NLO evolution equations for 4-point colorless operators

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8.09.2015

based on arXiv:1507.08622

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Definitions

Introduce the light cone vectors n_1 and n_2

$$n_1 = (1,0,0,1)\,, \quad n_2 = \frac{1}{2}(1,0,0,-1)\,, \quad n_1^+ = n_2^- = n_1 n_2 = 1$$

For any p define $p^{\pm}$

$$p^+ = pn_2 = \frac{1}{2}\left(p^0 + p^3\right), \qquad p^- = pn_1 = p^0 - p^3,$$

$$p^2 = 2p^+p^- - \vec{p}^{\,\mathbf{2}};$$

The scalar products:

$$p = p^+ n_1 + p^- n_2 + p_\perp, \quad (p\,k) = p^\mu k_\mu = p^+k^- + p^-k^+ - \vec{p}\vec{k}.$$

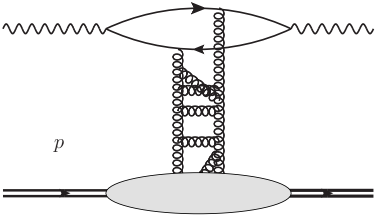
Definitions

Wilson line describing interaction with external field b_η^- made of slow gluons with $p^+ < e^\eta$

$$U_{\vec{z}}^\eta = Pe^{ig\int_{-\infty}^{+\infty} dz^+ b_\eta^-(z^+,\vec{z})}, \quad b_\eta^- = \int \frac{d^4p}{(2\pi)^4} e^{-ipz} b^-(p)\,\theta(e^\eta - p^+).$$

Introduction

Dipole picture $s \gg Q^2 \gg \Lambda^2_{QCD}$



$$\sigma_{\gamma^*}(s, Q^2) = \int d^2\mathbf{r} |\Psi_{\gamma^*}(\mathbf{r}, Q^2)|^2 \sigma_{dip}(\mathbf{r}, s), \quad \sigma_{dip}(\mathbf{r}, s) = 2 \int d\mathbf{b}(1 - \frac{1}{N_c} F(\mathbf{b}, \mathbf{r}, s))$$

$\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ — dipole size, $\mathbf{b} = \frac{1}{2}(\mathbf{r_1} + \mathbf{r_2})$ — impact parameter, $F = tr(U_1 U_2^\dagger)$, — dipole Green function, $U_i = U_i^\eta$ — Wilson lines, describing fast moving quarks interacting with the target.

η — rapidity divide, gluons with $p^+ > e^\eta$ belong to photon wavefunction, gluons with $p^+ < e^\eta$ belong to Wilson lines, describing the field of the target.

Introduction

$tr(U_1 U_2^\dagger)$ obeys the LO Balitsky-Kovchegov evolution equation

$$\frac{\partial tr(U_1 U_2^\dagger)}{\partial \eta} = \frac{\alpha_s}{2\pi^2} \int d\vec{r}_4 \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{14}^{\,2}\vec{r}_{42}^{\,2}} \left[tr(U_1 U_4^\dagger) tr(U_4 U_2^\dagger) - N_c tr(U_1 U_2^\dagger) \right].$$

$\vec{r}_{ij} \equiv \vec{r}_i - \vec{r}_j$
LO equation was obtained in 1996 (Balitsky) - 99 (Kovchegov), NLO — in 2007-2010 (Balitsky and Chirilli).

Shock wave

For a fast moving particle with the velocity $-\beta$ and the field strength tensor $\mathbb{F}(x^+, x^-, \vec{x})$ in its rest frame, in the observer's frame the field will look like

$$\mathfrak{F}^{-i}\left(y^+, y^-, \vec{y}\right) = \lambda \mathbb{F}^{-i}(\lambda y^+, \frac{1}{\lambda} y^-, \vec{y}) \to \delta\left(y^+\right) \mathfrak{F}^i\left(\vec{y}\right),$$

$$\mathfrak{F}^{-i} \gg \mathfrak{F}^{\cdots}$$

in the Regge limit $\lambda \to +\infty$, $\lambda = \sqrt{\frac{1+\beta}{1-\beta}}$.

Therefore the natural choice for the gauge is $b^{i,+} = 0$,

b^- is the solution of the equations

$$\frac{\partial b^-}{\partial y^i} = \delta(y^+) \mathfrak{F}^i\left(\vec{y}\right), i.e.$$

$$b^\mu\left(y\right) = \delta(y^+) B\left(\vec{y}\right) n_2^\mu$$

It is the shock-wave field.

Propagator in the shock wave background

Choose the gluon field $\mathcal{A}$ in the gauge $\mathcal{A}n_2 = 0$ as a sum of external classical b and quantum A.

$$\mathcal{A} = A + b, \quad b^{\mu}(x) = \delta(x^+) B(\vec{x})\, n_2^{\mu}.$$

The A-b interaction lagrangian has only one vertex

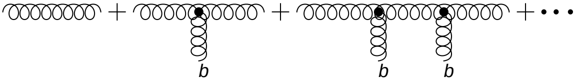
$$\mathcal{L}_i = \frac{g}{2} f^{acb} (b^-)^c g_{\perp}^{\alpha\beta} \left[A_{\alpha}^a \frac{\overleftrightarrow{\partial}}{\partial x^-} A_{\beta}^b \right].$$

The free propagator $G_0^{\mu\nu}(x^+, p^+, \vec{p}) =$

$$= \frac{-d_0^{\mu\nu}(p^+, p_{\perp})}{2p^+} e^{-i\frac{\vec{p}^{\,2} x^+}{2p^+}} \left(\theta(x^+)\theta(p^+) - \theta(-x^+)\theta(-p^+)\right) + n_2^{\mu} n_2^{\nu} \ldots,$$

$$d_0^{\mu\nu}(p) = g_{\perp}^{\mu\nu} - \frac{p_{\perp}^{\mu} n_2^{\nu} + p_{\perp}^{\nu} n_2^{\mu}}{p^+} - \frac{n_2^{\mu} n_2^{\nu} \vec{p}^{\,2}}{(p^+)^2}.$$

Propagator in the shock-wave background



Sum the diagrams

- b does not depend on x^-, hence the conservation of p^+,
- $b \sim \delta(x^+)$, hence $e^{-i\frac{\vec{p}^{\,2}(x_1^+ - x_2^+)}{2p^+}} \to 1$ in every internal vertex,
- $g_\perp^{\mu\nu} d_{0\nu\rho} g_\perp^{\rho\sigma} = g_\perp^{\mu\sigma}$, hence no dependence on $\vec{p} \Longrightarrow$ conservation of $\vec{x}$ in every internal vertex

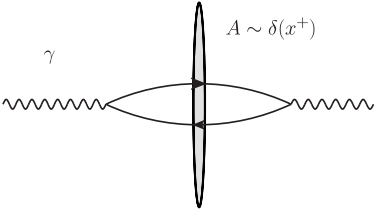
Propagator in the shock-wave background:

$$G_{\mu\nu}(x,y)|_{x^+>0>y^+} = 2i\, \overbrace{A^{\mu}(x) \int d^4 z\, \delta(z^+) F}^{}{}^{+i}(z) \frac{U_{\vec{z}}}{\overrightarrow{\frac{\partial}{\partial z^-}}} \overbrace{F^{+i}(z)\, A}^{}{}^{\nu}(y)\,.$$

where the interaction with b is through Wilson line

$$U_{\vec{z}} = P e^{ig\int_{y^+}^{x^+} dz^+ b^-(z^+, \vec{z})}.$$

Dipole picture



Color field of a fast moving particle $A^- \sim \delta(z^+) A^\eta(z_\perp)$
$A^\eta(z_\perp)$ contains slow components with rapidities $< \eta$

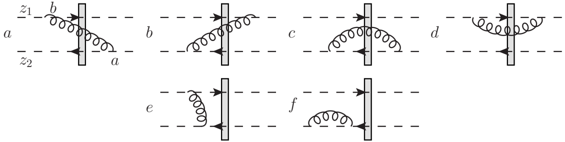
Quark propagator in such an external field $G(x, y) \sim U^\eta(z_\perp)$

DIS matrix element contains a Wilson loop = color dipole operator $U_{12}^\eta = tr(U^\eta(z_{1\perp}) U^{\eta\dagger}(z_{2\perp}))$. Balitsky 1996

Balitsky derivation of the BK equation

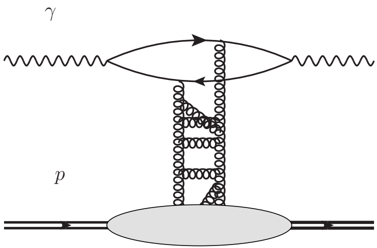
To derive the evolution equation we have to change $\eta \to \eta + \Delta\eta$ and integrate over the fields with the rapidities in the strip $\Delta\eta$

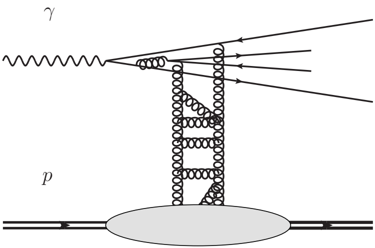
$$U_{12}^{\eta+\Delta\eta} = U_{12}^{\eta} + \frac{\langle 0|T(U_{12}^{\Delta\eta} e^{i\int \mathcal{L}(z)dz})|0\rangle}{\langle 0|T(e^{i\int \mathcal{L}(z)dz})|0\rangle}.$$



$$\frac{\partial U_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s}{2\pi^2}\int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2}\vec{z}_{42}^{\,2}}\left[U_{14}^{\eta}U_{42}^{\eta} - N_c U_{12}^{\eta}\right].$$

Motivation





Dipole picture,
BK equation for dipole
$tr(U_1 U_2^\dagger)$

Evolution equation for
quadrupole operator
$tr(U_1 U_2^\dagger U_3 U_4^\dagger)$
and double dipole operator
$tr(U_1 U_2^\dagger) tr(U_3 U_4^\dagger)$

LO Evolution equation for quadrupole

$$tr(U_i U_j^\dagger ... U_k U_l^\dagger) \equiv \mathbf{U}_{ij^\dagger...kl^\dagger},$$

Jalilian-Marian Kovchegov Dumitru 2004, 2010
Dominguez Mueller Munier Xiao 2011

$$\frac{\partial \mathbf{U}_{12^\dagger 34^\dagger}}{\partial \eta} = \frac{\alpha_s}{4\pi^2}\int d\vec{r}_0 \left\{ \frac{\vec{r}_{14}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{40}^{\,2}} (\mathbf{U}_{10^\dagger}\mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{4^\dagger 0}\mathbf{U}_{12^\dagger 30^\dagger} - (0 \to 1)) \right.$$
$$+ \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{20}^{\,2}} (\mathbf{U}_{10^\dagger}\mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{2^\dagger 0}\mathbf{U}_{10^\dagger 34^\dagger} - (0 \to 1))$$
$$- \frac{\vec{r}_{24}^{\,2}}{2\vec{r}_{20}^{\,2}\vec{r}_{40}^{\,2}} (\mathbf{U}_{10^\dagger}\mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{30^\dagger}\mathbf{U}_{04^\dagger 12^\dagger} - (0 \to 4))$$
$$- \frac{\vec{r}_{13}^{\,2}}{2\vec{r}_{10}^{\,2}\vec{r}_{30}^{\,2}} (\mathbf{U}_{4^\dagger 0}\mathbf{U}_{12^\dagger 30^\dagger} + \mathbf{U}_{2^\dagger 0}\mathbf{U}_{34^\dagger 10^\dagger} - (0 \to 1))$$
$$\left. + (1 \leftrightarrow 3, 2 \leftrightarrow 4) \right\}.$$

LO Evolution equation for double dipole

$$tr(U_i U_j^\dagger ... U_k U_l^\dagger) \equiv \mathbf{U}_{ij^\dagger ... kl^\dagger},$$

Follows from Balitsky 1996

$$\begin{aligned}\frac{\partial \mathbf{U}_{12^\dagger}\mathbf{U}_{34^\dagger}}{\partial\eta} =& \mathbf{U}_{4^\dagger 3}\frac{\partial \mathbf{U}_{12^\dagger}}{\partial\eta} + \mathbf{U}_{2^\dagger 1}\frac{\partial \mathbf{U}_{4^\dagger 3}}{\partial\eta} \\ &+ \frac{\alpha_s}{4\pi^2}\int d\vec{r}_0 \left(\frac{\vec{r}_{13}{}^2}{\vec{r}_{10}{}^2\vec{r}_{30}{}^2} - \frac{\vec{r}_{23}{}^2}{\vec{r}_{20}{}^2\vec{r}_{30}{}^2} - \frac{\vec{r}_{14}{}^2}{\vec{r}_{10}{}^2\vec{r}_{40}{}^2} + \frac{\vec{r}_{24}{}^2}{\vec{r}_{20}{}^2\vec{r}_{40}{}^2}\right) \\ &\times(\mathbf{U}_{2^\dagger 14^\dagger 3} + \mathbf{U}_{2^\dagger 34^\dagger 1} - \mathbf{U}_{2^\dagger 10^\dagger 34^\dagger 0} - \mathbf{U}_{2^\dagger 04^\dagger 30^\dagger 1}).\end{aligned}$$

Color algebra relations

Dipole limits, e.g. $\vec{r}_4 \to \vec{r}_3$, etc:

$$\mathbf{U}_{12^\dagger}\mathbf{U}_{34^\dagger} \to N_c\mathbf{U}_{12^\dagger}\,, \qquad \mathbf{U}_{12^\dagger 34^\dagger} \to \mathbf{U}_{12^\dagger}$$

$SU(3)$ relation at $N_c = 3$:

$$B_{123} \equiv \mathbf{U}_{12^\dagger}\mathbf{U}_{32^\dagger} - \mathbf{U}_{12^\dagger 32^\dagger}$$

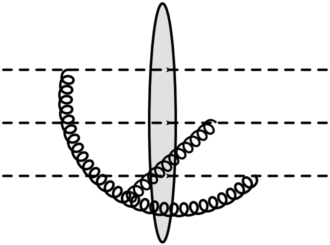
where B_{123} is the 3-quark Wilson loop (baryon) operator defined as

$$B_{123} \equiv \varepsilon^{i'j'h'}\varepsilon_{ijh}U^i_{1\,i'}U^j_{2j'}U^h_{3h'}.$$

All these relations are satisfied by the corresponding evolution equations.
Evolution equation for B: LO Gerasimov Grabovsky 2012, NLO Balitsky Grabovsky 2014

NLO corrections

NLO evolution of 1 and 2 Wilson lines with open indices from Balitsky and Chirilli 2013
NLO evolution of 3 Wilson lines Grabovsky 2013



$$\langle K_{NLO} \otimes \mathbf{U}_{12^\dagger 34^\dagger} \rangle = \frac{\alpha_s^2}{8\pi^4} \int d\vec{r}_0 d\vec{r}_5 \; (\mathbf{G}_s + \mathbf{G}_a) + \frac{\alpha_s^2}{8\pi^3} \int d\vec{r}_0 \; (\mathbf{G}_\beta + \mathbf{G}),$$

NLO corrections: symmetric part

$$\mathbf{G}_s = \mathbf{G}_{s1} + n_f \mathbf{G}_q + \mathbf{G}_{s2} + (1 \leftrightarrow 3, 2 \leftrightarrow 4).$$

$$\begin{aligned}
\mathbf{G}_{s1} =& (\{\mathbf{U}_{0\dagger34\dagger15\dagger02\dagger5} - \mathbf{U}_{5\dagger0}\mathbf{U}_{2\dagger5}\mathbf{U}_{0\dagger34\dagger1} - (5 \to 0)\} + (5 \leftrightarrow 0))\,(L_{12} + L_{32} - L_{13}) \\
&+ (\{\mathbf{U}_{0\dagger15\dagger02\dagger34\dagger5} - \mathbf{U}_{0\dagger5}\mathbf{U}_{5\dagger1}\mathbf{U}_{2\dagger34\dagger0} - (5 \to 0)\} + (5 \leftrightarrow 0))\,(L_{12} + L_{14} - L_{42}),
\end{aligned}$$

$$\begin{aligned}
\mathbf{G}_q =& (\{\frac{\mathbf{U}_{0\dagger34\dagger12\dagger5} + \mathbf{U}_{2\dagger34\dagger15\dagger0}}{N_c} - \frac{\mathbf{U}_{0\dagger5}\mathbf{U}_{2\dagger34\dagger1}}{N_c^2} - \mathbf{U}_{2\dagger5}\mathbf{U}_{0\dagger34\dagger1} - (5 \to 0)\} + (5 \leftrightarrow 0)) \\
&\times \frac{1}{2}\left(L^q_{12} + L^q_{32} - L^q_{13}\right) + \frac{1}{2}\left(L^q_{12} + L^q_{14} - L^q_{42}\right) \\
&\times (\{\frac{\mathbf{U}_{0\dagger12\dagger34\dagger5} + \mathbf{U}_{2\dagger34\dagger15\dagger0}}{N_c} - \frac{\mathbf{U}_{0\dagger5}\mathbf{U}_{2\dagger34\dagger1}}{N_c^2} - \mathbf{U}_{5\dagger1}\mathbf{U}_{2\dagger34\dagger0} - (5 \to 0)\} + (5 \leftrightarrow 0)),
\end{aligned}$$

$$\begin{aligned}
2\mathbf{G}_{s2} =& (\mathbf{U}_{0\dagger15\dagger02\dagger34\dagger5} - \mathbf{U}_{0\dagger5}\mathbf{U}_{5\dagger1}\mathbf{U}_{2\dagger34\dagger0} + (5 \leftrightarrow 0))\,(M_2^{14} + M_4^{12} + (5 \leftrightarrow 0)) \\
&+ (\mathbf{U}_{0\dagger34\dagger15\dagger02\dagger5} - \mathbf{U}_{5\dagger0}\mathbf{U}_{2\dagger5}\mathbf{U}_{0\dagger34\dagger1} + (5 \leftrightarrow 0))\,(M_1^{23} + M_3^{21} + (5 \leftrightarrow 0)) \\
&+ (\mathbf{U}_{0\dagger34\dagger52\dagger05\dagger1} - \mathbf{U}_{0\dagger1}\mathbf{U}_{2\dagger5}\mathbf{U}_{4\dagger05\dagger3} + (5 \leftrightarrow 0))\,(M_1^{34} - M_1^{24} + M_2^{43} - M_2^{13} + (5 \leftrightarrow 0)) \\
&+ (\mathbf{U}_{0\dagger35\dagger02\dagger54\dagger1} - \mathbf{U}_{0\dagger3}\mathbf{U}_{2\dagger5}\mathbf{U}_{4\dagger15\dagger0} + (5 \leftrightarrow 0))\,(M_3^{14} - M_3^{24} + M_2^{41} - M_2^{31} + (5 \leftrightarrow 0))
\end{aligned}$$

NLO corrections: antisymmetric part

$$\mathbf{G}_a = \mathbf{G}_{a1} + \mathbf{G}_{a2} + \mathbf{G}_{a3}.$$

$$\mathbf{G}_{a1} = (\mathbf{U}_{0^\dagger 1}\mathbf{U}_{2^\dagger 5}\mathbf{U}_{4^\dagger 05^\dagger 3} + \mathbf{U}_{0^\dagger 34^\dagger 52^\dagger 05^\dagger 1} - (5 \leftrightarrow 0))(M_2^{31} - M_2^{34} - M_1^{42} + M_1^{43})$$
$$+ (\mathbf{U}_{0^\dagger 3}\mathbf{U}_{2^\dagger 5}\mathbf{U}_{4^\dagger 15^\dagger 0} + \mathbf{U}_{0^\dagger 35^\dagger 02^\dagger 54^\dagger 1} - (5 \leftrightarrow 0))(M_2^{13} - M_2^{14} - M_3^{42} + M_3^{41})$$
$$+(1 \leftrightarrow 3, 2 \leftrightarrow 4).$$

$$\mathbf{G}_{a2} = \frac{1}{2}(\mathbf{U}_{0^\dagger 34^\dagger 15^\dagger 02^\dagger 5} - (5 \leftrightarrow 0))(\tilde{L}_{13} + 2M_{21} - 2M_{23} - M_1^{23} + M_3^{21} - (5 \leftrightarrow 0))$$
$$+ \frac{1}{2}(\mathbf{U}_{0^\dagger 15^\dagger 02^\dagger 34^\dagger 5} - (5 \leftrightarrow 0))(\tilde{L}_{42} - 2M_{12} + 2M_{14} + M_2^{14} - M_4^{12} - (5 \leftrightarrow 0))$$
$$+(1 \leftrightarrow 3, 2 \leftrightarrow 4).$$

$$\mathbf{G}_{a3} = \frac{1}{2}(\mathbf{U}_{0^\dagger 5}\mathbf{U}_{5^\dagger 1}\mathbf{U}_{2^\dagger 34^\dagger 0} - (5 \leftrightarrow 0))(\tilde{L}_{12} + \tilde{L}_{14} - 2M_{24} + M_2^{14} + M_4^{12} - (5 \leftrightarrow 0))$$
$$+ \frac{1}{2}(\mathbf{U}_{5^\dagger 0}\mathbf{U}_{2^\dagger 5}\mathbf{U}_{0^\dagger 34^\dagger 1} - (5 \leftrightarrow 0))(\tilde{L}_{21} + \tilde{L}_{23} - 2M_{13} + M_1^{23} + M_3^{21} - (5 \leftrightarrow 0))$$
$$+(1 \leftrightarrow 3, 2 \leftrightarrow 4).$$

NLO corrections

Pomeron contribution $L_{12}(0 \leftrightarrow 5) = L_{12}$

$$L_{12} = \left[\frac{1}{\vec{r}_{01}{}^2\vec{r}_{25}{}^2 - \vec{r}_{02}{}^2\vec{r}_{15}{}^2}\left(-\frac{\vec{r}_{12}{}^4}{8}\left(\frac{1}{\vec{r}_{01}{}^2\vec{r}_{25}{}^2} + \frac{1}{\vec{r}_{02}{}^2\vec{r}_{15}{}^2}\right) + \frac{\vec{r}_{12}{}^2}{\vec{r}_{05}{}^2} - \frac{\vec{r}_{02}{}^2\vec{r}_{15}{}^2 + \vec{r}_{01}{}^2\vec{r}_{25}{}^2}{4\vec{r}_{05}{}^4}\right)\right.$$
$$\left. + \frac{\vec{r}_{12}{}^2}{8\vec{r}_{05}{}^2}\left(\frac{1}{\vec{r}_{02}{}^2\vec{r}_{15}{}^2} - \frac{1}{\vec{r}_{01}{}^2\vec{r}_{25}{}^2}\right)\right] \ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{25}{}^2}{\vec{r}_{15}{}^2\vec{r}_{02}{}^2}\right) + \frac{1}{2\vec{r}_{05}{}^4}.$$

2-point contribution to odderon $\tilde{L}_{12}(0 \leftrightarrow 5) = -\tilde{L}_{12}$

$$\tilde{L}_{12} = \frac{\vec{r}_{12}{}^2}{8}\left[\frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{15}{}^2\vec{r}_{25}{}^2} - \frac{1}{\vec{r}_{01}{}^2\vec{r}_{05}{}^2\vec{r}_{25}{}^2} - \frac{1}{\vec{r}_{02}{}^2\vec{r}_{05}{}^2\vec{r}_{15}{}^2}\right] \ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{25}{}^2}{\vec{r}_{15}{}^2\vec{r}_{02}{}^2}\right).$$

Nonconformal structure

$$M_2^{13} = \left(\frac{\vec{r}_{12}{}^2\vec{r}_{23}{}^2}{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{25}{}^2\vec{r}_{35}{}^2} - \frac{\vec{r}_{15}{}^2\vec{r}_{23}{}^2}{\vec{r}_{01}{}^2\vec{r}_{05}{}^2\vec{r}_{25}{}^2\vec{r}_{35}{}^2} - \frac{\vec{r}_{03}{}^2\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{05}{}^2\vec{r}_{35}{}^2} + \frac{\vec{r}_{13}{}^2}{\vec{r}_{01}{}^2\vec{r}_{05}{}^2\vec{r}_{35}{}^2}\right)$$
$$\times \frac{1}{4}\ln\left(\frac{\vec{r}_{02}{}^2}{\vec{r}_{25}{}^2}\right).$$

NLO corrections: β-functional contribution

$$
\begin{aligned}
\mathbf{G}_\beta =& \frac{\vec{r}_{14}{}^2}{\vec{r}_{10}{}^2\vec{r}_{40}{}^2} M^\beta_{14} \left(\mathbf{U}_{10^\dagger}\mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{4^\dagger 0}\mathbf{U}_{12^\dagger 30^\dagger} - (0 \to 1)\right) \\
&+ \frac{\vec{r}_{12}{}^2}{\vec{r}_{10}{}^2\vec{r}_{20}{}^2} M^\beta_{12} \left(\mathbf{U}_{10^\dagger}\mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{2^\dagger 0}\mathbf{U}_{10^\dagger 34^\dagger} - (0 \to 1)\right) \\
&- \frac{\vec{r}_{24}{}^2}{2\vec{r}_{20}{}^2\vec{r}_{40}{}^2} M^\beta_{24} \left(\mathbf{U}_{10^\dagger}\mathbf{U}_{02^\dagger 34^\dagger} + \mathbf{U}_{30^\dagger}\mathbf{U}_{04^\dagger 12^\dagger} - (0 \to 4)\right) \\
&- \frac{\vec{r}_{13}{}^2}{2\vec{r}_{10}{}^2\vec{r}_{30}{}^2} M^\beta_{13} \left(\mathbf{U}_{4^\dagger 0}\mathbf{U}_{12^\dagger 30^\dagger} + \mathbf{U}_{2^\dagger 0}\mathbf{U}_{34^\dagger 10^\dagger} - (0 \to 1)\right) \\
&+ (1 \leftrightarrow 3, 2 \leftrightarrow 4).
\end{aligned}
$$

$$
M^\beta_{12} = \frac{N_c \beta}{2} \left\{ \ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right) + \frac{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}{\vec{r}_{12}^{\,2}} \ln\left(\frac{\vec{r}_{02}^{\,2}}{\vec{r}_{01}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}}\right)\right\}.
$$

$$
\beta = \left(\frac{11}{3} - \frac{2}{3}\frac{n_f}{N_c}\right), \quad \beta \ln\frac{1}{\tilde{\mu}^2} = \left(\frac{11}{3} - \frac{2}{3}\frac{n_f}{N_c}\right)\ln\left(\frac{\mu^2}{4e^{2\psi(1)}}\right) + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{n_f}{N_c}
$$

NLO corrections: 1gluon contribution

$$\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_0.$$

$$\mathbf{G}_0 = \frac{N_c}{4}(\mathbf{U}_{4^\dagger 1}\mathbf{U}_{2^\dagger 3} - \mathbf{U}_{4^\dagger 3}\mathbf{U}_{2^\dagger 1})\left\{\left(\frac{\vec{r}_{14}{}^2}{\vec{r}_{10}{}^2\vec{r}_{40}{}^2} + \frac{\vec{r}_{23}{}^2}{\vec{r}_{20}{}^2\vec{r}_{30}{}^2} - \frac{\vec{r}_{13}{}^2}{\vec{r}_{10}{}^2\vec{r}_{30}{}^2} - \frac{\vec{r}_{24}{}^2}{\vec{r}_{20}{}^2\vec{r}_{40}{}^2}\right)\right.$$
$$\times \ln\left(\frac{\vec{r}_{10}{}^2}{\vec{r}_{12}{}^2}\right)\ln\left(\frac{\vec{r}_{20}{}^2}{\vec{r}_{12}{}^2}\right) + \left(\frac{\vec{r}_{13}{}^2}{\vec{r}_{10}{}^2\vec{r}_{30}{}^2} + \frac{\vec{r}_{24}{}^2}{\vec{r}_{20}{}^2\vec{r}_{40}{}^2} - \frac{\vec{r}_{34}{}^2}{\vec{r}_{30}{}^2\vec{r}_{40}{}^2} - \frac{\vec{r}_{12}{}^2}{\vec{r}_{20}{}^2\vec{r}_{10}{}^2}\right)$$
$$\times \ln\left(\frac{\vec{r}_{10}{}^2}{\vec{r}_{14}{}^2}\right)\ln\left(\frac{\vec{r}_{40}{}^2}{\vec{r}_{14}{}^2}\right) + \left(\ln\left(\frac{\vec{r}_{20}{}^2}{\vec{r}_{24}{}^2}\right)\ln\left(\frac{\vec{r}_{40}{}^2}{\vec{r}_{24}{}^2}\right) + \ln\left(\frac{\vec{r}_{10}{}^2}{\vec{r}_{13}{}^2}\right)\ln\left(\frac{\vec{r}_{30}{}^2}{\vec{r}_{13}{}^2}\right)\right)$$
$$\left.\times\left(\frac{\vec{r}_{12}{}^2}{\vec{r}_{10}{}^2\vec{r}_{20}{}^2} - \frac{\vec{r}_{14}{}^2}{\vec{r}_{10}{}^2\vec{r}_{40}{}^2}\right)\right\} + (1 \leftrightarrow 3, 2 \leftrightarrow 4).$$

$$\mathbf{G} = \frac{\vec{r}_{12}{}^2}{\vec{r}_{10}{}^2\vec{r}_{20}{}^2}\ln\left(\frac{\vec{r}_{10}{}^2}{\vec{r}_{12}{}^2}\right)\ln\left(\frac{\vec{r}_{20}{}^2}{\vec{r}_{12}{}^2}\right)\{\frac{N_c}{2}(2N_c\mathbf{U}_{2^\dagger 34^\dagger 1} - \mathbf{U}_{0^\dagger 1}\mathbf{U}_{2^\dagger 34^\dagger 0} - \mathbf{U}_{2^\dagger 0}\mathbf{U}_{4^\dagger 10^\dagger 3})$$
$$+ (\mathbf{U}_{2^\dagger 10^\dagger 34^\dagger 0} - \mathbf{U}_{2^\dagger 0}\mathbf{U}_{4^\dagger 3}\mathbf{U}_{0^\dagger 1} - (0 \to 1))\}$$
$$+ \frac{\vec{r}_{14}{}^2}{\vec{r}_{10}{}^2\vec{r}_{40}{}^2}\ln\left(\frac{\vec{r}_{10}{}^2}{\vec{r}_{14}{}^2}\right)\ln\left(\frac{\vec{r}_{40}{}^2}{\vec{r}_{14}{}^2}\right)\{\frac{N_c}{2}(2N_c\mathbf{U}_{2^\dagger 34^\dagger 1} - \mathbf{U}_{0^\dagger 1}\mathbf{U}_{2^\dagger 34^\dagger 0} - \mathbf{U}_{4^\dagger 0}\mathbf{U}_{2^\dagger 30^\dagger 1})$$
$$+ (\mathbf{U}_{2^\dagger 30^\dagger 14^\dagger 0} - \mathbf{U}_{4^\dagger 0}\mathbf{U}_{2^\dagger 3}\mathbf{U}_{0^\dagger 1} - (0 \to 1))\}$$

NLO corrections: 1gluon contribution

$$+\frac{1}{2}\left(\frac{\vec{r}_{13}{}^2}{\vec{r}_{10}{}^2\vec{r}_{30}{}^2}\ln\frac{\vec{r}_{10}{}^2}{\vec{r}_{13}{}^2}\ln\frac{\vec{r}_{30}{}^2}{\vec{r}_{13}{}^2}+\frac{\vec{r}_{24}{}^2}{\vec{r}_{20}{}^2\vec{r}_{40}{}^2}\ln\frac{\vec{r}_{20}{}^2}{\vec{r}_{24}{}^2}\ln\frac{\vec{r}_{40}{}^2}{\vec{r}_{24}{}^2}\right)$$
$$\times\{(\mathbf{U}_{4\dagger 0}\mathbf{U}_{2\dagger 1}+\mathbf{U}_{4\dagger 1}\mathbf{U}_{2\dagger 0})\mathbf{U}_{0\dagger 3}-\mathbf{U}_{2\dagger 04\dagger 10\dagger 3}-\mathbf{U}_{2\dagger 04\dagger 30\dagger 1}-(0\to 3)\}$$
$$+\{\mathbf{U}_{2\dagger 0}\mathbf{U}_{4\dagger 1}\mathbf{U}_{0\dagger 3}-\mathbf{U}_{2\dagger 0}\mathbf{U}_{0\dagger 1}\mathbf{U}_{34\dagger}+\mathbf{U}_{2\dagger 10\dagger 34\dagger 0}-\mathbf{U}_{0\dagger 32\dagger 04\dagger 1}\}$$
$$\times\frac{1}{2\vec{r}_{20}{}^2}\left(\frac{\vec{r}_{23}{}^2}{\vec{r}_{30}{}^2}-\frac{\vec{r}_{12}{}^2}{\vec{r}_{10}{}^2}\right)\ln\frac{\vec{r}_{10}{}^2}{\vec{r}_{13}{}^2}\ln\frac{\vec{r}_{30}{}^2}{\vec{r}_{13}{}^2}+\frac{1}{2\vec{r}_{10}{}^2}\left(\frac{\vec{r}_{14}{}^2}{\vec{r}_{40}{}^2}-\frac{\vec{r}_{12}{}^2}{\vec{r}_{20}{}^2}\right)\ln\frac{\vec{r}_{20}{}^2}{\vec{r}_{24}{}^2}\ln\frac{\vec{r}_{40}{}^2}{\vec{r}_{24}{}^2}$$
$$\times\{\mathbf{U}_{2\dagger 3}\mathbf{U}_{4\dagger 0}\mathbf{U}_{0\dagger 1}-\mathbf{U}_{2\dagger 0}\mathbf{U}_{0\dagger 1}\mathbf{U}_{34\dagger}+\mathbf{U}_{2\dagger 10\dagger 34\dagger 0}-\mathbf{U}_{0\dagger 14\dagger 02\dagger 3}\}$$

$$+\{\mathbf{U}_{4\dagger 0}\mathbf{U}_{2\dagger 1}\mathbf{U}_{0\dagger 3}-N_c\mathbf{U}_{2\dagger 0}\mathbf{U}_{4\dagger 10\dagger 3}+\mathbf{U}_{2\dagger 34\dagger 1}-\mathbf{U}_{2\dagger 04\dagger 30\dagger 1}\}$$
$$\times\frac{1}{2\vec{r}_{30}{}^2}\left(\frac{\vec{r}_{23}{}^2}{\vec{r}_{20}{}^2}-\frac{\vec{r}_{13}{}^2}{\vec{r}_{10}{}^2}\right)\ln\frac{\vec{r}_{10}{}^2}{\vec{r}_{12}{}^2}\ln\frac{\vec{r}_{20}{}^2}{\vec{r}_{12}{}^2}+\frac{1}{2\vec{r}_{40}{}^2}\left(\frac{\vec{r}_{14}{}^2}{\vec{r}_{10}{}^2}-\frac{\vec{r}_{24}{}^2}{\vec{r}_{20}{}^2}\right)\ln\frac{\vec{r}_{10}{}^2}{\vec{r}_{12}{}^2}\ln\frac{\vec{r}_{20}{}^2}{\vec{r}_{12}{}^2}$$
$$\times\{\mathbf{U}_{4\dagger 0}\mathbf{U}_{2\dagger 1}\mathbf{U}_{0\dagger 3}-N_c\mathbf{U}_{0\dagger 1}\mathbf{U}_{2\dagger 34\dagger 0}+\mathbf{U}_{2\dagger 34\dagger 1}-\mathbf{U}_{2\dagger 04\dagger 30\dagger 1}\}$$

$$+\{\mathbf{U}_{2\dagger 0}\mathbf{U}_{4\dagger 1}\mathbf{U}_{0\dagger 3}-N_c\mathbf{U}_{4\dagger 0}\mathbf{U}_{12\dagger 30\dagger}+\mathbf{U}_{2\dagger 34\dagger 1}-\mathbf{U}_{2\dagger 04\dagger 10\dagger 3}\}$$
$$\times\frac{1}{2\vec{r}_{30}{}^2}\left(\frac{\vec{r}_{34}{}^2}{\vec{r}_{40}{}^2}-\frac{\vec{r}_{13}{}^2}{\vec{r}_{10}{}^2}\right)\ln\frac{\vec{r}_{10}{}^2}{\vec{r}_{14}{}^2}\ln\frac{\vec{r}_{40}{}^2}{\vec{r}_{14}{}^2}+\frac{1}{2\vec{r}_{20}{}^2}\left(\frac{\vec{r}_{12}{}^2}{\vec{r}_{10}{}^2}-\frac{\vec{r}_{24}{}^2}{\vec{r}_{40}{}^2}\right)\ln\frac{\vec{r}_{10}{}^2}{\vec{r}_{14}{}^2}\ln\frac{\vec{r}_{40}{}^2}{\vec{r}_{14}{}^2}$$
$$\times\{\mathbf{U}_{2\dagger 0}\mathbf{U}_{4\dagger 1}\mathbf{U}_{0\dagger 3}-N_c\mathbf{U}_{0\dagger 1}\mathbf{U}_{02\dagger 34\dagger}+\mathbf{U}_{2\dagger 34\dagger 1}-\mathbf{U}_{2\dagger 04\dagger 10\dagger 3}\}+(1\leftrightarrow 3, 2\leftrightarrow 4).$$

Conformal basis

Rewrite the results via the conformal operators (Balitsky Chirilli 2009, Kovner Lublinsky Mullian 2014)

$$O^{conf} = O + \frac{1}{2}\frac{\partial O}{\partial \eta}\Bigg|_{\frac{\vec{r}_{mn}^{\,2}}{\vec{r}_{im}^{\,2}\vec{r}_{in}^{\,2}} \to \frac{\vec{r}_{mn}^{\,2}}{\vec{r}_{im}^{\,2}\vec{r}_{in}^{\,2}} \ln\left(\frac{\vec{r}_{mn}^{\,2} a}{\vec{r}_{im}^{\,2}\vec{r}_{in}^{\,2}}\right)},$$

E. g. the conformal dipole reads

$$\mathbf{U}^{conf}_{12^\dagger} = \mathbf{U}_{2^\dagger 1} + \frac{\alpha_s}{4\pi^2}\int d\vec{r}_0 \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{20}^{\,2}} \ln\left(\frac{a\vec{r}_{12}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{20}^{\,2}}\right)\left(\mathbf{U}_{2^\dagger 0}\mathbf{U}_{0^\dagger 1} - N_c \mathbf{U}_{2^\dagger 1}\right).$$

The evolution equation in the conformal basis is quasi-conformal, i.e. has nonconformal terms only $\sim \beta$.

Summary

Results

- The nonlinear NLO low-x evolution equation for a quadrupole and a double dipole Green functions satisfying all the color identities.
- Transformation of the NLO equations to the quasi-conformal form.

Plans

- NLO evolution equation for Weizsäcker-Williams gluon distribution $xG \sim tr(\frac{\partial U_1}{\partial r_1^i} U_3^\dagger \frac{\partial U_3}{\partial r_{3i}} U_1^\dagger)$.

Thank you for your attention