SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL
FROM QUARKS AND GLUONS TO HADRONS AND NUCLEI

PRISMA

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Proton Radius Puzzle: the Status

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Special thanks to my collaborators:

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Felipe LLanes-Estrada (U. Madrid)

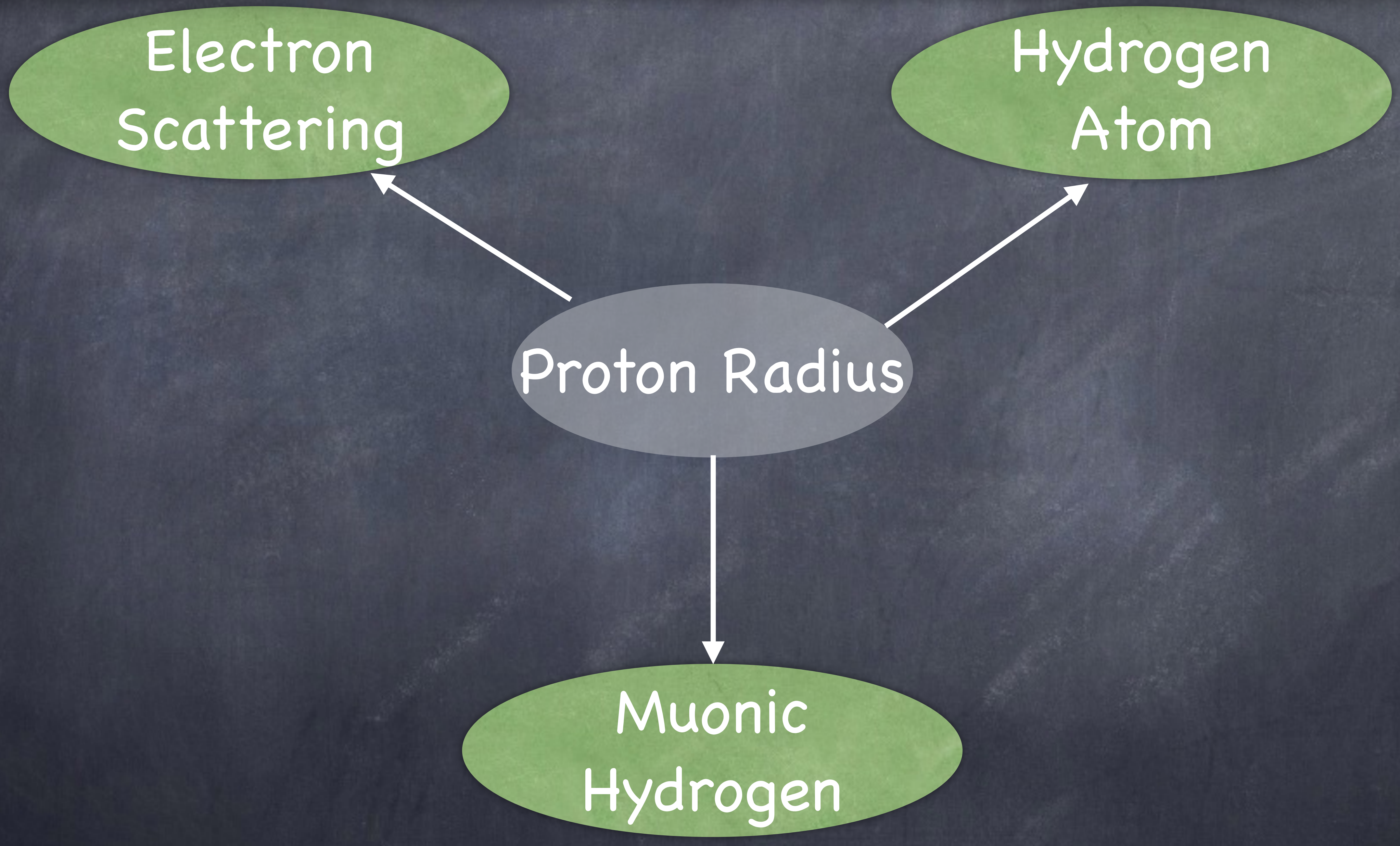
MG, Llanes-Estrada, Szczepaniak, Phys.Rev. A87 (2013) 052501, [arXiv:1302.2807]

Carlson, MG, Vanderhaeghen, Phys.Rev. A89 (2014) 022504, [arXiv:1311.6512]

MG, Phys.Rev. C90 (2014) 052201, [arXiv:1406.1612]

MG, [arXiv:1508.02509]

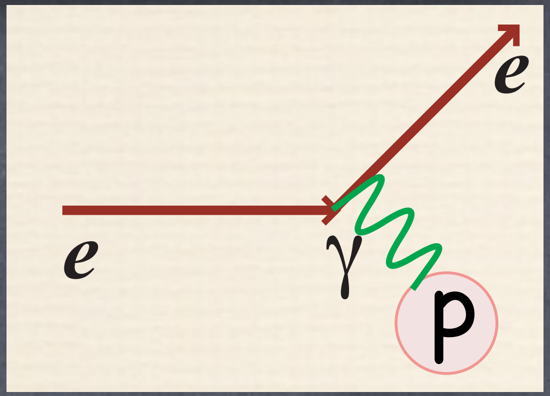
Proton radius puzzle



Elastic Electron Scattering

Unpolarized cross section

$$\left(\frac{d\sigma}{d\Omega}\right)^{unpol} = \sigma_{\text{Mott}} \frac{\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\epsilon(1+\tau)}$$



Momentum Transfer $Q^2 \to \tau = Q^2/(4M^2)$

Energy E -> ϵ: $0 < \epsilon < 1$ for $E_{min} < E < \infty$

$G_{E,M}(Q^2)$ – electric and magnetic form factors

FFs encode charge, magnetic moment, RMS radii, ...

$$G_E(Q^2) = 1 - (1/6)\, R_{Ch}^2\, Q^2 + \ldots$$

$$G_M(Q^2) = \mu_p[1 - (1/6)\, R_M^2\, Q^2 + \ldots]$$

Proton Radius from e-scattering

Measure cross section down to low Q^2

$$\frac{d\sigma^{exp}}{d\Omega} / \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\Bigg|_{Q^2\to 0} = 1 + Q^2\left[\frac{\mu_p^2 - 1}{4M^2} - \frac{1}{3}R_{Ch}^2\right] + \ldots$$

The radius is defined as the slope of the FF at origin, data are at finite Q^2: extrapolation is unavoidable

How low in Q^2 should/can one go?
up to now $Q_{min}^2 = 4 \times 10^{-3}$ GeV^2

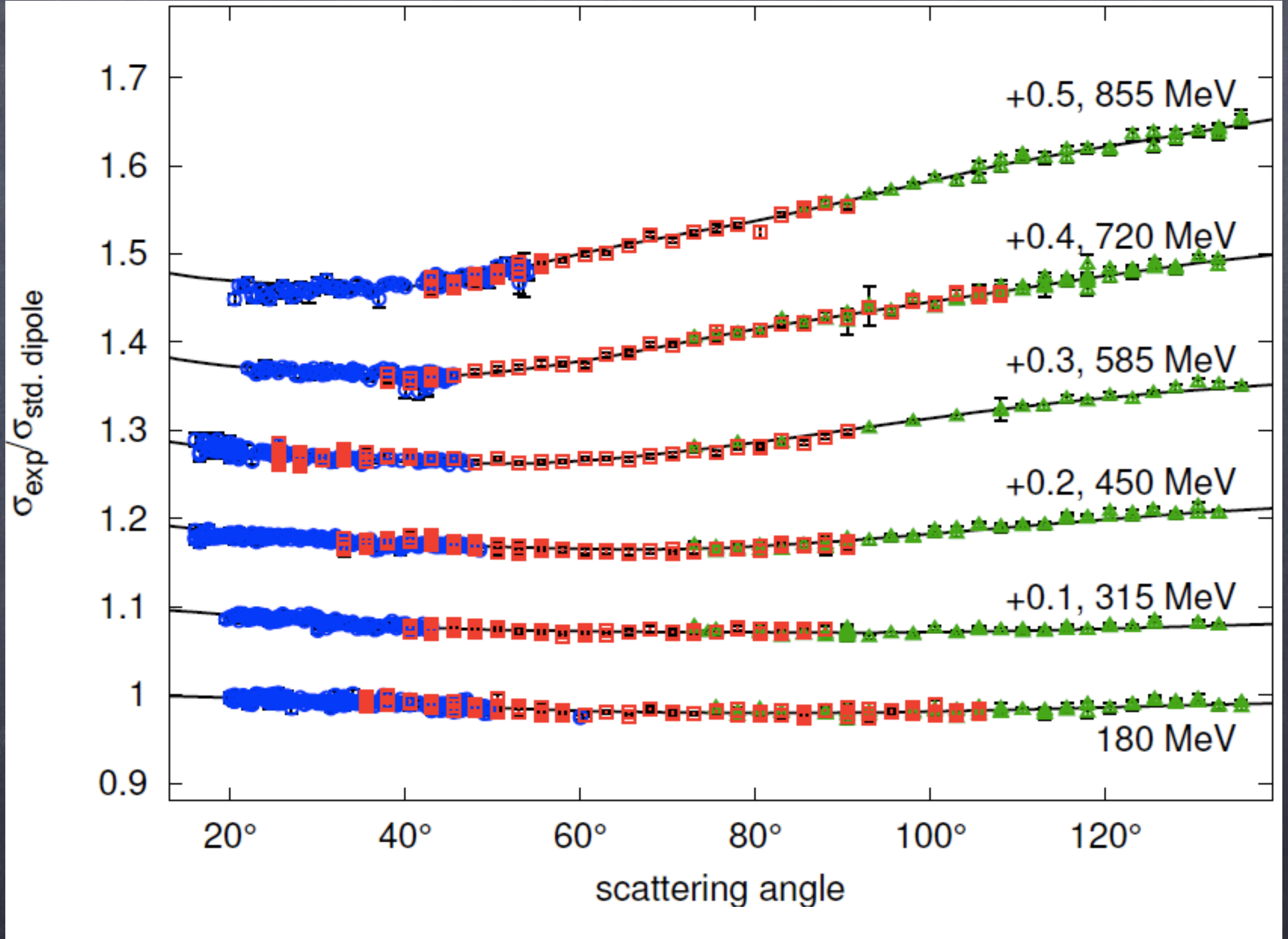
1% uncertainty in R_{Ch} - measure 1 to few x 10^{-4} precision!

Proton Radius from e-scattering

A1 @ MAMI

$R_{Ch} = 0.879(8)$

Bernauer et al., '10



Bernauer et al. (2010)

Proton Radius from e-scattering

- Individual data points - per cent level accuracy;
- Need large angle coverage to extract the radius to 1%
- Large statistics serves as a lever arm for extracting "1" to 0.05% precision;
- Higher Q^2 data influence the extracted radius
- The lower in Q^2 one goes, the lesser are higher order terms important - plans with ISR @ Mainz, PRad @ JLab, $Q^2 \geq 10^{-4}$ GeV^2

Proton Radius from e-scattering

- Bernauer et al.: used full statistics (low and moderate Q^2)
  studied systematics due to different fit functions
  (polynomial, splines, dipole, double dipole etc.)
  χ^2 close to 1 with 1400 d.o.f.

R_E^p = 0.879(8) fm

- Lorenz '12,13: Dispersion relation fit $G_{E,M}(Q^2) = \int_{4m_\pi^2}^{\infty} \frac{dt\, \rho_{E,M}(t)}{t + Q^2}$

Model of the spectral function: 2π continuum + VDM + QCD asymptotics
Radius mainly sensitive to the lowest states (2π, 3π) which are taken as exact -> fit function might not be flexible enough, χ^2 > 1.1
Consistent with previous DR fits (Höhler '76, Mergell '96, ...)

R_E^p = 0.84(1) fm

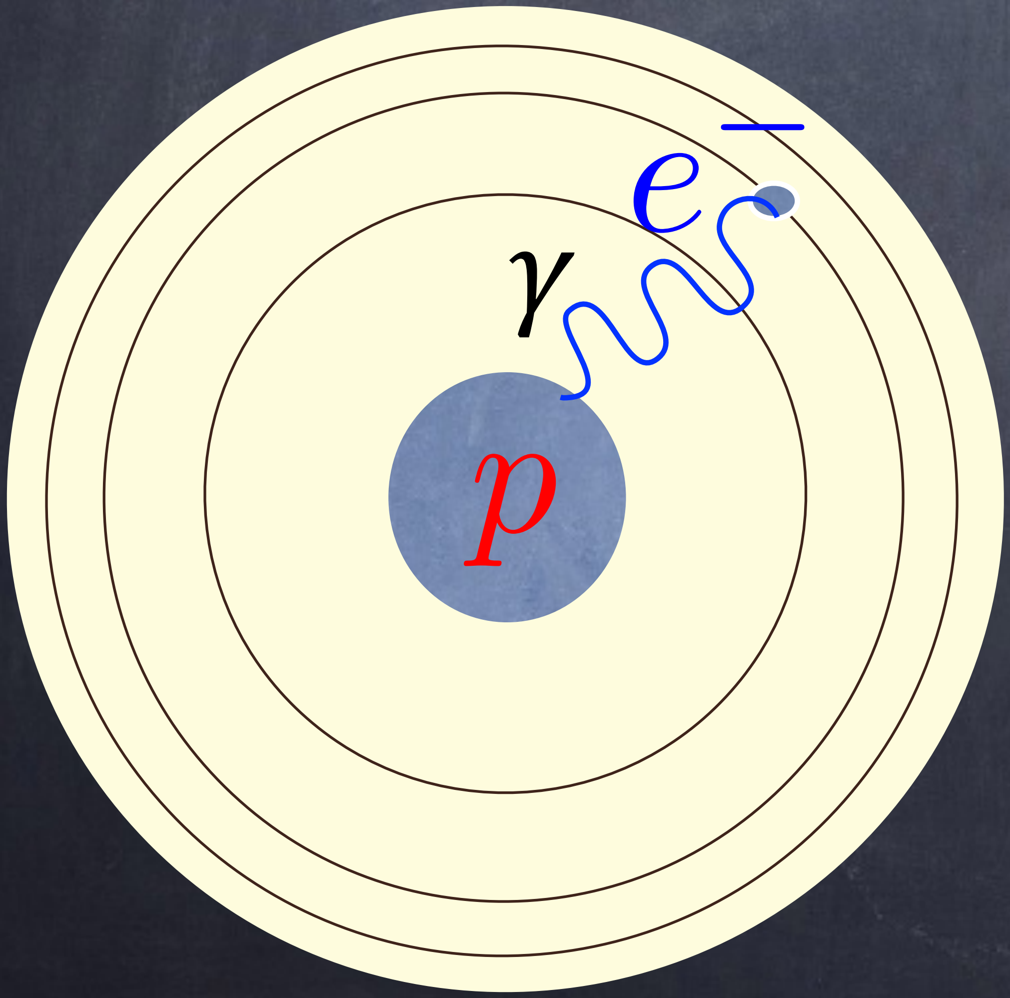
- Hill, Paz '10: Conformal mapping + Fourier series for the spectral fn.

R_E^p = 0.87(2) fm

Data tend to larger radii; Need extra input to get smaller radii

R_E^p from Lamb Shift in Hydrogen

No extrapolation problem in atoms;
typical momentum transfer in H-atom:
keV^2 in e-H, MeV^2 μ-H



Electrons occupy stationary orbits

Energy levels E_{NL}

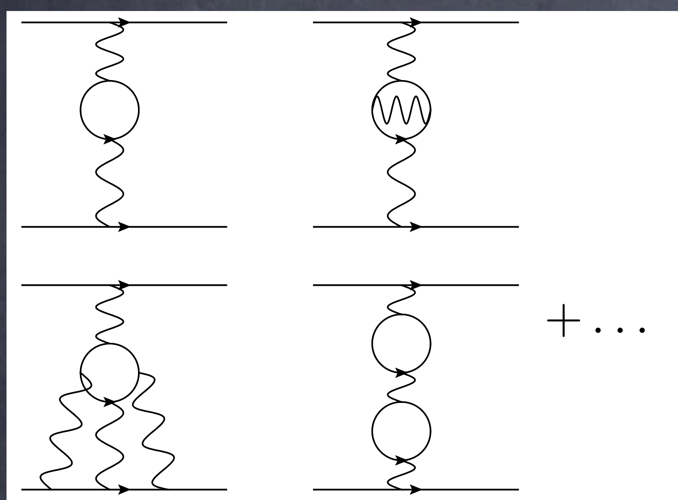
Principal (energy) Q.N.: N=1,2,3...;
Orbital momentum Q.N.: L=S,P,D...;

If only one photon were exchanged:

$$E_{2S} = E_{2P}$$

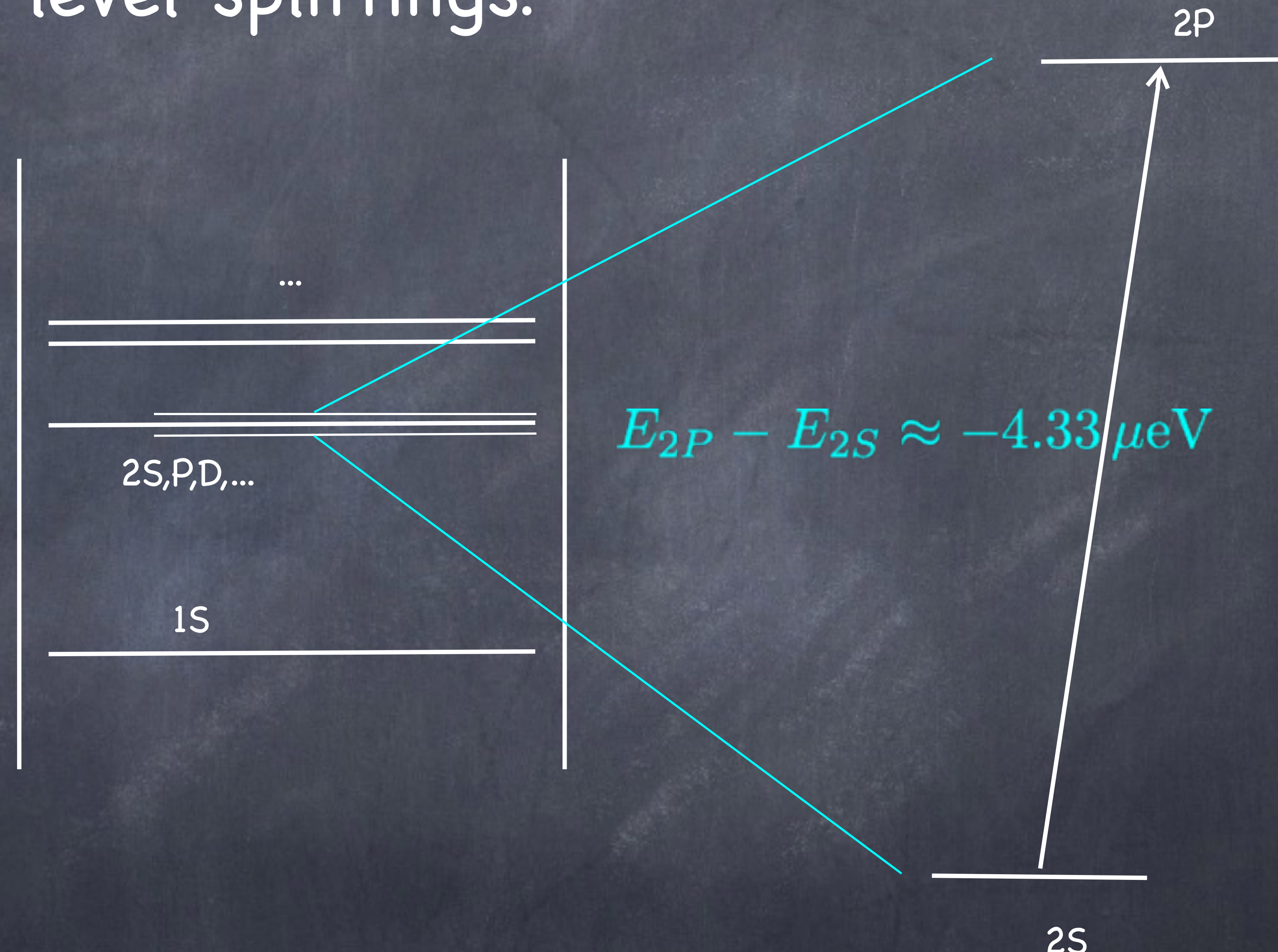
R_E^p from Lamb Shift in Hydrogen

Radiative corrections: level splittings!



$$E_{2S} - E_{1S} \approx 10.2\,\mathrm{eV}$$

$$E_{1S} \approx -13.6\,\mathrm{eV} = -hc\,R_\infty$$



$$E_{2P} - E_{2S} \approx -4.33\,\mu\mathrm{eV}$$

nS-nP splitting (Lamb shift) - authentic prediction of SM (QED)

Precise calculations of QED corrections: p.p.m. level precision

R_E^p from Lamb Shift in Hydrogen

- The proton is not a point-like charge - has a finite size - Lamb shift is sensitive to the proton radius

$$\Delta E_{nP-nS} = \Delta E^{QED}_{nP-nS} - \frac{2(Z\alpha)^4}{3n^3} m_r^3 R_E^2 + \mathcal{O}(\alpha_{em}^5)$$

- few p.p.m. correction
  - exceeds the QED precision
  - can be extracted

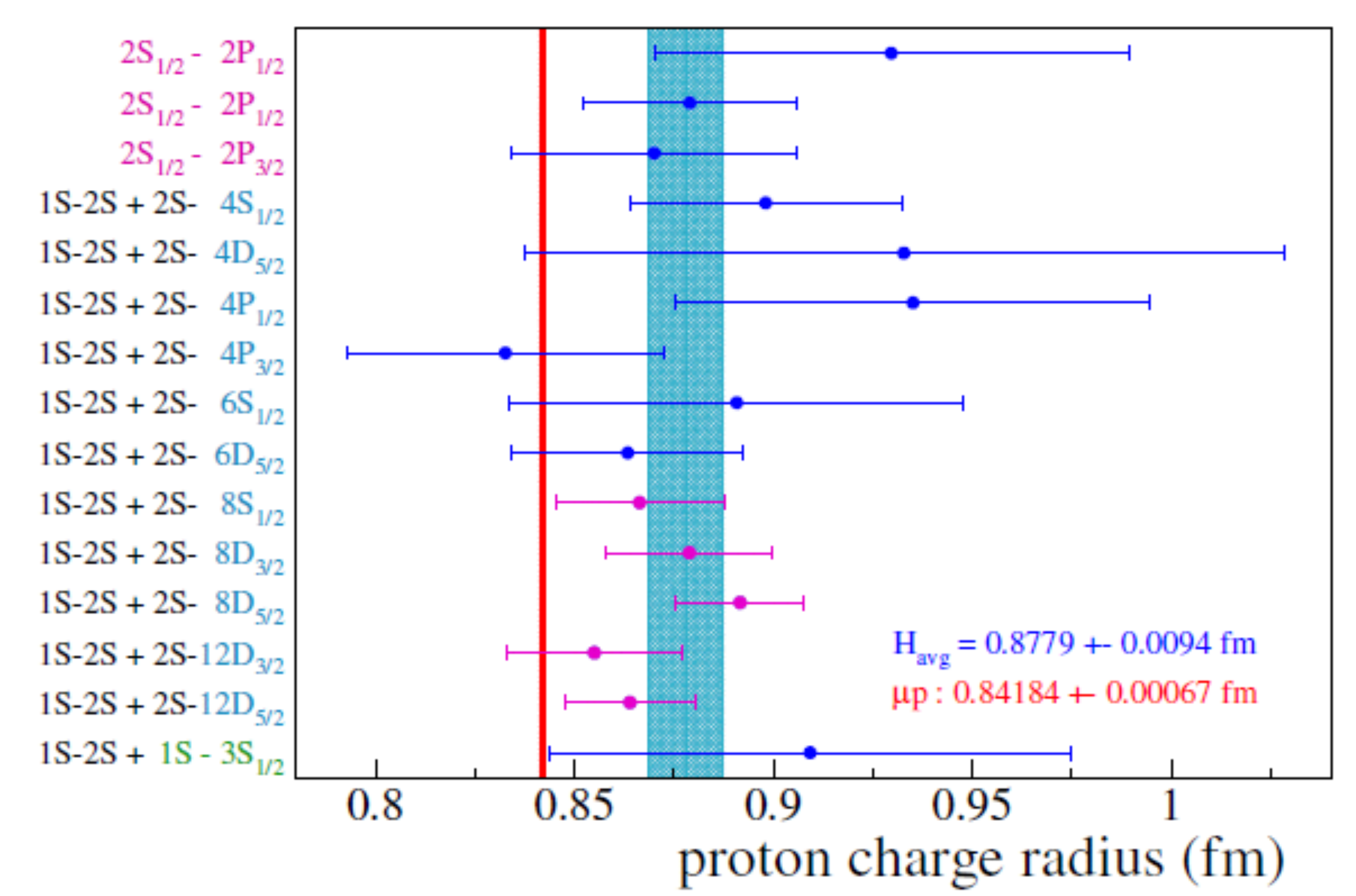
$$E_{2S} - E_{2P} = 33.7808(1)\,\mu\text{eV} + 0.0008 {R_E^p}^2\,\mu\text{eV}$$

QED Finite Size

R_E^p from Lamb Shift in Hydrogen

CODATA

$R_{Ch} = 0.8779(94)$ fm



Pohl et al [CREMA Coll.] '10, Antognini et al. '13

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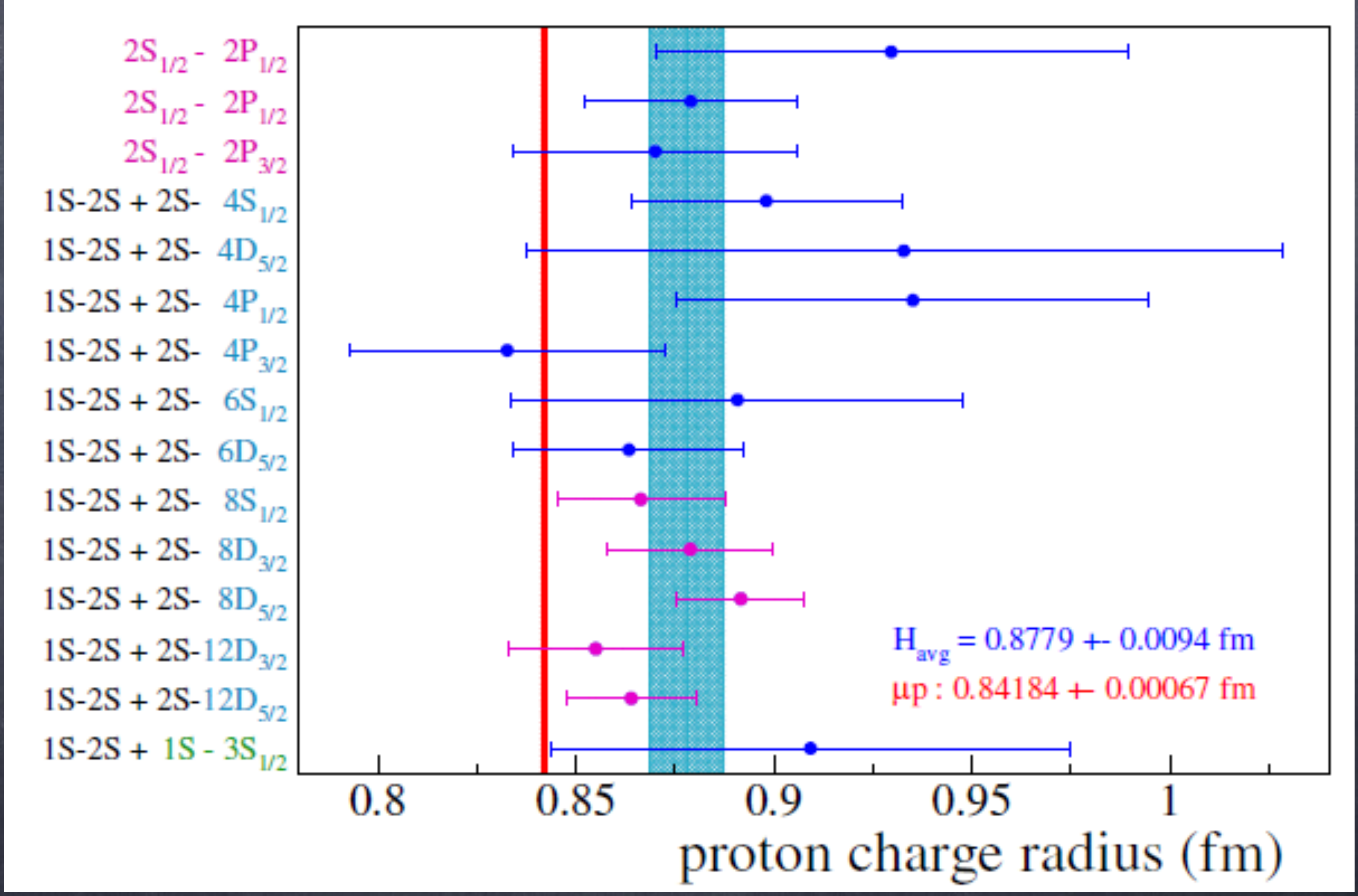

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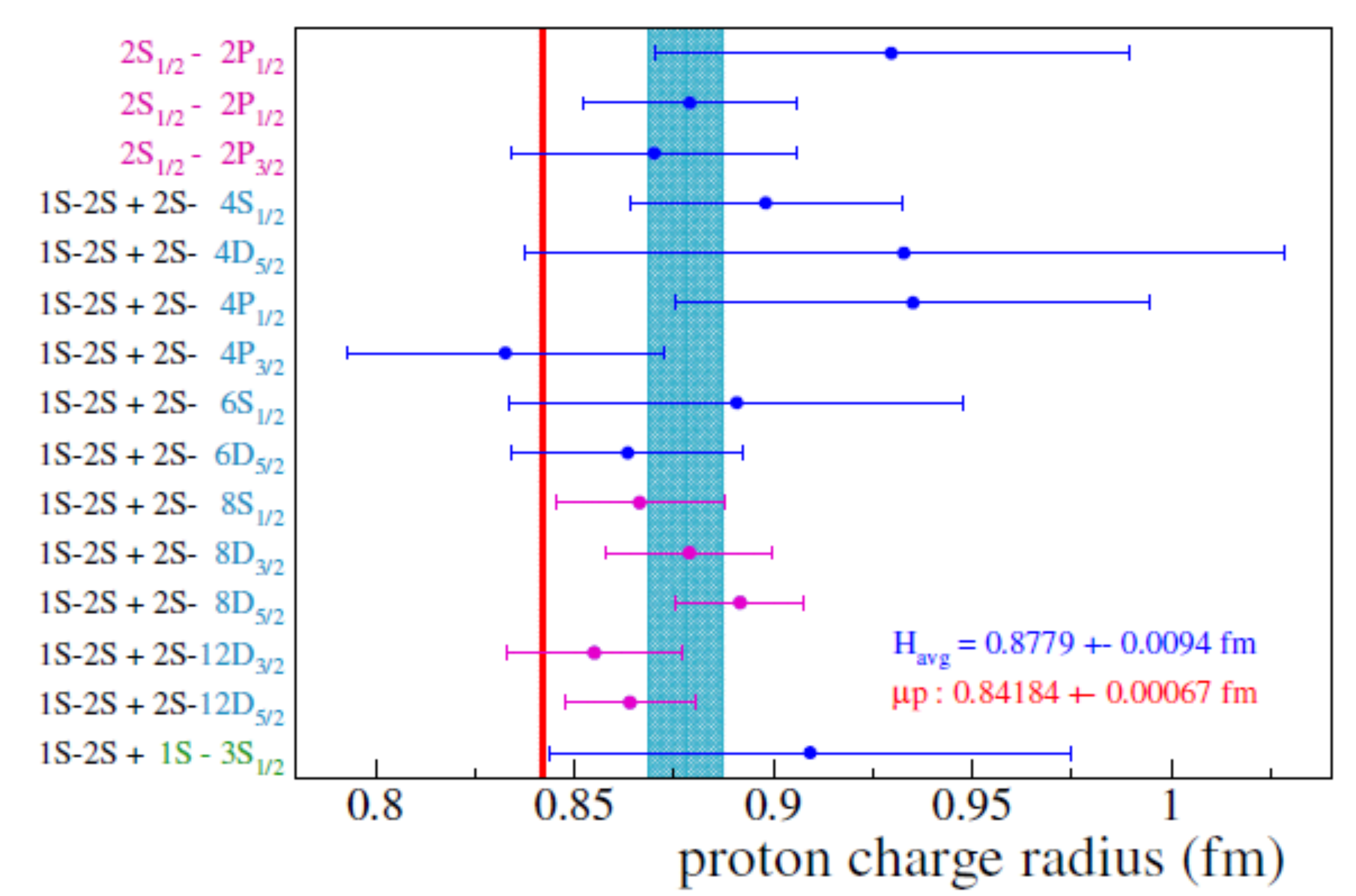
Combined

R_{Ch} = 0.8775(51) fm



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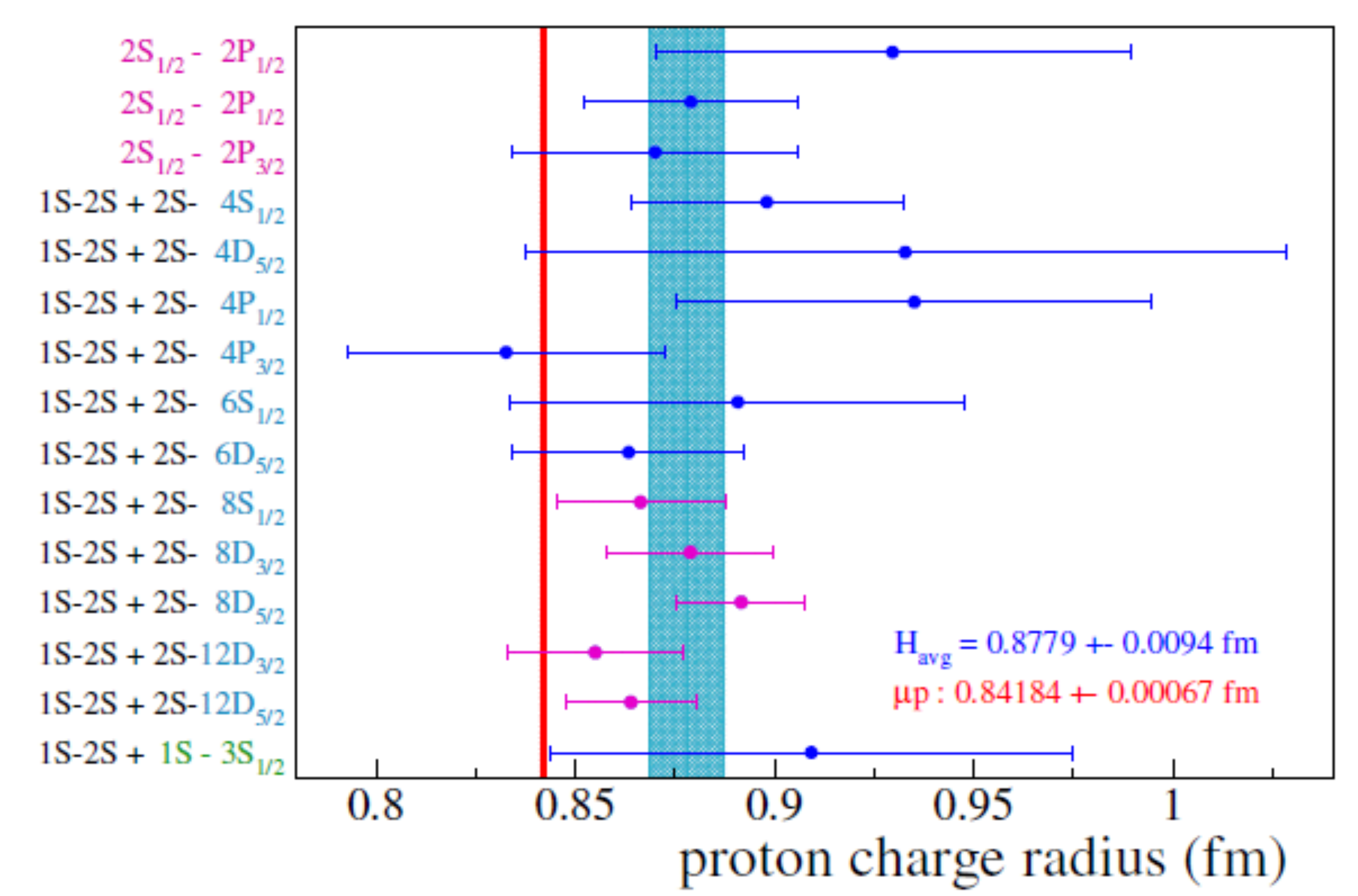
μH data @ PSI

$$R_E^p = 0.84087(39)\ \mathrm{fm}$$



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4% discrepancy for R_{Ch} (0.6% precision from e-p) - 7σ away!

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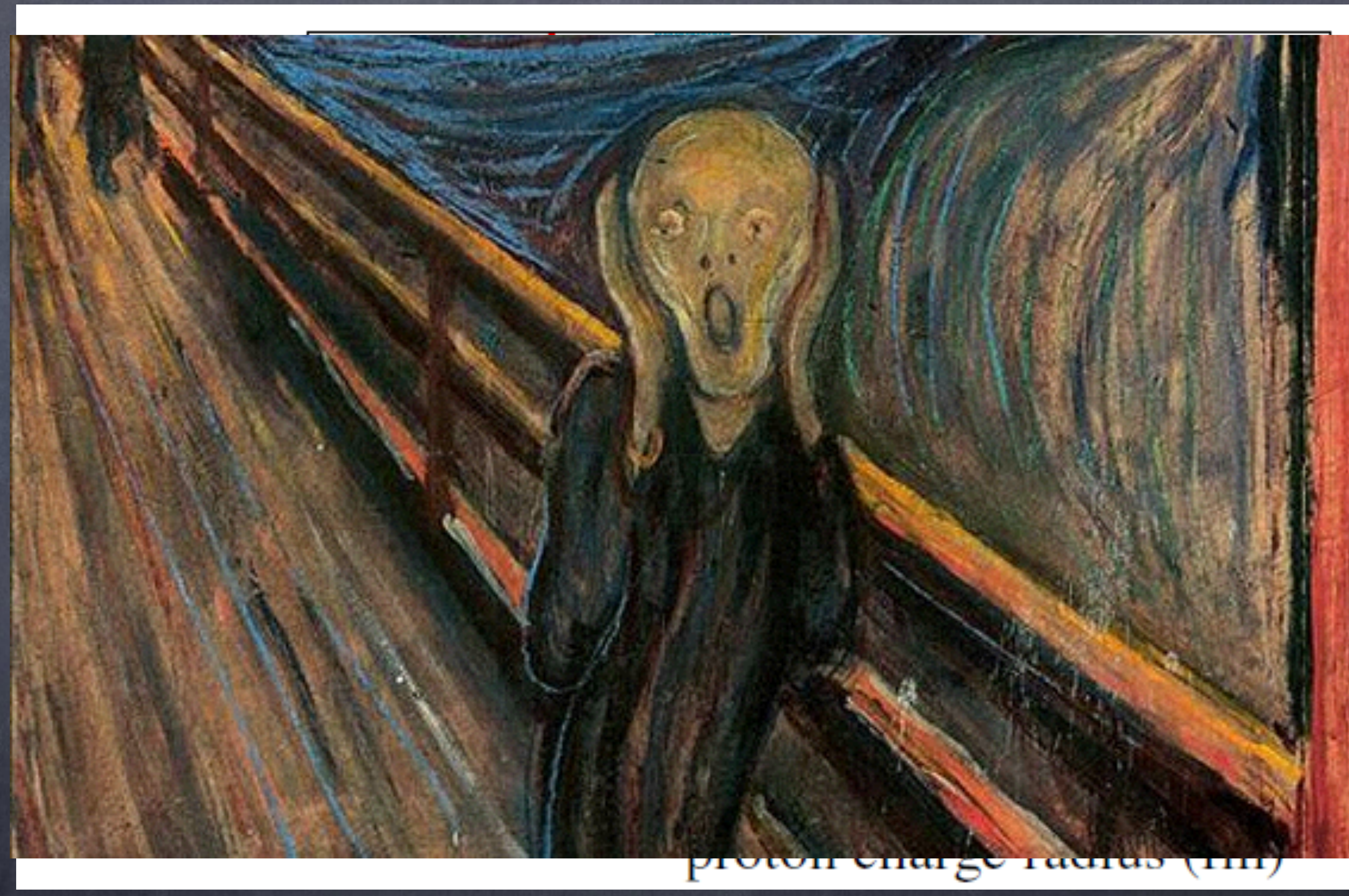

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R_E^p from e-H

Almost all individual e-H points are within 1.5σ from the muonic point

BUT they all lie systematically at larger radii - correlated systematics?

All QED corrections have been studied up to α^6 - under control

Electron scattering is the most precise single measurement and is in nice agreement with the statistical average of the e-H data.

Most of the measurements are old - may be a good idea to remeasure

New experiments with projected 1% radius extraction - under way:

2S-2P measurement - York U. (Canada);

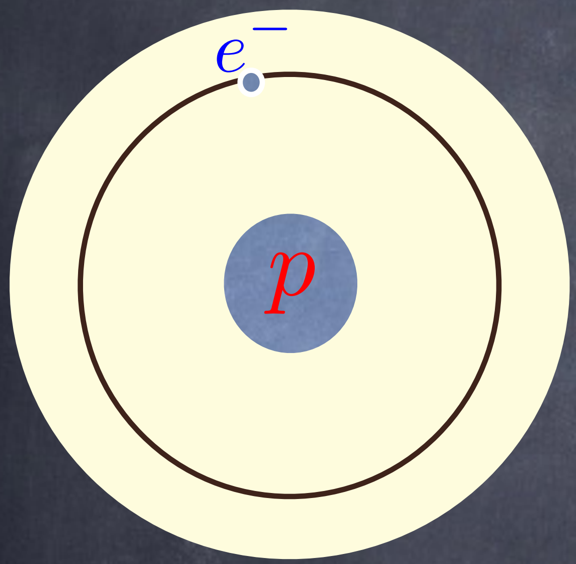
2S-4S measurement - MPI Garching;

1S-3S measurement - Laboratoire Kastler Brossel (Paris);

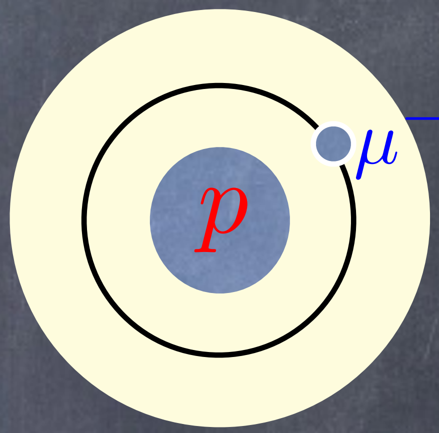
What's special about μ–H?

QED: the only difference is the mass $m_\mu \approx 200\, m_e$

Hydrogen atom



muonic Hydrogen



Bohr radius

$$R_B \sim \frac{1}{\alpha m_r}$$

Fine structure constant $\alpha \approx 1/137$

Reduced lepton-proton mass $m_r = \dfrac{mM}{m+M}$

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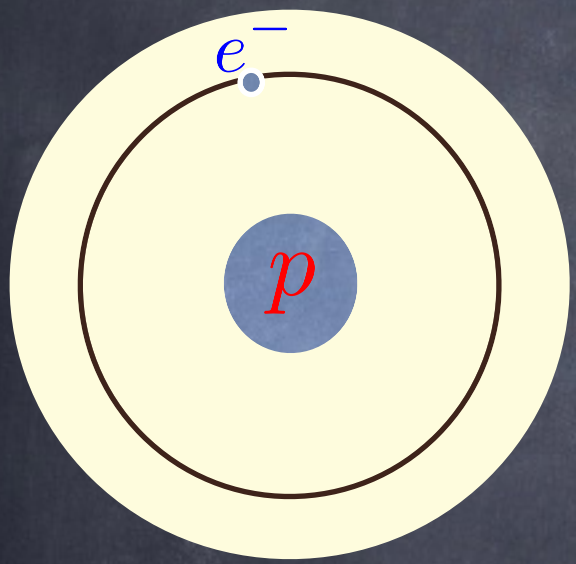
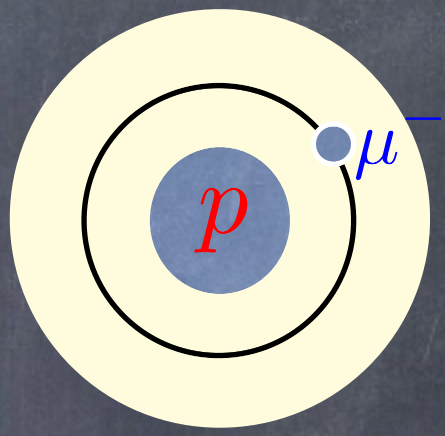


muonic Hydrogen



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Reduced lepton-proton mass $m_r = \dfrac{mM}{m+M}$

Finite size Lamb shift: $\Delta E^{R_E^p}_{2P-2S} \propto \alpha^4 m_r^3$

$$\Delta E^{eH}_{2P-2S} = -8.1 \times 10^{-7}\, R_E^2\, \mathrm{meV} \qquad \Delta E^{\mu H}_{2P-2S} = -5.2275(10)\, R_E^2\, \mathrm{meV}$$

μH unstable (τ_{2S} ~ μs) - 7 o.o.m. still make it 10 times more precise

R_E^p from μ-H

Using the proton radius from eH and scattering, expect

$$\left[\Delta E_{2P-2S}^{\text{Measured}} - \Delta E_{2P-2S}^{QED}\right]^{\text{Expected}} \approx -4.0\,\text{meV}$$

Observed splitting - off by 8%, radius off by 4%

$$\left[\Delta E_{2P-2S}^{\text{Measured}} - \Delta E_{2P-2S}^{QED}\right]^{\text{Measured}} \approx -3.7\,\text{meV}$$

What if the μH experiment is wrong?

Exp. precision: μeV, much smaller than missing 300 μeV;

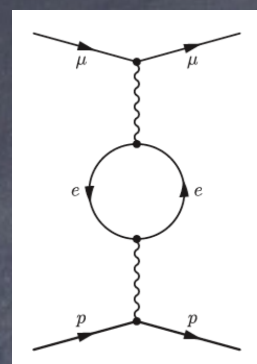
Pohl et al. and Antognini et al. measured $2P_{1/2}$ - 2S and $2P_{3/2}$ - 2S transitions, found consistency;

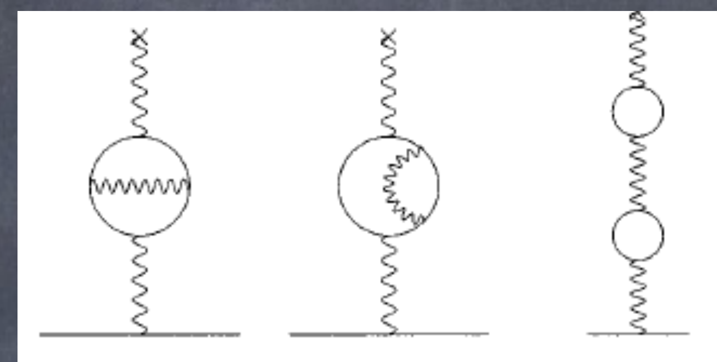
No other facility able to redo the μH experiment exists at the moment.

What has gone wrong?

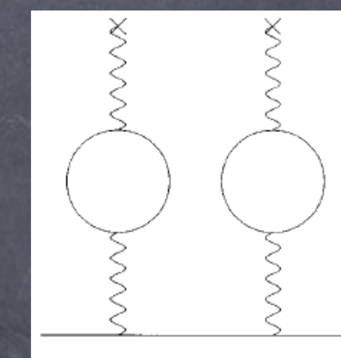
QED corrections?

1-loop eVP





2-loop eVP



Δ E = 205.0073 meV Δ E = 1.5081 meV Δ E = 0.1509 meV

Muon SE + VP Δ E = -0.6703 meV

QED corrections up to α^6 calculated: all < 0.005 meV

Further hadronic structure corrections - start at $(Z\alpha)^5$

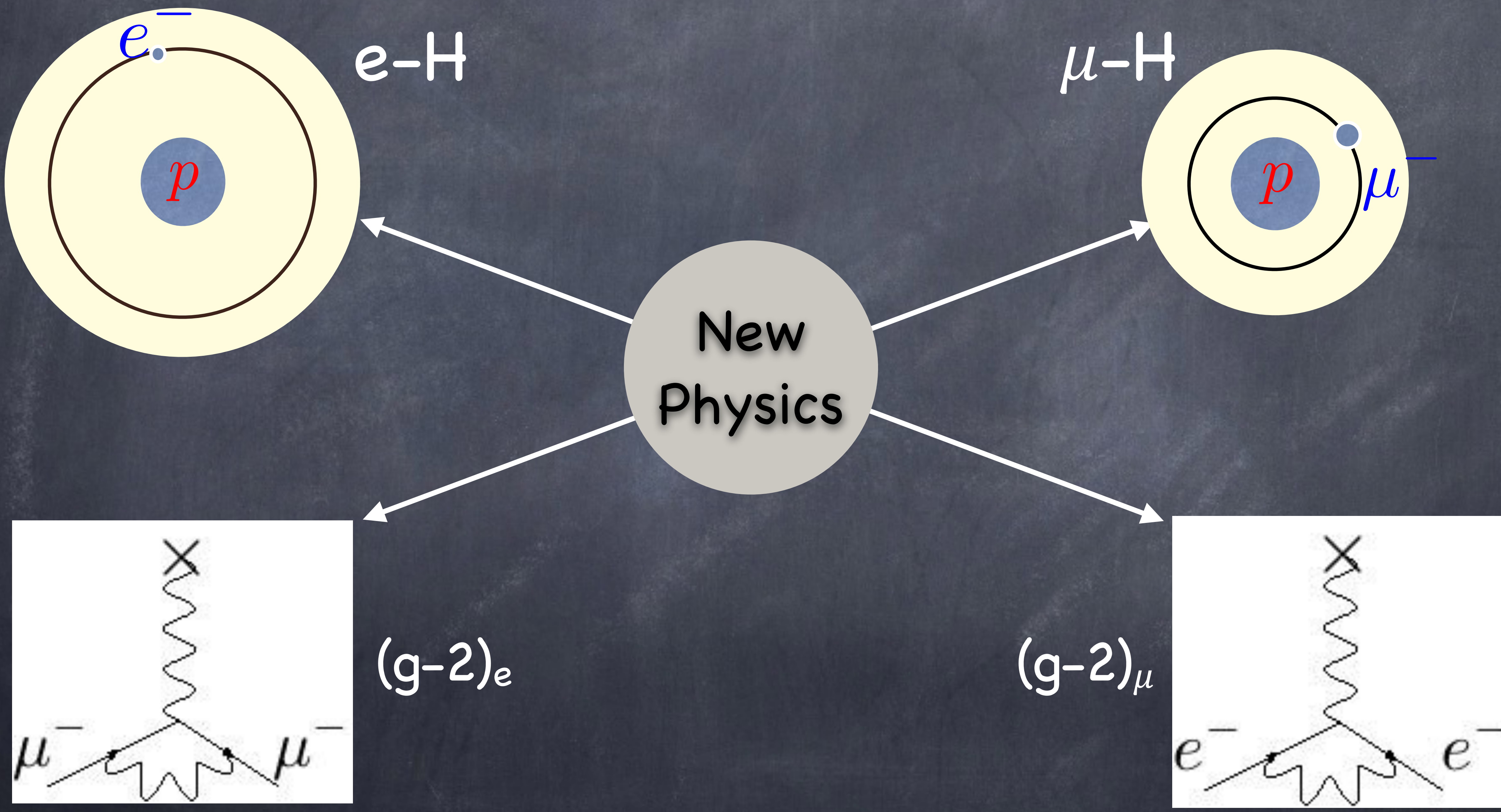
Include the third Zemach radius:

$$\Delta E_{2P-2S}^{\text{Measured}} - \Delta E_{2P-2S}^{QED} = -\frac{(Z\alpha)^4 m_r^3}{12}\left[R_p^2 - \frac{Z\alpha}{2} R_{(2)}^3\right]$$

Correction 0.03 meV - 10 times smaller than the discrepancy

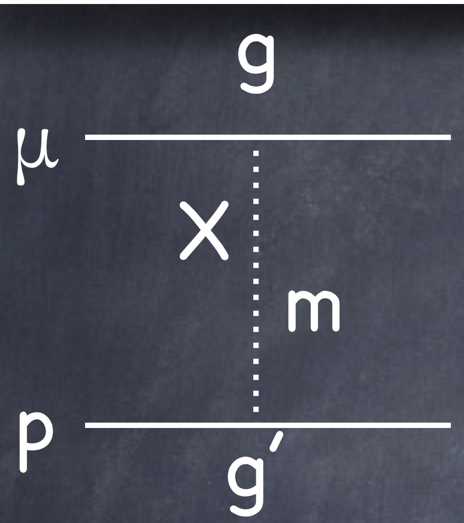
Proton Radius Puzzle: New Physics?

- Account for all constraints!



Stringent constraints from $(g-2)_e$: substantial μ-e non-universality

Proton Radius Puzzle: New Physics?



$$\Delta R_E^2 \sim -\frac{6gg'}{4\pi\alpha m^2}$$

Attractive scenario:
scalar exchange would
naturally pick up mass (Yukawa)

Tucker-Smith, Yavin '11; Batell et al, '11;
Brax, Burrage '11; Rislow, Carlson '12, '14; ...

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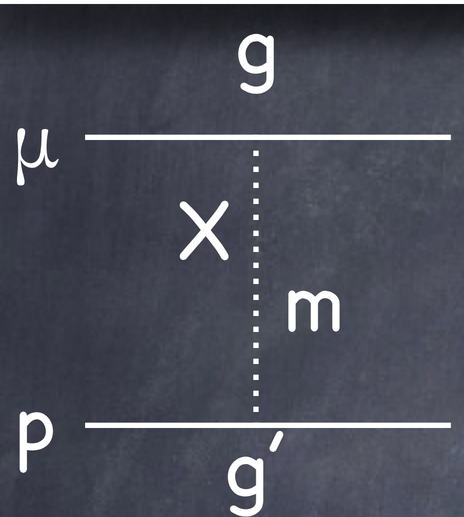

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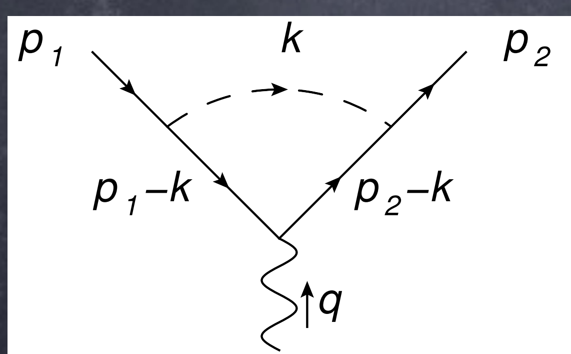
Would contribute to the muon a.m.m.

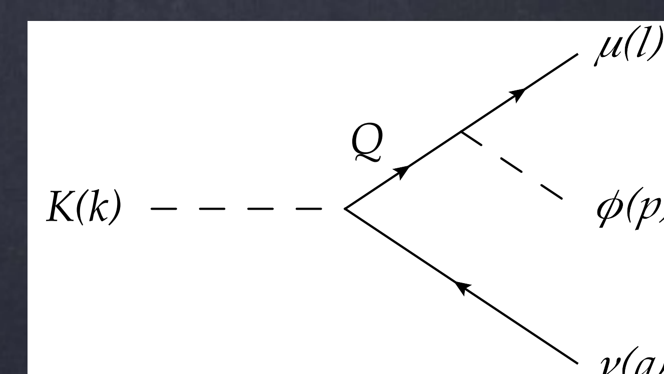


Muon $a_\mu = (g-2)_\mu/2$ has 2 ppm discrepancy

$$a_\mu(\text{data}) = (116\ 592\ 089 \pm 63) \times 10^{-11} \quad [0.5 \text{ ppm}],$$
$$a_\mu(\text{thy.}) = (116\ 591\ 840 \pm 59) \times 10^{-11} \quad [0.5 \text{ ppm}],$$
$$\delta a_\mu = (249 \pm 87) \times 10^{-11} \quad [2.1 \text{ ppm} \pm 0.7 \text{ ppm}].$$

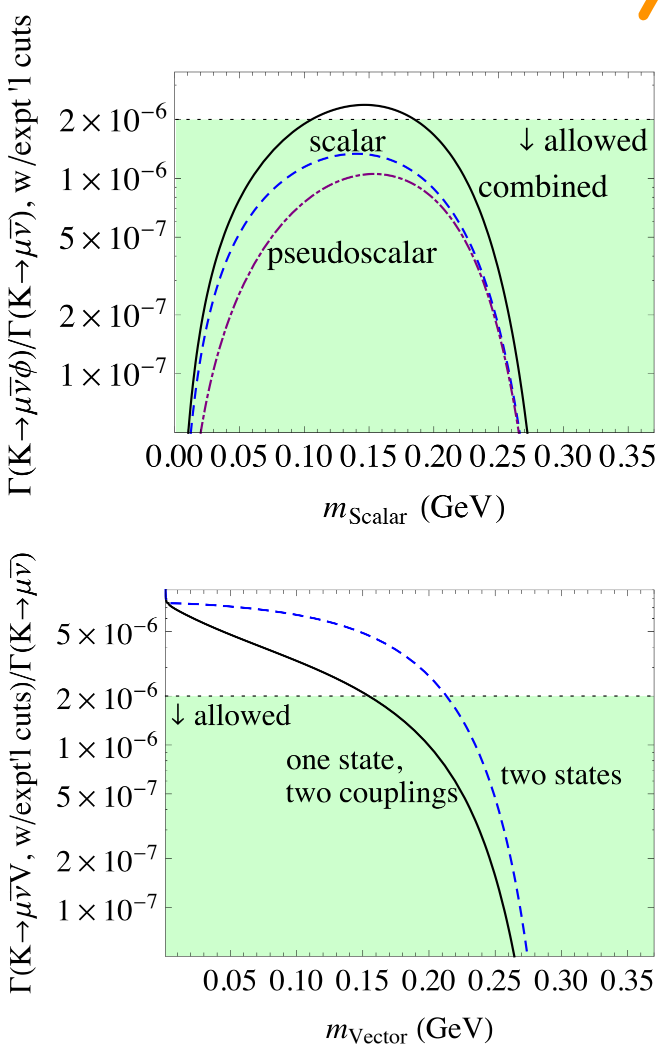
Requires fine-tuned S + PS or V + A exchanges

Would contribute to decays K→μ + invisible



Proton Radius Puzzle: New Physics?

K-decay constraints



- Solid line is sum of scalar and pseudoscalar couplings.
- Lower mass or higher mass o.k., but 90-200 MeV excluded.

- Same for polar and axial vectors.
- Solid is one particle with both V and A couplings.
- Dashed line is two particles, one polar and one axial vector.
- Lower masses excluded, 160 MeV for PV case, 210 for other case.

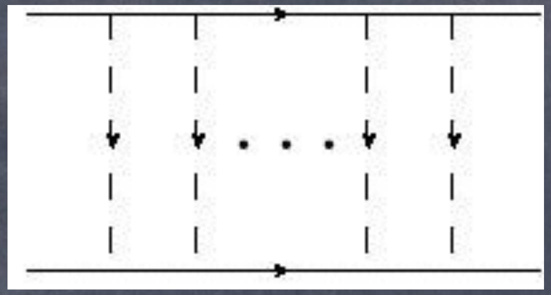
Carlson, Rislow, ‘12

Conclusion: BSM explanation possible, requires lepton non-universality, but fine tuned to evade the g-2 constraints

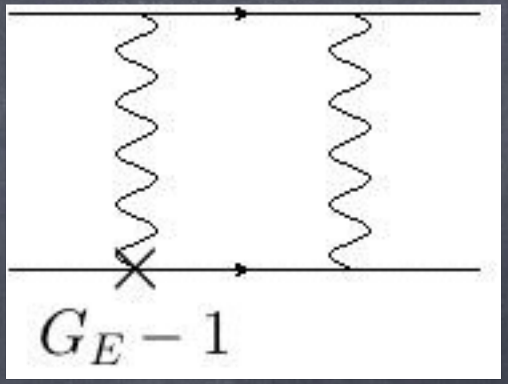
Further hadronic effects?

Hadronic correction at $(Z\alpha)^5$ - included partially!

Soft Coulomb: included in Schrödinger WF



Hard box: only part of it included (3rd Zemach m.)



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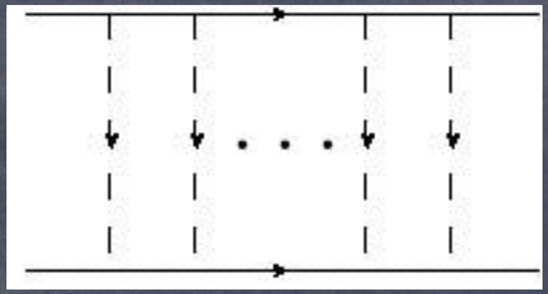

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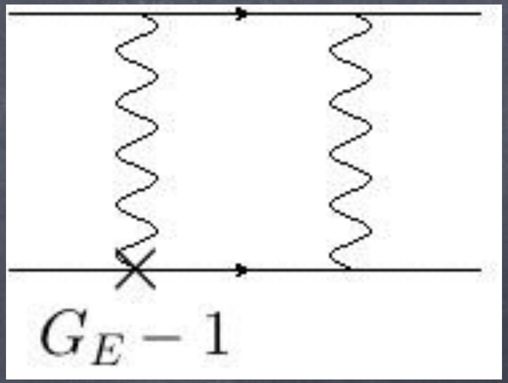
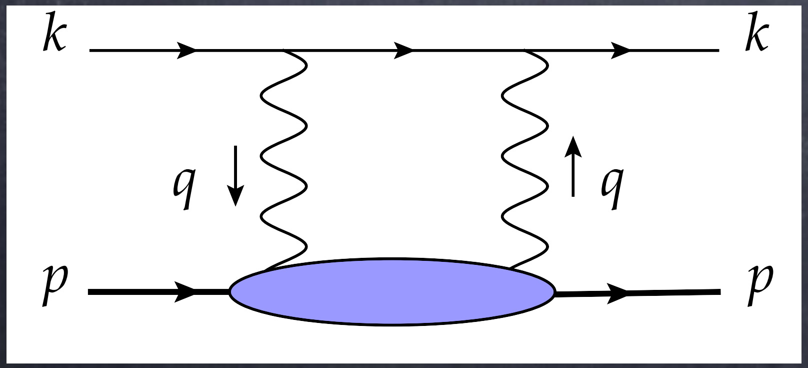


Do the full calculation



Blob: forward virtual Compton tensor

$$T_{\mu\nu} = \frac{i}{8\pi M}\int d^4x e^{iqx}\langle p|T\, j_\mu(x) j_\nu(0)|p\rangle$$

$$M_{2\gamma} = e^4 \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^4}\bar{u}(k)\left[\gamma^\nu \frac{1}{\not k - \not q - m_l + i\epsilon}\gamma^\mu + \gamma^\mu \frac{1}{\not k + \not q - m_l + i\epsilon}\gamma^\nu\right] u(k)\, T_{\mu\nu}$$

Polarizability Correction from DR

$$T_{\mu\nu} = \frac{i}{8\pi M}\int d^4x e^{iqx}\langle p|T\, j_\mu(x) j_\nu(0)|p\rangle$$

T-ordered non-local product of two vector currents - complicated!

Gauge, Lorentz inv. $T^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{\hat{p}^\mu \hat{p}^\nu}{M^2} T_2(\nu, Q^2)$

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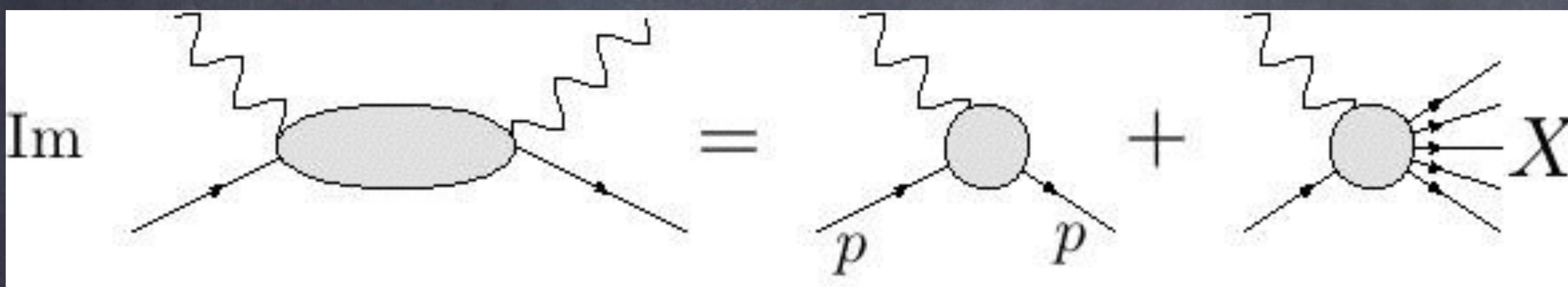
(nP - nS) splitting

$$\Delta E = -\frac{\alpha^2}{2\pi m_l M_d}\phi_n^2(0)\int d^4q \frac{(q^2+2\nu^2)T_1(\nu,q^2) - (q^2-\nu^2)T_2(\nu,q^2)}{q^4[(q^2/2m_l)^2 - \nu^2]}$$

Polarizability Correction from DR

Optical theorem: absorptive part of $T_{1,2}$ related to data



Form factors

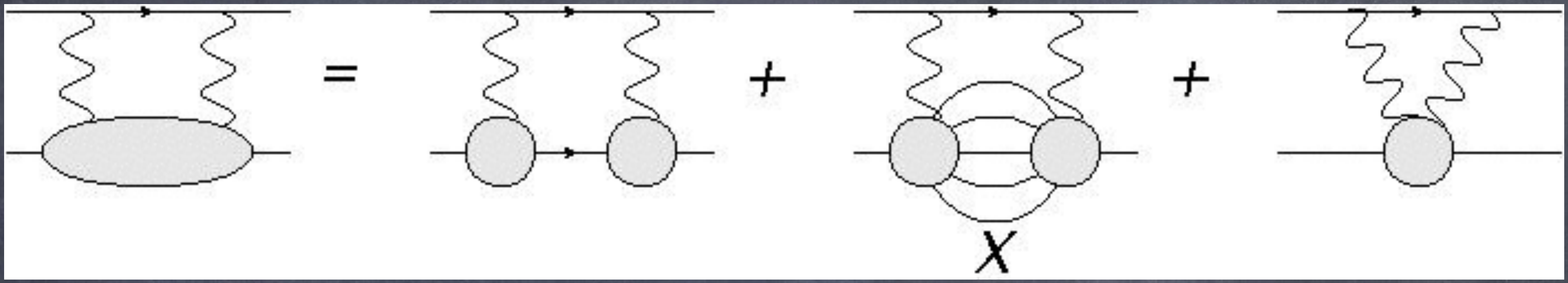
Unpolarized structure functions $F_{1,2}$

Dispersion relations (subtracted for T_1)

$$\mathrm{Re}\,T_1(\nu, Q^2) = T_1(0, Q^2) + \frac{\nu^2}{2\pi M}\mathcal{P}\int_0^\infty d\nu' \,\frac{F_1(\nu', Q^2)}{\nu'(\nu'^2 - \nu^2)}$$

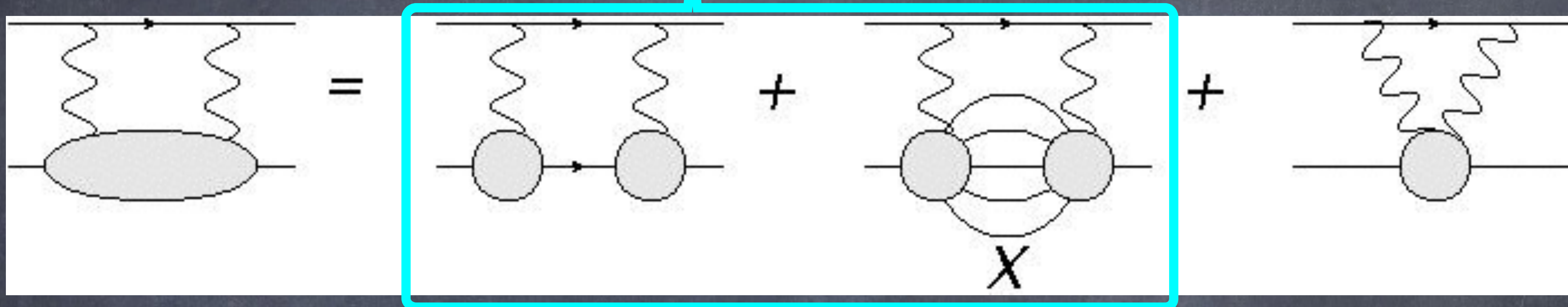
$$\mathrm{Re}\,T_2(\nu, Q^2) = \frac{1}{2\pi}\mathcal{P}\int_0^\infty d\nu' \,\frac{F_2(\nu', Q^2)}{(\nu'^2 - \nu^2)}$$

Polarizability Correction



Polarizability Correction

Dispersion Relation + Data

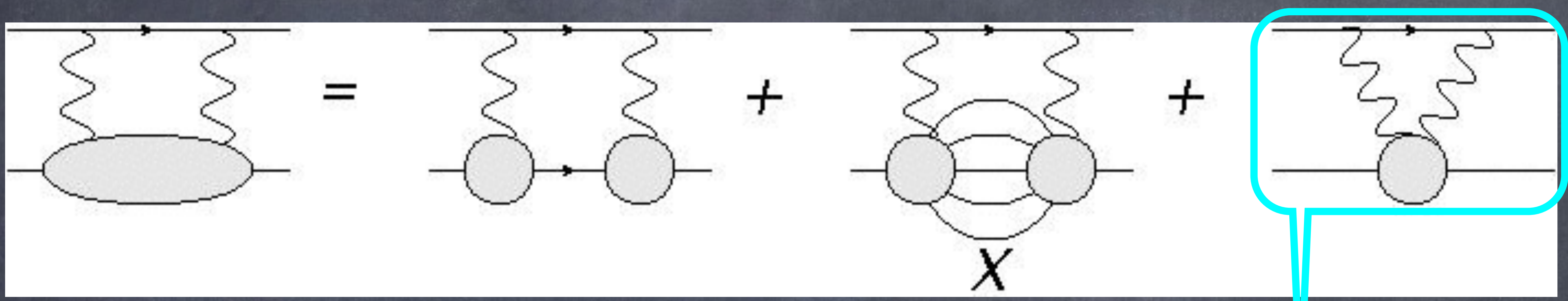


Lamb shift is obtained as

$$\Delta E \sim \alpha_{em}^5 \int_0^\infty dQ^2 \int_0^\infty d\nu \, \{A(\nu, Q^2)\, F_1 + B(\nu, Q^2)\, F_2\}$$

Good quality data (e.g., JLab) on $F_{1,2}$ $0 < Q^2 < 3$ GeV^2, $W < 4$ GeV

Polarizability Correction



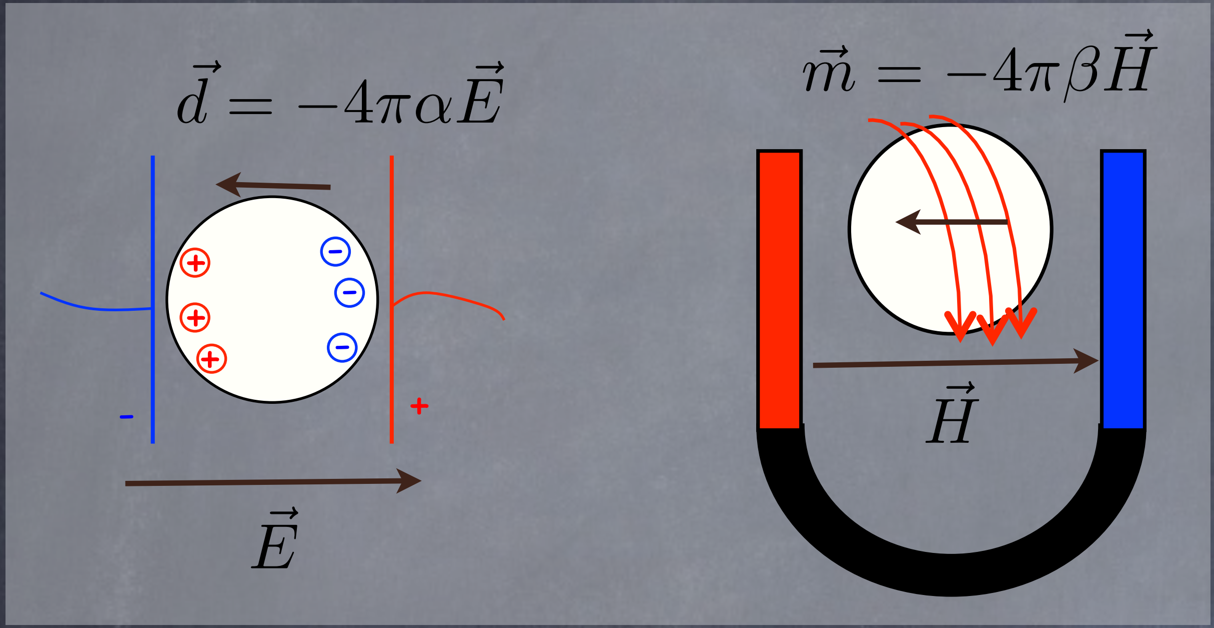
Subtraction function related to proton's magnetic polarizability β_M

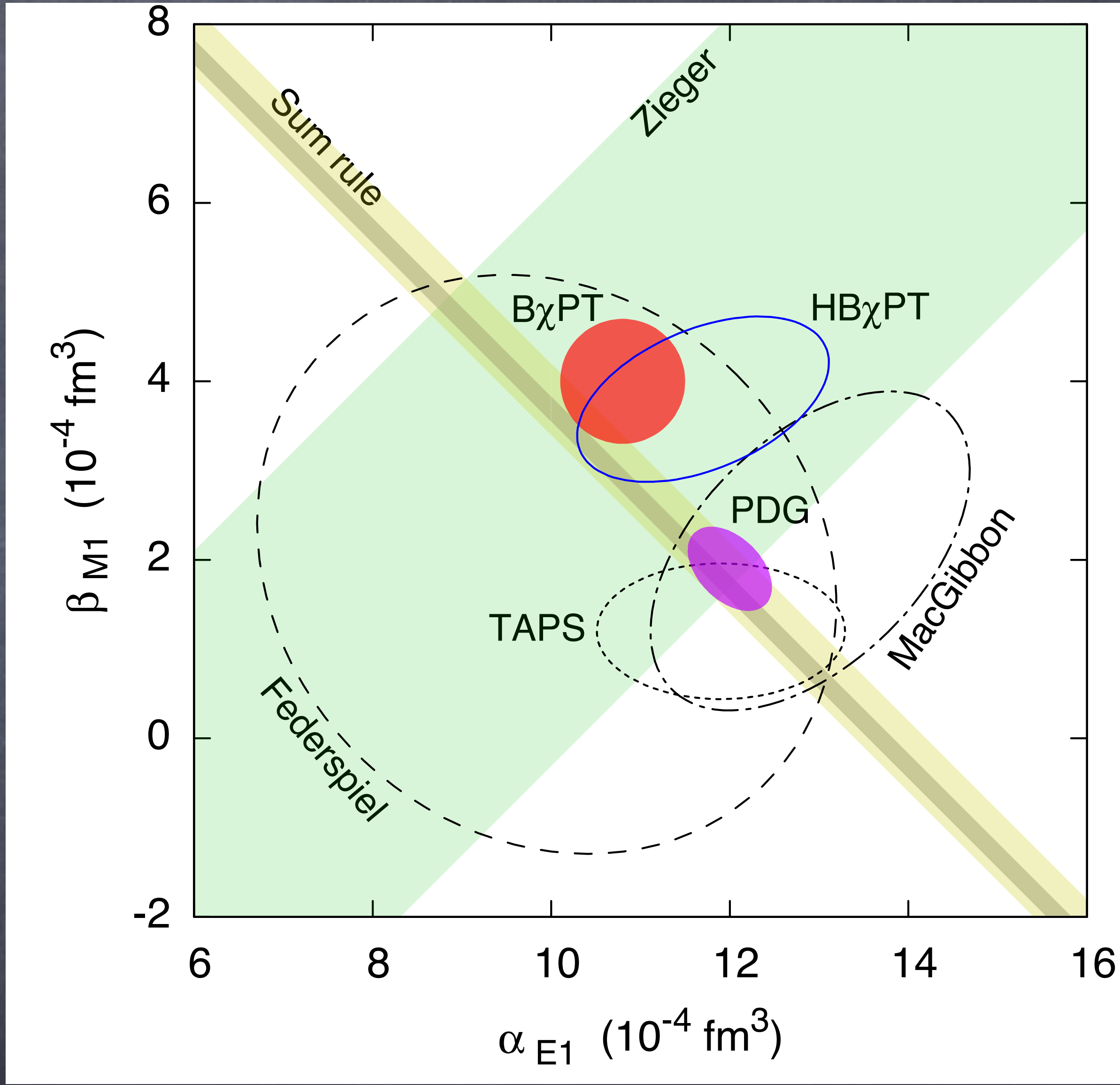
Low-Energy Theorem: $T_1(0, Q^2) = Q^2\ \beta_M$

Lamb shift is obtained as $\Delta E^{Sub} \sim \alpha_{em}^5 \int_0^\infty dQ^2 C(Q^2)\, \beta_M F_\beta(Q^2)$

Subtraction Constant

Proton (dipole) polarizabilities





PDG 2012

$$\alpha_E = 11.2(0.4) \times 10^{-4}\mathrm{fm}^3$$

$$\beta_M = 2.5(0.4) \times 10^{-4}\mathrm{fm}^3$$

MG et al, 1999: Proton polarizabilities from fixed-t DR

Total polarizability correction

Different approaches to estimate $F_\beta(Q^2)$

Dipole (like FF): Pachucki, 1996

Pion loops: Vanderhaeghen & Carlson, 2011

HBChPT + dipole: Birse & McGovern, 2012

BChPT: Alarcón, Pascalutsa,Lenski 2014

Finite Energy Sum Rule: MG, Llanes-Estrada, Szczepaniak, 2013

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Hadronic structure corrections to proton radius puzzle are constrained

$$\Delta E_{2P-2S} = -40 \pm 5\,\mu\text{eV}$$

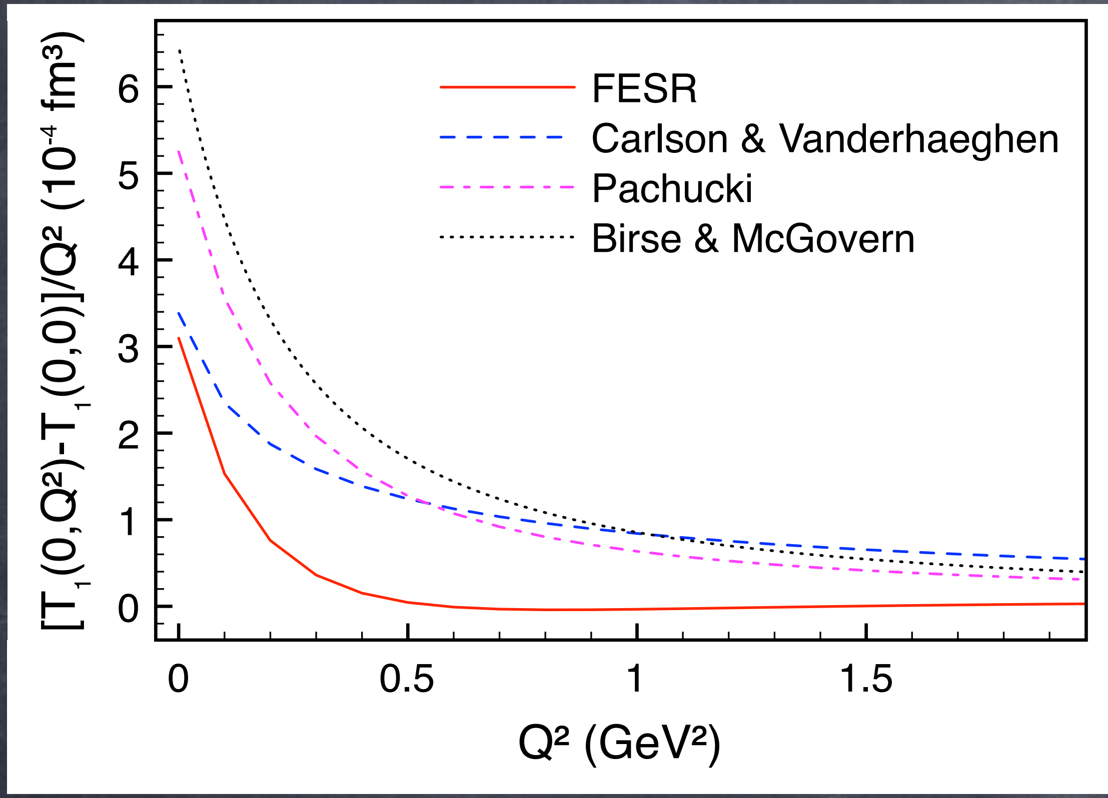
↕

$$\Delta E_{\text{Missing}} \approx -300\,\mu\text{eV}$$

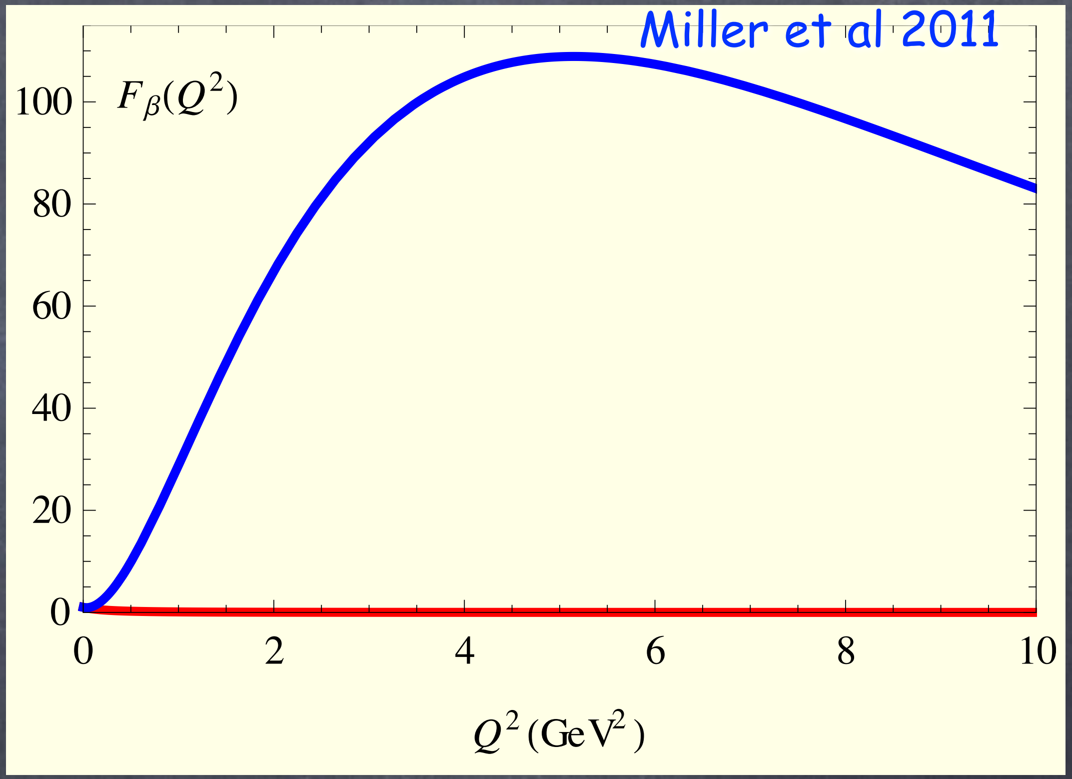
All known constraints built in!

Exotic Hadronic Contributions?

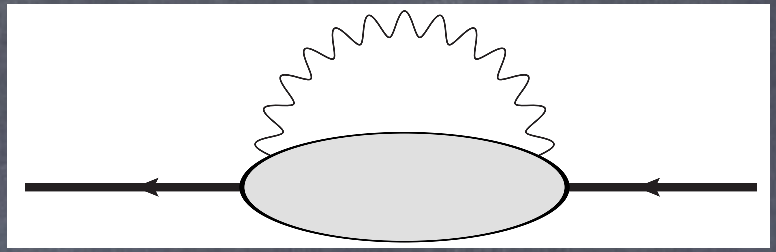
Reasonable hadronic models



To get ~300 μeV Lamb shift: need something like this



Exotic Hadronic Contributions?



Cottingham formula (p-n mass difference)

$$M_p - M_n = \frac{\alpha}{2M(2\pi)^3} \int \frac{d^4q}{q^2} \left[T^{p\mu}_{\mu}(\nu, q^2) - T^{n\mu}_{\mu}(\nu, q^2)\right)$$

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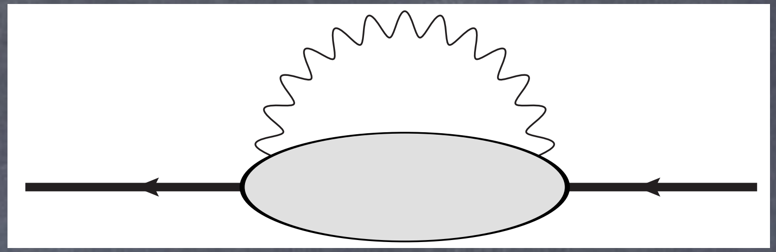

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Subtraction function contribution

$$[M_p - M_n]^{Subt} = -\frac{\beta_M^p - \beta_M^n}{(8\pi)^2 M} \int_0^{\Lambda^2} dQ^2 Q^2 F_\beta(Q^2)$$

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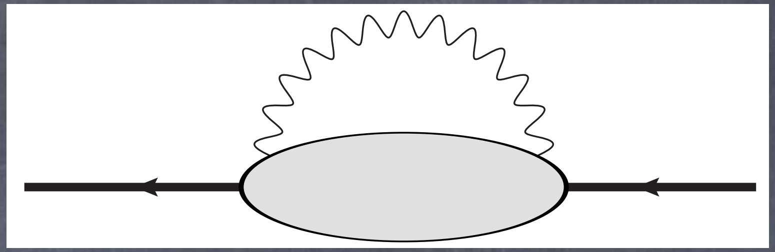

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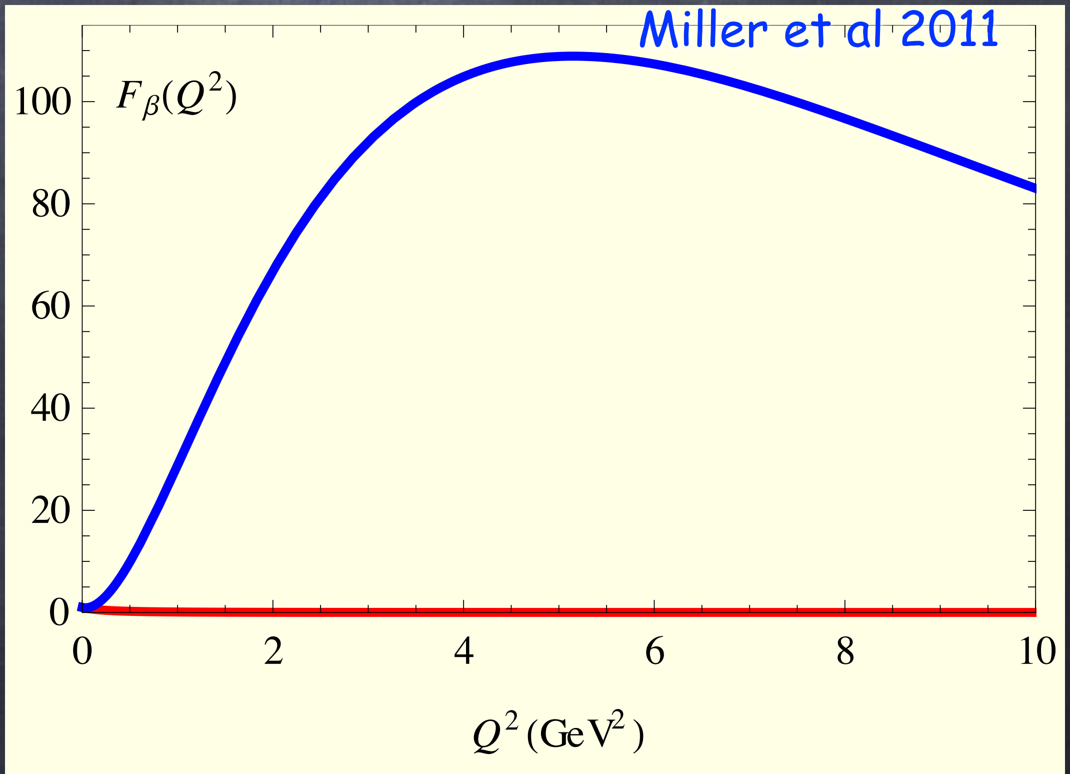
$$[M_p - M_n]^{Subt} = -\frac{\beta^p_M - \beta^n_M}{(8\pi)^2 M}\int_0^{\Lambda^2} dQ^2 Q^2 F_\beta(Q^2)$$

If the proton radius puzzle
is due to subtraction contribution

$$\delta M^p_{em} \sim 600\, MeV$$

Could be purely isoscalar but...
VERY unnatural!
Should be seen in Deuteron (I=0)



Muonic deuterium

One further piece of information available - isotope shift:
simultaneous 1S-2S splitting measurement in eH and eD

$$R_d^2 - R_p^2 = 3.82007(65)\,\mathrm{fm}^2$$

R_d^2-R_p^2 from μH, μD @ PSI - in agreement (preliminary)
Exotic hadronic contributions excluded by this finding

Extraction from μD relies on nuclear structure-dependent polarizability correction.

Nuclear models vs dispersion relations:

$$\delta_{pol}^{Nucl.} \approx -1.680(16)\,meV$$

Leidemann, '90; Pachucki '13;
Ji et al, '14; Friar, '14;

$$\delta_{pol}^{DR} \approx -2.1(7) meV$$

Carlson et al. '14

Lacking Input to DR for μD

$$\Delta E \sim \alpha_{em}^5 \int_0^\infty dQ^2 \int_0^\infty d\nu \, \{A(\nu, Q^2)\, F_1 + B(\nu, Q^2)\, F_2\}$$

All kinematics contribute to the dispersive integral;
Not all of them are equally important

The bulk of the correction - quasi elastic data
from $\nu \simeq$ 6-10 MeV and $Q^2 < 0.005$ GeV^2
- just below the kinematics of available QE data

New D(e,e')pn data down to $Q^2 = 0.002$ GeV^2 A1@MAMI
taken and under analysis;
2% measurement will reduce the uncertainty by a factor 2-4

Summary

- Proton radius puzzle - inconsistency between the e-scattering and eH on one hand, and μH data on the other hand.
- Each part has subtleties but no clear solution found - the puzzle persists
- Scattering experiments: extrapolation issue
- Electronic hydrogen: sensitivity issue
- Muonic hydrogen: no experimental issues found to date further muonic atoms consistent with μH (preliminary)
- BSM explanation possible but requires both lepton non-universality and fine tuning to evade known constraints from other observables

Proton Radius Puzzle: what's next?

- More precise eH experiments coming (2S-2P, 1S-3S, 2S-4S);
- e-p scattering: Q^2 down to 2×10^{-4} GeV^2 @ Mainz, JLab
- Deuteron radius from e-D scattering: new data at Mainz under analysis
  $Q^2 > 0.002$ GeV^2, radius under 0.25%
- To push Q^2 down and get the radius under 1%:
  improved radiative corrections (TPE) necessary.
  Recent works: MG '14; Tomalak, Vanderhaeghen '14, '15(2)
- Study lepton non-universality with μ-p scattering:
  MUSE @ PSI - elastic μ-p scattering at $Q^2 > 0.002$ GeV^2 (2017/18);
  $\gamma p \to \mu^+\mu^- p / \gamma p \to e^+e^- p$ measurement may be more sensitive
  Pauk, Vanderhaeghen '15 - proposal under consideration in Mainz

Proton Radius Puzzle: what's next?

- Further muonic atoms: μD, μHe-3, μHe-4 - data taken at PSI, now analyzed or finalized

- μD - more precise DR calculation needed:
  new QE data on deuteron analyzed at Mainz
  - to reduce the uncertainty of dispersion integrals by factor 2-4
  sum rule for the nuclear magnetic polarizability derived (MG, '15)
  - to reduce model dependence of the subtraction contribution
  DR treatment of hyperfine splitting in μD underway
  - with Carlson and Vanderhaeghen

- μHe-3,4 - DR analysis underway (with Carlson and Vanderhaeghen)
  potential model calculation by Bacca and Co underway

EXTRA SLIDES

Sum rule for nuclear magnetic polarizability

Levinger-Bethe sum rule - nucleus with Z protons and N neutrons

Thomson terms for Z free protons

Nuclear Thomson term

Total CS integrated over nuclear range

$$-Z\frac{\alpha}{M} = -\frac{Z^2\alpha}{(Z+N)M} - \frac{1}{2\pi^2}\int_{\nu_{\rm thr}}^{30\,MeV} d\omega\sigma_T(\omega)$$

Generalize to finite Q^2: charge form factor + magnetic pol.

$$T_1(0,Q^2) = -\frac{Z^2\alpha}{(Z+N)M}F^2(Q^2) + Q^2\beta_M^{Nucl.}(Q^2)$$

The Q^2-slope of the Levinger-Bethe sum rule:

$$\beta_M^{Nucl}(0) = -\frac{Z^2\alpha}{3(Z+N)M}R^2_{Ch} + \frac{1}{2\pi^2}\int_{\nu_{\rm thr}}^{30\,MeV} d\omega\;\frac{d}{dQ^2}\sigma_T(\omega,Q^2)\Big|_{Q^2\to 0}$$

Consistent with data for D;

Can predict β_M for any nucleus from data

MG, [arXiv:1508.02509]

Sum rule for nuclear magnetic polarizability

Calculate the subtraction function $T_1(0,Q^2)$ from data

$$T_1(0,Q^2) - T_1(0,0) = \frac{1}{2\pi^2}\int_{\nu_{\rm thr}}^{30\,MeV} [\sigma_T(\omega,Q^2) - \sigma_T(\omega,0)] + \text{hadr. corr.}$$

Can be used e.g. for calculating the subtraction contribution to Lamb shift in muonic atoms

Hadronic corrections can be neglected for low enough Q^2