CGC beyond eikonal accuracy: finite width target effects

Tolga Altinoluk

Departamento de Física de Partículas and IGFAE
Universidade de Santiago de Compostela

POETIC 6, Palaiseau, France

September 8, 2015

[T. A. , N. Armesto, G. Beuf, A. Moscoso, arXiv:1505.01400]
[T. A. , N. Armesto, G. Beuf, M. Martinez, C. A. Salgado, JHEP 1407 (2014) 068]

Dilute-Dense Scattering within CGC

High energy scattering within the CGC :

- Semi-classical approximation :
  - dense target $\equiv$ classical background field $\mathcal{A}_a^\mu(x) = O\left(\frac{1}{g}\right)$ at weak coupling g
  - dilute projectile $\equiv$ color charge $J_a^\mu(x) = O(g)$
- Eikonal approximation:
  - take the high energy limit $s \to \infty$.
  - drop power-suppressed contributions.

Coupling between the projectile and the target $\to \int d^4x\, J_\mu^a(x)\, \mathcal{A}_a^\mu(x)$

In the semi-classical approximation, the eikonal limit can be obtained by either boosting the projectile or the target or both...

Dilute-Dense Scattering

Boosting the target:

$$A^\mu_a(x) \mapsto \begin{cases} \gamma_t\, A^-_a\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ \frac{1}{\gamma_t} A^+_a\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ A^i_a\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \end{cases}$$

- $A^-_a \gg A^i_a \gg A^+_a$ in a generic gauge
- in the light-cone gauge:

$A^\mu_a(x) = \delta^{\mu -}\delta(x^+)A^-_a(\mathbf{x})$

target is localized at $x^+ = 0$

independent of x^-

Boosting the projectile :

$$J^\mu_a(x) \mapsto \begin{cases} \frac{1}{\gamma_p}\, J^-_a\left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \\ \gamma_p\, J^+_a\left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \\ J^i_a\left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \end{cases}$$

- $J^+_a \gg J^i_a \gg J^-_a$
- slow x^+ dependence due to Lorentz time dilation

$J^\mu_a(x) \propto \delta^{\mu +}\delta(x^-)\rho^a(\mathbf{x})$

projectile is localized at $x^- = 0$

Corrections beyond eikonal accuracy

At least three sources of corrections to eikonal approximation :

1. other components of the target background field $\mathcal{A}_a^\mu(x)$
2. dynamics of the target : x^- dependence of $\mathcal{A}_a^\mu(x)$
3. Finite width L^+ of the target along x^+

- In the context of jet quenching and in-medium energy loss:

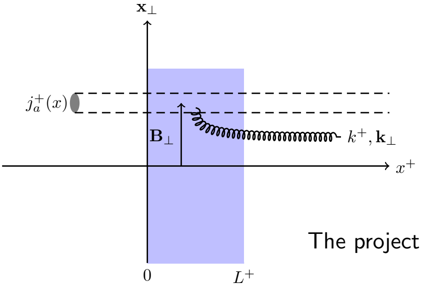
  full finite width effects are included, but not the other effects.

  $\rightarrow$ further approximation (like harmonic potential for BDMPS-Z) required to deal with quantum diffusion of the projectile inside the target.

- For scattering in the high-energy limit, power-suppressed finite L^+ correction can be calculated systematically without further approximation.

$$\boxed{\mathcal{A}^\mu = \delta^{\mu -}\delta(x^+)\mathcal{A}^-(\mathbf{x}) \rightarrow \mathcal{A}^\mu = \delta^{\mu -}\mathcal{A}^-(x^+, \mathbf{x})}$$

Consider a finite width target :



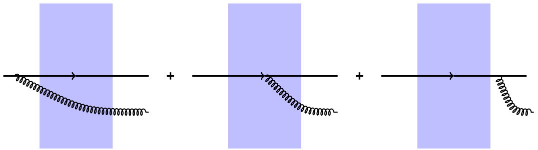
The target $\to \mathcal{A}^\mu(x) \equiv \delta^{\mu -} \mathcal{A}_a^-(x^+, \mathbf{x})$

The projectile $\to j_a^\mu(x) \propto \delta^{\mu +} \delta(x^-)\, \rho^b(\mathbf{x} - \mathbf{B})$

The single inclusive gluon cross section for pA:

$$(2\pi)^3\, (2k^+)\, \frac{d\sigma}{dk^+\, d^2\mathbf{k}} = \int d^2\mathbf{B} \sum_{\lambda\,\mathrm{phys.}} \left\langle \left\langle |\mathcal{M}^a_\lambda(\underline{k}, \mathbf{B})|^2 \right\rangle_p \right\rangle_A$$

gluon production amplitude

$\mathcal{M}^a_\lambda(\underline{k},\mathbf{B}) \;=$ 

at LO in g, LSZ reduction formula $\Rightarrow \mathcal{M}^a_\lambda(\underline{k},\mathbf{B}) = \varepsilon^{\mu *}_\lambda \int d^4x\; e^{ik\cdot x}\; \Box_x A^a_\mu(x)$

power counting : dense target $\Rightarrow \mathcal{A}^-_a(x^+,\mathbf{x}) = \mathcal{O}(1/g)$

dilute projectile $\Rightarrow j^+_a(\underline{x}) = \mathcal{O}(g)$

perturbative expansion of the classical field : $A^\mu_a(x) = \mathcal{A}^\mu_a(x) + a^\mu_a(x) + \mathcal{O}(g^3)$

$$a^\mu_a(x) \downarrow \; -i\int d^4y\; G^{\mu -}_R(x,y)_{ab}\; j^+_b(y)$$

In the LC gauge : $A^+_a = 0$ & $\varepsilon^{+*}_\lambda = 0 \Rightarrow \varepsilon^{\mu *}_\lambda \mathcal{A}^a_\mu(x) = -\varepsilon^{i*}_\lambda a^i_a$

$$\boxed{\mathcal{M}^a_\lambda(\underline{k},\mathbf{B}) = \varepsilon^{i*}_\lambda\,(2k^+) \lim_{x^+\to+\infty} \int d^2\mathbf{x} \int dx^-\; e^{ik\cdot x} \int d^4y\; G^{i-}_R(x,y)_{ab}\; j^+_b(y)}$$

$G^{\mu\nu}_R(x,y)_{ab}$ is the background retarded gluon propagator

Properties of the background retarded gluon propagator

$G_R^{\mu\nu}(x,y)_{ab}$ is the retarded solution of the linearized Yang-Mills equations around the classical field $\mathcal{A}_a^-(x^+,\mathbf{x})$:

$$\left[g_{\mu\nu}\left(\delta_{ab}\Box_x - 2ig\left(\mathcal{A}^-(x^+,\mathbf{x})\cdot T\right)_{ab}\partial_{x^-}\right) - \delta_{ab}\partial_{x^\mu}\partial_{x^\nu}\right] G_R^{\nu\rho}(x,y)_{bc} = i\, g_\mu{}^\rho\,\delta_{ac}\,\delta^{(4)}(x-y)$$

$\mathcal{A}_a^-(x^+,\mathbf{x})$ is independent of the x^-, so it is convenient to introduce the 1-d Fourier transform of $G_R^{\mu\nu}(x,y)_{ab}$

$$G_R^{\mu\nu}(x,y)_{ab} = \int \frac{\mathrm{d}k^+}{2\pi}\, e^{-ik^+(x^- - y^-)}\, \frac{1}{2(k^+ + i\epsilon)}\, \mathcal{G}_{k^+}^{\mu\nu}(\underline{x};\underline{y})_{ab}$$

The $(i-)$ component of the $\mathcal{G}_{k^+}^{\mu\nu}(\underline{x};\underline{y})_{ab}$ can be written as

$$\boxed{\mathcal{G}_{k^+}^{i-}(\underline{x};\underline{y})^{ab} = \frac{i}{k^+ + i\epsilon}\,\partial_{\mathbf{y}^i}\,\mathcal{G}_{k^+}^{ab}(\underline{x};\underline{y})}$$

$\mathcal{G}_{k^+}^{ab}(\underline{x};\underline{y})$ is the background scalar propagator.

Properties of the background retarded gluon propagator

The background scalar propagator $\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y})$ satisfies the scalar Green's equation :

$$\left[\delta^{ab}\left(i\partial_{x^+}+\frac{\partial_{\mathbf{x}}^2}{2(k^+ + i\epsilon)}\right)+g\Big(\mathcal{A}^-(\underline{x})\cdot T\Big)^{ab}\right]\mathcal{G}^{bc}_{k^+}(\underline{x};\underline{y})=i\,\delta^{ac}\,\delta^{(3)}(\underline{x}-\underline{y})$$

Schrödinger equation in $2+1$ dimensions in a space-time dependent matrix potential $-g\Big(\mathcal{A}^-(\underline{x})\cdot T\Big)^{ab}$.

The solution can be written formally as a path integral

$$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y})=\theta(x^+-y^+)\int_{\mathbf{z}(y^+)=\mathbf{y}}^{\mathbf{z}(x^+)=\mathbf{x}}\mathcal{D}\mathbf{z}(z^+)\ \exp\left[\frac{ik^+}{2}\int_{y^+}^{x^+}dz^+\ \dot{\mathbf{z}}^2(z^+)\right]$$
$$\times\ \mathcal{U}^{ab}\Big(x^+,y^+,\big[\mathbf{z}(z^+)\big]\Big)$$

with the Wilson line

$$\mathcal{U}^{ab}\Big(x^+,y^+,\big[\mathbf{z}(z^+)\big]\Big)\ =\ \mathcal{P}_+\exp\left\{ig\int_{y^+}^{x^+}dz^+\ T\cdot\mathcal{A}^-\Big(z^+,\mathbf{z}(z^+)\Big)\right\}^{ab}$$

following the Brownian trajectory $\mathbf{z}(z^+)$.

Discretization of the background propagator

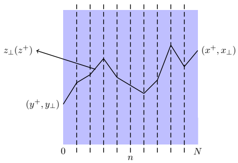
Discretized form of the background propagator :

$$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \lim_{N\to+\infty} \theta(x^+ - y^+) \int \left(\prod_{n=1}^{N-1} \mathrm{d}^2\mathbf{z}_n\right) \left(\frac{-i(k^+ + i\epsilon)N}{2\pi(x^+ - y^+)}\right)^N$$
$$\times \exp\left[\frac{i(k^+ + i\epsilon)N}{2(x^+ - y^+)} \sum_{n=0}^{N-1} (\mathbf{z}_{n+1} - \mathbf{z}_n)^2\right] \mathcal{U}^{ab}(x^+, y^+, \{\mathbf{z}_n\})$$

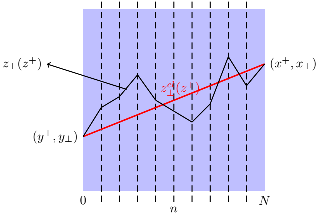
with the discertized Wilson line

$$\mathcal{U}^{ab}(x^+, y^+, \{\mathbf{z}_n\}) = \mathcal{P}_+ \left\{ \prod_{n=0}^{N-1} \exp\left[ig \frac{(x^+ - y^+)}{N} \left(\mathcal{A}^-(z_n^+, \mathbf{z}_n) \cdot T\right)\right]\right\}^{ab}$$

with $z_n^+ = y^+ + \frac{n}{N}(x^+ - y^+)$



(i) Perturbative expansion around free classical path:



eikonal limit : $\frac{k^+}{(x^+ - y^+)} \gg Q_\perp^2$ in the problem

$\mapsto$ large k^+ limit (classical free path!)

$\Rightarrow$ perturbative expansion around the free classical path:

$\mathbf{z}_n = \mathbf{z}_n^{\rm cl} + \mathbf{u}_n$ with $\mathbf{z}_n^{\rm cl} = \mathbf{y} + \frac{n}{N}(\mathbf{x} - \mathbf{y})$

Then

$$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = 2\pi i \mathcal{G}_{0,k^+}(\underline{x};\underline{y}) \frac{(x^+ - y^+)}{k^+} \lim_{N\to+\infty} \int \left(\prod_{n=1}^{N-1} \mathrm{d}^2 \mathbf{u}_n \right)$$

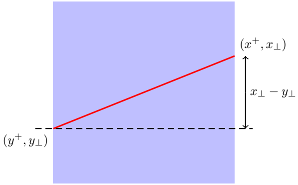
$$\times \mathcal{P}_+ \prod_{n=0}^{N-1} \left\{ \mathcal{G}_{0,k^+}\left(z^+_{n+1}, \mathbf{u}_{n+1}; z^+_n, \mathbf{u}_n\right) \exp\left[\frac{(x^+ - y^+)}{N} igT \cdot \mathcal{A}^-\left(z^+_n, \mathbf{z}_n^{\rm cl} + \mathbf{u}_n\right) \right] \right\}$$

- Taylor expand the gauge link around $\mathbf{u}_n = 0$ at each discretization step.
- Take the product of the Taylor expansion of all links and collect terms according to power of $\mathbf{u}_n$.

(ii) Expansion around the initial transverse position :

The first expansion is performed for fixed initial and final positions.

In the large k^+ limit , the result has to be re-expanded since $\mathbf{z}^{\mathrm{cl}}(z^+) - \mathbf{y}$ is small at each step.



After all:

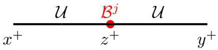
$$\int d^2\mathbf{x}\; e^{-i\mathbf{k}\cdot\mathbf{x}}\; \mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \theta(x^+ - y^+)\; e^{-i\mathbf{k}\cdot\mathbf{y}}\; e^{-ik^-(x^+-y^+)} \Bigg\{ \mathcal{U}(x^+, y^+, \mathbf{y})$$

$$+ \frac{(x^+ - y^+)}{k^+}\Big[\mathbf{k}^i\, \mathcal{U}^i_{[0,1]}(x^+, y^+, \mathbf{y}) + \frac{i}{2}\,\mathcal{U}_{[1,0]}(x^+, y^+, \mathbf{y})\Big]$$

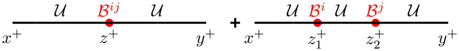
$$+ \frac{(x^+ - y^+)^2}{(k^+)^2}\Big[\mathbf{k}^i\mathbf{k}^j\mathcal{U}^{ij}_{[0,2]}(x^+, y^+; \mathbf{y}) + \frac{i}{2}\mathbf{k}^i\mathcal{U}^i_{[1,1]}(x^+, y^+; \mathbf{y}) - \frac{1}{4}\mathcal{U}_{[2,0]}(x^+, y^+; \mathbf{y})\Big]\Bigg\}^{ab}$$

Structure of the decorated Wilson lines

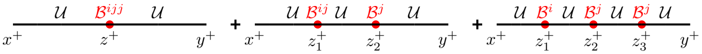
$\mathcal{U}^{j}_{[0,1]} \propto$



$\mathcal{U}^{ij}_{[0,2]} \propto$



$\mathcal{U}^{i}_{[1,1]} \propto$



with

$$
\begin{aligned}
\mathcal{B}^{i}(z^+,\mathbf{y}) &\equiv igT\cdot\partial_{\mathbf{y}^i}\mathcal{A}^-(z^+,\mathbf{y}),\\
\mathcal{B}^{ij}(z^+,\mathbf{y}) &\equiv igT\cdot\partial_{\mathbf{y}^i}\partial_{\mathbf{y}^j}\mathcal{A}^-(z^+,\mathbf{y}),\\
\mathcal{B}^{ijl}(z^+,\mathbf{y}) &\equiv igT\cdot\partial_{\mathbf{y}^i}\partial_{\mathbf{y}^j}\partial_{\mathbf{y}^l}\mathcal{A}^-(z^+,\mathbf{y}),
\end{aligned}
$$

Reduced amplitude and the observable

Defining Fourier transform of the color charge density

$$\rho^a(\mathbf{y}-\mathbf{B}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2}\, e^{i\mathbf{q}\cdot(\mathbf{y}-\mathbf{B})}\, \tilde{\rho}^a(\mathbf{q})$$

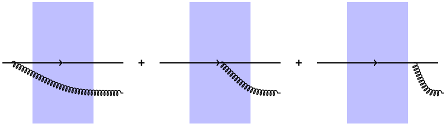
and gluon-nucleus reduced amplitude

$$\mathcal{M}^a_\lambda(\underline{k},\mathbf{B}) \;=\; \int \frac{d^2\mathbf{q}}{(2\pi)^2}\, e^{-i\mathbf{q}\cdot\mathbf{B}}\; \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})\; \tilde{\rho}^b(\mathbf{q})$$

The cross section:

$$k^+\frac{d\sigma}{dk^+\,d^2\mathbf{k}} = \int \frac{d^2\mathbf{q}}{(2\pi)^2}\, \varphi_p(\mathbf{q})\, \frac{\mathbf{q}^2}{4}\, \frac{1}{N_c^2-1} \sum_{\lambda\,\mathrm{phys.}} \Big\langle \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})^\dagger\, \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q}) \Big\rangle_A .$$

Total reduced amplitude at next-to-eikonal accuracy



strict eikonal term!

$$\overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k};\mathbf{q}) = i\,\varepsilon^{i*}_{\lambda}\int d^2\mathbf{y}\, e^{i\mathbf{y}\cdot(\mathbf{q}-\mathbf{k})}\Bigg\{2\left(\frac{\mathbf{k}^i}{\mathbf{k}^2}-\frac{\mathbf{q}^i}{\mathbf{q}^2}\right)\mathcal{U}(L^+,0;\mathbf{y})$$

$$+\left(\frac{L^+}{k^+}\right)\left[\left(\delta^{ij}-2\,\frac{\mathbf{q}^i\mathbf{k}^j}{\mathbf{q}^2}\right)\mathcal{U}^j_{[0,1]}(L^+,0;\mathbf{y}) - i\frac{\mathbf{q}^i}{\mathbf{q}^2}\,\mathcal{U}_{[1,0]}(L^+,0;\mathbf{y})\right]$$

$$+\left(\frac{L^+}{k^+}\right)^2\Bigg[-2\frac{\mathbf{q}^i}{\mathbf{q}^2}\mathbf{k}^j\mathbf{k}^l\,\mathcal{U}^{jl}_{[0,2]}(L^+,0;\mathbf{y}) - i\frac{\mathbf{q}^i\mathbf{k}^j}{\mathbf{q}^2}\,\mathcal{U}^j_{[1,1]}(L^+,0;\mathbf{y}) + \frac{1}{2}\frac{\mathbf{q}^i}{\mathbf{q}^2}\,\mathcal{U}_{[2,0]}(L^+,0;\mathbf{y})$$

$$+\frac{i}{4}\left(\mathbf{k}^2\delta^{ij}-2\mathbf{k}^i\mathbf{k}^j\right)\mathcal{U}^j_{(\mathrm{A})}(L^+,0;\mathbf{y}) + \frac{\mathbf{k}^j}{4}\,\mathcal{U}^{ij}_{(\mathrm{B})}(L^+,0;\mathbf{y}) + \frac{i}{4}\,\mathcal{U}^i_{(\mathrm{C})}(L^+,0;\mathbf{y})\Bigg]\Bigg\}^{ab}$$

Dipole operators

The usual adjoint dipole

$$\mathcal{O}(\mathbf{r}) = \int d^2\mathbf{b}\, \frac{1}{N_c^2-1} \left\langle \mathrm{tr}\left[\mathcal{U}\left(\mathbf{b}+\frac{\mathbf{r}}{2}\right)\mathcal{U}^\dagger\left(\mathbf{b}-\frac{\mathbf{r}}{2}\right)\right]\right\rangle_A$$

The decorated adjoint dipoles with one decorated Wilson line $\mathcal{U}^{i\cdots j}_{[\alpha,\beta]}$

$$\mathcal{O}^{i\cdots j}_{[\alpha,\beta]}(\mathbf{r}) = \int d^2\mathbf{b}\, \frac{1}{N_c^2-1} \left\langle \mathrm{tr}\left[\mathcal{U}^{i\cdots j}_{[\alpha,\beta]}\left(\mathbf{b}+\frac{\mathbf{r}}{2}\right)\mathcal{U}^\dagger\left(\mathbf{b}-\frac{\mathbf{r}}{2}\right)\right]\right\rangle_A$$

The decorated adjoint dipoles formed from two decorated Wilson lines

$$\mathcal{O}^{i\cdots j;l\cdots m}_{[\alpha,\beta];[\gamma,\delta]}(\mathbf{r}) = \int d^2\mathbf{b}\, \frac{1}{N_c^2-1} \left\langle \mathrm{tr}\left[\mathcal{U}^{i\cdots j}_{[\alpha,\beta]}\left(\mathbf{b}+\frac{\mathbf{r}}{2}\right)\mathcal{U}^{l\cdots m\,\dagger}_{[\gamma,\delta]}\left(\mathbf{b}-\frac{\mathbf{r}}{2}\right)\right]\right\rangle_A$$

Squared Amplitude

Strict eikonal:

$$\frac{1}{N_c^2-1}\left\langle \overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k},\mathbf{q})^{\dagger}\overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k},\mathbf{q})\right\rangle\Big|_{\mathrm{E}} = \varepsilon^{i*}_{\lambda}\varepsilon^{j}_{\lambda}\; f^{ij}(\mathbf{k},\mathbf{q}) \times (\text{adjoint dipole})$$

The next-to-eikonal contribution:

$$\frac{1}{N_c^2-1}\left\langle \overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k},\mathbf{q})^{\dagger}\overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k},\mathbf{q})\right\rangle\Big|_{\mathrm{NE}} = \varepsilon^{i*}_{\lambda}\varepsilon^{j}_{\lambda}\,\frac{L^+}{k^+}\,\mathrm{g}^{ij}(\mathbf{k},\mathbf{q}) \times (\text{decorated dipoles})$$

The next-to-next-to-eikonal contribution:

$$\frac{1}{N_c^2-1}\left\langle \overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k},\mathbf{q})^{\dagger}\overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k},\mathbf{q})\right\rangle\Big|_{\mathrm{NNE}} = \varepsilon^{i*}_{\lambda}\varepsilon^{j}_{\lambda}\left(\frac{L^+}{k^+}\right)^2\,\mathrm{l}^{ij}(\mathbf{k},\mathbf{q}) \;\times\; (\text{decorated dipoles})$$

$\mathrm{f}^{ij}(\mathbf{k},\mathbf{q})$ and $\mathrm{l}^{ij}(\mathbf{k},\mathbf{q})$ are symmetric under $i \to j$

$\mathrm{g}^{ij}(\mathbf{k},\mathbf{q})$ is anti-symmetric under $i \to j$

Back to the cross-section

For the single inclusive gluon cross section:

$$\sum_{\lambda} \varepsilon_{\lambda}^{i*} \varepsilon_{\lambda}^{j} = \delta^{ij}$$

The next-to-eikonal contributions vanish!

The next-to-next-to-eikonal terms give contribution.

$$k^+ \frac{d\sigma}{dk^+ \, d^2\mathbf{k}} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \, \varphi_p(\mathbf{q}) \, \frac{\mathbf{q}^2}{4} \int d^2\mathbf{r} \; e^{i\mathbf{r}\cdot(\mathbf{q}-\mathbf{k})} \Bigg\{ 4\, \mathcal{C}^i(\mathbf{k},\mathbf{q}) \, \mathcal{C}^i(\mathbf{k},\mathbf{q}) \, \mathcal{O}(\mathbf{r})$$

$$+ \left(\frac{L^+}{k^+}\right)^2 \Bigg[2\, \frac{\mathbf{q}^i}{\mathbf{q}^2} \, \mathcal{C}^i(\mathbf{k},\mathbf{q}) \Big[-4\, \mathbf{k}^l \mathbf{k}^m \, \mathcal{O}^{l,m}_{[0,2]}(\mathbf{r}) - 2i\, \mathbf{k}^l \, \mathcal{O}^l_{[1,1]}(\mathbf{r}) + \mathcal{O}_{[2,0]}(\mathbf{r}) \Big]$$

$$+ \mathcal{C}^i(\mathbf{k},\mathbf{q}) \Big[i(\mathbf{k}^2 \delta^{il} - 2\mathbf{k}^i \mathbf{k}^l) \, \mathcal{O}^l_{\mathrm{(A)}}(\mathbf{r}) + \mathbf{k}^m \, \mathcal{O}^{im}_{\mathrm{(B)}}(\mathbf{r}) + i\, \mathcal{O}^i_{\mathrm{(C)}}(\mathbf{r}) \Big]$$

$$+ \tilde{\mathcal{C}}^{li}(\mathbf{k},\mathbf{q}) \Big[\tilde{\mathcal{C}}^{mi}(\mathbf{k},\mathbf{q}) \, \mathcal{O}^{l;m}_{[0,1];[0,1]}(\mathbf{r}) + 2i\, \frac{\mathbf{q}^i}{\mathbf{q}^2} \, \mathcal{O}^l_{[0,1];[1,0]}(\mathbf{r}) \Big] + \frac{1}{\mathbf{q}^2} \, \mathcal{O}_{[1,0];[1,0]}(\mathbf{r}) \Bigg] \Bigg\}$$

with

$\mathcal{C}^i(\mathbf{k},\mathbf{q}) = \left(\frac{\mathbf{k}^i}{\mathbf{k}^2} - \frac{\mathbf{q}^i}{\mathbf{q}^2} \right)$ and $\tilde{\mathcal{C}}^{ij}(\mathbf{k},\mathbf{q}) = \left(\delta^{ij} - 2\, \mathbf{k}^i \frac{\mathbf{q}^j}{\mathbf{q}^2} \right)$

Single transverse spin asymmetry : polarized target

Consider the process $p + A^{\uparrow} \to g + X$. The SSA is defined as

$$A_N = \frac{k^+ \frac{d\sigma^{\uparrow}}{dk^+ \, d^2\mathbf{k}} - k^+ \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\mathbf{k}}}{k^+ \frac{d\sigma^{\uparrow}}{dk^+ \, d^2\mathbf{k}} + k^+ \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\mathbf{k}}} = \frac{k^+ \frac{d\sigma^{\uparrow}}{dk^+ \, d^2\mathbf{k}} - k^+ \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\mathbf{k}}}{2k^+ \frac{d\sigma}{dk^+ \, d^2\mathbf{k}}}$$

Transversely polarized target $\Rightarrow$ **s** dependent **standard adjoint** and **decorated dipoles**!

Due to transverse rotational symmetry around the center of the target:

$$\mathcal{O}(-\mathbf{r}, -\mathbf{b}, -\mathbf{s}) = \mathcal{O}(\mathbf{r}, \mathbf{b}, \mathbf{s})$$
$$\mathcal{O}^{i\cdots j}_{[\alpha,\beta]}(-\mathbf{r}, -\mathbf{b}, -\mathbf{s}) = (-1)^n \mathcal{O}^{j}_{[\alpha,\beta]}(\mathbf{r}, \mathbf{b}, \mathbf{s})$$
$$\mathcal{O}^{i\cdots j; l\cdots m}_{[\alpha,\beta];[\gamma,\delta]}(-\mathbf{r}, -\mathbf{b}, -\mathbf{s}) = (-1)^n \mathcal{O}_{[\alpha,\beta];[\gamma,\delta]}(\mathbf{r}, \mathbf{b}, \mathbf{s})$$

$n \equiv$ number of indicies.Then:

$$k^+ \left(\frac{d\sigma^{\uparrow}}{dk^+ \, d^2\mathbf{k}} - \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\mathbf{k}} \right) = \frac{L^+}{k^+} \int_{\mathbf{qrb}} \varphi_p(\mathbf{q}) \Big\{ f(\mathbf{k}^j, \mathbf{q}^j, \mathbf{r}) \mathcal{O}^{j}_{[0,1]}(\mathbf{r}, \mathbf{b}, \mathbf{s}) + g(\mathbf{k}, \mathbf{q}, \mathbf{r}) \mathcal{O}_{[1,0]}(\mathbf{r}, \mathbf{b}, \mathbf{s}) \Big\}$$

The strict eikonal and next-to-next-to-eikonal terms vanishes!!

Next-to-eikonal terms in the expansion become leading terms!!

Anology between twist-3 contributions to hard processes and next-to-eikonal contributions to high-energy processes??

Conclusions and Outlook

- We develop a systematic eikonal expansion of the retarded gluon propagator in a background field.
- We apply this method to single inclusive gluon cross section in order to study the corrections to the CGC beyond eikonal limit. The strict eikonal term provides the usual $k_\perp$-factorization formula, whereas the next-to-eikonal corrections vanish for this particular observable. The first non vanishing correction to the strict eikonal limit appears at next-to-next-to-eikonal accuracy.
- The same expansion can be applied to DIS and "Hybrid Formula" by considering the quark background propagator [work in progress!]
- Rapidity evolution of the decorated dipoles?[work in progress!]

Back up slides - Decorated Wilson lines

$$\mathcal{U}^{i}_{[0,1]}(x^+,y^+;\mathbf{y}) = \mathbb{P}_+\,\mathcal{U}(x^+,y^+;\mathbf{y})\int_{y^+}^{x^+} dz_1^+\,\frac{(z_1^+ - y^+)}{(x^+ - y^+)}\,\mathcal{B}^i(z_1^+,\mathbf{y})$$

$$\mathcal{U}^{ij}_{[0,2]}(x^+,y^+;\mathbf{y}) = \mathbb{P}_+\,\mathcal{U}(x^+,y^+;\mathbf{y})\Bigg\{\int_{y^+}^{x^+} dz_1^+\,\left(\frac{z_1^+ - y^+}{x^+ - y^+}\right)^2\,\mathcal{B}^{ij}(z_1^+,\mathbf{y})$$
$$+2\int_{y^+}^{x^+} dz_1^+\int_{z_1^+}^{x^+} dz_2^+\,\frac{(z_2^+ - y^+)(z_1^+ - y^+)}{(x^+ - y^+)^2}\,\mathcal{B}^i(z_2^+,\mathbf{y})\;\;\mathcal{B}^j(z_1^+,\mathbf{y})\Bigg\}$$

$$\mathcal{U}_{[1,0]}(x^+,y^+;\mathbf{y}) = \mathbb{P}_+\,\mathcal{U}(x^+,y^+;\mathbf{y})\Bigg\{\int_{y^+}^{x^+} dz_1^+\,\frac{(z_1^+ - y^+)}{(x^+ - y^+)}\delta^{ij}\,\mathcal{B}^{ij}(z_1^+,\mathbf{y})$$
$$+2\int_{y^+}^{x^+} dz_1^+\int_{y^+}^{z_1^+} dz_2^+\,\frac{(z_2^+ - y^+)}{(x^+ - y^+)}\,\mathcal{B}^i(z_1^+,\mathbf{y})\;\;\mathcal{B}^i(z_2^+,\mathbf{y})\Bigg\}$$

Back up slides - Decorated Wilson lines

$$\mathcal{U}^i_{[1,1]}(x^+,y^+;\mathbf{y}) = \mathbb{P}_+\,\mathcal{U}(x^+,y^+;\mathbf{y})\Bigg\{\int_{y^+}^{x^+}dz_1^+\,\left(\frac{z_1^+-y^+}{x^+-y^+}\right)^2\delta^{jl}\,\mathcal{B}^{ijl}(z_1^+,\mathbf{y})$$

$$+2\int_{y^+}^{x^+}dz_1^+\int_{z_1^+}^{x^+}dz_2^+\Bigg[\left(\frac{z_1^+-y^+}{x^+-y^+}\right)^2\mathcal{B}^j(z_2^+,\mathbf{y})\;\mathcal{B}^{ij}(z_1^+,\mathbf{y})$$

$$+\frac{(z_1^+-y^+)(z_2^+-y^+)}{(x^+-y^+)^2}\Bigg(\mathcal{B}^{ij}(z_2^+,\mathbf{y})\;\mathcal{B}^j(z_1^+,\mathbf{y})$$

$$+\frac{1}{2}\mathcal{B}^i(z_2^+,\mathbf{y})\;\delta^{jl}\,\mathcal{B}^{jl}(z_1^+,\mathbf{y})+\frac{1}{2}\delta^{jl}\,\mathcal{B}^{jl}(z_2^+,\mathbf{y})\;\mathcal{B}^i(z_1^+,\mathbf{y})\Bigg)\Bigg]$$

$$+2\int_{y^+}^{x^+}dz_1^+\int_{z_1^+}^{x^+}dz_2^+\int_{z_2^+}^{x^+}dz_3^+\Bigg[\frac{(z_3^+-y^+)(z_1^+-y^+)}{(x^+-y^+)^2}\mathcal{B}^i(z_3^+,\mathbf{y})\;\mathcal{B}^j(z_2^+,\mathbf{y})\;\mathcal{B}^j(z_1^+,\mathbf{y})$$

$$+\frac{(z_2^+-y^+)(z_1^+-y^+)}{(x^+-y^+)^2}\mathcal{B}^j(z_3^+,\mathbf{y})\left(\mathcal{B}^i(z_2^+,\mathbf{y})\,\mathcal{B}^j(z_1^+,\mathbf{y})+\mathcal{B}^j(z_2^+,\mathbf{y})\,\mathcal{B}^i(z_1^+,\mathbf{y})\right)\Bigg]\Bigg\}$$

$$\frac{1}{N_c^2-1}\left\langle \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})^\dagger \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})\right\rangle\Big|_{\mathrm{E}} = \varepsilon^{i*}_\lambda \varepsilon^{j}_\lambda \int d^2\mathbf{r}\; e^{i\mathbf{r}\cdot(\mathbf{q}-\mathbf{k})}\, 4\,\mathcal{C}^i(\mathbf{k},\mathbf{q})\,\mathcal{C}^j(\mathbf{k},\mathbf{q})\,\mathcal{O}(\mathbf{r})\,. \qquad (1)$$

Similarly, the next-to-eikonal contribution can be written as

$$\frac{1}{N_c^2-1}\left\langle \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})^\dagger \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})\right\rangle\Big|_{\mathrm{NE}} = \varepsilon^{i*}_\lambda \varepsilon^{j}_\lambda \int d^2\mathbf{r}\; e^{i\mathbf{r}\cdot(\mathbf{q}-\mathbf{k})}\, 2\,\frac{L^+}{k^+} \qquad (2)$$
$$\times\Big\{\Big[\mathcal{C}^j(\mathbf{k},\mathbf{q})\,\tilde{\mathcal{C}}^{li}(\mathbf{k},\mathbf{q}) - \mathcal{C}^i(\mathbf{k},\mathbf{q})\,\tilde{\mathcal{C}}^{lj}(\mathbf{k},\mathbf{q})\Big]\,\mathcal{O}^l_{[0,1]}(\mathbf{r}) - i\Big[\mathcal{C}^j(\mathbf{k},\mathbf{q})\,\frac{\mathbf{q}^i}{\mathbf{q}^2} - \mathcal{C}^i(\mathbf{k},\mathbf{q})\,\frac{\mathbf{q}^j}{\mathbf{q}^2}\Big]\,\mathcal{O}_{[1,0]}(\mathbf{r})\Big\}$$

and

$$\frac{1}{N_c^2-1}\left\langle \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})^\dagger \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})\right\rangle\Big|_{\mathrm{NNE}} = \varepsilon^{i*}_\lambda \varepsilon^{j}_\lambda \int d^2\mathbf{r}\; e^{i\mathbf{r}\cdot(\mathbf{q}-\mathbf{k})}\left(\frac{L^+}{k^+}\right)^2$$
$$\times\Big\{\Big[\mathcal{C}^j(\mathbf{k},\mathbf{q})\,\frac{\mathbf{q}^i}{\mathbf{q}^2} + \mathcal{C}^i(\mathbf{k},\mathbf{q})\,\frac{\mathbf{q}^j}{\mathbf{q}^2}\Big]\Big[-4\,\mathbf{k}^l\mathbf{k}^m\,\mathcal{O}^{l,m}_{[0,2]}(\mathbf{r}) - 2i\,\mathbf{k}^l\,\mathcal{O}^l_{[1,1]}(\mathbf{r}) + \mathcal{O}_{[2,0]}(\mathbf{r})\Big]$$
$$+\frac{i}{2}\Big[\mathcal{C}^j(\mathbf{k},\mathbf{q})\,(\mathbf{k}^2\delta^{il} - 2\mathbf{k}^i\mathbf{k}^l) + \mathcal{C}^i(\mathbf{k},\mathbf{q})\,(\mathbf{k}^2\delta^{jl} - 2\mathbf{k}^j\mathbf{k}^l)\Big]\,\mathcal{O}^l_{(\mathrm{A})}(\mathbf{r})$$
$$+\frac{1}{2}\Big[\mathcal{C}^j(\mathbf{k},\mathbf{q})\,\delta^{il} + \mathcal{C}^i(\mathbf{k},\mathbf{q})\,\delta^{jl}\Big]\Big[\mathbf{k}^m\,\mathcal{O}^{lm}_{(\mathrm{B})}(\mathbf{r}) + i\,\mathcal{O}^l_{(\mathrm{C})}(\mathbf{r})\Big]$$
$$+\Big[\tilde{\mathcal{C}}^{li}(\mathbf{k},\mathbf{q})\,\tilde{\mathcal{C}}^{mj}(\mathbf{k},\mathbf{q})\,\mathcal{O}^{l;m}_{[0,1];[0,1]}(\mathbf{r}) + \frac{\mathbf{q}^i}{\mathbf{q}^2}\,\frac{\mathbf{q}^j}{\mathbf{q}^2}\,\mathcal{O}_{[1,0];[1,0]}(\mathbf{r})\Big]$$
$$+i\Big[\frac{\mathbf{q}^i}{\mathbf{q}^2}\,\tilde{\mathcal{C}}^{lj}(\mathbf{k},\mathbf{q}) + \frac{\mathbf{q}^j}{\mathbf{q}^2}\,\tilde{\mathcal{C}}^{li}(\mathbf{k},\mathbf{q})\Big]\,\mathcal{O}^l_{[0,1];[1,0]}(\mathbf{r})\Big\}\,. \qquad (3)$$