Phenomenological status of GPDs

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- *Accessing GPDs in deeply virtual processes*
- *GPD models*
- *Data description/predictions (partially includes DVMP)*
- *Prospects*
- *Conclusions*

see also talks of V. Braun (A. Manashov, B. Pirnay)
K. Kumerički (KK)
K. Semenov-Tian-Shansky (M. Polyakov)

some new work in collaboration with G. Duplančić and K. Passek-Kumerički (P-K)

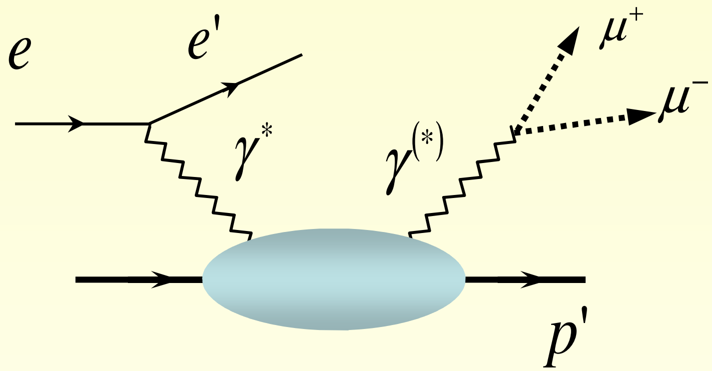
GPD related hard exclusive processes

- Deeply virtual Compton scattering (clean probe)

$$ep \to e'p'\gamma$$

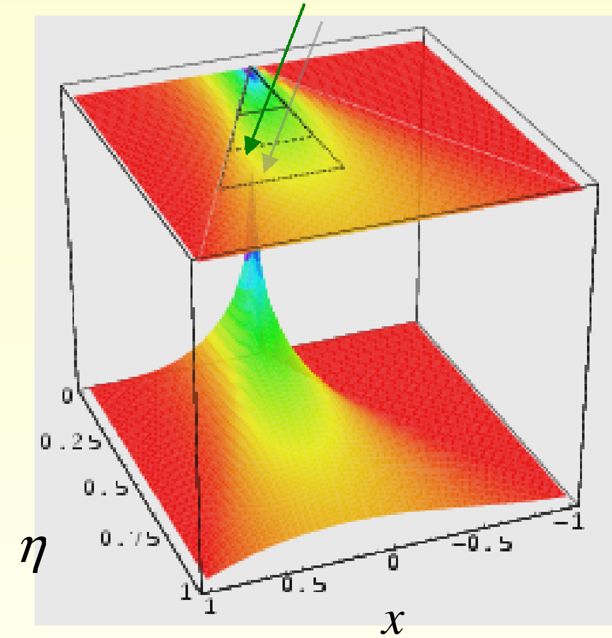
$$ep \to e'p'\mu^+\mu^-$$

$$\gamma p \to p'e^-e^+$$



factorization proof for transversal cross sections
[Collins Freund (99)]

scanned area of the surface as a functions of lepton energy



$ep \to e'\,p'\,\mu^+\mu^-$

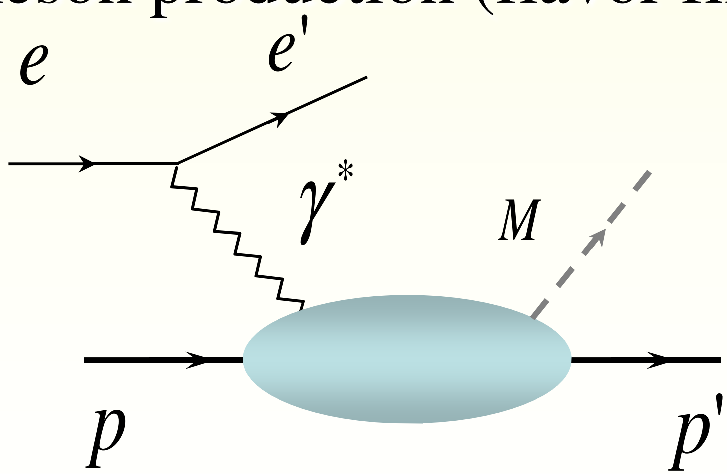
- Deeply virtual meson production (flavor filter)

$$ep \to e'p'\pi$$

$$ep \to e'p'\rho$$

$$ep \to e'n\pi^+$$

$$ep \to e'n\rho^+$$



<u>twist-two observables:</u>

longitudinal cross sections

transverse target spin asymmetries

- etc.

factorization proof for longitudinal cross sections
[Collins, Frankfurt, Strikman (96)]

GPDs embed non-perturbative physics

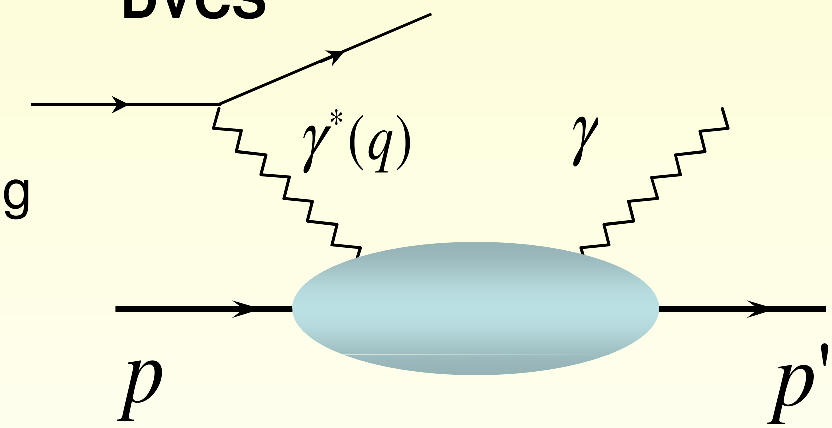
GPDs appear in various hard exclusive processes,

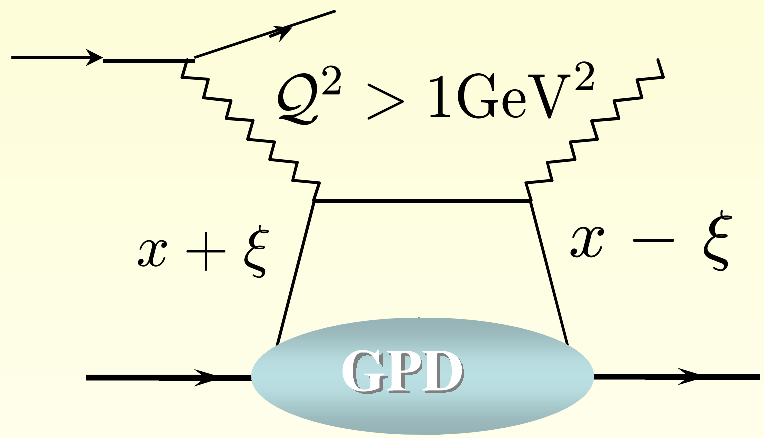
e.g., hard electroproduction of photons (DVCS)

[DM et. al (91/94)
Radyushkin (96); Ji (96);
Collins, Frankfurt,
Strikman (96)]

BH **+** **DVCS**

Bethe Heitler
Bremsstrahlung





$$\mathcal{Q}^2 \gg -t = -\Delta^2 \ (\text{fix})$$

$\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}$ photon helicity conserved CFFs (twist-2 associated)

$\mathcal{H}_{0+}, \mathcal{E}_{0+}, \widetilde{\mathcal{H}}_{0+}, \widetilde{\mathcal{E}}_{0+}$ twist-3 associated CFFs

$\mathcal{H}_{-+}, \mathcal{E}_{-+}, \widetilde{\mathcal{H}}_{-+}, \widetilde{\mathcal{E}}_{-+}$ twist-2 (gluon transversity) + twist-4 contamination

$$\mathcal{F}(\xi, \mathcal{Q}^2, t) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, t, \mu) + O(\tfrac{1}{\mathcal{Q}^2})$$

4+4+4 CFFs
Compton form factors
observables

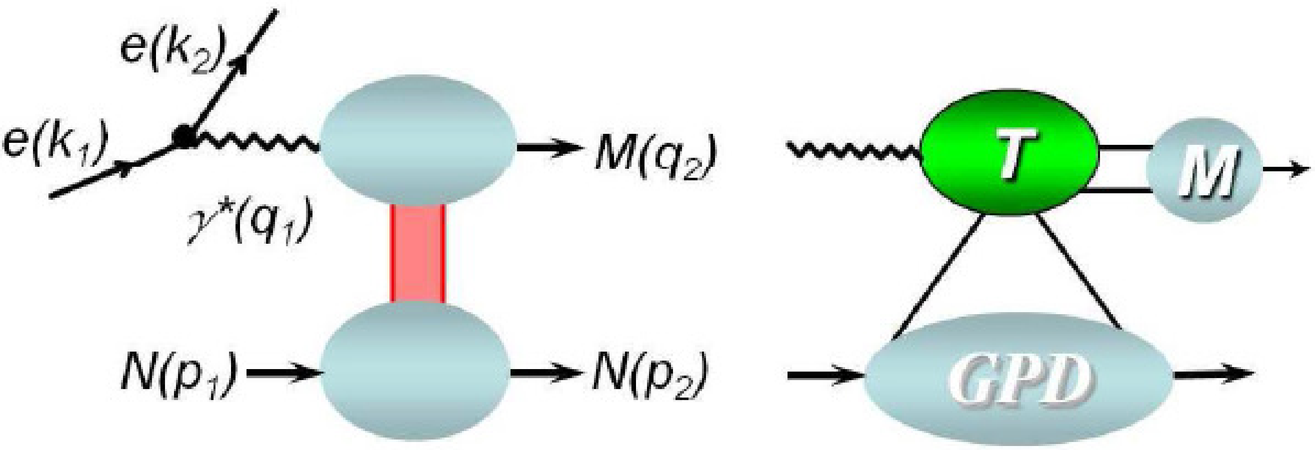
hard scattering part
perturbation theory
(our conventions/microscope)

GPD
universal
(conventional)

higher twist
depends on
approximation

Deeply virtual meson production (DVMP)

GPDs are universally defined within the collinear framework

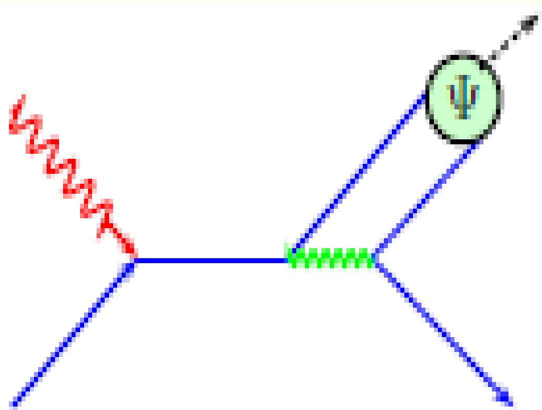


skepticisms that pQCD is applicable (e.g., large NLO corrections within fixed GPDs)

consequently, one would be left with DVCS or one might use hand-bag model, **i.e., universality is lost**

Goloskokov and Kroll (GK) provide a systematic analysis

$$\frac{1}{u(\xi - x) - i\epsilon} \Rightarrow \frac{1}{k_\perp^2 + u(\xi - x) - i\epsilon}$$



Sudakov resumation in impact space

freezing coupling constant

`integrating out' transverse degrees of freedom by hand

meson WF width is used to fit normalization

RDDA + NLO PDF (CTEQ6) + PDF evolution

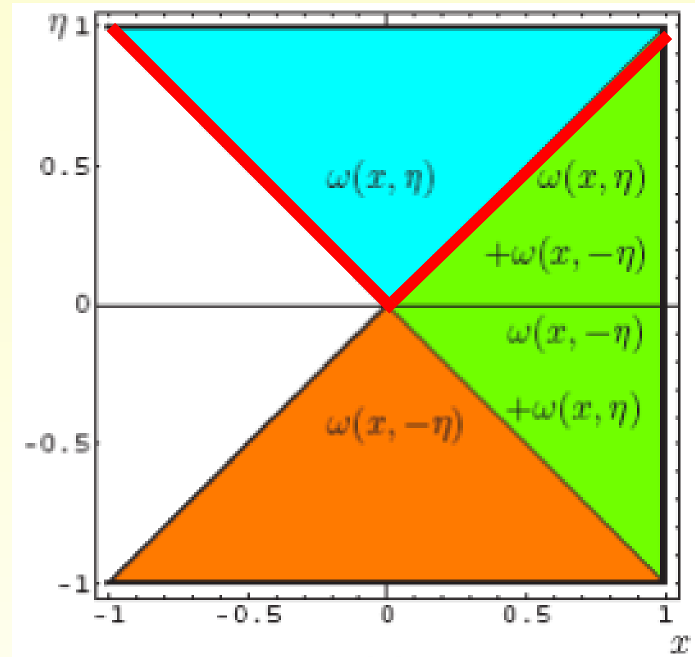
A partonic duality interpretation

quark GPD (anti-quark *x* → -*x*):

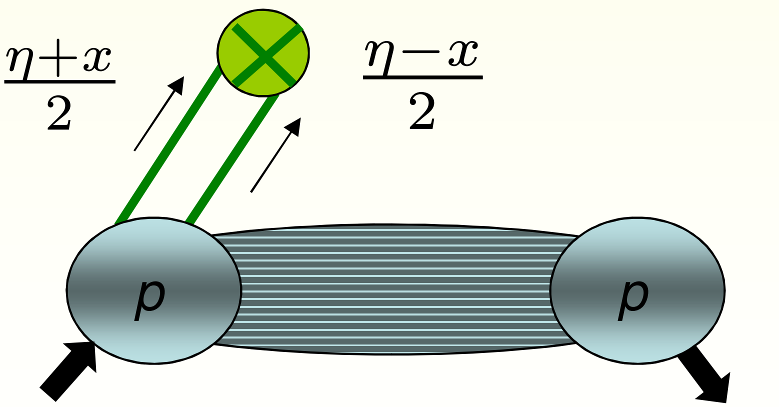
$$F(x,\eta,t) \quad =$$

$$\theta(-\eta \leq x \leq 1)\,\omega(x,\eta,t) + \theta(\eta \leq x \leq 1)\,\omega(x,-\eta,t)$$

$$\omega\,(x,\eta,t) = \frac{1}{\eta}\int_0^{\frac{x+\eta}{1+\eta}} dy\,(a+bx)f(y,(x-y)/\eta,t)$$

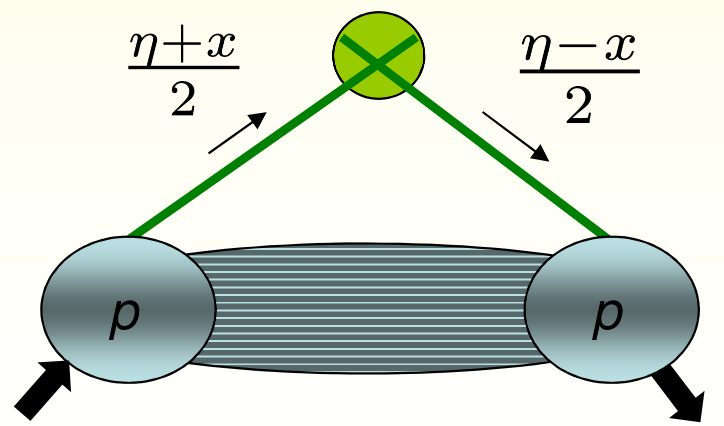


dual interpretation on partonic level:



support extension
is unique [DM et al. 92]

[DM, A. Schäfer (05)
KMP-K (07)]



central region - $\eta < x < \eta$

mesonic exchange in *t*-channel

?ambiguous (*D*-term)
Polyakov Weiss (99)

outer region $\eta < x$

partonic exchange in *s*-channel

GPD models

VGG model (99) $H^q(x,\eta,t) = F_q(t)q(x,\mu^2)$ **[Vanderhaeghen, Guidal, Guichon]**

Radyushkin (99)
RDDA
(a holographic
model **[KM (10)]**)

$$H^q(x,\eta,t=0) = \int_{-1}^{1} dy \int_{-1+|y|}^{1-|y|} dz\, \delta(x-y-z\eta)\frac{q(y)}{1-y}\Pi\left(\frac{z}{1-y}\right)$$

$$\Pi(z) \propto \left(1-z^2\right)^b$$

D-term added to complete polynomiality **[Polyakov & Weiss (99)]**

any GPD can be represented as DD part + D-term **[Belitsky, DM et al. (00), Teryaev (01)]**

? Are DD and D-term dependent? (YES, if J=0-fixe pole is universal; NO otherwise)
see Kirill`s talk

assumption for GK model (same for models that are implemented in VGG code)

$$q(x) \Rightarrow q(x,t) = q(x)e^{g(x)t} \overset{\text{simplified to}}{\Rightarrow} q(x,t) \propto e^{-\beta t}x^{-\alpha-\alpha' t}(1-x)^{\beta}(1+\cdots)$$

KM valence quark model (suited for dispersion relation or LO analysis)

$$H(x,x,t) = \frac{n\,r}{1+x}\left(\frac{2x}{1+x}\right)^{-\alpha-\alpha' t}\left(\frac{1-x}{1+x}\right)^{b}\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{-1}$$

+ double partial wave expansion of GPDs see Kreso`s and Kirill`s talk

Status of theory

[Belitsky, DM (97); Mankiewicz et. al (97); Ji,Osborne (97/98); Pire, Szymanowski, Wagner (11); DM, Pire, Szymanowski, Wagner (11)]

✓ ***twist-two*** DVCS coefficients at **NLO**

✓ twist-two DVMP coefficients at **NLO** [Belitsky, DM (01); Ivanov, Szymanowski,Krasnikov (04)] checked & extended Duplancic, P-K, DM (15)

NLO effects are well understood generically
large-ξ: logarithmical enhancement
valence region: weak evolution implies moderate effects
small-ξ: model dependence

DM, T. Lautenschlager, P-K. A. Schäfer (13)

✓ anomalous dimensions and evolution kernels at **NLO** [Belitsky, DM (98) + Freund (01) Braun, Manashov (14)]

evolution effects can be called moderate, except for H/E at small- ξ
NLO analyses have to include NLO evolution

✓ gluon transversity at **NLO** [Belitsky, DM (00)]

[DM (06); DM.KK,P-K, Schäfer (06)]

✓ ***next-to-next-to-leading*** order for DVCS in a specific conformal subtraction scheme

NLO → NNLO corrections can be called moderate w.r.t. LO → NLO [DM.KK,P-K 07]

✓ ***twist-three*** including quark-gluon-quark correlation at LO

✓ partially, ***twist-three*** sector at **NLO** [Kivel, Mankiewicz (03)]

[Anikin,Teryaev, Pire (00); Polyakov et. al (00), Belitsky DM (00); Kivel et. al, Weiss, Radyushkin (00)]

? \`target mass corrections' (not understood) [Belitsky DM (01)]

✓ **kinematical twist-four** corrections [Braun, Manashov (11)] (complicated, see Volodya\`s talk)

Strategies to analyze DVCS data

(ad hoc) modeling: **VGG** code [**Goeke et. al (01), Guidal et. al (05)** based on RDDA] (dead
BMK model [**Belitsky, DM , Kirchner (01)** based on RDDA] end reached ~05)
`aligned jet' model [**Freund, McDermott, Strikman (02)**] (immediately dead)
Goloskokov/Kroll (05) based on RDDA (pinned down by DVMP)
`dual' model [**Polyakov,Shuvaev 02;Guzey,Teckentrup 06;Polyakov 07**]
" -- " [**KMP-K (07)** in MBs-representation] (lo-SO(3)-PWE is dead)
polynomials [**Belitsky et al. (98), Liuti et. al (07), Moutarde (09)**] (dead end)

closing the loop after ~1 decade

dynamical models not applied **[Radyushkin et.al (02); Tiburzi et.al (04); Hwang DM (07,14)]**
(respecting Lorentz symmetry) (might be something for the future)

flexible models: any representation by including ***unconstrained*** degrees of freedom
(for fitting) **KMP-K (07/08)** for H1/ZEUS in MBs-integral-representation

CFFs (real and imaginary parts) and GPD fits/predictions

i. CFF extraction (local) **[BMK (01), HALL-A (06,15)]** and **[KK,DM, Murray (13)]**
least square fits (model independent ☺) **[Guidal, Moutarde (08…)]**
neural networks – a start up **[KMS (11)]** **see Michel`s talk**

ii. `dispersion integral' fits **[KMP-K (08),KM (08…)]**

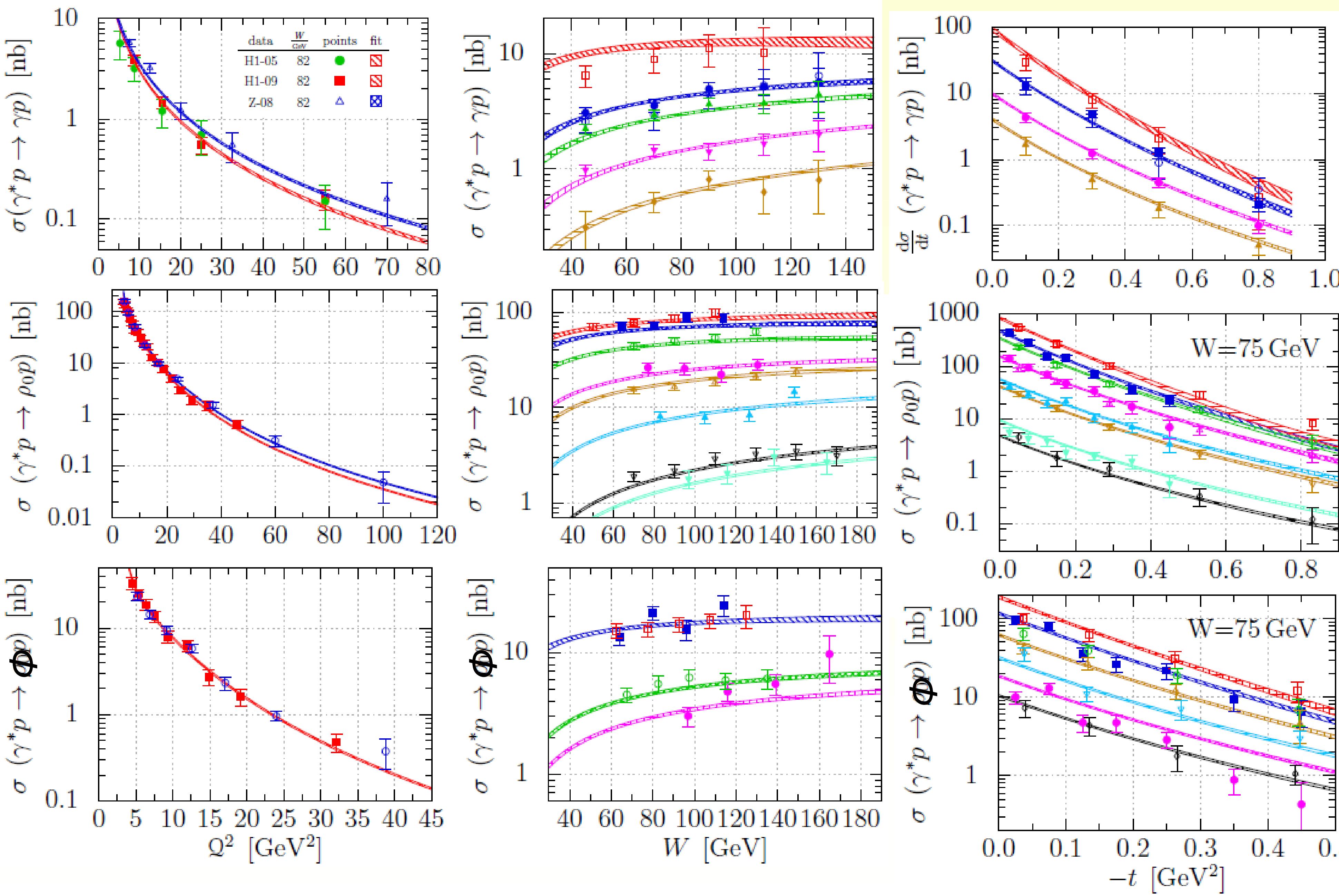
iii. **flexible GPD model fits** **[KM (08…), AFKM (13), KMM (13), LSM (13)]**

vi. model comparisons & predictions **VGG code, however also BMK01 (up to ~05)**
Goloskokov/Kroll model based on RDDA
[DVCS: by `us' (12) also by Kroll,Moutarde,Sabatie (13)]

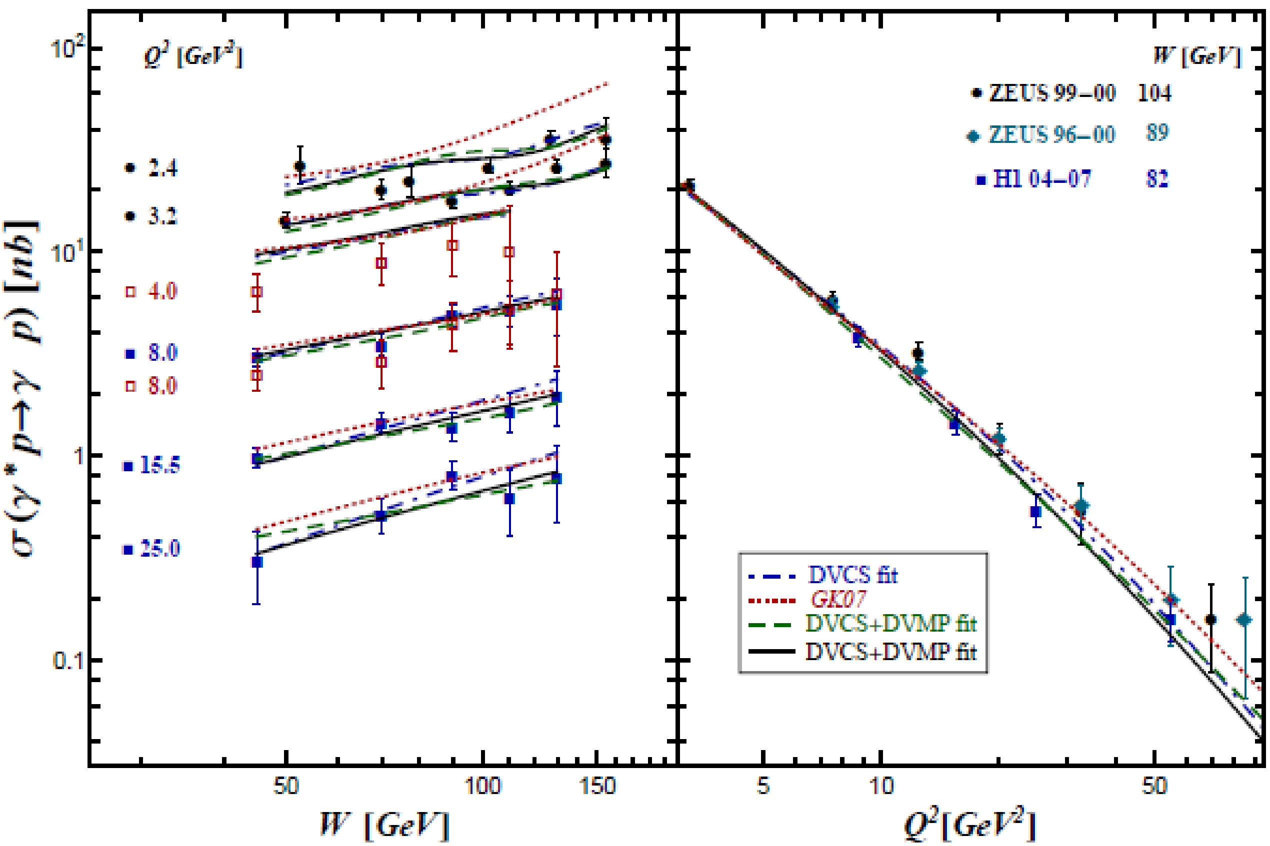
DIS+DVCS+DVMP phenomenology at small-x_B (H1,ZEUS)

works somehow without DIS at LO **[T. Lautenschlager, DM, A. Schäfer (13)]**

works at NLO ($Q^2 > 4$ GeV^2), done with Bayes theorem (probability distribution function)



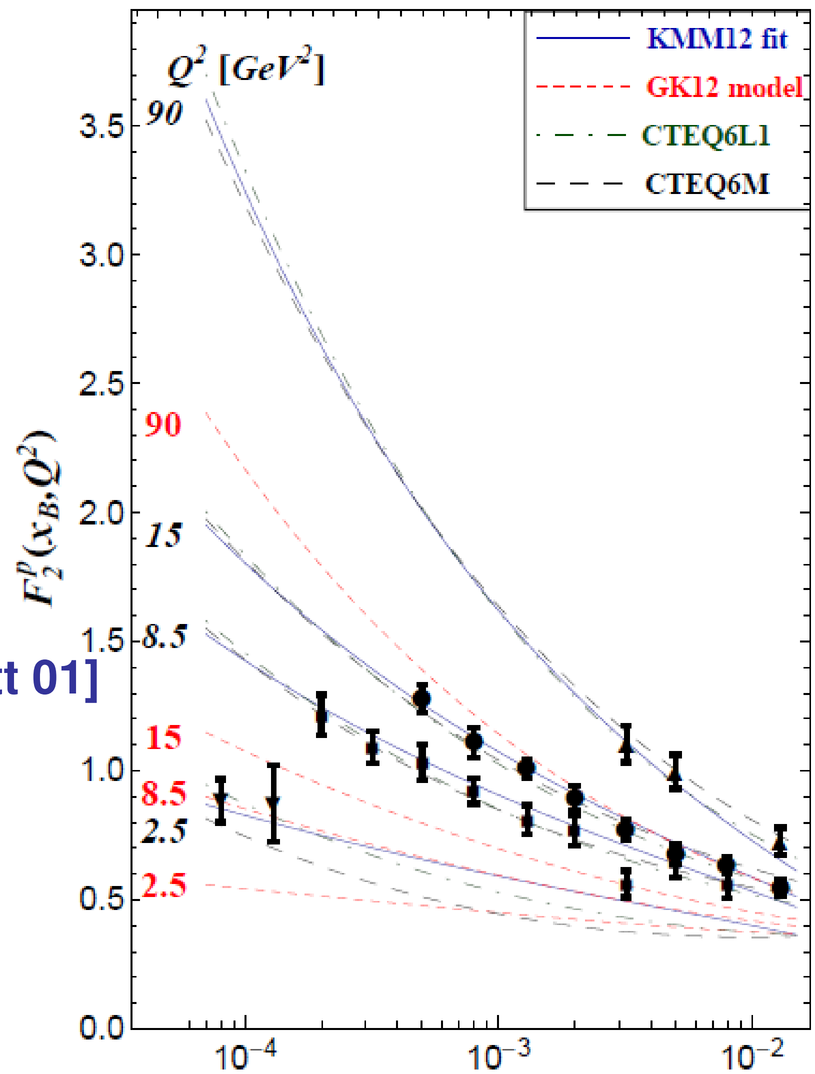
GK model versus DVCS measurements H1 & ZEUS
[Meskauskas, DM (11), Kroll, Moutarde, Sabatie (12)]



as in our flexible GPD LO analysis

also GK model does not describe

DVMP(handbag) + DVCS + DIS

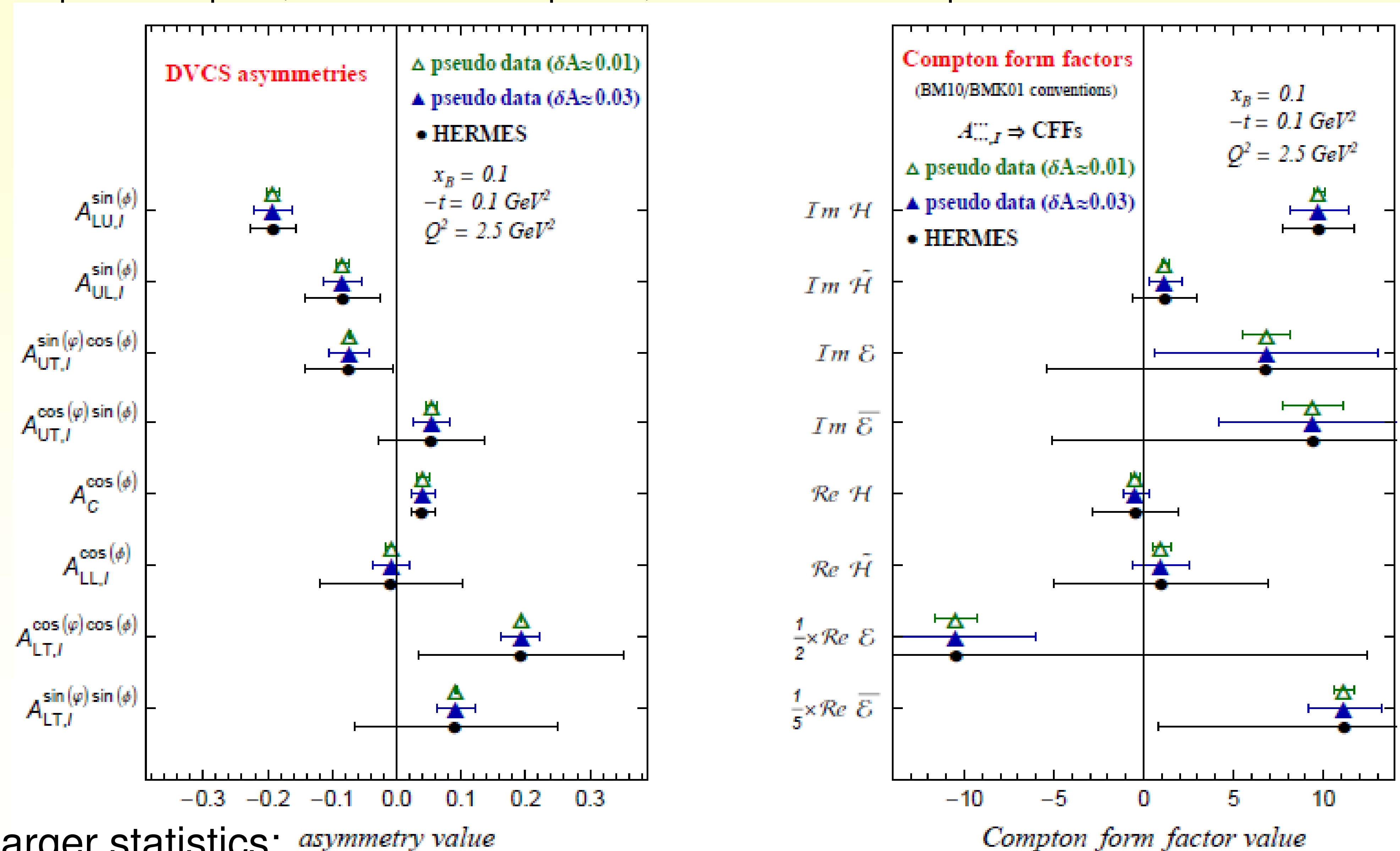


on the other hand it is known from **[Freund & McDermott 01]**
that RDDA based models do not describe H1 & ZEUS
DVCS data

Claims **[Kroll, Moutarde, Sabatie (2012)] :**
GK model is better than older RDDA based models
GPD universality shows up (DVMP, DVCS)

➢ a complete measurement allows in principle to pin down all CFFs **KK, DM, Murray (13)**

➢ adopting twist-two hypothesis together with certain conventions (4 CFFs, 8 parameters)
(Michel`s philosophy: use noise together with hypotheses and model constraints,
except for one point, which was not reported, our results are compatible for HERMES

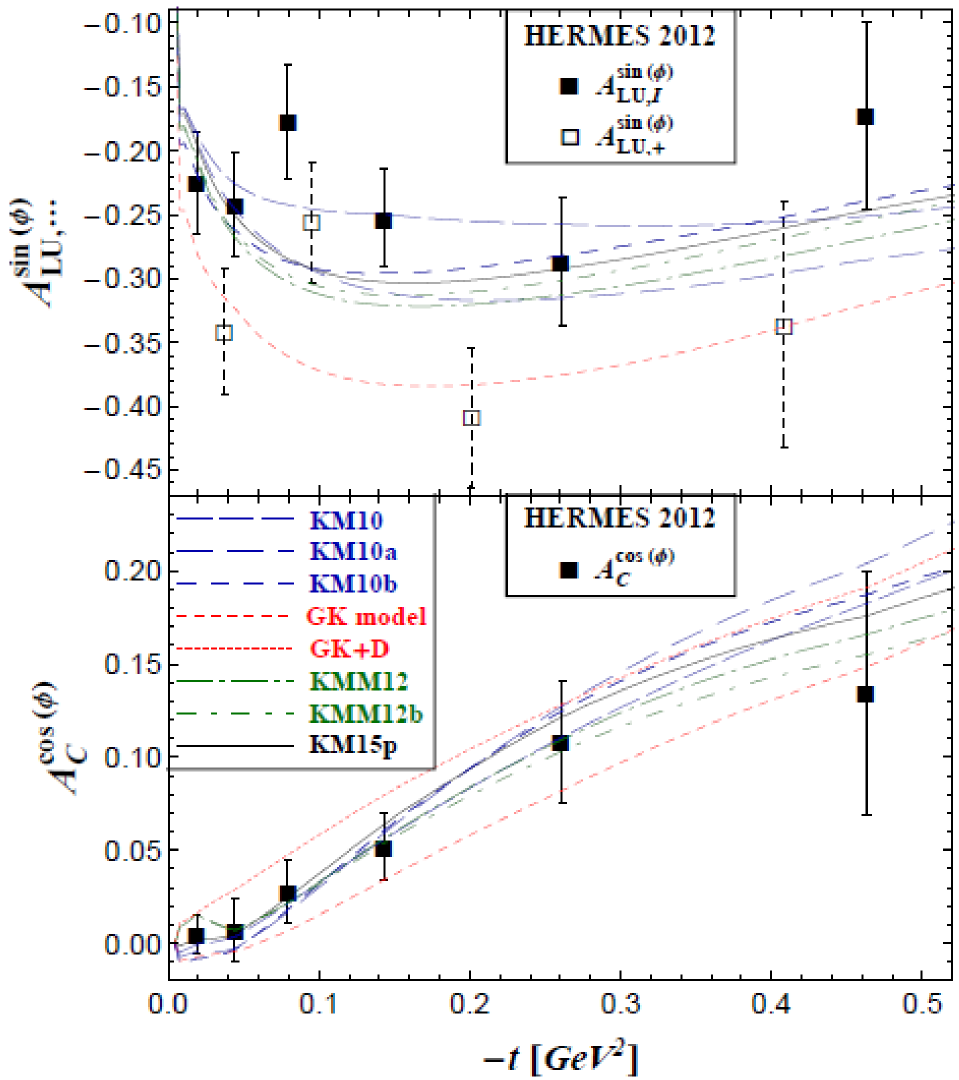


➢ larger statistics:
some CFF *E* constraint might have been obtained by HERMES

HERMES recoil detector data for beam spin asymmetry

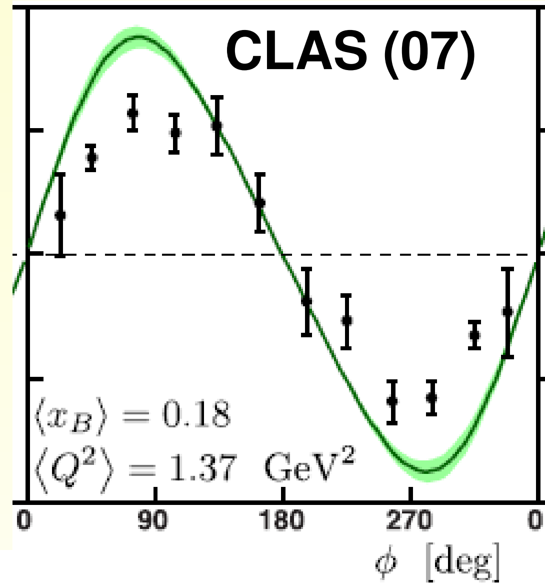
GK and VGG models are **compatible** with HERMES data, only if **recoil detector data** are used

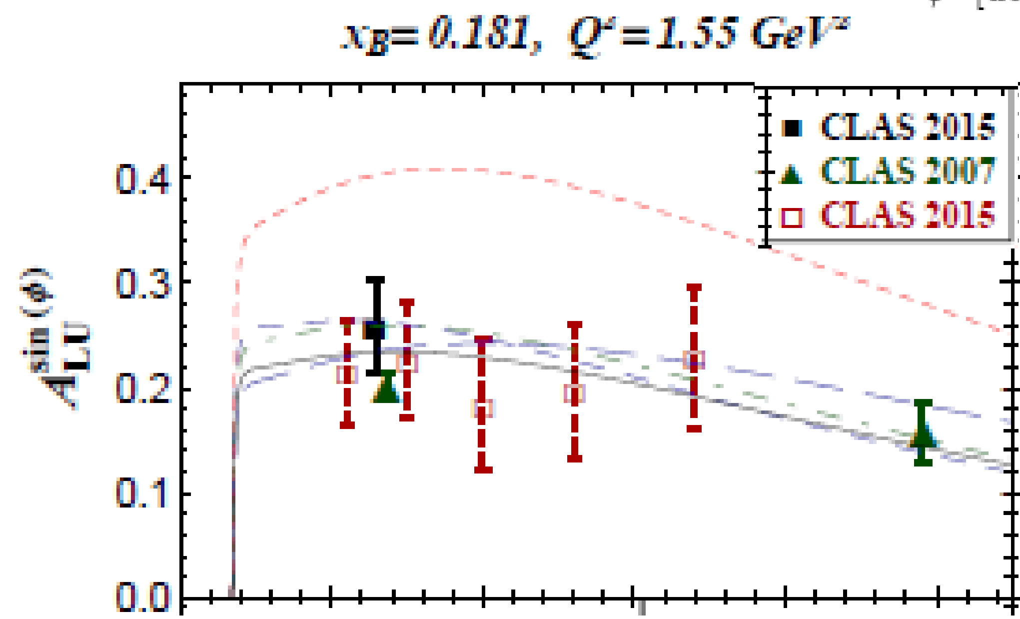
they were and are **incompatible** with old and new CLAS/HALL A data



claim [Kroll et. al (13)] that discrepancy is on the same order as in HERMES kinematics

disproved by Fourier transform [Kumericki et. al (11)]





Tension in longitudinally polarized proton data

GPD H is the big player, however, also $\hat{H}$ is accessible

tension between HERMES and old CLAS single spin asymmetry measurements

$$A_{\mathrm{UL}}^{\sin(\phi)} = -0.73 \pm 0.032(\mathrm{sys}) \pm 0.008(\mathrm{sta})(\mathrm{HERMES\ overall})$$

tension is perhaps gone with new CLAS data but not on GPD level

tension for the second harmonic remains

$$A_{\mathrm{UL}}^{\sin(\phi)} \sim A_{\mathrm{UL}}^{\sin(2\phi)} = -0.106 \pm 0.032(\mathrm{sys}) \pm 0.008(\mathrm{sta})(\mathrm{overall})$$

- no significant twist-three contribution in all other DVCS measurements
- second harmonic is not describable with any reasonable GPD model
- tension in HERMES data set (see slide 27 of Kreso`s talk)

D-term form factor and J=0 fixed pole extraction

(see also Barbara`s talk)

- D-term form factor comes out negative
- J=0 fixed pole is in principle extractable
- How robust and how model biased is that?

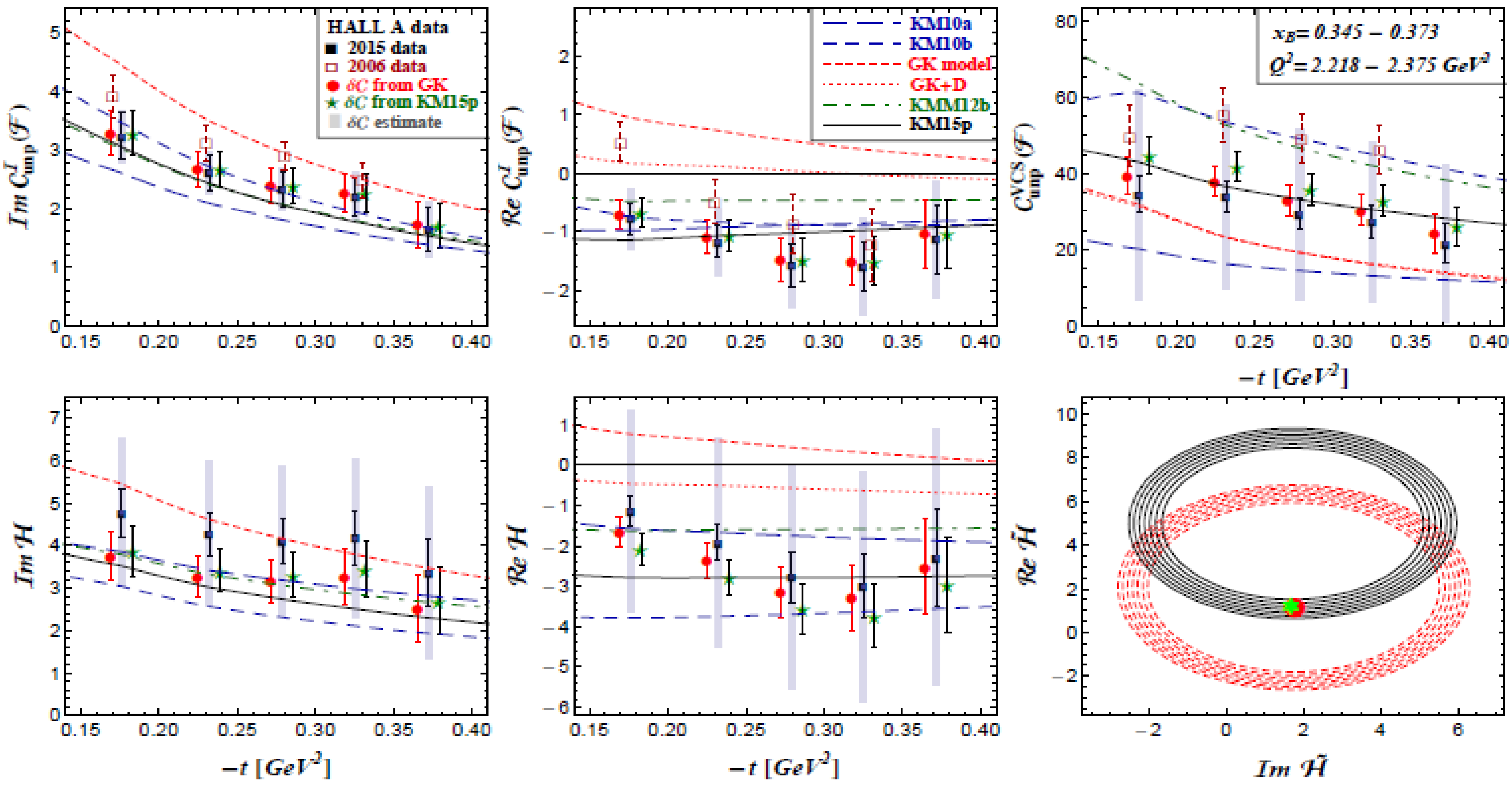
model:	KM10	KM10a	KM10b	KMM12	KMM12b	KM15p
$\mathcal{D}$	-6.0	-1.6	-4.4	-0.9	-2.5	-3.2
$\mathcal{H}_{-1}^{\mathrm{val}}$	-4.6	-5.8	-5.3	-6.0	-5.4	-4.0
$\mathcal{H}_{-1}^{\mathrm{sea}}$	15.9	16.4	13.8	15.9	13.6	18.8
$\mathcal{H}_{\infty}$	-17.3	-12.2	-12.8	-10.9	-9.8	-17.4

Role of old and new HALL A cross section measurements

$$\frac{d\Delta\sigma^{\sin(n\phi)}}{dx_B\, d\mathcal{Q}^2\, dt} = \frac{\mathcal{N}}{x_B c_0^{\mathcal{P}} t}\left[\frac{x_B}{y} s^{\mathrm{I}}_{1,\mathrm{unp}} + \frac{x_B^2 c_0^{\mathcal{P}} t}{\mathcal{Q}^2}\, \frac{2-w_2}{2}\, \cancel{s^{\mathrm{VCS}}_{1,\mathrm{unp}}}\right] \approx \frac{\mathcal{N}}{x_B c_0^{\mathcal{P}} t}\, \frac{x_B S_{++}(1)}{y}\, \Im\mathrm{m}\, \mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}(\mathcal{F}) \quad !$$

$$\frac{d\sigma^{\cos(0\phi)}}{dx_B\, d\mathcal{Q}^2\, dt} = \frac{\mathcal{N}}{x_B c_0^{\mathcal{P}} t}\left[\frac{c^{\mathrm{BH}}_{0,\mathrm{unp}}}{(1+\epsilon^2)^2} + \frac{x_B}{y}\, c^{\mathrm{I}}_{0,\mathrm{unp}} \; ! + \frac{x_B^2 c_0^{\mathcal{P}} t}{\mathcal{Q}^2}\left(c^{\mathrm{VCS}}_{0,\mathrm{unp}} + \frac{w_1}{2}\, \cancel{c^{\mathrm{VCS}}_{1,\mathrm{unp}}} + \frac{w_2}{2}\, \cancel{c^{\mathrm{VCS}}_{2,\mathrm{unp}}}\right)\right]$$

$$\frac{d\sigma^{\cos(\phi)}}{dx_B\, d\mathcal{Q}^2\, dt} = \frac{-\mathcal{N}}{x_B c_0^{\mathcal{P}} t}\left[\frac{c^{\mathrm{BH}}_{1,\mathrm{unp}}}{(1+\epsilon^2)^2} + \frac{x_B}{y}\, c^{\mathrm{I}}_{1,\mathrm{unp}} \; ! + \frac{x_B^2 c_0^{\mathcal{P}} t}{\mathcal{Q}^2}\left(\frac{2-w_2}{2}\, \cancel{c^{\mathrm{VCS}}_{1,\mathrm{unp}}} + w_1 c^{\mathrm{VCS}}_{0,\mathrm{unp}} + \frac{w_1}{2}\, \cancel{c^{\mathrm{VCS}}_{2,\mathrm{unp}}}\right)\right]$$



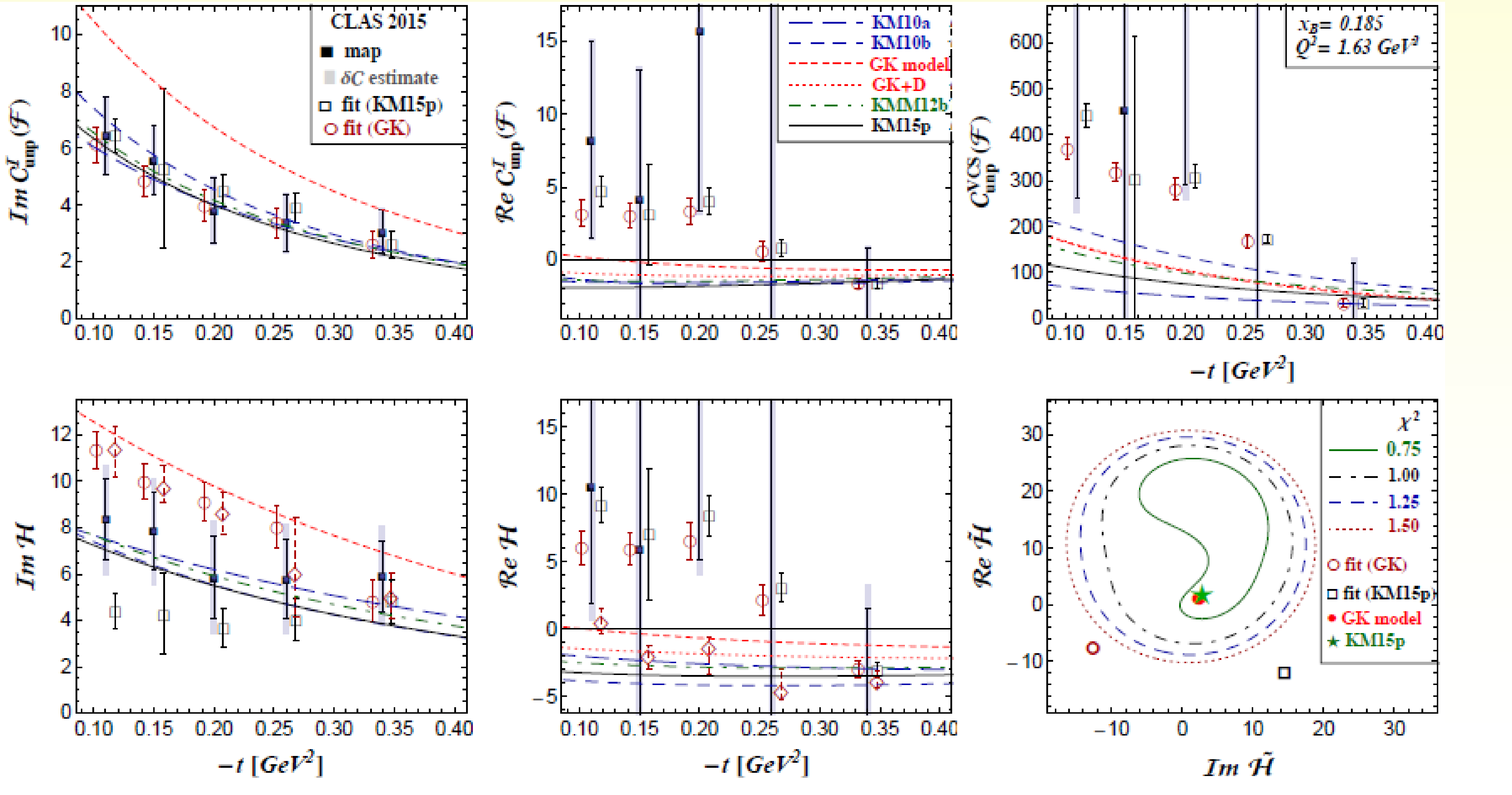
Ill defined fitting problem

HALL A (06,15) extraction of CFF combinations (shown on previous slide)

adopting **(any)** twist-two approximation yields to an underestimate of errors for real part of linear CFF combination and the bilinear CFF form

CLAS (15) uses `model independent' fitter code, referring to `VGG models'

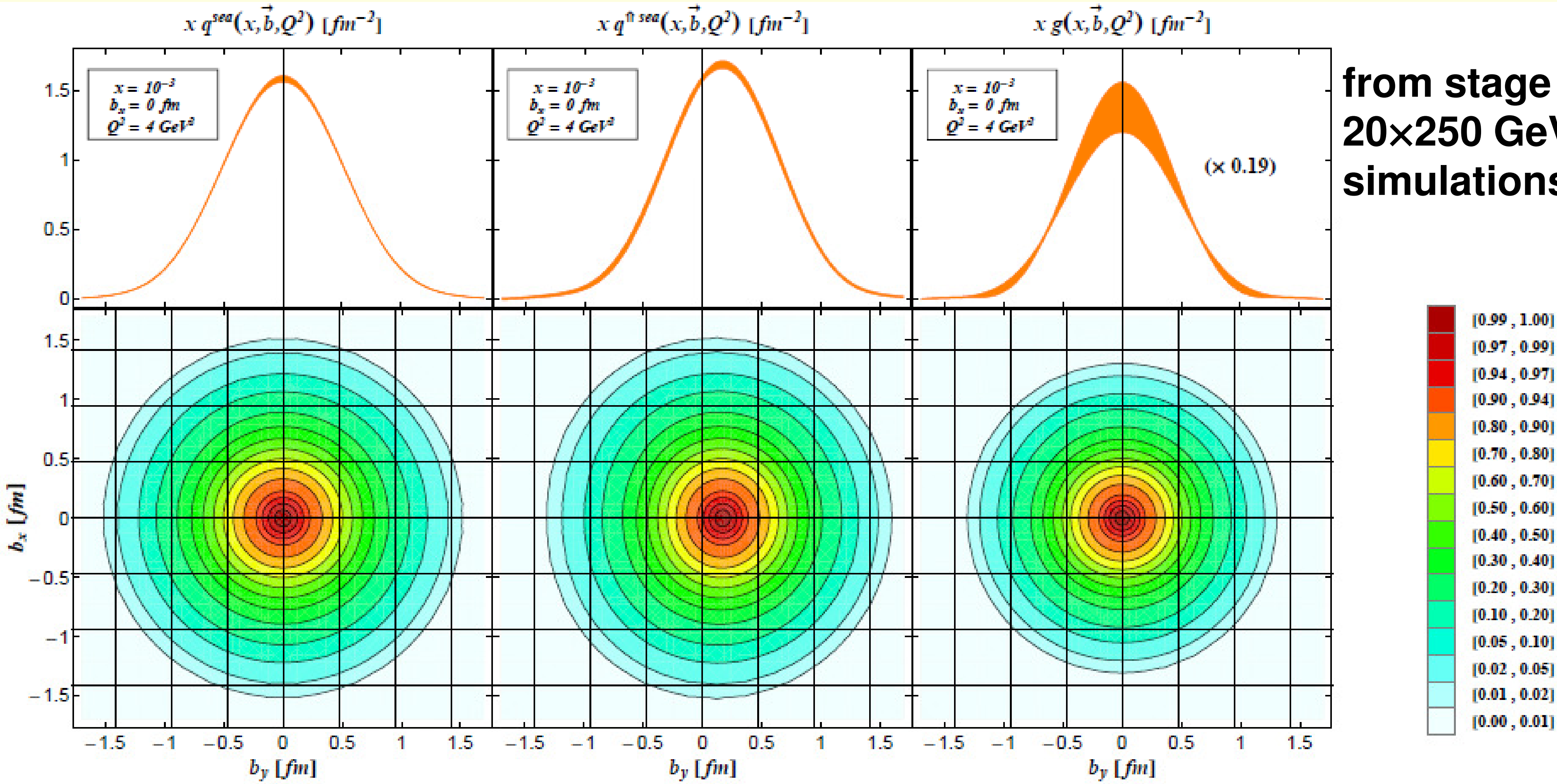
error and mean estimates are model dependent, it looks to me that results are human biased



The Future

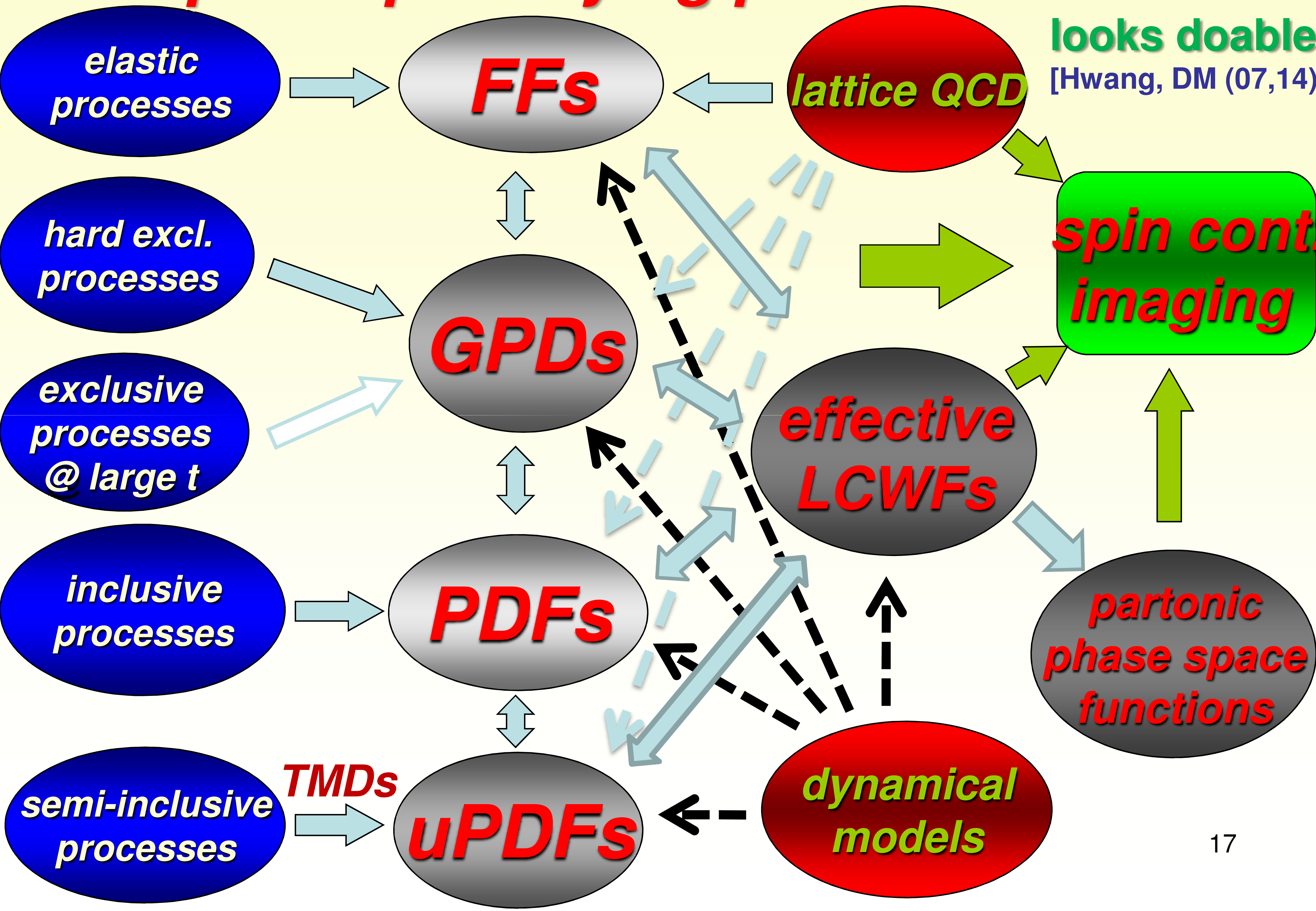
- ✓ COMPASS II
- ✓ JLAB@12 GeV
- ? ENC@GSI
- ? LHeC@CERN
- ? EIC@BNL or EIC@JLAB (also access to E^{sea}, i.e. J^{sea})

Aschenauer, Firzo
KK, DM (13)



from stage II 20×250 GeV² simulations

Prospect: quantifying partonic content



Summary

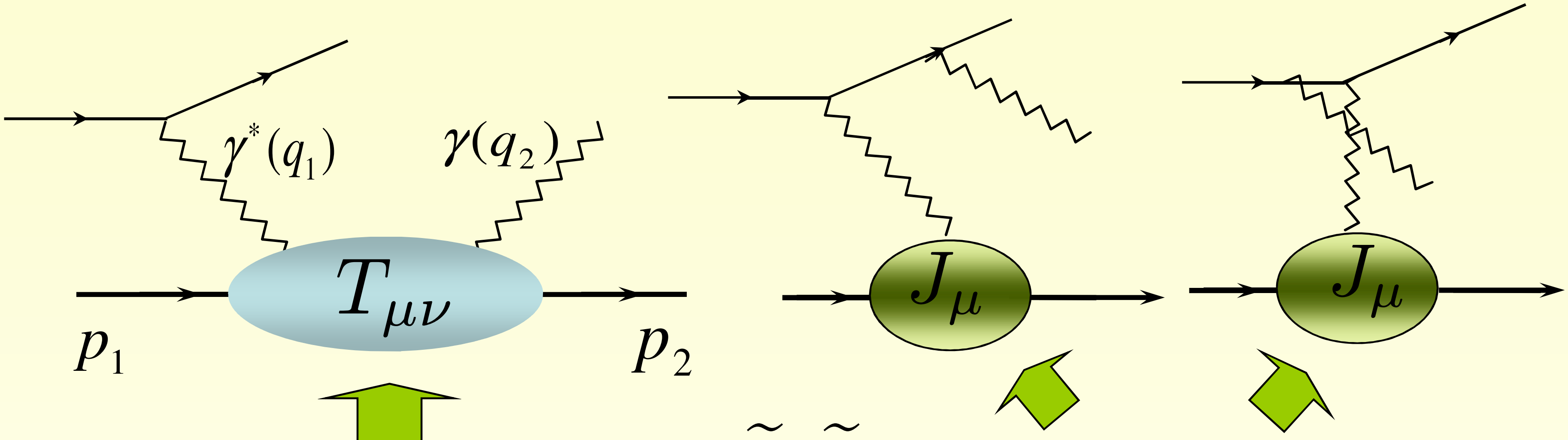
GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons (to some extend done at small x_B)
- to address the spin content of the nucleon (not possible at present in pheno.)
- providing a bridge to non-perturbative methods (lattice, also LCWFs models)
- modeling in terms of effective LCWFs seems t doable (requires efforts)

first decade of hard exclusive leptoproduction measurements

- CFFs have their own interest, bridging low and high virtuality regimes
- global model fits to DVCS can be straightforwardly improved
- DVCS and DVMP data are describable in global NLO fits at small x
- moving on: to NLO, kinematical twist, full GPD models, DVCS+DVMP+...
- covering the kinematical region between HERA (COMPASS) experiments within a high luminosity machine and dedicated detectors is needed to quantify exclusive and inclusive QCD phenomena: handle on GPD E & 3D
- support for theory is needed (otherwise no robust phenomenology will show up)
- some kind of education is desired before one can enter GPD phenomenology

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}, \cdots$ elastic form factors F_1, F_2
(helicity amplitudes)

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\rm Bj}^2 y^2 t\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) \right\},$$

exactly known
(LO, QED)

$$|\mathcal{T}_{\rm DVCS}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\},$$

harmonics ⇕ **1:1** helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm Bj} y^3 t \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}.$$

harmonics ⇕ **1:1** helicity ampl.

all harmonics are given by twist-2 and -3 GPDs: [Diehl et. al (97) Belitsky, DM, Kirchner (01)]

$$\left\{\begin{matrix} c_1 \\ s_1 \end{matrix}\right\}^{\mathcal{I}} \propto \frac{\Delta}{\mathcal{Q}} \text{ tw-2(GPDs)} + O(1/\mathcal{Q}^3), \qquad c_0^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2} \text{ tw-2(GPDs)} + O(1/\mathcal{Q}^4),$$

$$\left\{\begin{matrix} c_2 \\ s_2 \end{matrix}\right\}^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2} \text{ tw-3(GPDs)} + O(1/\mathcal{Q}^4), \qquad \left\{\begin{matrix} c_3 \\ s_3 \end{matrix}\right\}^{\mathcal{I}} \propto \frac{\Delta\alpha_s}{\mathcal{Q}} (\text{tw-2})^{\mathrm{T}} + O(1/\mathcal{Q}^3),$$

$$c_0^{\mathrm{CS}} \propto (\text{tw-2})^2, \qquad \left\{\begin{matrix} c_1 \\ s_1 \end{matrix}\right\}^{\mathrm{CS}} \propto \frac{\Delta}{Q} (\text{tw-2})\ (\text{tw-3}), \qquad \left\{\begin{matrix} c_2 \\ s_2 \end{matrix}\right\}^{\mathrm{CS}} \propto \alpha_s (\text{tw-2})(\text{tw-2})^{\mathrm{GT}}$$

e.g., *n=1* odd harmonic is approximately given by `CFF' combination

$$\left\{\begin{matrix} c^{\mathcal{I}}_{1,\mathrm{unp}} \\ s^{\mathcal{I}}_{1,\mathrm{unp}} \end{matrix}\right\} = 8K \left\{\begin{matrix} -(2-2y+y^2) \\ \lambda y(2-y) \end{matrix}\right\} \left\{\begin{matrix} \Re\mathrm{e} \\ \Im\mathrm{m} \end{matrix}\right\} \mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}(\mathcal{F}), \mathcal{C}^{\mathcal{I}}_{\mathrm{unp}} = F_1\mathcal{H} + \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}(F_1+F_2)\widetilde{\mathcal{H}} - \frac{\Delta^2}{4M^2}F_2\mathcal{E}$$

relations among **harmonics** and (helicity dependent) **CFFs** are not more based on a 1/*Q* expansion: [Belitsky, DM (10) -- Belitsky, DM, Ji (12), see also Braun et. al (14)]

$$s^{\mathcal{I}}_{1,\mathrm{unp}} = \frac{8\widetilde{K}\lambda\sqrt{1-y-\frac{y^2\gamma^2}{4}}(2-y)y}{Q(1+\gamma^2)} \Im\mathrm{m}\Big\{\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}\Big(\Big[1-\frac{\varkappa}{2Q^2}\frac{Q^2+t}{\sqrt{1+\gamma^2}}\Big]\mathcal{F}_{++} + \Big[1-\frac{2+\varkappa}{2Q^2}\frac{Q^2+t}{\sqrt{1+\gamma^2}}\Big]\mathcal{F}_{-+} + \frac{(Q^2+t)\varkappa_0}{Q^2\sqrt{1+\gamma^2}}\mathcal{F}_{0+}\Big)$$
$$+ \frac{-t(Q^2+t)}{\sqrt{1+\gamma^2}Q^4}\Delta\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}\Big(\mathcal{F}_{-+} + \frac{\varkappa}{2}[\mathcal{F}_{++}+\mathcal{F}_{-+}] - \varkappa_0\mathcal{F}_{0+}\Big)\Big\}, \qquad (70)$$

new improved C coefficients ensure the cancellation of kinematical singularities

relations among CFFs and GPDs are always based on a *1/Q* expansion

Conformal partial wave expansion

- GPD support is a consequence of Poincaré covariance (polynomiality)

$$H_j(\eta,t,\mu^2)=\int_{-1}^{1}dx\,c_j(x,\eta)H(x,\eta,t,\mu^2)\,,\qquad c_j(x,\eta)=\eta^j C_j^{3/2}(x/\eta)$$

- conformal moments evolve autonomously (to LO and beyond in a special scheme)

$$\mu\frac{d}{d\mu}H_j(\eta,t,\mu^2)=-\frac{\alpha_s(\mu)}{2\pi}\gamma_j^{(0)}H_j(\eta,t,\mu^2)$$

- inverse relation is given as series of (mathematical) generalized distributions:

$$H(x,\eta,t)=\sum_{j=0}^{\infty}(-1)^j p_j(x,\eta)H_j(\eta,t)\,,\ p_j(x,\eta)\propto\theta(|x|\leq\eta)\frac{\eta^2-x^2}{\eta^{j+3}}C_j^{3/2}(-x/\eta)$$

- various ways of resummation were proposed: see Kreso`s and Kirill`s talk

- smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
- mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
- `dual' parameterization [M. Polyakov, A. Shuvaev (02), Polyakov (07), Semenov-Tian-Shansky]
- based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
- ***Mellin-Barnes integral*** [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]