) + s_n^{\mathcal{I}}\sin(n\phi)\right] \right\}$$

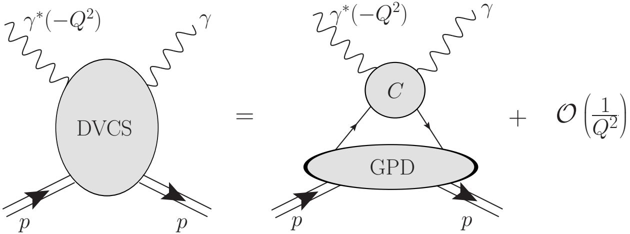
$$|\mathcal{T}_{\rm DVCS}|^2 \propto \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2}\left[c_n^{\rm DVCS}\cos(n\phi) + s_n^{\rm DVCS}\sin(n\phi)\right] \right\}$$

- We work at leading order accuracy where c_n can be expressed in terms of four complex Compton form factors (CFFs) [Belitsky, Müller et. al '01–'14].

$$\mathcal{H}(x_{\rm B}, t, \mathcal{Q}^2),\ \mathcal{E}(x_{\rm B}, t, \mathcal{Q}^2),\ \tilde{\mathcal{H}}(x_{\rm B}, t, \mathcal{Q}^2),\ \tilde{\mathcal{E}}(x_{\rm B}, t, \mathcal{Q}^2)$$

Factorization of DVCS $\longrightarrow$ GPDs

- [Collins et al. '98]



- Compton form factors factorize into a convolution:

$$^{a}\mathcal{H}(x_{\rm B} \approx \tfrac{2\xi}{1+\xi}, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mathcal{Q}_0^2) \; H^a(x, \eta = \xi, t, \mathcal{Q}_0^2)$$

$a = q, G$

- $H^a(x, \eta, t, \mathcal{Q}_0^2)$ — Generalized parton distribution (GPD)

Curse of dimensionality

- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*

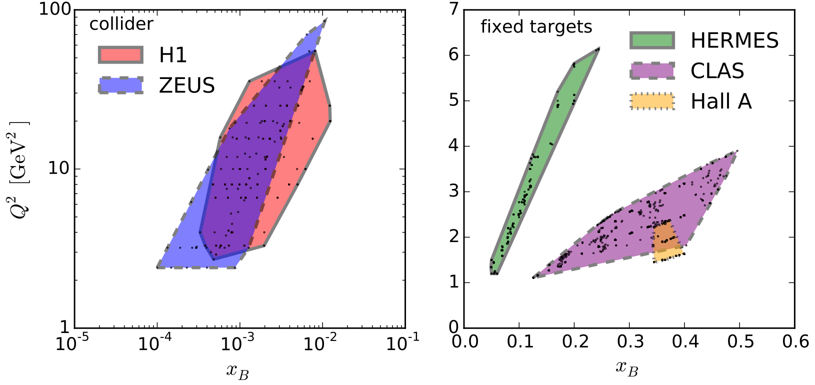
Curse of dimensionality

- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*
- When the dimensionality increases, the volume of the space increases so fast that the available data becomes sparse.
- Analogously, in contrast to *PDFs*(x), it is very difficult to perform truly model independent extraction of *GPDs*(x, η, t)

Curse of dimensionality

- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*
- When the dimensionality increases, the volume of the space increases so fast that the available data becomes sparse.
- Analogously, in contrast to $PDFs(x)$, it is very difficult to perform truly model independent extraction of $GPDs(x, \eta, t)$
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.
- As an intermediate step, one can attempt extraction of $CFFs(x_B, t)$
- (Dependence on additional variable, photon virtuality $\mathcal{Q}^2$, is in principle known — given by evolution equations.)

Experimental coverage (1/2)



- Coming soon: COMPASS, JLab12

Experimental coverage (2/2) — fixed target

Collab.	Year	Observables	Kinematics			No. of points	
			$x_{\rm B}$	Q^2 [GeV^2]	$\|t\|$ [GeV^2]	total	indep.
HERMES	2001	$A_{\rm LU}^{\sin\phi}$	0.11	2.6	0.27	1	1
CLAS	2001	$A_{\rm LU}^{\sin\phi}$	0.19	1.25	0.19	1	1
CLAS	2006	$A_{\rm UL}^{\sin\phi}$	0.2–0.4	1.82	0.15–0.44	6	3
HERMES	2006	$A_{\rm C}^{\cos\phi}$	0.08–0.12	2.0–3.7	0.03–0.42	4	4
Hall A	2006	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.36	1.5–2.3	0.17–0.33	4×24+12×24	4×24+12×24
CLAS	2007	$A_{\rm LU}(\phi)$	0.11–0.58	1.0–4.8	0.09–1.8	62×12	62×12
HERMES	2008	$A_{\rm C}^{\cos(0,1)\phi}$, $A_{\rm UT,DVCS}^{\sin(\phi-\phi_S)}$, $A_{\rm UT,I}^{\sin(\phi-\phi_S)\cos(0,1)\phi}$, $A_{\rm UT,I}^{\cos(\phi-\phi_S)\sin\phi}$	0.03–0.35	1–10	<0.7	*12+12+12* 12+12 *12*	*4+4+4* 4+4 *4*
CLAS	2008	$A_{\rm LU}(\phi)$	0.12–0.48	1.0–2.8	0.1–0.8	66	33
HERMES	2009	$A_{\rm LU,I}^{\sin(1,2)\phi}$, $A_{\rm LU,DVCS}^{\sin\phi}$, $A_{\rm C}^{\cos(0,1,2,3)\phi}$	0.05–0.24	1.2–5.75	<0.7	18+18+18 18+18+*18+18*	6+6+6 6+6+*6+6*
HERMES	2010	$A_{\rm UL}^{\sin(1,2,3)\phi}$, $A_{\rm LL}^{\cos(0,1,2)\phi}$	0.03–0.35	1–10	<0.7	12+12+*12* 12+*12*+12	4+4+*4* 4+*4*+4
HERMES	2011	$A_{\rm LT,I}^{\cos(\phi-\phi_S)\cos(0,1,2)\phi}$, $A_{\rm LT,I}^{\sin(\phi-\phi_S)\sin(1,2)\phi}$, $A_{\rm LT,BH+DVCS}^{\cos(\phi-\phi_S)\cos(0,1)\phi}$, $A_{\rm LT,BH+DVCS}^{\sin(\phi-\phi_S)\sin\phi}$	0.03–0.35	1–10	<0.7	*12+12+12* *12+12* *12*+12 12	*4+4+4* *4+4* *4*+4 4
HERMES	2012	$A_{\rm LU,I}^{\sin(1,2)\phi}$, $A_{\rm LU,DVCS}^{\sin\phi}$, $A_{\rm C}^{\cos(0,1,2,3)\phi}$	0.03–0.35	1–10	<0.7	18+*18+18* 18+18+*18+18*	6+*6+6* 6+6+*6+6*
CLAS	2015	$A_{LU}(\phi)$, $A_{UL}(\phi)$, $A_{LL}(\phi)$	0.17–0.47	1.3–3.5	0.1–1.4	166+166+166	166+166+166
CLAS	2015	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.1–0.58	1–4.6	0.09–0.52	2640+2640	2640+2640
Hall A	2015	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.33–0.40	1.5–2.6	0.17–0.37	480+600	240+360

Modelling sea quark and gluon GPDs

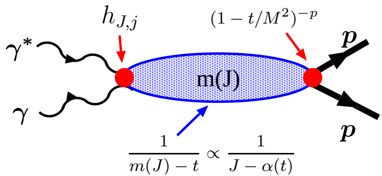
- Instead of considering momentum fraction dependence $H(x, \ldots)$
- ...it is convenient to make a transform into complementary space of conformal moments j:

$$H_j^q(\eta, \ldots) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \eta^j \, C_j^{3/2}(x/\eta) \, H^q(x, \eta, \ldots)$$

- They are analogous to Mellin moments in DIS: $x^j \to C_j^{3/2}(x)$
- $C_j^{3/2}(x)$ — Gegenbauer polynomials
- At LO easy multiplicative evolution (pQCD series behaviour and evolution of CFFs studied also to NNLO)

SO(3) partial wave expansion

- To model η-dependence of GPD's $H_j(\eta, t)$ consider crossed t-channel process $\gamma^*\gamma \to p\bar{p}$ and perform SO(3) partial wave expansion:



$$H_j(\eta, t) = \sum_{J=J_{\min}}^{j+1} h_{J,j} \frac{1}{J-\alpha(t)} \frac{1}{\left(1-\frac{t}{M^2}\right)^p} \eta^{j+1-J} d^J_{0,\nu}\left(\frac{1}{\eta}\right)$$

- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)
  $\nu = 0, \pm 1$ — depending on hadron helicities
- Similar to "dual" parametrization [Polyakov, Shuvaev '02] (see talk by [Semenov-Tian-Shansky])

Starting with leading SO(3) partial wave

- sea quarks $\approx$ flavour singlet $\equiv \Sigma$

$$\mathbf{H}_j(\xi, t, \mu_0^2) = \begin{pmatrix} N'_\Sigma F_\Sigma(t) \, \mathrm{B}\big(1 + j - \alpha_\Sigma(0), 8\big) \\ N'_G F_G(t) \, \mathrm{B}\big(1 + j - \alpha_G(0), 6\big) \end{pmatrix}$$

$$\alpha_a(t) = \alpha_a(0) + 0.15t \qquad F_a(t) = \frac{j + 1 - \alpha(0)}{j + 1 - \alpha(t)} \left(1 - \frac{t}{{M_a}^2}\right)^{-p_a}$$

...corresponding in forward case to PDFs of form

$$\Sigma(x) = N'_\Sigma \, x^{-\alpha_\Sigma(0)} \, (1 - x)^7 \; ; \qquad G(x) = N'_G \, x^{-\alpha_G(0)} \, (1 - x)^5$$

- $M_G = \sqrt{0.7}\,\mathrm{GeV}$ is fixed by the J/ψ production data
- Free parameter (for DVCS): M_Σ

Inclusion of subleading PW — flexible models

$$\mathbf{H}_j(\eta, t) = \underbrace{\begin{pmatrix} N'_{\rm sea}\, F_{\rm sea}(t)\, \mathrm{B}\big(1+j-\alpha_{\rm sea}(0), 8\big) \\ N'_{\rm G}\, F_{\rm G}(t)\, \mathrm{B}\big(1+j-\alpha_{\rm G}(0), 6\big) \end{pmatrix}}_{\text{skewness } r \approx 1.6 \text{ (too large)}} + \underbrace{\begin{pmatrix} s_{\rm sea} \\ s_{\rm G} \end{pmatrix} \begin{pmatrix} \text{subleading partial waves, } \eta\text{-dependence!} \end{pmatrix}}_{\substack{<0 \\ \text{negative skewness}}}$$

- Addition of second PW needed for good fits. Third PW gives more flexibility.
- two or four new parameters: $s_{\rm sea}^{(2,4)}$ and $s_{\rm G}^{(2,4)}$
- CFFs finally given by Mellin-Barnes integral

$$\mathcal{H}_S(\xi, t, \mathcal{Q}^2) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1} \left[i + \tan\left(\frac{\pi j}{2}\right) \right] \mathbf{C}_j \cdot \mathbf{H}_j \,.$$

Modelling valence quark GPDs

- **Hybrid models**
- Sea quarks and gluons modelled like just described (conformal moments + SO(3) partial wave expansion + LO $\mathcal{Q}^2$ evolution).
- Valence quarks model (ignoring $\mathcal{Q}^2$ evolution):

$$\mathfrak{Im}\,\mathcal{H}(\xi,t)=\pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t)+\frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t)+\frac{2}{9}H^{\rm sea}(\xi,\xi,t)\right]$$

$$H(x,x,t)=n\,r\,2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

- Fixed: n (from PDFs), $\alpha(t)$ (eff. Regge), p (counting rules)

$$\alpha^{\rm val}(t)=0.43+0.85\,t/\mathrm{GeV}^2\quad(\rho,\,\omega)$$

- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations

$$\mathfrak{Re}\,\mathcal{H}(\xi, t) = \frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right)\mathfrak{Im}\,\mathcal{H}(\xi', t) - \frac{C}{\left(1 - \frac{t}{M_C{}^2}\right)^2}$$

- Typical set of free parameters:

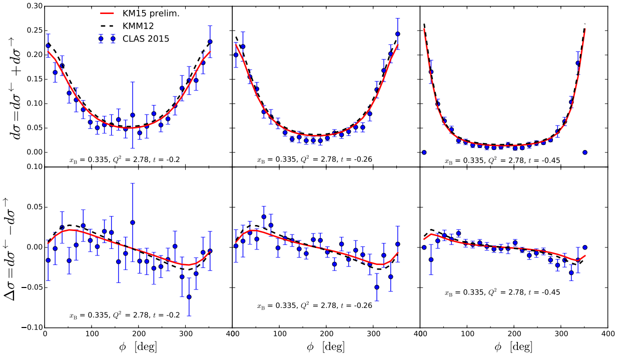
$M_{\rm sea}$, $s_{\rm sea}^{(2,4)}$, $s_G^{(2,4)}$	sea quarks and gluons H
$r^{\rm val}$, $M^{\rm val}$, $b^{\rm val}$	valence H
$\tilde{r}^{\rm val}$, $\tilde{M}^{\rm val}$, $\tilde{b}^{\rm val}$	valence $\widetilde{H}$
C, M_C	subtraction constant (H, E)
r_π, M_π	"pion pole" $\widetilde{E}$

Model	KM09a	KM09b	KM10	KM10a	KM10b	KMS11	KMM12
free params.	{3}+(3)+5	{3}+(3)+6	{3}+15	{3}+10	{3}+15	NNet	{3}+15
$\chi^2/\mathrm{d.o.f.}$	32.0/31	33.4/34	135.7/160	129.2/149	115.5/126	13.8/36	123.5/80
F_2	{85}	{85}	{85}	{85}	{85}		{85}
σ_{DVCS}	(45)	(45)	51	51	45		11
$d\sigma_{\mathrm{DVCS}}/dt$	(56)	(56)	56	56	56		24
$A_{LU}^{\sin\phi}$	12+12	12+12	12	16	12+12		4
$A_{LU,I}^{\sin\phi}$			18	18		18	6
$A_{C}^{\cos 0\phi}$							6
$A_{C}^{\cos\phi}$	12	12	18	18	12	18	6
$\Delta\sigma^{\sin\phi,w}$			12				12
$\sigma^{\cos 0\phi,w}$			4				4
$\sigma^{\cos\phi,w}$			4				4
$\sigma^{\cos\phi,w}/\sigma^{\cos 0\phi,w}$		4			4		
$A_{UL}^{\sin\phi}$							6
$A_{UL,+}^{\sin\phi}$							4
$A_{LL,+}^{\cos 0\phi}$							4
$A_{UT,I}^{\sin(\phi-\phi_S)\cos\phi}$							4

- `KM15 prelim.` — update of KMM12 using 2015 CLAS and Hall A cross-section data: $\chi^2/\mathrm{d.o.f.} = 238.2/259$
- These models are available at WWW : http://calculon.phy.hr/gpd/

2015 CLAS cross-sections (1/2)

- Restriction to kinematics where leading-order framework should be valid: $-t/Q^2 < 0.25$ with $Q^2 > 1.5\,\mathrm{GeV}^2$, means using 48 out of measured 110 $x_{\rm B}$–Q^2–t bins.



- $\chi^2/\mathrm{npts} = 1032.0/1014$ for $d\sigma$ and $936.1/1012$ for $\Delta\sigma$

ϕ-space vs. harmonics (1/3)

- ϕ-space figures and perfect χ^2 are not revealing the whole story
- Instead to $\sigma(\phi)$ it is favourable to work with harmonics like

$$\sigma^{\sin n\phi, w} \equiv \frac{1}{\pi}\int_{-\pi}^{\pi} dw \, \sin(n\phi)\, \sigma(\phi) \;,$$

with specialy weighted Fourier integral measure

$$dw \equiv \frac{2\pi \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}{\int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} d\phi \;,$$

thus cancelling strongly oscillating factors $1/(\mathcal{P}_1(\phi)\mathcal{P}_2(\phi))$ in Bethe-Heitler and interference terms in $d\sigma$. Series of such weighted harmonic terms converges then faster with increasing n than normal Fourier series.

ϕ-space vs. harmonics (2/3)

- Be careful with propagation of correlated systematic uncertainties from ϕ-space to harmonics
- If syst. uncertainty is ϕ-dependent it can influence differently different harmonics

ϕ-space vs. harmonics (2/3)

- Be careful with propagation of correlated systematic uncertainties from ϕ-space to harmonics
- If syst. uncertainty is ϕ-dependent it can influence differently different harmonics
- There is possible enhancement of uncertainty of subleading harmonics

ϕ-space vs. harmonics (2/3)

- Be careful with propagation of correlated systematic uncertainties from ϕ-space to harmonics
- If syst. uncertainty is ϕ-dependent it can influence differently different harmonics
- There is possible enhancement of uncertainty of subleading harmonics
- So although we presently mostly perform harmonic analysis of data ourselves, experimentalists who better understand their systematics should be able to do better job
- Extracting standard Fourier harmonics would also be good enough (they are related to weighted harmonics by simple linear transformation)

ϕ-space vs. harmonics (3/3)

- How many harmonics to extract?
- One approach:
  1. Fit harmonic expansion

     $$\sigma(\phi) = c_0 + c_1 \cos\phi + \cdots + s_1 \sin\phi + \cdots$$

     to randomly chosen subset of data in a bin, and calculate χ^2 error for description of the rest of data (so called *cross-validation* procedure)
  2. Increase the number of harmonics until $\chi^2/\mathrm{d.o.f}$ starts to fall

ϕ-space vs. harmonics (3/3)

- How many harmonics to extract?
- One approach:
  1. Fit harmonic expansion

$$\sigma(\phi) = c_0 + c_1 \cos\phi + \cdots + s_1 \sin\phi + \cdots$$

     to randomly chosen subset of data in a bin, and calculate χ^2 error for description of the rest of data (so called *cross-validation* procedure)
  2. Increase the number of harmonics until $\chi^2/\mathrm{d.o.f}$ starts to fall
- Highest extractable harmonics in 2015 cross-section data:

	CLAS		Hall A	
	sine	cosine	sine	cosine
$\Delta\sigma^w$	0.9 ± 0.4	0.1 ± 0.3	1.1 ± 0.3	0.1 ± 0.3
$d\sigma^w$	0.3 ± 0.6	0.7 ± 0.7	0.6 ± 0.8	1.5 ± 0.7

ϕ-space vs. harmonics (3/3)

- How many harmonics to extract?
- One approach:
  1. Fit harmonic expansion

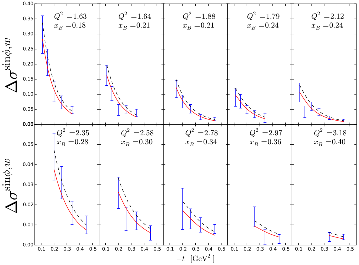
$$\sigma(\phi) = c_0 + c_1 \cos\phi + \cdots + s_1 \sin\phi + \cdots$$

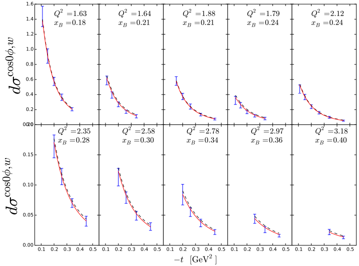
     to randomly chosen subset of data in a bin, and calculate χ^2 error for description of the rest of data (so called *cross-validation* procedure)
  2. Increase the number of harmonics until $\chi^2/\mathrm{d.o.f}$ starts to fall
- Highest extractable harmonics in 2015 cross-section data:

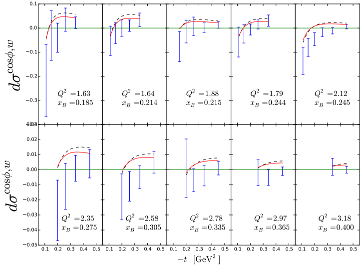
	CLAS		Hall A	
	sine	cosine	sine	cosine
$\Delta\sigma^w$	0.9 ± 0.4	0.1 ± 0.3	1.1 ± 0.3	0.1 ± 0.3
$d\sigma^w$	0.3 ± 0.6	0.7 ± 0.7	0.6 ± 0.8	1.5 ± 0.7

- (So $\Delta\sigma^w = s_1 \sin\phi$ and $d\sigma^w = c_0 + c_1 \cos\phi$ is enough.)

2015 CLAS cross-sections (2/2)



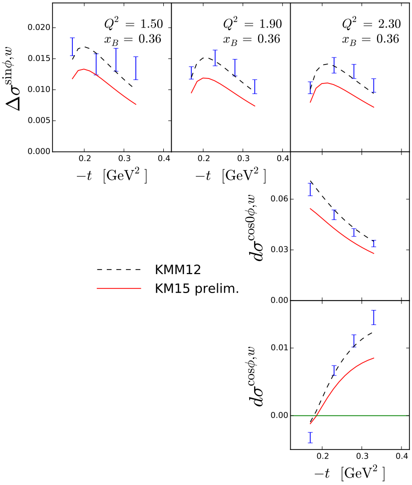


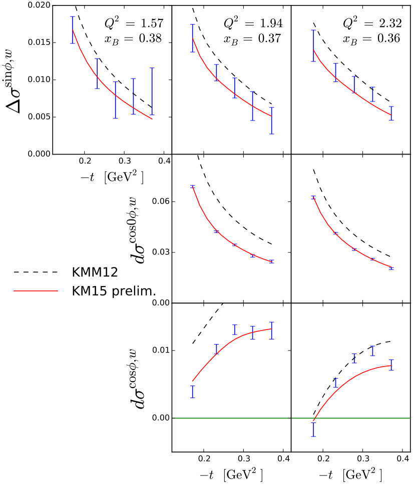


- $\chi^2/\mathrm{npts} = 62.2/48$ for $d\sigma^{\cos\phi,w}$

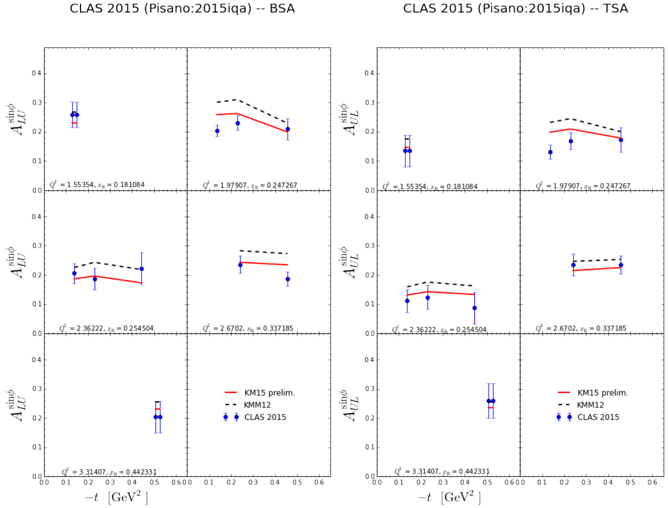
(O.K. but not so perfect as in ϕ-space)

2006 vs 2015 Hall A cross-sections

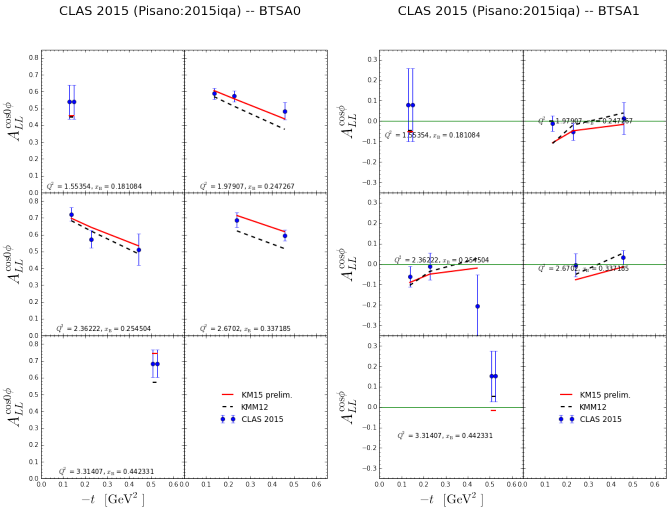




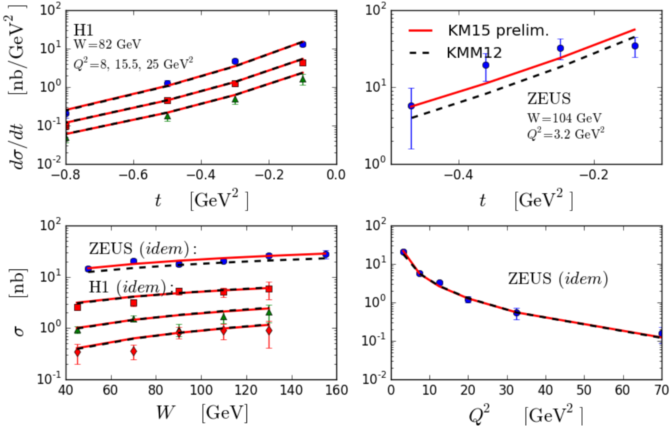
2015 CLAS asymmetries (1/2)



2015 CLAS asymmetries (2/2)



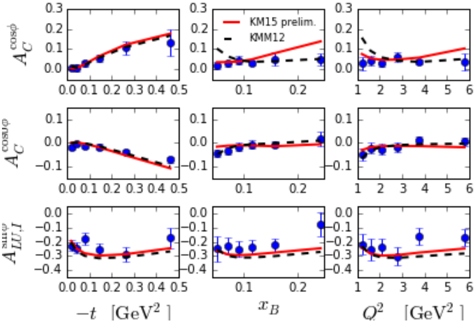
H1 (2007), ZEUS (2008)



HERMES (2012)

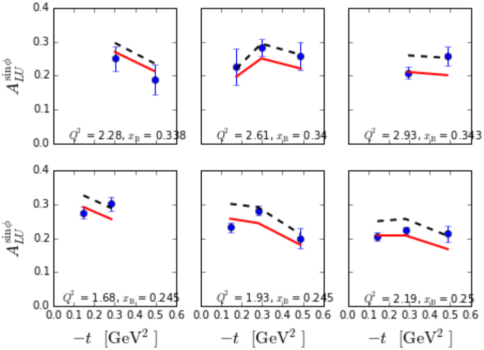
$$BCA \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} \sim A_C^{\cos 0\phi} + A_C^{\cos 1\phi} \cos\phi \sim \mathfrak{Re}\,\mathcal{H}$$

$$BSA \equiv \frac{d\sigma_{e\uparrow} - d\sigma_{e\downarrow}}{d\sigma_{e\uparrow} + d\sigma_{e\downarrow}} \sim A_{LU}^{\sin 1\phi} \sin\phi \sim \mathfrak{Im}\,\mathcal{H}$$

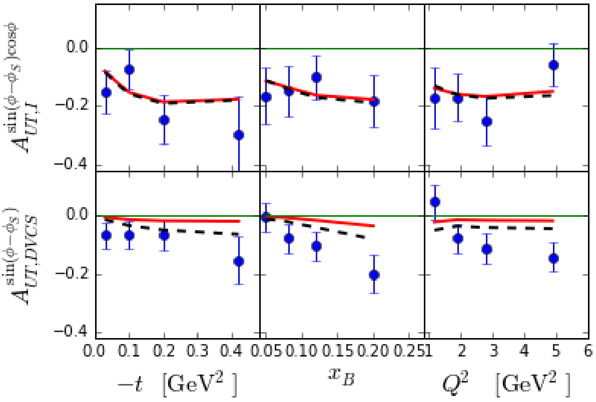


CLAS (2007)

- BSA. (Only data with $|t| \leq 0.3\,\mathrm{GeV}^2$ used for fits.)

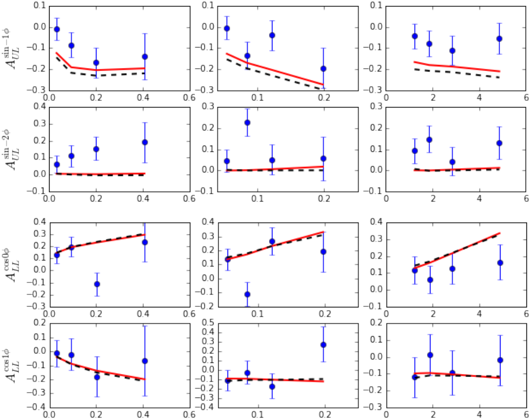


Transversally polarized target — HERMES (2008)



Longitudinally polarized target — HERMES (2010)

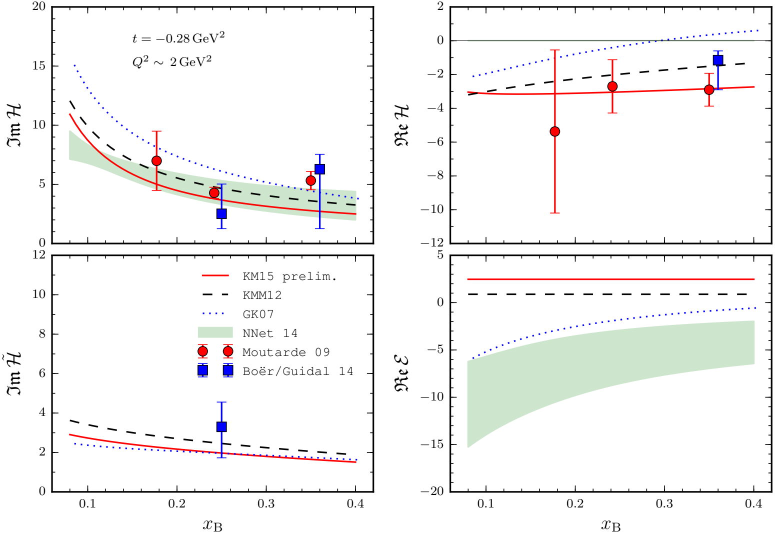
- Surprisingly large sin(2ϕ) harmonic of A_{UL} cannot be described within this leading twist framework



$\chi^2/npts$ values

		npts	KMM12	KM15 prelim.	
H1ZEUS	X_DVCS :	(35)	0.82	0.95	
HERMES	ALUI :	(6)	2.08	1.38	(one 2.7 sigma outlier)
HERMES	BCA :	(12)	0.78	1.11	
CLAS	BSA :	(12)	1.20	0.71	
HRM/CLS	AUL :	(7)	2.23	1.60	
HERMES	ALL :	(4)	3.44	3.33	(one 3.7 sigma outlier)
HERMES	AUTI :	(4)	0.90	0.91	
CLAS	BSDw_s1:	(48)	0.91	0.38	
CLAS	BSSw_c0:	(48)	0.84	0.21	
CLAS	BSSw_c1:	(48)	1.81	1.30	
Hall A	BSDw_s1:	(15)	2.11	0.42	
Hall A	BSSw_c0:	(10)	563.46	0.62	
Hall A	BSSw_c1:	(10)	31.19	4.36	(corr. syst. not included)
===	TOTAL :	(259)	24.12	0.92	

Comparison of various approaches



Summary

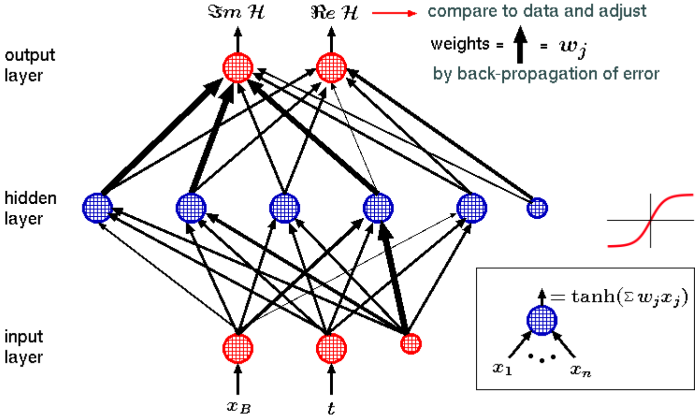
- Global fits of all proton DVCS data using flexible hybrid models are in healthy shape
- Data clearly restrict $H(x, x, t)$, and to some extent $\tilde{H}$, but any information about E is very model-dependent
- New 2015 data relieve some old tensions
- In future experiments, more details about systematic uncertainties would be welcome

Summary

- Global fits of all proton DVCS data using flexible hybrid models are in healthy shape
- Data clearly restrict $H(x, x, t)$, and to some extent $\tilde{H}$, but any information about E is very model-dependent
- New 2015 data relieve some old tensions
- In future experiments, more details about systematic uncertainties would be welcome

The End

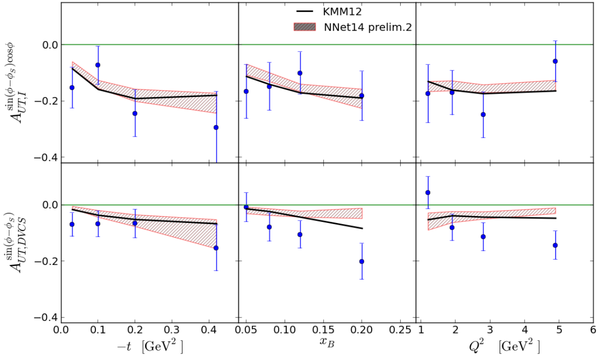
Appendix: Neural networks



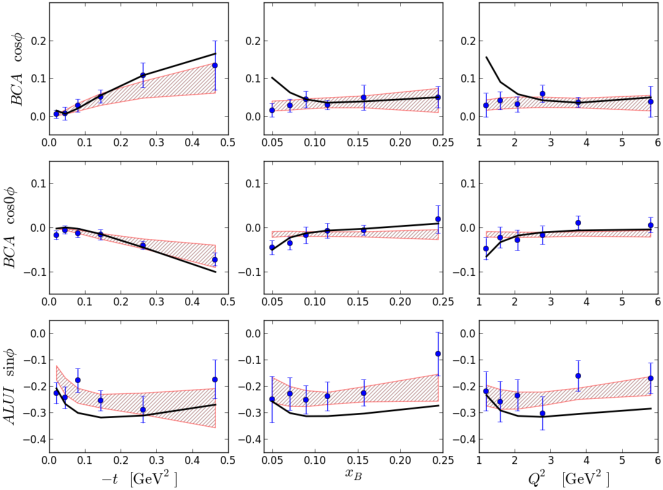
- Essentially a least-squares fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$
  $\Rightarrow$ no theory bias

Preliminary neural Net HERMES fit

- Fit to all HERMES DVCS data with two types of neural nets
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H},\ \mathfrak{Re}\,\mathcal{H},\ \mathfrak{Im}\,\tilde{\mathcal{H}})$: $\chi^2/n_{\rm pts} = 135.4/144$
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H},\ \mathfrak{Re}\,\mathcal{E})$: $\chi^2/n_{\rm pts} = 120.2/144$



Neural Net HERMES fit - BSA/BCA



Neural Net HERMES fit - CFFs